Non-perturbative gravitationally produced scalar particles and induced gravitational waves

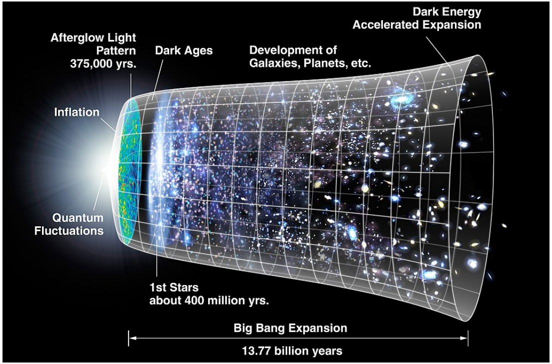
Ayan Chakraborty

IIT Guwahati
Department of Physics

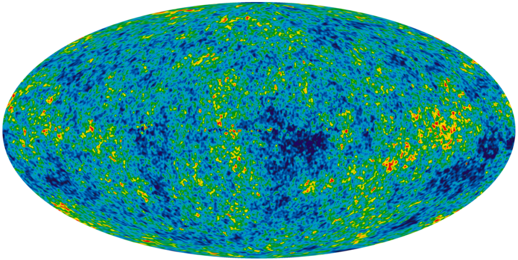
29th January 2026

Overview

Cosmic Evolution and Cosmic Microwave Background(CMB)



The full sky CMB temperature map



- The early inflationary era explains the tiny anisotropies observed over a uniform CMB temperature, $T_0 = 2.725$ K.

[1] NASA / WMAP Science Team
[2] NASA Universe Web Team

Observational challenges to probe the early era

CMB predictions of inflation

- **Energy scale:** $E_{\rm inf} \sim 10^{15}$ GeV
- **Time-scale:** $t_{\rm inf} \sim 10^{-36}$ sec

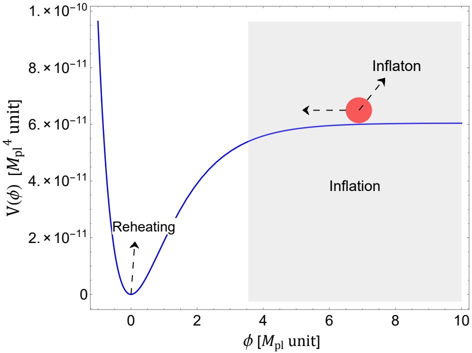
BBN prediction

- **Energy scale:** $E_{\rm BBN} \sim 4$ MeV
- **Time-scale:** $t_{\rm BBN} \sim 1$ sec

- Inflation $\xrightarrow{\text{enormous gap in energy and time scale}}$ Big-Bang nucleosynthesis(BBN) phase
- This intermediate phase is called *"Reheating"* that bridges the gap between inflation and BBN.
- This intermediate early universe phase is poorly understood from observational perspective.

Why do we need reheating phase?

- At the end of early accelerated expansion(Inflation), universe was left in a cold state of vanishing entropy, and particle no. density.
- To achieve successful nucleosynthesis, universe must transit to a hot, thermalized radiation-dominated phase.



Inflaton$\longrightarrow$ SM+BSM $\longrightarrow$ hot thermal bath $\longrightarrow$ reheating completes

Non-perturbative production of scalar fluctuations: General Formalism

Non-perturbative production of scalar fluctuations: Theoretical framework

- We shall work with spin-0 scalar fluctuations.
- **Lagrangian of the system:**

$$\mathcal{L}_{[\phi,\chi]} = -\underbrace{\sqrt{-g}}_{a^4(\eta)}\left(\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + V(\phi) + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi + \frac{1}{2}\underbrace{\left(m_\chi^2 + F(\phi) + \xi R\right)}_{m_{\text{eff}}^2}\chi^2\right)$$

$a \rightarrow$ scale factor; $R \rightarrow$ Ricci scalar; $\xi \rightarrow$ non-minimal coupling; $m_\chi \rightarrow$ bare mass of scalar field(χ);

$V(\phi) \rightarrow$ inflaton potential; $F(\phi) \rightarrow$ Generic interaction between inflaton and produced scalar

Non-perturbative production of scalar fluctuations: Theoretical framework

- We shall work with spin-0 scalar fluctuations.
- **Lagrangian of the system:**

$$\mathcal{L}_{[\phi,\chi]} = -\underbrace{\sqrt{-g}}_{a^4(\eta)}\left(\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + V(\phi) + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi + \frac{1}{2}\underbrace{\left(m_\chi^2 + F(\phi) + \xi R\right)}_{m_{\rm eff}^2}\chi^2\right)$$

$a \rightarrow$ scale factor; $R \rightarrow$ Ricci scalar; $\xi \rightarrow$ non-minimal coupling; $m_\chi \rightarrow$ bare mass of scalar field(χ);

$V(\phi) \rightarrow$ inflaton potential; $F(\phi) \rightarrow$ Generic interaction between inflaton and produced scalar

- **Fourier decomposition:** $\hat{\chi}(\eta,\vec{x}) = \int \frac{d^3k}{(2\pi)^3}\left(\chi_{\vec{k}}(\eta)\ \hat{a}_k + \chi^*_{\vec{k}}(\eta)\hat{a}^\dagger_{-k}\right)\ e^{i\vec{k}.\vec{x}}$
- **EoM of rescaled field mode(**$X_k = a(\eta)\chi_k(\eta)$**):**

$$X_k'' + \left[k^2 + a^2\left(m_\chi^2 + F(\phi)\right) + \frac{a^2R}{6}(6\xi - 1)\right]X_k = 0 \qquad (1)$$

$R = \left(6a''/a^3\right)$, $\omega_k^2 = k^2 + a^2\left(m_\chi^2 + F(\phi)\right) + \frac{a^2R}{6}(6\xi - 1)$

Defining a few important quantities

- **Power spectrum:** $\langle 0|\,|\hat{\chi}|^2\,|0\rangle = \int d(\ln k)\,\frac{k^3}{2\pi^2\,a^2}|X_k(\eta)|^2 \Rightarrow \boxed{\mathcal{P}_\chi \equiv \frac{k^3}{2\pi^2\,a^2}|X_k(\eta)|^2}$
- **Late-time number operator:** $\hat{N}(\eta) = \int \frac{d^3k}{(2\pi)^3}\,a^\dagger_{-\vec{k}}(\eta)a_{\vec{k}}(\eta)$
- **Number density spectrum:**

$$_{\rm BD}\langle 0|\hat{N}|0\rangle_{\rm BD} = a^3 n_\chi = \int \frac{d^3k}{(2\pi)^3}|\beta_k|^2, \quad \boxed{n_k = |\beta_k|^2 = \frac{1}{2\omega_k}|\omega_k X_k - iX_k'|^2}$$

- **Energy density spectrum:** $a^4\rho_\chi = \frac{1}{(2\pi)^3}\int d^3k\,\omega_k n_k, \quad \boxed{\rho_{\chi_k} = \omega_k\,n_k}$

▸ Energy-momentum tensor

Gravitational production of minimally coupled scalar fluctuation: Generalization of the Bogoliubov vs Boltzmann framework

- Based on the work: Generalizing the Bogoliubov vs Boltzmann approaches in gravitational production Phys. Rev. D 112 043511 (2025)

Motivation

- To study the massless and massive scalar spectrum in large(IR) and small(UV) scales generated by gravity during inflation and the general reheating phase in the Bogoliubov framework.

