Quantum entanglement phenomenon between uniformly accelerated detectors in a thermal bath

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References

- S. Barman and B. R. Majhi,"Radiative process of two entangled uniformly accelerated atoms in a thermal bath: a possible case of anti-Unruh event", JHEP 03, 245 (2021), arXiv:2101.08186 [gr-qc].

- D. Barman, S. Barman, B. R. Majhi, "Role of thermal field in entanglement harvesting between two accelerated Unruh-DeWitt detectors", JHEP 07 (2021) 124, arXiv:2104.11269 [gr-qc].

Other related works:

- S. Barman, D. Barman, B. R. Majhi, "Entanglement harvesting from conformal vacuums between two Unruh-DeWitt detectors moving along null paths", arXiv:2112.01308 [gr-qc].

- S. Barman, B. R. Majhi, L. Sriramkumar, "Radiative processes of single and entangled detectors on circular trajectories in (2+1) dimensional Minkowski spacetime", arXiv:2205.01305 [gr-qc].

- D. Barman, S. Barman, B. R. Majhi, "Entanglement harvesting between two inertial Unruh-DeWitt detectors from non-vacuum quantum fluctuations", Phys. Rev. D 106 (2022) 4, 045005, arXiv:2205.08505 [gr-qc].

Motivation and outline

- In the simplest terms, black holes (BHs) are astrophysical objects on which escape velocity is equal to the speed of light.
- Classically BHs only swallow particles and do not emit any.
- Using QFT in BH spacetime Stephen Hawking[1] showed that BHs can also emit particles, which has a Planckian spectrum.
- However, this spectrum only depends on the final BH parameters, like mass, charge, angular momentum, etc.
- There is no other information about the matter that collapsed into the BH, and this leads to the BH information loss paradox.
- There is no classical way to probe the inside structure, if any, of a BH.
- Here comes quantum entanglement, which says no two measurements on entangled particles are independent of each other.

[1] S. W. Hawking, Comm. Math. Phys. 43, 199 (1975).

Motivation and outline

- Keeping one of the entangled particles outside of a BH and letting another to fall into the BH may provide information about the one inside the BH horizon.
- Entanglement also helps to frame the BH information loss problem[2] in a mathematical manner.
- Our main goal is to understand the dynamics of entangled particles in a BH spacetime in realistic situations.
- In this regard, we first considered studying entanglement with accelerated observers in a thermal bath.
  - As according to equivalence principle local gravitational force is equivalent to the one experienced in an accelerated frame.
  - In nature it is impossible to achieve a zero-temperature background for experimental considerations.
- With this system, we have studied the radiative process of entangled atoms and also the entanglement harvesting condition and its characteristics.

[2] S. D. Mathur, Class. Quantum Gravity 26(22), 224001 (2009).

Radiative process of two entangled two-level atoms

Model set-up:

- For two level atomic detectors interacting with a scalar field, the Hamiltonian is $H = H_A + H_F + H_{int}$.
- The interaction Hamiltonian given by, $H_{int} = \sum_{j=1}^{2} \mu_j \kappa_j(\tau_j)\, m^j(\tau_j)\, \Phi\big(X_j(\tau_j)\big)$.
- The time translation operator is then, $U = \mathscr{T}\ \exp\left\{-i\int_{-\infty}^{\infty}\sum_{j=1}^{2} d\tau_j\ \mu_j \kappa_j(\tau_j)\, m^j(\tau_j)\, \Phi\big(X_j(\tau_j)\big)\right\}$.
- Transition amplitude is $\mathscr{A}_{|\omega,0_M\rangle\to|\bar{\omega},\Theta\rangle} = \langle\Theta,\bar{\omega}\,|\,\hat{U}\,|\,\omega,0_M\rangle$, and transition probability is,

$$\Gamma_{|\omega\rangle\to|\Omega\rangle} = \sum_{\{|\Theta\rangle\}} \mathscr{A}^{*}_{|\omega;0_M\rangle\to|\Omega;\Theta\rangle}\mathscr{A}_{|\omega;0_M\rangle\to|\Omega;\Theta\rangle} \approx \mu^2 \sum_{j,l=1}^{2} m^{j*}_{\Omega\omega} m^{l}_{\Omega\omega}\ F_{jl}(\Delta E).$$

- Here $m^j_{\Omega\omega} = \langle\Omega\,|\,m^j(0)\,|\,\omega\rangle$, $\Delta E = E_\Omega - E_\omega$, and $F_{jl}(\Delta E) = \int_{-\infty}^{\infty} d\tau_j\, d\tau'_l\ e^{-i(\tau_j-\tau'_l)\Delta E}\ G^{+}_{jl}\big(\tau_j,\tau'_l\big)\ \kappa_j\kappa_l$.
- The Wightman function, $G^{+}_{jl}\big(\tau_j,\tau'_l\big) = \langle 0_M\,|\,\Phi\big[X_j(\tau_j)\big]\Phi\big[X_l(\tau'_l)\big]\,|\,0_M\rangle$.

