# On Dark Matter Self Interactions, Viscosity and Cosmic Expansion

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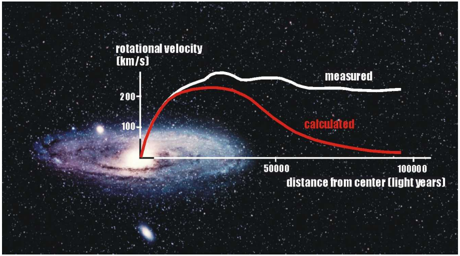
# Pre-Vera Era, Before 1970

- In pre 1970 cosmology, the compostition of the universe was thought to be composed of ordinary matter viz, stars, planets, ateroids, comets etc.
- Observations showed a linearly increasing rotational velocity of the stars as one moves away from the centre of the galaxy/cluster.
  - Consistent with Newtonian Gravity
- It was expected that at large distances the curve will turn and fall of as $r^{-1/2}$, as in Kepler's law.
- Zwicky[1] was the first to point out that Coma cluster doesn't follow the expectations: coined term 'dark matter'.

[1] F. Zwicky, Helvetica Physica Acta 6, 110 (1933)

# Confirmation: Vera Rubin[2]

- Rubin and Ford confirmed that velocities do not fall off but remain constant.




- There must be a large amount of matter in the galaxies that is not visible to us: Dark
- Independently confirmed by gravitational lensing and other observations.

[2] V. C. Rubin and W. K. Ford, Jr., Astrophys. J. 159, 379 (1970).

# Nature of Dark Matter

- Density is $\sim$ 5 times the luminous matter.
- Non Baryonic: Severe constraints from Nucleosysnthesis and astrophysical observations.
- No interactions within the standard model of particle physics known yet.
- Obervations like CMB, Large Scale Structures (LSS), Baryon Acoustic Oscillations (BAO) are consistent with a non interacting cold dark matter (CDM).
  - Specifically $\Lambda$CDM paradigm.

# Constraints from SN-IA + BAO + CMB[3]

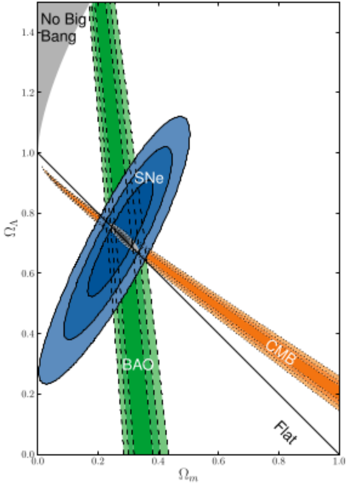



Figure: 68.3%, 95.4%, and 99.7% confidence regions of the $(\Omega_m, \Omega_\Lambda)$ plane.

[3] Suzuki et al, The Astrophysical Journal, 746:85, 2012

# Small Scale Issues[4]

- Core Cusp Problem:- Simulations predict that the densities in the core of CDM halo increases however obervations suggest a constant density core.
- Diversity Problem:- The structure formation in the dark matter halos is expected to be ‘self similar’ i.e. similar at all scales, however the DM halos with same maximum velocity show huge variation in the interior.
- Satellite Problem:- Due to self similarity, simulations predict a large number of subhalos within a DM halo which is contrary to observations.
- Too Big To Fail Problem:- Simulations suggest that the most luminous satellites would have most massive subhalos.

[4] S. Tulin and H. B. Yu, Phys.Rept. 730 (2018) 1-57

# Possible Resolutions

- Precise Measurements:- Missing satellite problem can be explained if we take in to account the detector efficiency. [5]
- Baryon Dissipation:- Star formation, Supernova explosion, gas cooling?
- Warm Dark Matter:- Maybe at the time of decoupling, DM was not completely non-relativistic.
  - Form halo later than the CDM candidates and thus they are less concentrated.
  - Can't explain the abundance of galaxies at large redshift.
  - May provide a solution to missing satellite and too-big-to-fail problem.
  - Fails to resolve the core-cusp problem.
- Self Interacting Dark Matter:- Maybe Self interactions are important at small scales.

[5] Kim, Peter and Hargis, Phys. Rev. Lett. 121 211302 (2018)

# Self Interacting Dark Matter[6]

- Retains the success of non-interacting CDM at large scales while tries to resolve the issues at small scales.
- The scattering rate of SIDM,

$$R_{\mathrm{scat}} = \frac{\langle \sigma v \rangle \rho_{\mathrm{SIDM}}}{m}$$

- At large scales the density is low thus the scattering rate goes to zero and SIDM behaves like the non-interacting CDM.
- At small scales like near the central region of dark matter halo $\rho$ is large and thus the scattering rate is non-zero.

