Gravity waves from Inflation and the Lyth bound

Aditya Aravind

Weinberg Theory Group
Department of Physics
The University of Texas at Austin

August 11, 2014

Motivation

- In March 2014, an experiment named BICEP2 located near the south pole claimed to have detected “Primordial Gravity Waves”.
- This was very big news in the field of cosmology and its validity is still being hotly debated.
- Why is it so important/exciting/controversial?
- We shall discuss the relevance of this discovery in the context of Cosmic Inflation and go over some of its implications.

Outline

1. Brief overview of Inflation.
2. Scalar and Tensor perturbations.
3. BICEP2 observation and implications.
4. The Lyth bound and excited states.

Why inflation?

- Cosmic Microwave Background (CMB) presents us a photograph of the universe as it was $\sim 380,000$ years after Big Bang.
- The photograph tells us that the universe was remarkably uniform at that time.
- If the universe was mostly made of matter or radiation, its expansion slows down with time.

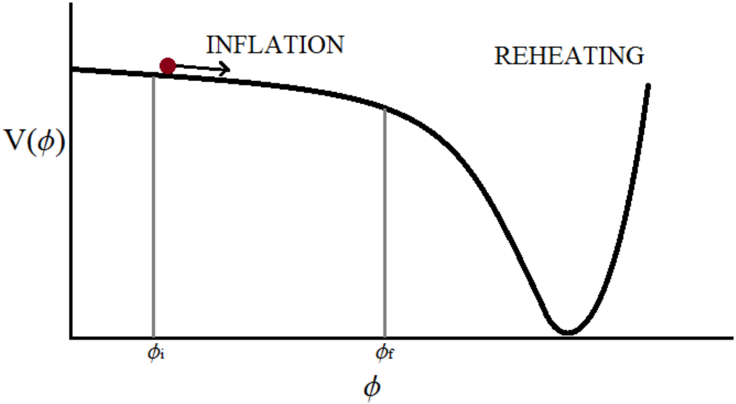
$$\frac{\ddot{a}}{a} = -\frac{1}{6}(\rho + 3P)$$

- Widely separated regions couldn't have "talked to each other" between Big Bang and CMB.
- Solution: an early period of accelerated expansion.

Single-field slow-roll inflation

Energy density in universe dominated by a single scalar field: "Inflaton"

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}R + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right]$$



Single-field slow-roll inflation

Energy Density:

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi) \approx V(\phi)$$

Pressure:

$$P = \frac{1}{2}\dot{\phi}^2 - V(\phi) \approx -V(\phi)$$

Hubble Parameter (Expansion rate):

$$H^2 = \frac{1}{3M_P^2}\rho \approx \frac{1}{3M_P^2}V(\phi)$$

Slow Roll Parameters:

$$\epsilon = \frac{1}{2}\frac{\dot{\phi}^2}{H^2 M_P^2} \qquad \eta = -\frac{\ddot{\phi}}{H\dot{\phi}}$$

What does this achieve?

- Since H is nearly constant, scale parameter a increases exponentially.
- For a large-enough value of H, this gives sufficiently accelerated expansion.
- The whole observable universe presumably came from a causally connected patch before inflation, which inflated into a large volume.
- Also addresses/alleviates "flatness" problem, "monopole" problem, etc.
- But the real reason for which Inflation is widely favoured is yet to come.

Fluctuations

Even if we begin with a homogeneous background, there will be quantum fluctuations of the inflaton and the metric.

- Inflaton fluctuations ($\delta\phi$):

$$\phi(x,t) = \bar{\phi}(t) + \delta\phi(x,t)$$

- Metric fluctuations (Φ, B_i, Ψ, E_{ij}):

$$ds^2 = -(1+2\Phi)dt^2 + 2a(t)B_i dx^i dt + a^2(t)\left[(1-2\Psi)\delta_{ij} + 2E_{ij}\right]dx^i dx^j$$

Gauge-invariant fluctuations

- All these fluctuations are not “physical”, because General Relativity has some gauge freedom.
- Only quantities that do not change from gauge to gauge are really useful to compute.
- Gauge invariant fluctuations: One scalar and two tensor degrees of freedom.
    1. Scalar (Comoving curvature perturbations):
    $$\mathcal{R}(x,t) = \Psi + \frac{H}{\dot{\phi}}\delta\phi$$
    2. Tensor:
    $$\gamma_{ij}(t): \quad \gamma_{ij,i}(t) = \gamma^i_i(t) = 0$$