Motivation

- To study the massless and massive scalar spectrum in large(IR) and small(UV) scales generated by gravity during inflation and the general reheating phase in the Bogoliubov framework.

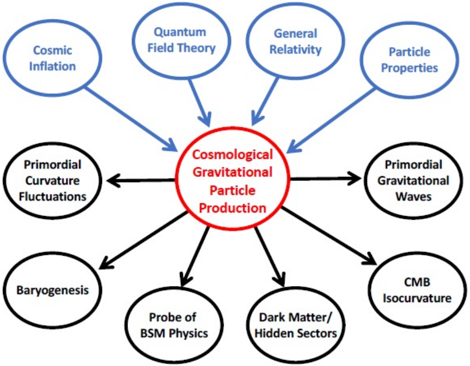
- To compute a limit of the IR scale below which the IR spectrum departs from the massless limit for a finite mass(m_χ) of the fluctuation.

Motivation

- To study the massless and massive scalar spectrum in large(IR) and small(UV) scales generated by gravity during inflation and the general reheating phase in the Bogoliubov framework.
- To compute a limit of the IR scale below which the IR spectrum departs from the massless limit for a finite mass(m_χ) of the fluctuation.
- To do a comparative analysis between gravity-mediated($h_{\mu\nu}$) process($\phi\phi \xrightarrow{h_{\mu\nu}} \chi\chi$) in the Boltzmann approach and the small-scale(UV) spectrum computed in the Bogoliubov treatment for a general reheating background with minute background(inflaton(ϕ)) oscillation effect.

CGPP: Significance and potential outputs

- CGPP ⟶ Quantum mechanical particle production in a time-dependent background
- Natural portal for the generation of elementary particles(SM and BSM) without any non-gravitational interaction.
- Testable link between early-universe dynamics and present-day cosmological observations.



Potential output of CGPP

- Provides the origin of primordial plasma through gravitational reheating
- Unavoidable production channel of primordial gravitational waves.
- the origin of Dark Matter(DM) and particles in a hidden sector.

General set up of minimally coupled($\xi = 0$) scalar field(χ) system

- **Lagrangian of the system:** $\mathcal{L}_{[\phi,\chi]} = -\underbrace{\sqrt{-g}}_{a^4(\eta)}\left(\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + V(\phi) + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi + \frac{1}{2}m_\chi^2\chi^2\right)$

General set up of minimally coupled($\xi = 0$) scalar field(χ) system

- **Lagrangian of the system:** $\mathcal{L}_{[\phi,\chi]} = -\underbrace{\sqrt{-g}}_{a^4(\eta)}\left(\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + V(\phi) + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi + \frac{1}{2}m_\chi^2\chi^2\right)$

- **EoM of rescaled field mode($X_k = a(\eta)\chi_k(\eta)$):**

$$X_k'' + \left[\underbrace{k^2 + a^2 m_\chi^2 - \frac{a''}{a}}_{\omega_k^2}\right] X_k = 0 \tag{2}$$

source term $\rightarrow (-a''/a)$, for a massless system

Dynamical equation during inflation and reheating

- **Form of scale factor:** $a(\eta) = a_{\rm end}\left(\frac{1+3w_\phi}{2|\eta_{\rm end}|}\right)^{\frac{2}{1+3w_\phi}}\left(\eta - \eta_{\rm end} + \frac{2|\eta_{\rm end}|}{1+3w_\phi}\right)^{\frac{2}{1+3w_\phi}};\ \eta_i < \eta \leq \eta$

Dynamical equation during inflation and reheating

- **Form of scale factor:** $a(\eta) = a_{\rm end}\left(\frac{1+3w_\phi}{2|\eta_{\rm end}|}\right)^{\frac{2}{1+3w_\phi}}\left(\eta - \eta_{\rm end} + \frac{2|\eta_{\rm end}|}{1+3w_\phi}\right)^{\frac{2}{1+3w_\phi}}; \; \eta_i < \eta \leq \eta$

- **Inflationary evolution:** $X_k'' + \left[k^2 - \frac{\left(2-\frac{m_\chi^2}{H_{\rm end}^2}\right)}{\eta^2}\right]X_k = 0$

- **Post-inflationary evolution:** $X_k'' + \left[k^2 + a^2(\eta)m_\chi^2 - \frac{2(1-3w_\phi)}{(1+3w_\phi)^2\left(\eta+\frac{3(1+w_\phi)}{a_{\rm end}H_{\rm end}(1+3w_\phi)}\right)^2}\right]X_k = 0$

Inflationary and post-inflationary vacuum solution for massless minimal system

Adiabatic vacuum solution

Inflationary vacuum solution: $X_k^{(\mathrm{inf})} = \frac{\sqrt{\pi|\eta|}}{2} e^{i(\pi/4+\pi\nu_1/2)} H^{(1)}_{\nu_1}(k|\eta|)$

Post-inflationary vacuum solution: $X_k^{(\mathrm{reh})} = \sqrt{\frac{\bar{\eta}}{\pi}} \exp\left[\frac{3ik\mu}{a_{\mathrm{end}} H_{\mathrm{end}}} + \frac{i\pi}{4}\right] K_{\nu_2}(ik\bar{\eta})$

- **EoS dependent indices:** $\nu_1 = 3/2$; $\mu = \frac{(1+w_\phi)}{(1+3w_\phi)}$; $\nu_2 = \frac{3}{2}\frac{(1-w_\phi)}{(1+3w_\phi)}$; $\bar{\eta} = (\eta + 3\mu/a_{\mathrm{end}} H_{\mathrm{end}})$

Introduction to Bogoliubov coefficients

Inflationary and post-inflationary vacuum solution for massless minimal system

Adiabatic vacuum solution

Inflationary vacuum solution: $X_k^{(\mathrm{inf})} = \frac{\sqrt{\pi|\eta|}}{2} e^{i(\pi/4+\pi\nu_1/2)} H_{\nu_1}^{(1)}(k|\eta|)$

Post-inflationary vacuum solution: $X_k^{(\mathrm{reh})} = \sqrt{\frac{\bar{\eta}}{\pi}} \exp\left[\frac{3ik\mu}{a_{\mathrm{end}} H_{\mathrm{end}}} + \frac{i\pi}{4}\right] K_{\nu_2}(ik\bar{\eta})$