[6] Spergel and Steinhardt, Phys.Rev.Lett. 84, 3760 (2000)

# Estimating Self Interactions[7]

- Scattering is more in the core of DM halo where the density is largest.
- Assume that collisions drive the DM to kinetic equillibrium.
- Jeans equation for relaxed halos ($\partial/\partial t \approx 0$) gives,

$$\sigma_0^2 \nabla^2 \ln \rho_{DM} = -4\pi G(\rho_{DM} + \rho_b)$$

  where $\sigma_0$ is the isotropic velocity dispersion, $\rho_b$ is the baryon mass density.
- Divide the halo into two regions, separated by a characteristic radius $r_1$, such that

$$R_{\rm scat}(r_1) \times t_{age} = \frac{\langle \sigma v \rangle \rho_{\rm DM}(r_1)}{m} \times t_{age} \approx 1$$

- $t_{age} \sim 10$ Gyr and 5 Gyr for galaxy and cluster size halo.

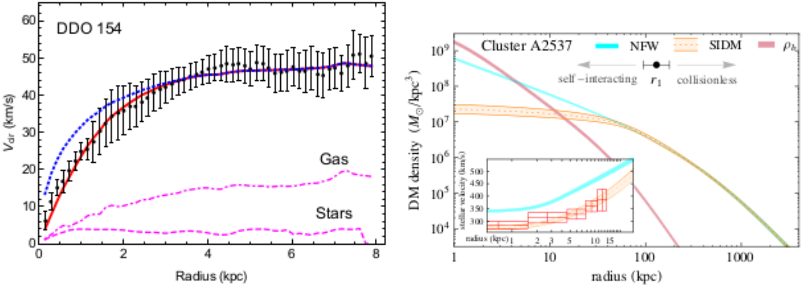
[7] Kaplinghat, Tulin and Yu, PRL 116, no. 4, 041302 (2016)

# Estimating Self interactions

- The profile of DM is a hybrid profile:

$$\rho(r) = \begin{cases} \rho_{\rm iso}, & r < r_1 \\ \rho_{\rm NFW}, & r > r_1 \end{cases}$$

- Markov chain Monte Carlo scan over the parameters $(\rho_0, \sigma_0, r_1)$



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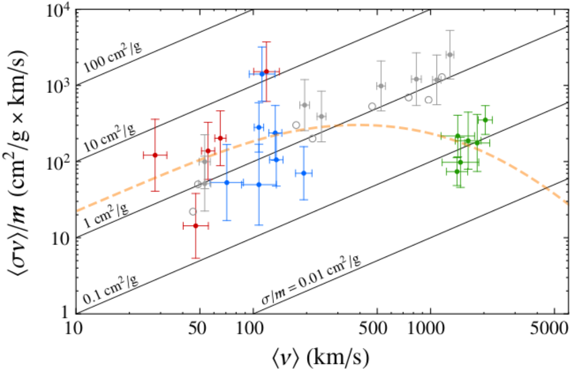


Figure: $\frac{\langle \sigma v \rangle}{m}$ vs $\langle v \rangle$ for Dwarf Galaxies, LSB galaxies, and Clusters.

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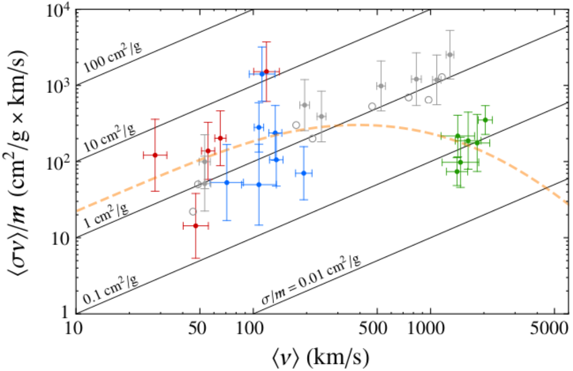


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Could it lead to Dissipation?

# Viscosity From Kinetic Theory

- The starting point is the Boltzmann equation,

$$\frac{\partial f_p}{\partial t} + v_p^i \frac{\partial f_p}{\partial x^i} = \underbrace{I\{f_p\}}_{\text{Collisional Integral}}$$

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  - Locally assign thermodynamic quantities like $T$ and $E$.
- Collision term can be approximated by

$$I\{f_p\} \simeq -\frac{\delta f_p}{\tau},$$

where $\delta f_p \equiv (f_p - f_p^0)$ is deviation from equillibrium.