Time evolution of fluctuations

Perturbative Action (up to second order in perturbations):

$$
\begin{aligned}
S_s &= \frac{1}{2}\int d^4x a^3 \frac{\dot{\phi}^2}{H^2}\left[\dot{\mathcal{R}}^2 - a^{-2}\left(\partial_i \mathcal{R}\right)^2\right] \\
S_t &= \frac{M_P^2}{8}\int d^4x a^3 \left[\dot{\gamma}_{ij}^2 - a^{-2}\left(\partial_l \gamma_{ij}\right)^2\right]
\end{aligned}
$$

- On going to Fourier space, the Lagrangian becomes diagonal.
- Therefore, each mode ($\mathcal{R}_k(t)$, $\gamma_k^{\pm}(t)$) evolves independently of every other mode ($\mathcal{R}_{k'}(t)$, $\gamma_{k'}^{\pm}(t)$).
- Hamiltonian obtained from this action determines time evolution of perturbations.
- From this, the spectrum of fluctuations can be calculated.

Perturbation Spectrum for scalars

Equation of motion for perturbations (for each k):

$$\ddot{\mathcal{R}} + 3H\dot{\mathcal{R}} + \frac{k^2}{a^2}\mathcal{R} = 0$$

- There are infinitely many solutions to this equation.
- Picking a state $|\psi\rangle$ for the fluctuations corresponds to choosing any one solution.
- Different choices are related through Bogoliubov transformations.
- This solution is known as the "mode function" $\mathcal{R}_{k,\psi}(t)$.
- The magnitude of $|\mathcal{R}_{k,\psi}(t)|^2$ determines the amplitude/power spectrum.

Perturbation Spectrum

- Standard choice of state: "Bunch Davies state"

$$\mathcal{R}_{k,BD}(\tau) = \frac{1}{\sqrt{2k^3}} \frac{H^2}{\dot{\phi}} (1 - ik\tau) e^{ik\tau}$$

$\Longrightarrow$ At late times, when $|k\tau| \ll 1$, the amplitude $\mathcal{R}_{k,BD}(\tau)$ becomes approximately constant.

- Bunch Davies Power spectrum:

$$\left\langle \hat{\mathcal{R}}_k \hat{\mathcal{R}}_{k'} \right\rangle_{BD} = (2\pi)^3 \delta^3(k + k') \frac{1}{2k^3} \frac{H^4}{\dot{\phi}^2}$$

H and $\dot{\phi}$ are approximately constant: evaluated at horizon exit ($k = aH$).

Spectrum: From Inflation to CMB

- Power spectrum goes as k^{-3} (approximately).
- This is termed as "nearly scale invariant" power spectrum.
- We define the amplitude of power spectrum:

$$\Delta_{\mathcal{R}}^2 = \frac{k^3}{2\pi^2} P_{\mathcal{R}} = \frac{1}{8\pi^2 \epsilon} \frac{H^2}{M_P^2}$$

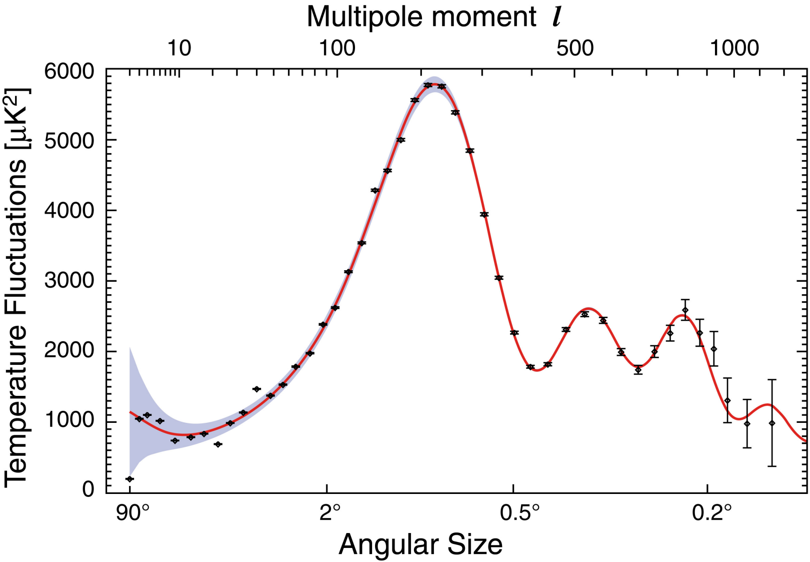
- This has a slight k-dependence parametrized by the spectral tilt n_S