- **EoS dependent indices:** $\nu_1 = 3/2$; $\mu = \frac{(1+w_\phi)}{(1+3w_\phi)}$; $\nu_2 = \frac{3}{2}\frac{(1-w_\phi)}{(1+3w_\phi)}$; $\bar{\eta} = (\eta + 3\mu/a_{\mathrm{end}} H_{\mathrm{end}})$
- **General reheating field solution:** $X_k(\eta) = \alpha_k X_k^{(\mathrm{reh})} + \beta_k X_k^{*(\mathrm{reh})}$ $\alpha_k,\ \beta_k \longrightarrow$ Bogoliubov coefficients

▸ Introduction to Bogoliubov coefficients

Effect of finite mass term(m_χ)

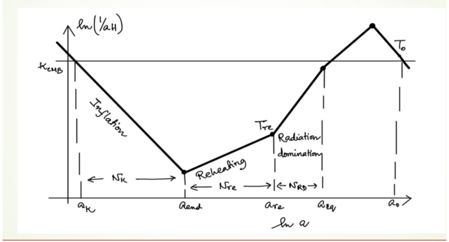
- **Frequency term for the massive minimally coupled system:**

$$\omega_k^2(\eta) = \left(k^2 + \underbrace{a^2 m_\chi^2 + \left(\frac{3w_\phi - 1}{2}\right) a^2 H^2}_{\text{comparable contribution}}\right)$$

- For a given mode k, bare mass term will be important when $a^2 m_\chi^2 \propto a^2 H^2$

$$\left(\frac{a_m}{a_{\rm end}}\right) = \begin{cases} \left(\sqrt{\frac{2}{|3w_\phi - 1|}}\frac{m_\chi}{H_e}\right)^{-\frac{2}{3(1+w_\phi)}} & w_\phi \neq 1/3 \\ \sqrt{\frac{H_{\rm end}}{m_\chi}} & w_\phi = 1/3 \end{cases}$$

[1] A.chakraborty, S.Cléry, M.R.Haque, D.Maity and Y.Mambrini [https://arxiv.org/abs/2503.21877]



Massless IR spectrum

$$|\beta_k|^2_{\text{IR},\, m_\chi=0} \propto \left(\frac{k_{\rm end}}{k}\right)^{(2\bar\nu+3)}$$

$\bar\nu = \frac{3}{2}\frac{(1-w_\phi)}{(1+3w_\phi)}$, "end"$\to$ end of inflation,
$w_\phi \to$ reheating equation of state(EoS)

Finite mass effect

$$\frac{k_m}{k_{\rm end}} = \begin{cases} \left(\sqrt{\frac{2}{|3w_\phi-1|}}\frac{m_\chi}{H_{\rm end}}\right)^{\frac{1+3w_\phi}{3(1+w_\phi)}} & w_\phi \neq 1/3 \\ \sqrt{\frac{m_\chi}{H_{\rm end}}} & w_\phi = 1/3 \end{cases}$$

- Any $k \lesssim k_m$ suffers from mass breaking effect.

$$|\beta_k|^2_{\text{IR},\, m_\chi\neq 0} \propto \begin{cases} \left(\frac{k_{\rm end}}{k}\right)^3 & \frac{m_\chi}{H_{\rm end}} < 3/2 \\ e^{-\frac{\pi\, m_\chi}{H_{\rm end}}} & \frac{m_\chi}{H_{\rm end}} > 3/2 \end{cases}$$

Importance of Hubble oscillations for small-scale UV modes $k > k_{\rm end}$

- **Hubble oscillation**: $H(a) \simeq$

$$\bar{H}\left(1+\underbrace{\frac{\mathcal{P}\sqrt{6(1-\mathcal{P}^{2n})}}{2(n+1)}\left(\frac{\phi_{\rm end}}{M_{\rm pl}}\right)\left(\frac{a}{a_{\rm end}}\right)^{-\frac{3}{n+1}}}_{\text{fast oscillatory term}}\right)$$

Nature of UV modes

- Never exited the horizon, always remains quantum.
- Never experiences non-adiabatic evolution.
- These small-scales can sense the tiny background oscillations.

Background quantities

Computation of UV spectrum

- $\beta_k(t) \simeq \int_{t_{\rm end}}^{t} dt' \, \frac{\dot{\omega}_k}{2\omega_k} e^{-2i\Omega_k(t')}$

  with $\Omega_k(t') = \int_{t_{\rm end}}^{t'} \frac{\omega_k(t)}{a(t)} dt$

- $\omega_k^2(t) = k^2 + a^2(m_\chi^2 - 2H^2 - \dot{H})$

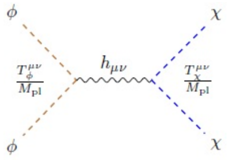
Small-scale UV Spectrum $k > k_{\rm end}$

UV spectrum

$$|\beta_k|^2_{\rm UV} \propto \begin{cases} (\bar{\mathcal{N}}_0)^2\, k^{\frac{9(w_\phi-1)}{2-6w_\phi}} + \underbrace{\bar{\mathcal{N}}_0\bar{\mathcal{N}}_2 k^{\frac{45w_\phi-21}{4(1-3w_\phi)}} \cos\psi}_{\text{interference term}} & w_\phi \leq 1/9 \\ (\bar{\mathcal{N}}_2)^2\, k^{-6} + \underbrace{\bar{\mathcal{N}}_0\bar{\mathcal{N}}_2 k^{\frac{45w_\phi-21}{4(1-3w_\phi)}} \cos\psi}_{\text{interference term}} & 1/9 < w_\phi < 1/3 \\ \frac{1}{16f^2(w_\phi)} k^{-6} \times (\text{oscillation coefficient}) & w_\phi \geq 1/3 \end{cases}$$

$f(w_\phi) = \left(-\frac{(1+3w_\phi)}{3(1+w_\phi)}\right)$, $\bar{\mathcal{N}}_0$ and $\bar{\mathcal{N}}_2 \rightarrow$ carrying information of background oscillation, $\psi \rightarrow$ phase factor(function of momentum mode k)

Boltzmann equation



Interaction Lagrangian

$$\mathcal{L}_{\rm int} = -\frac{h^{\mu\nu}}{M_{\rm pl}}\left(T^{\phi}_{\mu\nu} + T^{\chi}_{\mu\nu}\right)$$

$M_{\rm pl} \rightarrow$ Reduced Planck mass $h_{\mu\nu} \rightarrow$ graviton field, $T^{\phi}_{\mu\nu} \rightarrow$ Stress-energy tensor of inflaton, $T^{\chi}_{\mu\nu} \rightarrow$ Stress-energy tensor of the produced scalar