# Relaxation Time and Distribution Function

- We consider $2-2$ scattering process in DM.
- The relaxation time is then given by

$$\tau^{-1} = \sum_{b,c,d} \int \frac{d^3 p_b}{(2\pi)^3} \frac{d^3 p_c}{(2\pi)^3} \frac{d^3 p_d}{(2\pi)^3} W(a+b \to c+d) f_p^0$$

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  - Depends on the details of the underlying particle physics theory.
- One can use the averaged relaxation time

$$\bar{\tau}^{-1} \equiv n\langle \sigma v \rangle$$

  - The average is over the momentum distribution.

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- Rewrite the expression for $\tau^{-1}$ as

$$\tau^{-1} = 0.1\mathrm{Gyr}^{-1} \times \left(\frac{\rho_{DM}}{0.1\mathrm{M}_{\odot} pc^{-3}}\right)\left(\frac{v}{50\mathrm{kms}^{-1}}\right)\left(\frac{\sigma/m}{1\mathrm{cm}^2/\mathrm{g}}\right)$$

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  - For galaxies, $v \sim 10^2$ km/s
    $\rho \sim 10^{-2} - 10^{-1}$ $\mathrm{M}_{\odot}\mathrm{pc}^{-3}$, and
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- For cluster $v \sim 10^3$ km/s, $\rho \sim 2 \times 10^{-2} - 5 \times 10^{-1}$ $\mathrm{M}_\odot\mathrm{pc}^{-3}$ $v \sim 10^3$ km/s and $\sigma/m \sim 0.1$ cm$^2$/g, thus $t_{age}/\tau \sim 0.2 - 5$.

  Newman *et al*, ApJ. 765, 25 (2013) and ApJ. 765, 24 (2013)

# Viscosity in Kinetic Theory

- We may assume that DM halo are in local thermal equilibrium.
- Viscous coeffecients are then given by

$$\eta = \frac{1}{15T}\int \frac{d^3p}{(2\pi)^3}\tau(E_p)\frac{p^4}{E_p^2}f_p^0, \text{ and}$$

$$\zeta = \frac{1}{T}\int \frac{d^3p}{(2\pi)^3}\tau(E_p)\left[E_p C_n^2 - \frac{p^2}{3E_p}\right]^2 f_p^0$$

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- We use the Maxwell-Boltzmann distribution function.

$$f_p^0 = exp\left(-\frac{E_p - \mu}{T}\right)$$

# Viscosity of SIDM[8]

- The viscous coefficients are:

$$\eta = \frac{1.18 m\langle v\rangle^2}{3\langle\sigma v\rangle}; \quad \zeta = \frac{5.9\ m\langle v\rangle^2}{9\langle\sigma v\rangle}$$

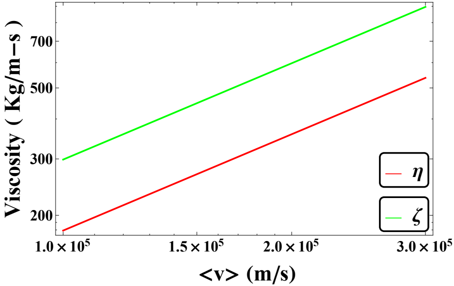


Figure: $\eta$ and $\zeta$ *vs* $\langle v\rangle$ at galactic scale.

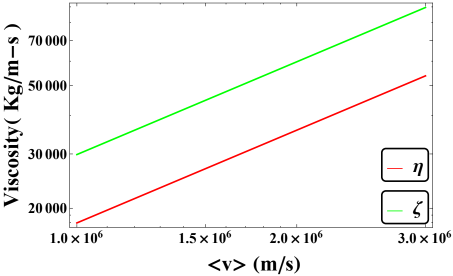


Figure: For cluster scale.

[8] Atreya, Bhatt and Mishra, JCAP 1802 (2018) no.02, 024

# Discussion So Far

- Small scale issues maybe resolved by self interactions of dark matter.
- Faithful reproduction of stellar velocities require $\sigma/m \sim 1$ $\text{cm}^2\text{gm}^{-1}$ for galactic halo and $0.1\text{cm}^2\text{gm}^{-1}$ for clusters.
- Assuming local thermalization we determined the relation between viscous coefficients $(\eta, \zeta)$, velocity weighted crossection to mass ratio $(\langle \sigma v \rangle / m)$ of dark matter and $\langle v \rangle$ of dark matter halos.
- $\eta$ and $\zeta$ change by roughly two orders of magnitude from the galactic to cluster scale.

# ***Story at Large Scales***

# Can Viscosity Affect Expansion Rate of Universe?

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# Yes!