$$\Delta_{\mathcal{R}}^2 \sim k^{n_S - 1}$$

Spectrum: From Inflation to CMB

- In single-field slow-roll inflation, modes “freeze out” after horizon exit.
- After the end of inflation, universe undergoes decelerating expansion.
- Modes re-enter horizon during this period as classical density perturbations.
- Density perturbations then evolve under the influence of gravity.
- After accounting for acoustic oscillations and other effects, the fluctuation spectrum during CMB can be predicted.

CMB Power Spectrum: WMAP 7-year Results



"Success" of inflation

- Working backwards from CMB observations indicate inflationary perturbations must have had nearly scale-invariant power spectrum.
- The power spectrum should have a slight red-tilt.
- The latest observations (Planck 2013) also indicate they should have very small non-Gaussianity.
- All of these observations neatly agree with the simplest inflationary models (and many more complicated ones too).
- However, it is possible to come up with non-inflationary explanations for these observations.
- It would be great if we observe new CMB features that could rule out alternatives to inflation and also narrow down inflationary landscape.

Tensor perturbations

We have seen inflationary predictions for scalar perturbations $\mathcal{R}$. What about the tensor perturbations of the metric γ_{ij}?

- The derivation of spectrum for tensors is very similar to that of scalars.
- The second-order action is different by a factor, while the equations of motion are identical.
- The mode functions are different from scalars by a normalization factor.
- Power spectrum (derived the same way) is different by a factor of 16ϵ.
- There are two tensor polarizations $\gamma_{ij}^{\pm}$ to be accounted for.

Tensor Spectrum for simplest Single field slow-roll models

Even for Tensors, the theory predicts a nearly scale invariant power spectrum.

$$\Delta_\gamma^2 = \frac{2k^3}{2\pi^2} P_\gamma = \frac{2}{\pi^2} \frac{H^2}{M_P^2}$$

- We don't have prior knowledge the values of H and ϵ during inflation (during horizon exit of the modes seen in CMB).
- Therefore, we cannot predict the values of and $\Delta_{\mathcal{R}}^2$ and Δ_γ^2.
- However, scalar perturbations have already been observed, so we know $\Delta_{\mathcal{R}}^2$ from observations.
- If we measure tensor modes, we can obtain the inflationary values for H and ϵ.
- Knowing the scale of inflation could help connect particle physics to cosmology.

"Discovery" of Tensor modes

In Cosmology, BICEP = Background Imaging of Cosmic Extragalactic Polarization!!

- In March 2014, BICEP2 announced that they observed a signal consistent with inflationary tensor modes.
- They claimed to have observed data consistent with a tensor-to-scalar ratio $r = \Delta_\gamma^2/\Delta_\mathcal{R}^2 = 0.2$.
- This discovery is still being hotly debated; we are waiting for more observational data.
- Planck satellite bound on r (from 2013): $r < 0.11$ at 95% CL.