Boltzmann Spectrum

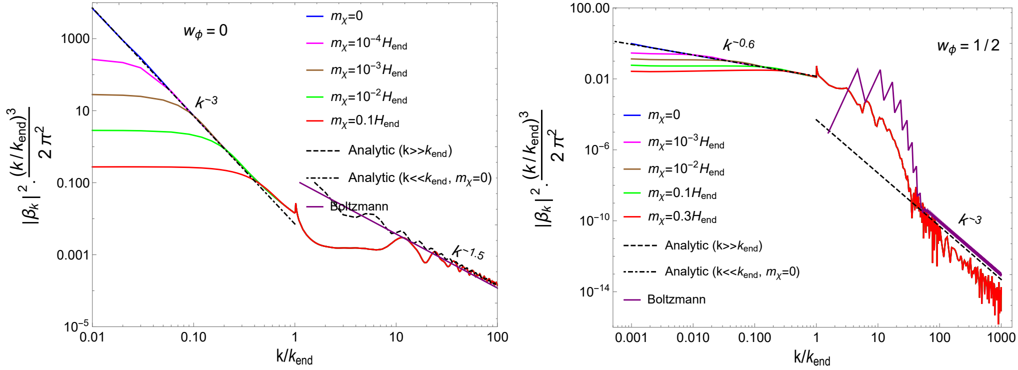
For $w_\phi \neq 1/3$:

$$f(k,a) \propto \left(\frac{k_{\rm end}}{k}\right)^{\frac{9(1-w_\phi)}{2(1-3w_\phi)}} \sum_{\nu=1}^{\infty} |\mathcal{P}_\nu^{2n}|^2 \left(\frac{\nu}{2}\right)^{\frac{3(1+3w_\phi)}{2(1-3w_\phi)}}$$
$$\times\, \theta\left(\left(\frac{2k}{\nu a_{\rm end}\, m_\phi^{\rm end}\gamma}\right)^{\frac{1}{1-3w_\phi}} - 1\right)$$

For $w_\phi = 1/3$:

$$f(k,a) \propto \left[1 - \left(\frac{a_{\rm end}}{a}\right)^3\right] \sum_{\nu=1}^{+\infty} \frac{|\mathcal{P}_\nu^{2n}|^2}{\nu^2}$$
$$\times\, \delta\left(k/k_{\rm end} - \nu\gamma m_\phi^{\rm end}/2H_{\rm end}\right)$$

Behavior of spectral density $|\beta_k|^2 \frac{(k/k_{\rm end})^3}{2\pi^2}$



Summary

- IR spectral index varies between -6 and -3, depending on the post-inflationary equation of state (EoS), $0 \leq w_\phi \leq 1$.

- For $m_\chi/H_{\rm end} \gtrsim 3/2$, the IR spectrum experiences exponential mass suppression, while for $m_\chi/H_{\rm end} < 3/2$, the spectrum remains flat in the IR regime regardless of the EoS.

- In the UV regime, for any w_ϕ, $1/9 \lesssim w_\phi \lesssim 1$, the spectral behavior turns out to be independent of the EoS, with a spectral index -6. Further, the oscillations of inflaton background lead to *interference terms explaining the high-frequency oscillations in the spectrum*.

- We found an interesting equivalence between Boltzmann and Bogoliubov approaches in the UV regime for any EoS, $0 \lesssim w_\phi \lesssim 1$.

Part II

Gravitational production of nonminimally coupled scalar fluctuation: Induced Gravitational Wave

Based on the work:

- Probing a nonminimal coupling through superhorizon instability and secondary gravitational waves Phys. Rev. D 111, 083505 (2025)

Motivation

- Scalar fluctuation, nonminimally coupled to gravity, can be treated as a potential source of secondary gravitational waves(SGWs). This is an unavoidable production channel of secondary or induced GWs.
- Significant post-inflationary long-wavelength(IR) instability of the source field beyond a certain coupling strength leaves a visible imprint on secondary gravitational wave spectrum, which can be probed by various future GW observatories.
- Constraining nonminimal coupling through *Planck* bound on tensor-to-scalar ratio and $\Delta N_{\rm eff}$.

Part I

Based on the work:

- Probing a nonminimal coupling through superhorizon instability and secondary gravitational waves

Dynamical equation and appearance of IR(Infrared) instability(Tachyonic instability) during inflation and reheating

- We are interested in IR modes ($k < a_{\rm end} H_{\rm end} = k_{\rm end}$) of very low mass case, $m_\chi \approx 0$

Dynamical equation and appearance of IR(Infrared) instability(Tachyonic instability) during inflation and reheating

- We are interested in IR modes ($k < a_{\rm end}H_{\rm end} = k_{\rm end}$) of very low mass case, $m_\chi \approx 0$

- **Inflationary evolution:** $$X_k'' + \underbrace{\left[k^2 - \frac{2(1-6\xi)}{\eta^2}\right]}_{\omega_k^2<0(\text{Instability})\rightarrow \text{ for } \xi<1/6} X_k = 0$$

- **Post-inflationary evolution:** $$X_k'' + \underbrace{\left[k^2 - \frac{2(1-3w_\phi)(1-6\xi)}{(1+3w_\phi)^2\left(\eta + \frac{3(1+w_\phi)}{a_{\rm end}H_{\rm end}(1+3w_\phi)}\right)^2}\right]}_{\omega_k^2<0\rightarrow \text{ for } w_\phi>1/3,\ \xi>1/6,\ \text{for } w_\phi<1/3,\ \xi<1/6} X_k = 0$$

Inflationary and post-inflationary vacuum solution

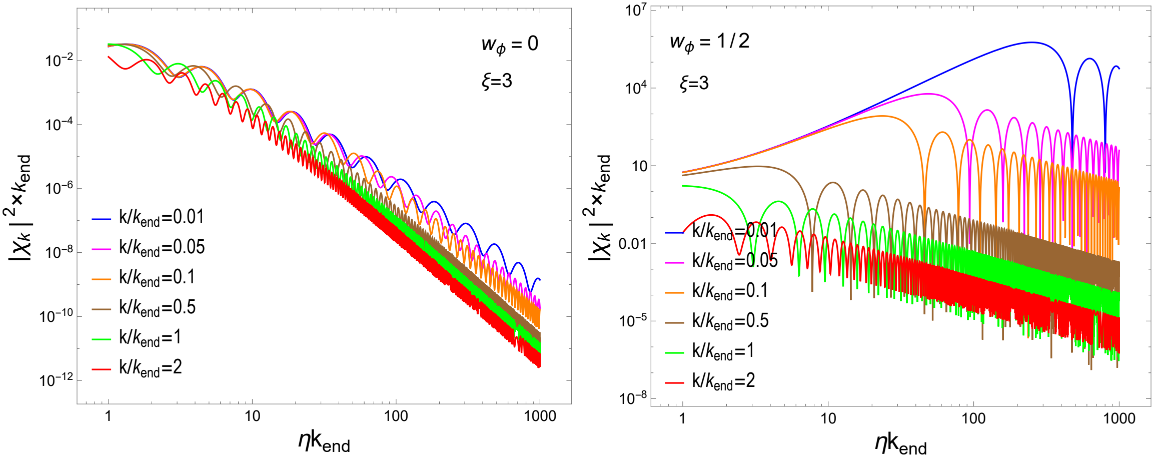
Adiabatic vacuum solution

Inflationary vacuum solution: $X_k^{(\mathrm{inf})} = \frac{\sqrt{\pi|\eta|}}{2} e^{i(\pi/4+\pi\nu_1/2)} H^{(1)}_{\nu_1}(k|\eta|)$