- Viscous Energy momentum tensor

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + (p + \Pi_b)\Delta^{\mu\nu} + \pi^{\mu\nu}, \quad \text{where}$$

  - $(p + \Pi_b) \equiv p_{eff}$ is the effective pressure.
  - $\Pi_b = -\zeta \nabla_\mu u^\mu$ is the bulk viscous tensor.
  - $\Delta^{\mu\nu} = (g^{\mu\nu} + u^\mu u^\nu)$ is the projection operator.
  - $\pi^{\mu\nu} = -\eta \left(\Delta^{\mu\alpha}\Delta^{\mu\beta} + \Delta^{\mu\beta}\Delta^{\mu\alpha} - \left(\frac{2}{3}\right)\Delta^{\alpha\beta}\Delta^{\mu\nu}\right)\nabla_\alpha u_\beta$ is shear stress tensor satisfying $\pi^\mu_\mu = 0$.

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- If $|\Pi_b| > p$, then $p_{eff} < 0$, then $\ddot{a} > 0$ is possible.

  Viscosity can drive accelerated cosmic expansion.

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$$\Pi_b = -\zeta \nabla_\mu u^\mu \approx -3\zeta H$$
$$H \sim \rho^{1/2}/m_{pl}$$

# Constraints

- Severe constraints from observations of the large scale structure formation[9].
- Large $\zeta$ leads to decay of the gravitational potential during structure formation.
- Also leads to large Integrated Sachs-Wolfe (ISW) effect.[10]
- Constraints on $\zeta$ within $\Lambda$CDM model from ISW and small scale structures.[11]

[9] Li and Barrow, Phys. Rev. D 79, 103521 (2009)
[10] Velten and Schwarz, JCAP 09 (2011)
[11] Velten and Schwarz, Phys. Rev. D 86, 083501 (2012)
[12] Anand, Chaubal, Mazumdar, Mohanty, JCAP 1711 (2017) no.11, 005

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- A small viscosity in CDM explains the mismatch in the values of $H_0$ and $\sigma_8$ from Planck CMB observation and LSS observations.[12]

[9] Li and Barrow, Phys. Rev. D 79, 103521 (2009)
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# Viscous Backreaction on Metric

- Can backreaction of the fluid perturbation on the metric, lead to accelerated expansion?[13]
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$$\frac{\dot{\epsilon}}{a} + 3H\left(\epsilon + p - 3\zeta H\right) = D,$$

$$D = \frac{1}{a^2}\left\langle \eta\left[\partial_i v_j \partial_i v_j + \partial_i v_j \partial_j v_i - \frac{2}{3}\partial_i v_i \partial_j v_j\right]\right\rangle + \frac{1}{a^2}\langle \zeta[\vec{\nabla}.\vec{v}]^2\rangle$$
$$+ \frac{1}{a}\langle \vec{v}.\vec{\nabla}\left(p - 6\zeta H\right)\rangle$$

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$$+ \frac{1}{a}\langle \vec{v}.\vec{\nabla}(p - 6\zeta H) \rangle$$

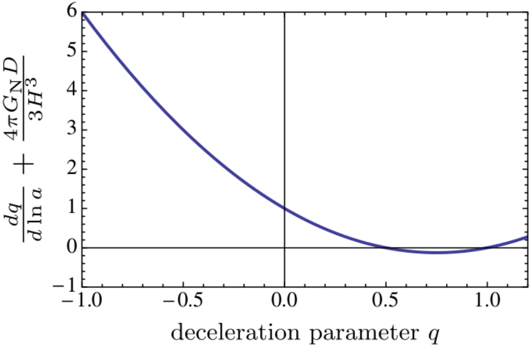
- Trace of averaged Einstein's equation $\langle R \rangle = 8\pi G \langle T^{\mu}_{\mu} \rangle$ gives

$$\frac{\ddot{a}}{a^3} = \frac{\dot{H}}{a} + 2H^2 = \frac{4\pi G}{3}(\epsilon - 3p - 3\Pi_b)$$

[13] Floerchinger et al PRL 114, 091301 (2015)

# Quantifying the Dissipation

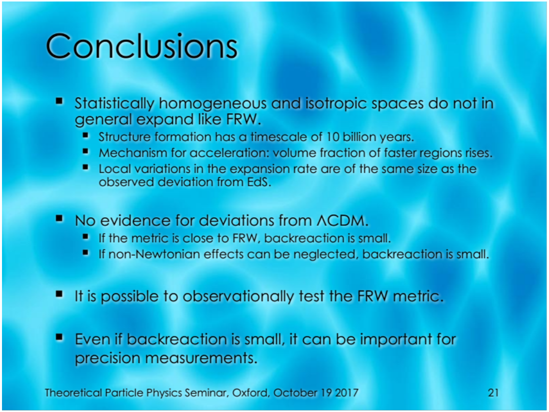
$$-\frac{dq}{d\ln a} + 2(q-1)\left(q - \frac{(1+3\hat{\omega_{eff}})}{2}\right) = \frac{4\pi GD(1-3\hat{\omega_{eff}})}{3H^3}$$




