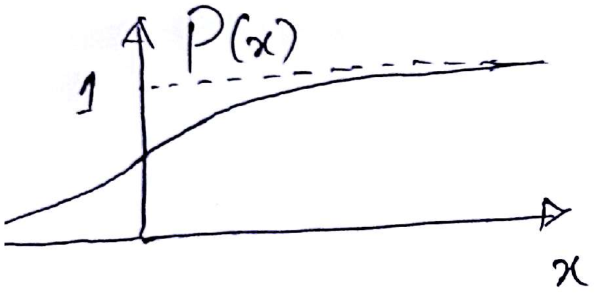
One random variable: The complete set of outcomes is defined as $S_x = \{-\infty < x < +\infty\}$

Cumulative probability function (CPF): It is defined as the probability of finding an event in the range $\{-\infty, x\}$



$$P(-\infty) = 0$$

$$P(+\infty) = 1.$$

$$P(x) = \text{prob}\left(E \subset [-\infty, x]\right)$$

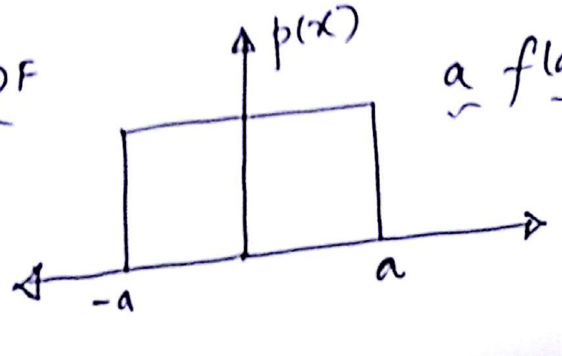
Probability density function (PDF): $p(x)$

It is defined as $p(x) = \frac{d}{dx} P(x)$

Hence $p(x)dx = \text{prob}\left(E \in [x, x+dx]\right)$.

$p(x)$ is properly normalized such that $\int_{-\infty}^{+\infty} dx\, p(x) = 1$

a typical PDF



a flat PDF

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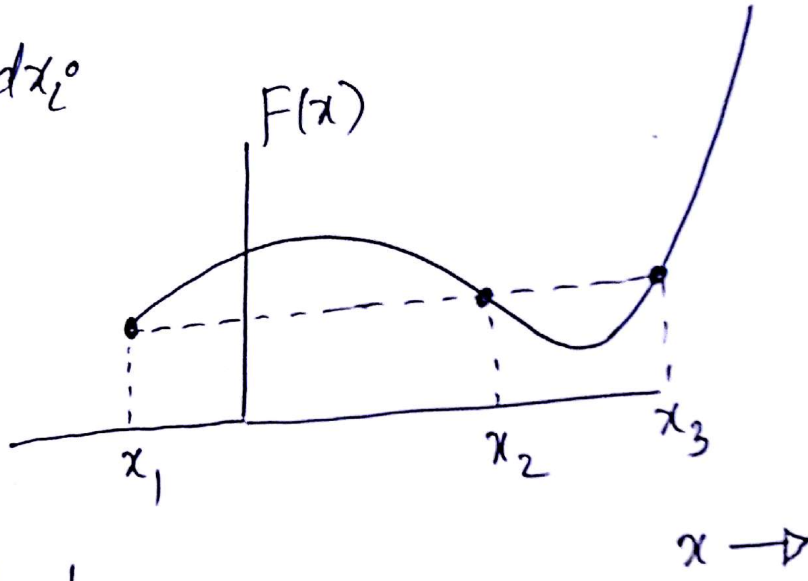
*. Expectation value of a function of x.

$$\langle F(x)\rangle = \int_{-\infty}^{+\infty} dx \; p(x)\, F(x)$$

Question: If x is a continuas random variable with PDF $p(x)$. What is the PDF of $F(x)$?

Ans. Since $F(x)$ is also a random variable, its PDF has the following property

$$p(F)\,dF = \sum_{i=1}^{3} p(x_i)\, dx_i$$



$$\therefore \; p(F) = \sum_{i} p(x_i) \left| \frac{dx}{dF} \right|_{x = x_i} \qquad \text{—— (1)}$$

Here $\left|\frac{dx}{dF}\right|$ are the Jacobians associated with the variable transformation.