Post-inflationary vacuum solution: $X_k^{(\mathrm{reh})} = \sqrt{\frac{\bar{\eta}}{\pi}} \exp\left[\frac{3ik\mu}{a_{\mathrm{end}}H_{\mathrm{end}}} + \frac{i\pi}{4}\right] K_{\nu_2}(ik\bar{\eta})$

- **EoS and ξ dependent indices:** $\nu_1 = \sqrt{9-48\xi}/2$; $\mu = \frac{(1+w_\phi)}{(1+3w_\phi)}$;
  $\nu_2 = \frac{\sqrt{3(1+w_\phi)\left(3(1-w_\phi)^2+16\xi(3w_\phi-1)\right)}}{2\sqrt{1+3w_\phi}\sqrt{1+4w_\phi+3w_\phi^2}}$; $\bar{\eta} = (\eta + 3\mu/a_{\mathrm{end}}H_{\mathrm{end}})$
- **General reheating field solution:** $X_k(\eta) = \alpha_k X_k^{(\mathrm{reh})} + \beta_k X_k^{*(\mathrm{reh})}$ $\alpha_k,\ \beta_k \longrightarrow$ Bogoliubov coefficients

Time-evolution of long-wavelength(IR) modes of scalar fluctuations

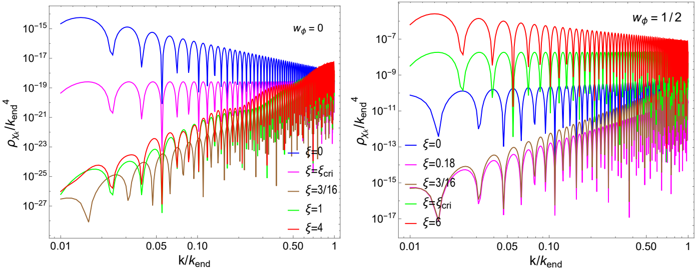


[1] A.chakraborty, S.Maiti, and D.Maity [arxiv: 2408.07767]

Behavior of energy density spectrum

- **Field power spectrum:**
  $\mathcal{P}_\chi(k,\eta) = \frac{k^3}{2\pi^2 a^2}|X_k|^2$
- **Field energy-density spectrum:**

  $\rho_{\chi_k}(\eta) = \frac{k^3}{4\pi^2 a^4}(|X_k'|^2 + k^2|X_k|^2) = (k^2/a^2)\mathcal{P}_\chi(k,\eta)$



- $w_\phi = 0 \rightarrow \xi_{\rm cri} \approx 5/48$

  $w_\phi = 1/2 \rightarrow \xi_{\rm cri} \approx 4.073$

▸ Energy-spectrum for $0 \leq w_\phi < 1/3$

Model independent definition of reheating parameters($N_{\rm re}$, $T_{\rm re}$)

Reheating point: $\rho_{\rm R}(a_{\rm re}) = \rho_\phi(a_{\rm re})$

Reheating e-folding number: $N_{\rm re} = \frac{1}{3(1+w_\phi)} \ln\left(\frac{90 H_{\rm end}^2 M_{\rm pl}^2}{\pi^2 g_{\rm re} T_{\rm re}^4}\right)$

Defining $k_{\rm end}$ and $k_{\rm re}$:

$(k_{\rm end}/a_0) = \left(\frac{43}{11 g_{\rm re}}\right)^{1/3} \left(\frac{\pi^2 g_{\rm re}}{90}\right)^{\alpha} \frac{H_{\rm end}^{1-2\alpha} T_{\rm re}^{4\alpha-1} T_0}{M_{\rm pl}^{2\alpha}}$, $(k_{\rm end}/k_{\rm re}) = \exp\left(\frac{N_{\rm re}(1+3w_\phi)}{2}\right)$, $\alpha = 1/3(1+w_\phi)$, $a_0 \rightarrow$ present scale factor, and $T_0 = 2.725$ K is the present CMB temperature

[1] L. Dai, M. Kamionkowski and J. Wang, PRL. 113, 041302 (2014)

[2] J. L. Cook, et al. JCAP 04 (2015) 047

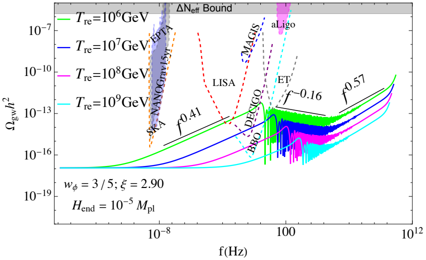
Generation of secondary(induced) gravitational wave(SGW)

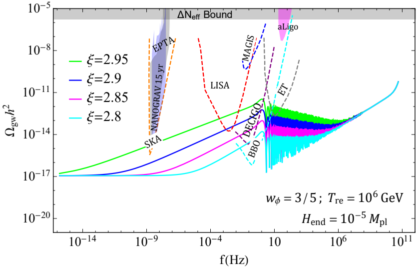
- **Perturbed FLRW metric:** $ds^2 = a^2(\eta)\left[-d\eta^2 + (\delta_{ij} + h_{ij})dx^i dx^j\right]$, transverse-traceless tensor $\rightarrow \partial^i h_{ij} = h^i_i = 0$
- **anisotropic stress tensor:** $\Pi_{ij} \sim (1-2\xi)\partial_i\chi\partial_j\chi - 2\xi\chi\partial_i\partial_j\chi + \xi\chi^2 G_{ij}$
- **Evolution equation:** $h^{\lambda\prime\prime}_{\mathbf{k}} + 2\frac{a'}{a}h^{\lambda\prime}_{\mathbf{k}} + k^2 h^{\lambda}_{\mathbf{k}} = \frac{2}{M^2_{\rm pl}} e^{ij}_{\lambda}(k) P^{lm}_{ij}(\hat{k}) T_{lm}(k,\eta)$, $P^{lm}_{ij}(\hat{k}) \rightarrow$ transverse-traceless projector