Dissipation can account for today's accelerated expansion

# Something I Will Ignore In The Rest of My Talk

***Can the metric perturbations "backreact" sufficiently to change the expansion rate?*** [14]




[14] Buchert and Rasanen; Ann. Rev. of Nuc. and Par. Sc. 62 (2012) 57-79 arXiv:1112.5335 [astro-ph.CO]

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EoS for SIDM? $\hat{\omega}_{eff} = \frac{\langle P \rangle}{\langle \epsilon \rangle} + \frac{\langle \Pi_b \rangle}{\langle \epsilon \rangle}.$

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- For Maxwell-Boltzmann distribution $P = nT$.
- Equipartition of energy $\Rightarrow mv^2/2 = 3T/2$
- $P/\epsilon = v^2/3 \sim 10^{-5}$ for cluster.

- $\Pi_b = -\zeta(\vec{\nabla} \cdot \vec{v} + 3H_0)$.
- $\zeta = (5.9v/9)(\sigma/m)$. Neglect $\zeta H_0$.
- $\partial_i \sim 1/L \Rightarrow \Pi_b \sim -\zeta v/L$.
- $\Pi_b/\epsilon \sim 10^{-7}$ for cluster size halo.

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$\hat{\omega}_{eff} = 0 \Rightarrow$ Cold Dark Matter.

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- For a dilute gas $\eta = \rho v \lambda / 3$,

$$\Rightarrow \lambda \sim 10^{10} \left( \frac{m}{\sigma} \right) \frac{1}{\rho} \;\; \text{(in kpc)}$$

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- For galaxies $\lambda \sim 10$ kpc, $\sigma/m \sim 1$ cm$^2$/g $\Rightarrow \rho \sim \times 10^9$ $M_\odot kpc^{-3}$.
- $\rho \sim 10^7 - 10^8$ $M_\odot kpc^{-3}$
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Cluster is the smallest appropriate scale for estimating dissipation.
Hydro description is valid at larger scales

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Validity of form of $T^{\mu\nu}$

$$T^{\mu\nu} = \epsilon u^{\mu} u^{\nu} + (p + \Pi_b)\Delta^{\mu\nu} + \pi^{\mu\nu}$$

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First order expansion is valid.

# Estimating Dissipation

$$D = \frac{1}{a^2}\left\langle \eta \left[ \partial_i v_j \partial_i v_j + \partial_i v_j \partial_j v_i - \frac{2}{3}\partial_i v_i \partial_j v_j \right] \right\rangle + \frac{1}{a^2}\langle \zeta [\vec{\nabla}.\vec{v}]^2 \rangle$$

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- $\eta, \zeta$ are constant in space. Estimated for cluster scales.
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- $R_H = H^{-1}$ is the horizon size, $\alpha = L/R_H$ is the fraction of horizon

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$$D = \left(\frac{H\langle v\rangle}{\alpha}\right)^2 \left[\frac{4}{3}\eta + 2\zeta\right]$$

# Extracting Crossection

- We look at $z = 0$, *i.e.* $H = H_0, \; a = 1$

$$D = \frac{16.32\langle v\rangle^4}{9}\left(\frac{m}{\langle\sigma v\rangle}\right)\left(\frac{H_0}{\alpha}\right)^2.$$

- The dissipation parameter is:

$$-\frac{dq}{d\ln a} + (q-1)(2q-1) = \frac{4\pi G D}{3H^3}$$

- Assuming $\left|\frac{dq}{d\ln a}\right| \ll 1$, we get

$$\frac{\sigma}{m} = \frac{65.28\pi v^3}{27(q-1)(2q-1)H_0 m_{pl}^2\alpha^2}, \quad \text{where } m_{pl}^2 \equiv \frac{1}{G}.$$

# Extracting Crossection[15]

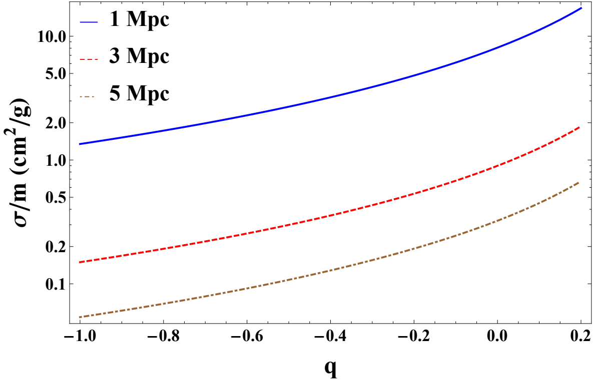


Figure: $\sigma/m$ vs $q$ for cluster scale.