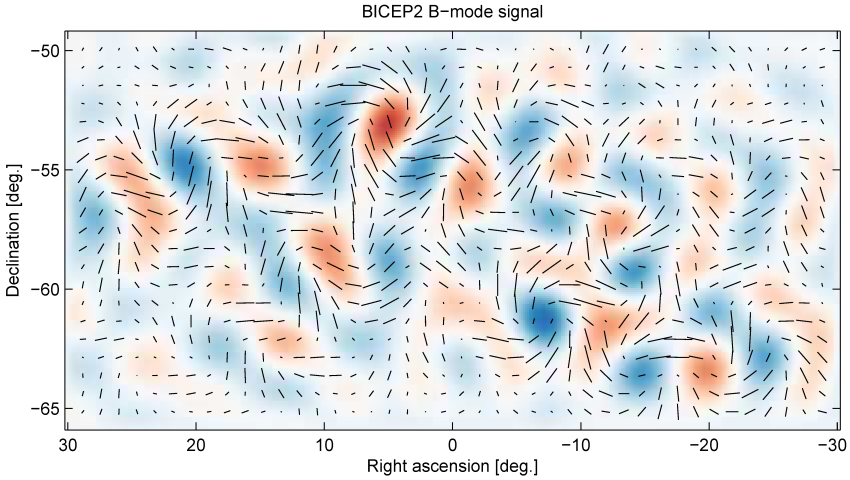
BICEP2 Telescope

More about BICEP observations

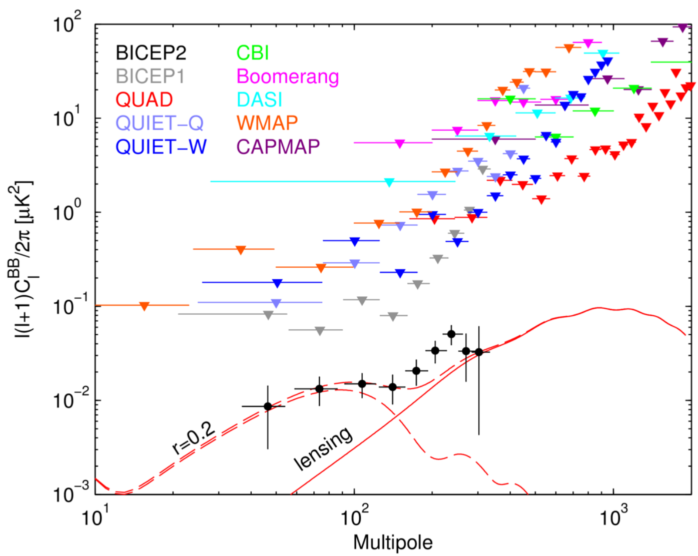
What is the present-day signal produced by tensor perturbations/gravity waves from early universe??

- Light from Cosmic Microwave background is slightly polarized.
- It is possible to map the polarization pattern using sensitive observations.
- There are two kinds of patterns we see in the polarization map: E-modes and B-modes (inspired by Electric and Magnetic fields).
- Most of the patterns are E-modes, produced by many sources including gravity waves; we don't know enough to resolve them.
- B-modes are much harder to detect, but have fewer sources.

BICEP2 B-mode Signal



BICEP2 Plot: Signal and Background



What does this mean?

Assuming BICEP2 signal did come from primordial gravity waves:

- The case gets stronger: Another prediction of the simplest inflation models has been observed.
- We know better: Many non-inflationary models as well as a majority of inflation models can be thrown away.
- Exciting times ahead: Lot more information to be gained through more detailed observations of tensor spectrum in near future.
- A gateway into HEP(?): Assuming the simplest models of inflation, we know the scale of inflation: $\sim 10^{16}$GeV - around the GUT scale.

The Lyth bound

A potential implication of such an observation was discussed by D. H. Lyth in 1997.

- Given a measurement of $\Delta^2_{\mathcal{R}}$ and Δ^2_{γ}, we can calculate H and ϵ during horizon exit.
- ϵ gives us a measure of how "far" the inflaton field rolls (in field space) during one e-folding of inflation.
- This makes it possible to compute the distance covered by the inflaton field during horizon exit of the few decades of modes we observe in CMB.
- An observable value of r typically means this distance is going to be large.

The Lyth bound

The field distance covered in units of Planck Mass:

$$\left(\frac{\Delta\phi}{M_P}\right) = \int_0^N dN' \sqrt{2\epsilon} = \frac{1}{\sqrt{8}} \int_0^N dN' \sqrt{r}$$

- To be conservative, let us assume $N \sim 7$ (3 decades of observed modes).
- BICEP value of tensor-to-scalar ratio $r = 0.2$
- This gives us $\left(\frac{\Delta\phi}{M_P}\right) \approx 1.1 \implies$ inflaton evolves through super-Planckian distances during inflation.
- Higher-order operators could possibly become relevant.

Is there a way around this?