▸ outline of evolution Equation

Defining Gravitational wave(GW) energy spectrum

- **GW energy spectrum:** $\Omega_{\rm gw}(k,\eta) = (\Omega_{\rm gw}^{\rm pri} + \Omega_{\rm gw}^{\rm sec}) = \frac{(1+k^2/k_{\rm re}^2)}{24}(\mathcal{P}_{\rm T}^{\rm pri}(k,\eta_{\rm re}) + \mathcal{P}_{\rm T}^{\rm sec}(k,\eta_{\rm re}))$
- **Energy spectrum for today:** $\Omega_{\rm gw}(k)h^2 \approx \left(\frac{g_{r,0}}{g_{r,eq}}\right)^{1/3} \Omega_R h^2 \Omega_{\rm gw}(k,\eta),\ \ \Omega_R h^2 = 4.3\times 10^{-5}$





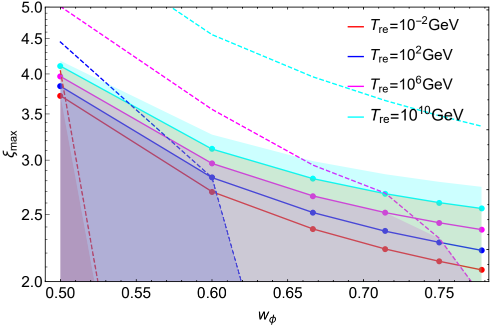
▸ Defining secondary tensor power spectrum

Constraining ξ through tensor-to-scalar ratio($r_{0.05}$) and $\Delta N_{\rm eff}$(for the source field)

- For $w_\phi > 1/3$, in the regime $k < k_{\rm re}$;

$$r_{0.05} \propto \left(\frac{90 H_{\rm end}^2 M_{\rm pl}^2}{\pi^2 g_{\rm re} T_{\rm re}^4}\right)^{\frac{2(3w_\phi - 1)}{3(1+w_\phi)}} \left(\frac{k_*}{k_{\rm end}}\right)^{4(2-\nu_2)} \leq 0.036;$$

$$(k_*/a_0) \equiv (k_{\rm CMB}/a_0) = 0.05\ {\rm Mpc}^{-1}$$



[1] N. Aghanim et al. (Planck)[arXiv: 1807.06209[astro-ph.CO]]

Constraining ξ through tensor-to-scalar ratio($r_{0.05}$) and $\Delta N_{\rm eff}$(for the source field)

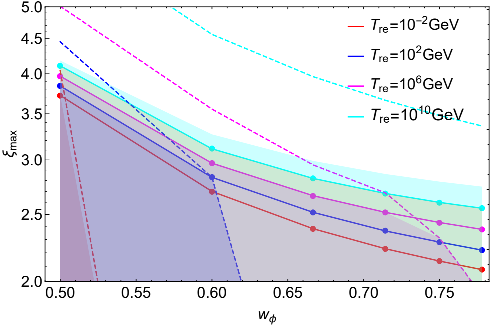
- For $w_\phi > 1/3$, in the regime $k < k_{\rm re}$;

$$r_{0.05} \propto \left(\frac{90 H_{\rm end}^2 M_{\rm pl}^2}{\pi^2 g_{\rm re} T_{\rm re}^4}\right)^{\frac{2(3w_\phi - 1)}{3(1+w_\phi)}} \left(\frac{k_*}{k_{\rm end}}\right)^{4(2-\nu_2)} \leq 0.036;$$

$$(k_*/a_0) \equiv (k_{\rm CMB}/a_0) = 0.05\ {\rm Mpc}^{-1}$$

- This massless scalar, possible candidate for dark radiation(cosmological relic), solely contributes to $\Delta N_{\rm eff}$ then,

$$\Delta N_{\rm eff} \to \left(\frac{g_{r,0}}{g_{r,eq}}\right)^{1/3} \Omega_{\rm R} h^2\, \Omega_\chi(\eta_{\rm re}) \simeq 1.6 \times 10^{-6} \left(\frac{\Delta N_{\rm eff}}{0.284}\right)$$

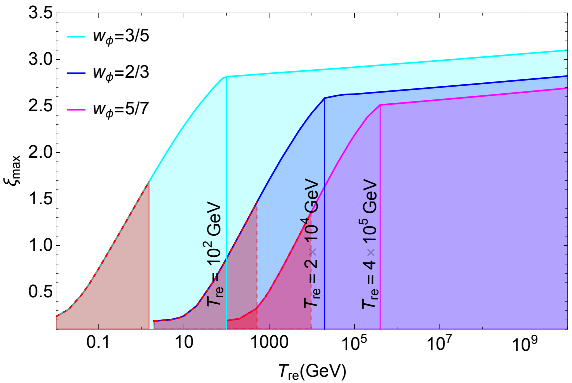


[1] N. Aghanim et al. (Planck)[arXiv: 1807.06209[astro-ph.CO]]

- If GWs(PGW+SGW) solely contributes to $\Delta N_{\rm eff}$ then

$$\Omega_{\rm gw} h^2 \leq 1.6 \times 10^{-6} \left(\frac{\Delta N_{\rm eff}}{0.284}\right),$$

$$\Omega_{\rm gw} h^2 = \int_{k_{\rm min}}^{k_{\rm end}} \frac{dk}{k} \Omega_{\rm gw}(k) h^2$$



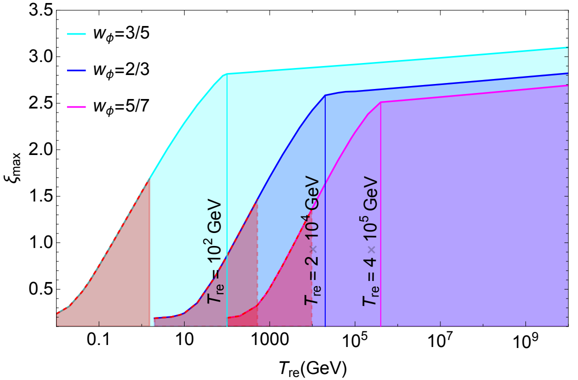
[1] S. Maiti, D. Maity, and L. Sriramkumar, (2024)[arXiV:2401.01864[gr-qc]]

Contribution of GWs to $\Delta N_{\rm eff}$

- If GWs(PGW+SGW) solely contributes to $\Delta N_{\rm eff}$ then

$$\Omega_{\rm gw} h^2 \leq 1.6 \times 10^{-6} \left(\frac{\Delta N_{\rm eff}}{0.284}\right),$$

$$\Omega_{\rm gw} h^2 = \int_{k_{\rm min}}^{k_{\rm end}} \frac{dk}{k} \Omega_{\rm gw}(k) h^2$$



- **Minimum bound on $T_{\rm re}$(avoiding overproduction of extra degrees of freedom):**

$$T_{\rm re}^{\rm min} \geq \left(\frac{90 H_{\rm end}^2 M_{\rm pl}^2}{\pi^2 g_{\rm re}}\right)^{1/4} \beta^{\frac{3(1+w_\phi)}{4(3w_\phi-1)}} \left(\frac{0.284}{\Delta N_{\rm eff}}\right)^{\frac{3(1+w_\phi)}{4(3w_\phi-1)}}, \ \beta \to \left(1.43 \times 10^{-11}/n_w\right) \left(H_{\rm end}/10^{-5} M_{\rm pl}\right)^2$$

[1] S. Maiti, D. Maity, and L. Sriramkumar, (2024)[arXiV:2401.01864[gr-qc]]

Summary

- Post-inflationary instability effect is dominant for higher EoS $w_\phi > 1/3$. The longer the wavelength, the more the enhancement owing to prolonged instability for a larger coupling strength ξ.