Larger negative value of $q$ is supported by smaller values of $\sigma/m$.

[15] Atreya, Bhatt and Mishra, JCAP 1802 (2018) no.02, 024

# Evolution of Viscous Universe [16]

## Assumptions

- Dark matter thermalised for redshift $z \geq 2.5$.
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  - This assumption breaks down as one goes to the epoch of structure formation.

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- Dark matter thermalised for redshift $z \geq 2.5$.
  - Thus $\eta$ and $\zeta$ are constant.
  - This assumption breaks down as one goes to the epoch of structure formation.
- Velocity gradients evolve at a scale $L > 1$Mpc.
- Ansatz:

$$\partial v(z) = \frac{\partial v|_{z=0}}{(1+z)^n} \sim \left(\frac{v_0}{L}\right) \frac{1}{(1+z)^n}$$

  where $n \geq 0$ is the free parameter.
- $\partial v(z=0) \sim v_0/L$.
- $v_0$ is the velocity at scale $L$.

[16] Atreya, Bhatt and Mishra JCAP 1902 (2019) 045

# Relevant Equations

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$$\frac{dH}{dz} = \frac{(q+1)H}{(1+z)}$$

- Coupled differential equation in $q(z)$ and $H(z)$ that need to solve numerically.
- The initial conditions are $H(z=0) = 100h$ and $q_0 = -0.60$.
- Two free model parameters: $n$ and $L$.

# $\chi^2$ Analysis

$$\chi^2(z,n,L) = \sum_{i=1}^{N} \left[ \frac{H_{obs}(z_i) - H_{th}(z_i,n,L)}{\sigma_i} \right]^2,$$

- We use data for Hubble expansion rate from the cosmic chronometer data [17].

[17] Farooq *et al*; ApJ. 835, no. 1, 26 (2017)

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| Best Fit | $\chi^2_{min}$ | $\chi^2_{\mathrm{d.o.f}}$ |
|---|---|---|
| $n = 0.5770$ <br> $L = 20.1265$ Mpc | 22.0207 | 0.6116 |

Table: The best fit values for $\chi^2_{\mathrm{min}}$

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Table: The best fit values for $\chi^2_{\text{min}}$

Gradients evolve at scales approximately 20 times larger than the dissipative scales ($\sim$ Mpc).

[17] Farooq *et al*; ApJ. 835, no. 1, 26 (2017)

# $\chi^2$ Analysis

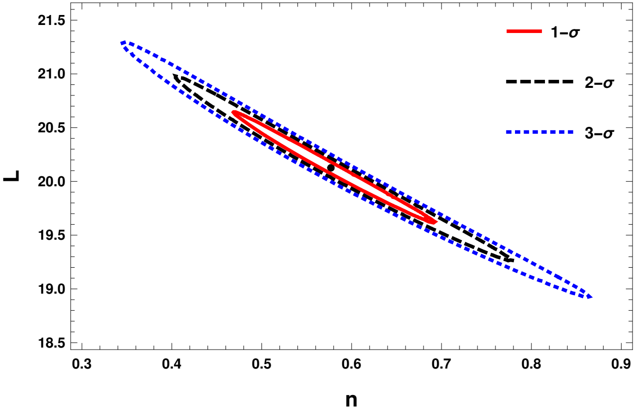



Figure: The joint confidence region of model parameters $n$ and $L$. The best fit value is shown as a point.

# Behaviour of Dissipation

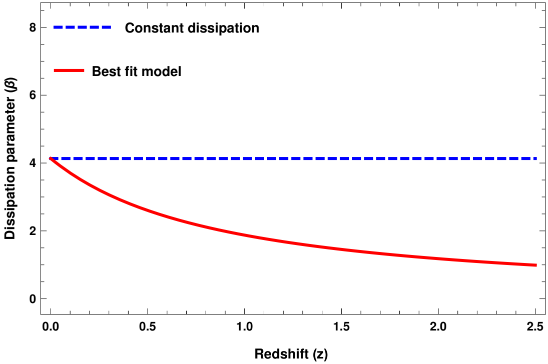



Figure: Dissipation term ($\frac{4\pi GD}{3H^3}$) for the best fit and constant dissipation.