- Suppose the perturbations are in an excited state (and not the Bunch-Davies state) during inflation.
- This affects the spectrum and it will affect the Lyth bound argument as well.

Mode functions for Bogoliubov transformed (excited) states:

$$\begin{aligned}
\mathcal{R}_{k,ex}(t) &= \alpha(k)\mathcal{R}_{k,BD}(t) + \beta(k)\mathcal{R}^*_{k,BD}(t) \\
\gamma^s_{k,ex}(t) &= \tilde{\alpha}(k)\gamma^s_{k,BD}(t) + \tilde{\beta}(k)\gamma^{s*}_{k,BD}(t) \\
|\alpha|^2 - |\beta|^2 &= |\tilde{\alpha}|^2 - |\tilde{\beta}|^2 = 1
\end{aligned}$$

Modified Lyth Bound

- From observations, we know that scalar spectrum is nearly scale invariant over the 4 decades of modes observed in CMB.
- This means that $\alpha(k)$ and $\beta(k)$ are nearly constant over the observed range of k.
- For tensor spectrum, we don't have enough data to say anything about scale invariance.
- For simplicity, we shall assume the tensor spectrum is also scale invariant over the 4 decades of k.

Modified Lyth Bound:

$$\left(\frac{\Delta\phi}{M_P}\right) = \frac{|\alpha+\beta|}{|\tilde{\alpha}+\tilde{\beta}|}\frac{1}{\sqrt{8}}\int_0^N dN'\sqrt{r}$$

Constraints on β and $\tilde{\beta}$

- Though we have some freedom in choosing the excited state of fluctuations, there are constraints we must satisfy.
- Two major constraints that are relevant for us: Subhorizon constraint and Backreaction constraint.
- We will now look how these constraints affect scalar modes (similar argument for tensor modes).

Subhorizon Constraint

- Life of a mode: Physical momentum $p = k/a$ monotonically decreases with time (wavelength increases with time).
- Going back to the original motivations of inflation: we expect inflation to solve the horizon problem.
- Therefore, if inflation had a beginning, at the start of inflation, all the modes we observe should have been "inside the horizon" ($p > H$).
- During inflation, as H stays constant and universe expands, these modes exit the horizon at some point.
- After inflation, H decreases faster than k/a (decelerating expansion) and eventually catches up, meaning modes re-enter the horizon.
- This must have happened a little before CMB was created.

Backreaction Constraint

- When perturbations are in an excited state, they carry more energy than they do in the ground state.
- However, if this energy is too large, then the fluctuations are big enough to overwhelm the background evolution/inflation.
- In fact, even before they are large enough to do this, they begin to affect the perturbative expansion of the inflationary action.
- To avoid this problem, we enforce a "back-reaction constraint": it is most stringent at early times like the beginning of inflation.
- It constrains the excited modes to have a certain maximum p value.

$$\langle \rho_{\mathcal{R}} \rangle \sim \frac{|\beta|^2}{8\pi^2} p_{UV}^4 \ll 3\epsilon M_P^2 H^2$$

Combining the constraints

- Combining the two constraints put stringent restrictions on β (and similarly $\tilde{\beta}$).

- At the beginning of inflation, after some of the modes were excited due to some unknown physics, we must have had a time when:

  1. All the 4 decades of observed modes were sub-horizon $p \gg H$.
  2. All the 4 decades of observed modes satisfied the backreaction constraint $p < p_{UV}$

- Satisfying these constraints gives us $|\beta| \leq 0.02$ and $|\tilde{\beta}| \leq 0.02$.

- Net result: There is at best a 4% difference in the Lyth bound RHS: $\Longrightarrow$ STILL HAVE SUPER-PLANCKIAN EVOLUTION!!

Summary

- The reported observation of primordial gravity waves by BICEP2 is favourable to the inflationary paradigm.
- If confirmed, it points to more observational data around the corner and promise of a much better understanding of the very-early universe.
- In the context of standard single-field slow-roll inflation, this observation indicates a super-Planckian excursion of the Inflaton field.
- Even if we allow scalar and tensor modes to be in Bogoliubov transformed (excited) states over the adiabatic vacuum, this conclusion is not significantly affected.
- This is mostly because of the stringent limits coming from the sub-horizon and back-reaction constraints.

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