Summary

- Post-inflationary instability effect is dominant for higher EoS $w_\phi > 1/3$. The longer the wavelength, the more the enhancement owing to prolonged instability for a larger coupling strength ξ.
- For $w_\phi > 1/3$, significant IR instability beyond a certain large ξ leaves a visible imprint on the SGW spectrum overcoming the PGW strength at the low and intermediate frequency ranges.

Summary

- Post-inflationary instability effect is dominant for higher EoS $w_\phi > 1/3$. The longer the wavelength, the more the enhancement owing to prolonged instability for a larger coupling strength ξ.
- For $w_\phi > 1/3$, significant IR instability beyond a certain large ξ leaves a visible imprint on the SGW spectrum overcoming the PGW strength at the low and intermediate frequency ranges.
- Combining two strong observational bounds, $r_{0.05}$ and $\Delta N_{\rm eff}$, to prevent the overproduction of tensor fluctuations at the CMB scale and the overproduction of extra relativistic degrees of freedom, we have found a tight constraint on coupling strength. We find that $\xi_{\rm max} \lesssim 4$ for any $w_\phi \geq 1/2$ for a wide range of reheating temperatures.

Conclusions

- Reheating is a very important early universe phase, which is poorly understood due to the absence of direct observational evidence. We can indirectly constrain this phase by studying its imprint on cosmic relics.

- We have focused on the rich dynamics of the large-scale fluctuations of the massless and massive scalar fields in the early reheating era.

- We decode the nonperturbative features of nonminimally coupled scalar field system through induced gravitational wave.

Thank you!

Energy-momentum tensor

Back

- **Energy-momentum tensor:** $T^{\chi}_{\mu\nu} = (1-2\xi)\partial_\mu\chi\partial_\nu\chi + \left(2\xi - \frac{1}{2}\right) g_{\mu\nu}(\partial_\alpha\chi\partial^\alpha\chi) + 2\xi\left(g_{\mu\nu}\chi\Box\chi - \chi\nabla_\mu\partial_\nu\chi\right) + \xi G_{\mu\nu}\chi^2 - \frac{1}{2}g_{\mu\nu}\left(m_\chi^2 + F(\phi)\right)\chi^2.$

- **Energy density:**
$_{\rm BD}\langle 0|\, T^{\chi\,0}_{0}\, |0\rangle_{\rm BD} = a^{-4}(\eta)\int \frac{d^3k}{(2\pi)^3}\left[\frac{1}{2}|X_k'|^2 + \frac{1}{2}\omega_k^2|X_k|^2 + \frac{1}{2}(1-6\xi)\left(\mathcal{H}^2 - \frac{1}{6}a^2 R\right)|X_k|^2\right] - a^{-4}(\eta)\int \frac{d^3k}{(2\pi)^3}(1-6\xi)\mathcal{H}\left(X_k X_k'^{\,*} + X_k^* X_k'\right)$

- **Late-time Ladder operators:** $a_{\vec{k}}(\eta) = \alpha_{\vec{k}}\hat{a}_{\vec{k}} + \beta^*_{\vec{k}}\hat{a}^\dagger_{-\vec{k}}, \quad a^\dagger_{-\vec{k}}(\eta) = \alpha^*_{\vec{k}}\hat{a}^\dagger_{-\vec{k}} + \beta_{\vec{k}}\hat{a}_{\vec{k}}$

- $(\alpha_k, \beta_k) \rightarrow$ Bogoliuvbov coefficients, satisfying $|\alpha_k|^2 - |\beta_k|^2 = 1$.

- **Bunch-Davies(BD) Vacuum:** $X_{\vec{k}}(\eta\rightarrow -\infty) = \frac{1}{\sqrt{2k}}e^{-i\,k\,\eta}$

Introduction to Bogoliubov coefficients

Back

- **Bogoliubov coefficients** (α_k, β_k)**:** Making the adiabatic vacuum solutions $X_k^{(\mathrm{inf})}$ and $X_k^{(\mathrm{reh})}$, and their first derivatives continuous at the junction $\eta = \eta_{\mathrm{end}}$, we compute the Bogoliubov coefficients as follows [1]:

$$\alpha_k = i\left(X_k^{(\mathrm{inf})'}(\eta_{\mathrm{end}})X_k^{(\mathrm{reh})^*}(\eta_{\mathrm{end}}) - X_k^{(\mathrm{inf})}(\eta_{\mathrm{end}})X_k^{(\mathrm{reh})^{*\prime}}(\eta_{\mathrm{end}})\right)$$

$$\beta_k = -i\left(X_k^{(\mathrm{inf})'}(\eta_{\mathrm{end}})X_k^{(\mathrm{reh})}(\eta_{\mathrm{end}}) - X_k^{(\mathrm{reh})'}(\eta_{\mathrm{end}})X_k^{(\mathrm{inf})}(\eta_{\mathrm{end}})\right) \tag{3}$$

where $(')$ denotes the derivative with respect to conformal time.

[1] M. R. de Garcia Maia Phys. Rev. D 48, 647 (1993)

Introduction to Bogoliubov coefficients

Back

- **Bogoliubov coefficients** (α_k, β_k)**:** Making the adiabatic vacuum solutions $X_k^{(\mathrm{inf})}$ and $X_k^{(\mathrm{reh})}$, and their first derivatives continuous at the junction $\eta = \eta_{\mathrm{end}}$, we compute the Bogoliubov coefficients as follows [2]:

$$\alpha_k = i\left(X_k^{(\mathrm{inf})\prime}(\eta_{\mathrm{end}})X_k^{(\mathrm{reh})*}(\eta_{\mathrm{end}}) - X_k^{(\mathrm{inf})}(\eta_{\mathrm{end}})X_k^{(\mathrm{reh})*\prime}(\eta_{\mathrm{end}})\right)$$

$$\beta_k = -i\left(X_k^{(\mathrm{inf})\prime}(\eta_{\mathrm{end}})X_k^{(\mathrm{reh})}(\eta_{\mathrm{end}}) - X_k^{(\mathrm{reh})\prime}(\eta_{\mathrm{end}})X_k^{(\mathrm{inf})}(\eta_{\mathrm{end}})\right) \quad (4)$$

where $(')$ denotes the derivative with respect to conformal time.