$$\frac{4\pi GD}{3H_0^3} \sim 4.1 \neq 3.5$$

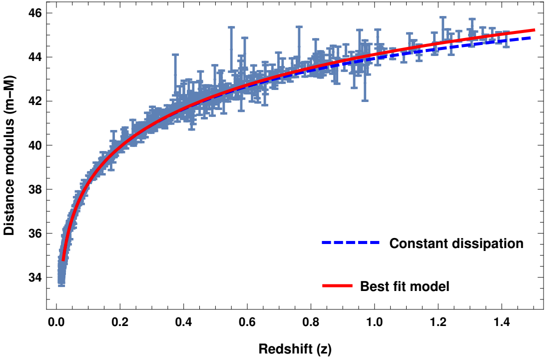
# Supernovae data

- We measure distance modulus $\mu$,

$$\mu(z) = 5\log_{10}\left(\frac{d_L(z)}{Mpc}\right) + 25$$

- Luminosity distance, $d_L$,

$$d_L(z) = (1+z)\int_0^z \frac{dz}{H(z)}.$$



# Deceleration Parameter

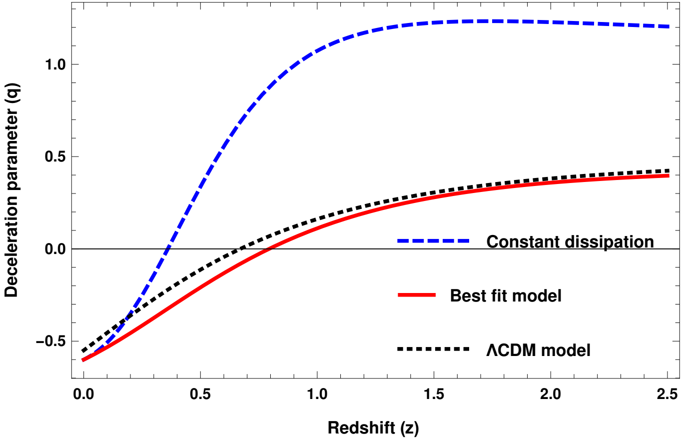



Figure: Deceleration parameter ($q$) for best fit, ΛCDM and constant dissipation.

# Hubble Parameter

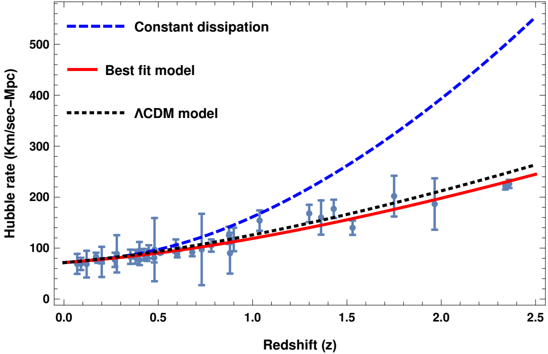



Figure: $H(z)$ for the best fit, constant dissipation and ΛCDM model.

- Constant dissipation case doesn't explain the data.

# Summary

- Discussed a viable viscous dark matter scenario consistent with observations.
- With some simplifying assumptions we estimated the dissipation due to the dark matter viscosity.
- We make a power law ansatz for the evolution of the disspation with redshift.
- Perform a $\chi^2$ fit with latest Hubble chronometer data to fix the power law exponent and the length scale at which the gradients dominate.
- Best fit parameters explains Supernova data as well.
- We conclude that to explain the data, the dissipation must be smaller at earlier times.

***Thank You !***

# Small Scale Issues[18]

- Core Cusp Problem:- Simulations predict that the densities in the core of CDM halo increases however obervations suggest a constant density core.

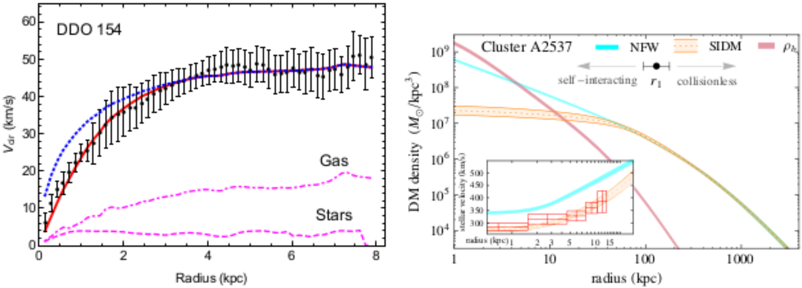



Figure: Left: Galaxy rotation curve for NFW (blue) and cored (red) profiles. Right: Density profile for the Cluster. Index: Velocity Profile for inner core.

[18] S. Tulin and H. B. Yu, arXiv:1705.02358 [hep-ph]

# Small Scale Issues

- Diversity Problem:- The structure formation in the dark matter halos is expected to be 'self similar' i.e. similar at all scales, however the DM halos with same maximum velocity show huge variation in the interior.