[2] M. R. de Garcia Maia Phys. Rev. D 48, 647 (1993)

Background quantities

Back

- **Slow-varying Hubble scale term:** $\bar{H} = H_{\rm end}\left(a/a_{\rm end}\right)^{-\frac{3n}{n+1}}$ with $H_{\rm end} = \sqrt{V_{\rm end}/2M_{\rm pl}^2}$
- **Defining inflaton field:** $\phi(t) = \phi_0(t)\mathcal{P}(t)$; $\mathcal{P} \to$ oscillatory part, and $\phi_0(a) = \phi_{\rm end}\left(\frac{a}{a_{\rm end}}\right)^{-\frac{3}{n+1}}$
- **Adiabatic approximation:**
$\left(X''_{\vec{k}} + \omega_k^2(\eta)X_{\vec{k}}\right) = 0 \Rightarrow X_{\vec{k}}(\eta) = \frac{1}{\sqrt{2\omega_k}}\left[\alpha_{\vec{k}}e^{-i\int^\eta \omega_k(\eta')d\eta'} + \beta_{\vec{k}}e^{i\int^\eta \omega_k(\eta')d\eta'}\right]$ with $\left|\frac{\omega_k'}{\omega_k^2}\right| < 1$

Boltzmann equation

Back

- **Boltzmann transport equation**: $\frac{\partial f_\chi}{\partial t} - H|\vec{p}|\frac{\partial f_\chi}{\partial|\vec{p}|} = \underbrace{C\left[f_\chi(|\vec{p}|, t)\right]}_{\text{collision term}}$

- **Collision term:** $C\left[f_\chi(|\vec{p}|, t)\right] =$
$$\frac{1}{2p^0}\sum_{\nu=1}^{\infty}\int\frac{d^3\vec{k}'_\nu}{(2\pi)^3 n_\phi}\frac{d^3\vec{p}'}{(2\pi)^3 p^{0\prime}}(2\pi)^4\delta^{(4)}(k'_\nu - p - p')|\overline{\mathcal{M}_\nu}|^2_{\phi\to\chi\chi}\Big[f_\phi(k')(1+f_\chi(p))(1+f_\chi(p'))\Big]$$

- **Fourier components:** $\mathcal{P}_\nu = \frac{1}{T}\int_{t_{\rm end}}^{T}\mathcal{P}(t)\,e^{-i\nu\omega t}\,dt$ with inflaton oscillation frequency
$$\to \omega = m_\phi\underbrace{\sqrt{\frac{\pi n}{(2n-1)}}\frac{\Gamma\left(\frac{1}{2}+\frac{1}{2n}\right)}{\Gamma\left(\frac{1}{2n}\right)}}_{\gamma}$$

Energy-spectrum for $0 \leq w_\phi < 1/3$

◂ Back

Energy spectrum of IR modes for $1/3 \leq w_\phi \leq 1$

$$\rho_{\chi_k}(\eta > \eta_{\rm end}) \propto \begin{cases} (k/k_{\rm end})^{2(2-\nu_1-\nu_2)} & \text{for} \quad 0 \leq \xi < 3/16 \\ (k/k_{\rm end})^{2(2-\nu_2)} & \text{for} \quad \xi = 3/16 \\ (k/k_{\rm end})^{2(2-\nu_2)} & \text{for} \quad \xi > 3/16 \end{cases} \tag{5}$$

Energy spectrum of IR modes for $0 \leq w_\phi < 1/3$

$$\rho_{\chi_k}(\eta > \eta_{\rm end}) \propto \begin{cases} (k/k_{\rm end})^{2(2-\nu_1-\nu_2)} & \text{for} \quad 0 \leq \xi < 3/16 \\ (k/k_{\rm end})^{2(2-\nu_2)} & \text{for} \quad \xi = 3/16 \\ (k/k_{\rm end})^{4} & \text{for} \quad \xi > 3/16 \end{cases} \tag{6}$$

outline of evolution Equation

Back

- **Action with anisotropic stress:** $S_{GW} = \int dx^4 \sqrt{-g} \left[-\frac{g^{\mu\nu}}{64\pi G} \partial_\mu h_{ij} \partial_\nu h^{ij} + \frac{1}{2} \Pi^{ij} h_{ij} \right]$
- **Fourier decomposition:** $h_{ij}(\eta, \mathbf{x}) = \sum_{\lambda=(+,\times)} \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} e^{\lambda}_{ij}(\mathbf{k}) h^{\lambda}_{\mathbf{k}}(\eta) \mathrm{e}^{i\mathbf{k}\cdot\mathbf{x}}, \quad e^{\lambda}_{ij}(\mathbf{k}) \rightarrow$ polarization tensor

Defining secondary tensor power spectrum

Back

- **Tensor power spectrum:**
  $\mathcal{P}_{\rm T}(k,\eta) = 4\frac{k^3}{2\pi^2}|h_{\mathbf{k}}(\eta)|^2,\;\; h_{\mathbf{k}}(\eta) = h_{\mathbf{k}}^{\rm vac} + \frac{2e^{ij}(\mathbf{k})}{M_{\rm pl}^2}\int d\eta_1 \mathcal{G}_k(\eta,\eta_1)\Pi_{ij}^{\rm TT}(\mathbf{k},\eta_1)$
- **Secondary tensor power spectrum:**
  $\mathcal{P}_{\rm T}^{\rm sec}(k,\eta_{\rm re}) \propto \frac{1}{M_{\rm pl}^4}\left(\int_{x_{\rm e}}^{x_{\rm re}} dx_1 \frac{\mathcal{G}_k^{\rm re}(x_{\rm re},x_1)}{a^2(x_1)}\right)^2 \times \int_{k_{\rm min}}^{k_{\rm end}} \frac{dq}{k}\int_{-1}^{1} d\gamma (1-\gamma^2)^2 \times \frac{(q/k)^3\mathcal{P}_X(q,\eta_1)\mathcal{P}_X(|\mathbf{k}-\mathbf{q}|\eta_1)}{|1-q/k|^3},\;\; x = k\eta,\;\; \cos\gamma = \hat{k}.\hat{q}$

For $w_\phi > 1/3$, $\xi > 3/16$

$$\mathcal{P}_{\rm T}^{\rm sec}(k<k_{\rm re},\eta_{\rm re}) \propto \left(\frac{k_{\rm end}}{k_{\rm re}}\right)^{4-2\delta}\left(\frac{k}{k_{\rm end}}\right)^{4(2-\nu_2)};\delta = 4/(1+3w_\phi) \quad (7)$$

$$\mathcal{P}_{\rm T}^{\rm sec}(k>k_{\rm re},\eta_{\rm re}) \propto \left(\frac{k_{\rm re}}{k_{\rm end}}\right)^{\delta}\left(\frac{k}{k_{\rm end}}\right)^{4+\delta-4\nu_2} \quad (8)$$