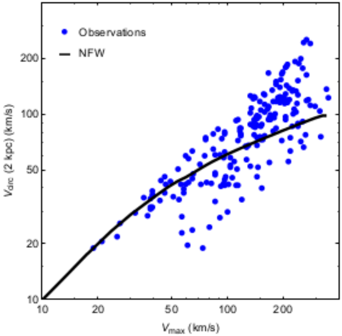



Figure: Velocity at 2 kpc versus $V_{max}$ for observed galaxies.

# Small Scale Issues

- Satellite Problem:- Due to self similarity, simulations predict a large number of subhalos within a DM halo which is contrary to observations.

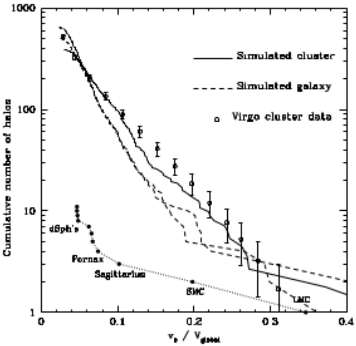



Figure: Observed MW satellites (solid) and galaxies in the Virgo cluster (empty) with subhalos predicted by the ΛCDM simulations for MW (dashed) and Virgo cluster (solid).

# Small Scale Issues

- Too Big To Fail Problem:- Simulations suggest that the most luminous satellites would have most massive subhalos. The densities inferred from stellar dynamics don't match simulations.
  - Related to the core-cusp problem.

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## Possible Resolutions

- Baryon Dissipation:- Star formation, Supernova explosion, gas cooling?
- Warm Dark Matter:- Maybe at the time of decoupling, DM was not completely non-relativistic.
  - Can't explain the abundance of galaxies at large redshift.
  - Fails to resolve the core-cusp problem.
- Self Interacting Dark Matter.

# Calculation of Viscosity

- Boltzmann Equation:

$$\frac{\partial f_p}{\partial t} + v_p^i \frac{\partial f_p}{\partial x^i} = I\{f_p\}$$

- In relaxation time approximation

$$I\{f_p\} \approx \frac{\delta f_p}{\tau}$$

$$\Rightarrow \delta f_p = -\tau \left( \frac{\partial f_p^0}{\partial t} + v_p^i \frac{\partial f_p^0}{\partial x^i} \right).$$

- Since we assume local equilibrium, we can define the average energy density ($T^{00}$) and momentum density ($T^{0i}$) of the system.
- Extending the definition to $i - j$ components

$$T^{ij} = \int \frac{d^3p}{(2\pi)^3}\, v^i p^j f_p.$$

# Calculation of Viscosity

- Using $f_p = f_p^0 + \delta f_p$ and $T^{ij} = T^{ij}_{ideal} + T^{ij}_{\rm diss}$

$$T^{ij}_{\rm diss} = -\int \frac{d^3p}{(2\pi)^3}\, \tau v^i p^j \left( \frac{\partial f_p^0}{\partial t} + v_p^l \frac{\partial f_p^0}{\partial x^l} \right).$$

- Within the hydrodynamics, in a local Lorentz frame,

$$T^{ij}_{\rm diss} = -\eta \left( \frac{\partial u^i}{\partial x^j} + \frac{\partial u^j}{\partial x^i} - \frac{2}{3}\frac{\partial u^k}{\partial x^k}\delta^{ij} \right) - \zeta \frac{\partial u^k}{\partial x^k}\delta^{ij}$$

- Consider the fluid motion along, say, $x$ axis, $\mathbf{u} = (u_x(y), 0, 0)$.

$$T^{xy} = -\eta \frac{\partial u_x}{\partial y}.$$

# Calculation of Viscosity[19]

- Using $f_p^0 = \exp(-p^\mu u_\mu / T)$ and comparing the two forms

$$\eta = \frac{1}{15T} \int \frac{d^3p}{(2\pi)^3} \tau \frac{p^4}{E_p^2} \frac{\partial f_p^0}{\partial E_p}.$$

- For bulk viscosity we have to compare the trace of $T^{ij}$ obtained above.
- Use $\partial_\mu T^{\mu\nu} = 0$

$$\zeta = \frac{1}{T} \int \frac{d^3p}{(2\pi)^3} \tau \left[E_p C_n^2 - \frac{p^2}{3E_p}\right]^2 f_p^0,$$

where $C_n = \frac{\partial P}{\partial \epsilon}|_n$ is the speed of sound at constant number density.

[19] S. Gavin, Nucl. Phys. A 435, 826 (1985)