Accelerated observers in a thermal bath:

- First we consider (1+3) dimensional Minkowski line element, given by $ds^2 = -dT^2 + dX^2 + dY^2 + dZ^2$.

- The coordinates of an accelerated observer are confined to specific regions of a Minkowski spacetime known as the Rindler wedges.

- For an object accelerated along the '+'-ve X-axis the transformation of coordinates in the right Rindler wedge is

$$T = \frac{e^{a\xi}}{a}\sinh a\eta = \frac{1}{b}\sinh b\tau;$$
$$X = \frac{e^{a\xi}}{a}\cosh a\eta = \frac{1}{b}\cosh b\tau$$

Here $\tau = e^{a\xi}\eta$ and $b = ae^{-a\xi}$ respectively denote the proper time and acceleration of the observer.

- Then the line element reads $ds^2 = e^{2a\xi}\left[-d\eta^2 + d\xi^2\right] + dY^2 + dZ^2$.

- For an observer in equilibrium with a thermal bath characterized by $\beta = 1/(k_B T)$, the Wightman function is given by $G^+_\beta(X_2; X_1) = \langle \Phi(X_2)\Phi(X_1)\rangle_\beta = \frac{1}{Z}\,\mathrm{Tr}\left[e^{-\beta H}\Phi(X_2)\Phi(X_1)\right]$.

Accelerated observers in a thermal bath:

- One can evaluate the previous Green's function in Minkowski spacetime and then put the Rindler coordinate transformation into it.

- Or evaluate it in terms of Rindler mode functions with the help of Unruh operators[3].

- The ladder operators $(\hat{a}_k, \hat{a}_k^{\dagger})$ corresponding to (T, X) and the ones $(\hat{b}_k^R, \hat{b}_k^{R\dagger})$ corresponding to Rindler modes are not the same.

- In particular, $N_k = \langle 0_M | \hat{b}_k^{R\dagger} \hat{b}_k^R | 0_M \rangle$ gives the number density of Unruh effect.

- According to Unruh one can represent $(\hat{b}_k^R, \hat{b}_k^{R\dagger})$ in terms of ladder operators which correspond to the Minkowski vacuum.

[3] N. D. Birrell and P. C. W. Davies, Quantum Fields in Curved Space.

Radiative process using Minkowski modes with Rindler transformation:

- In (1+1) dimensions $G^+_\beta(X_j;X_l) = \int_0^\infty \frac{d\omega_k}{4\pi\omega_k}\left[\frac{e^{i\omega_k(\Delta T_{jl}-\Delta X_{jl})}+e^{i\omega_k(\Delta T_{jl}+\Delta X_{jl})}}{e^{\beta\omega_k}-1}+\frac{e^{-i\omega_k(\Delta T_{jl}-\Delta X_{jl})}+e^{-i\omega_k(\Delta T_{jl}+\Delta X_{jl})}}{1-e^{-\beta\omega_k}}\right]$,

  where one has, $\Delta T_{jl}-\Delta X_{jl} = -\frac{1}{b_j}e^{-b_j\tau_j}+\frac{1}{b_l}e^{-b_l\tau_l}$, $\Delta T_{jl}+\Delta X_{jl} = \frac{1}{b_j}e^{b_j\tau_j}-\frac{1}{b_l}e^{b_l\tau_l}$,

  with $\Delta T_{jl} = T_{j,2}-T_{l,1}$ and $\Delta X_{jl} = X_{j,2}-X_{l,1}$.

- In(1+3) dimensions $G^+_\beta(X_j;X_l) = \int_0^\pi \sin\theta d\theta \int_0^\infty \frac{\omega_k d\omega_k}{2(2\pi)^2}\left[\frac{e^{i\omega_k(\Delta X_{jl}\cos\theta+\Delta T_{jl})}}{e^{\beta\omega_k}-1}+\frac{e^{i\omega_k(\Delta X_{jl}\cos\theta-\Delta T_{jl})}}{1-e^{-\beta\omega_k}}\right]$,

  where $X_j\cos\theta+T_j = \frac{1}{2b_j}\left(\delta_1\, e^{b_j\tau_j}-\delta_2\, e^{-b_j\tau_j}\right)$, $X_j\cos\theta - T_j = \frac{1}{2b_j}\left(-\delta_2\, e^{b_j\tau_j}+\delta_1\, e^{-b_j\tau_j}\right)$,

  with, $\delta_1 = 1+\cos\theta$ and $\delta_2 = 1-\cos\theta$.

- These Green's functions are not time translational invariant.

- We use these Green's functions to find transition probability for a certain field mode frequency[4]

  $F_{jl}(\Delta E) = \int_0^\infty d\omega_k\, \mathscr{F}_{jl}\left(\Delta E,\omega_k\right)$.

[4] M. O. Scully, S. Fulling, D. Lee, D. N. Page, W. Schleich, and A. Svidzinsky, Proc. Nat. Acad. Sci. 115, 8131 (2018), arXiv:1709.00481.
S. Kolekar and T. Padmanabhan, Phys. Rev. D 89, 064055 (2014), arXiv:1309.4424.

Radiative process using Minkowski modes with Rindler transformation:

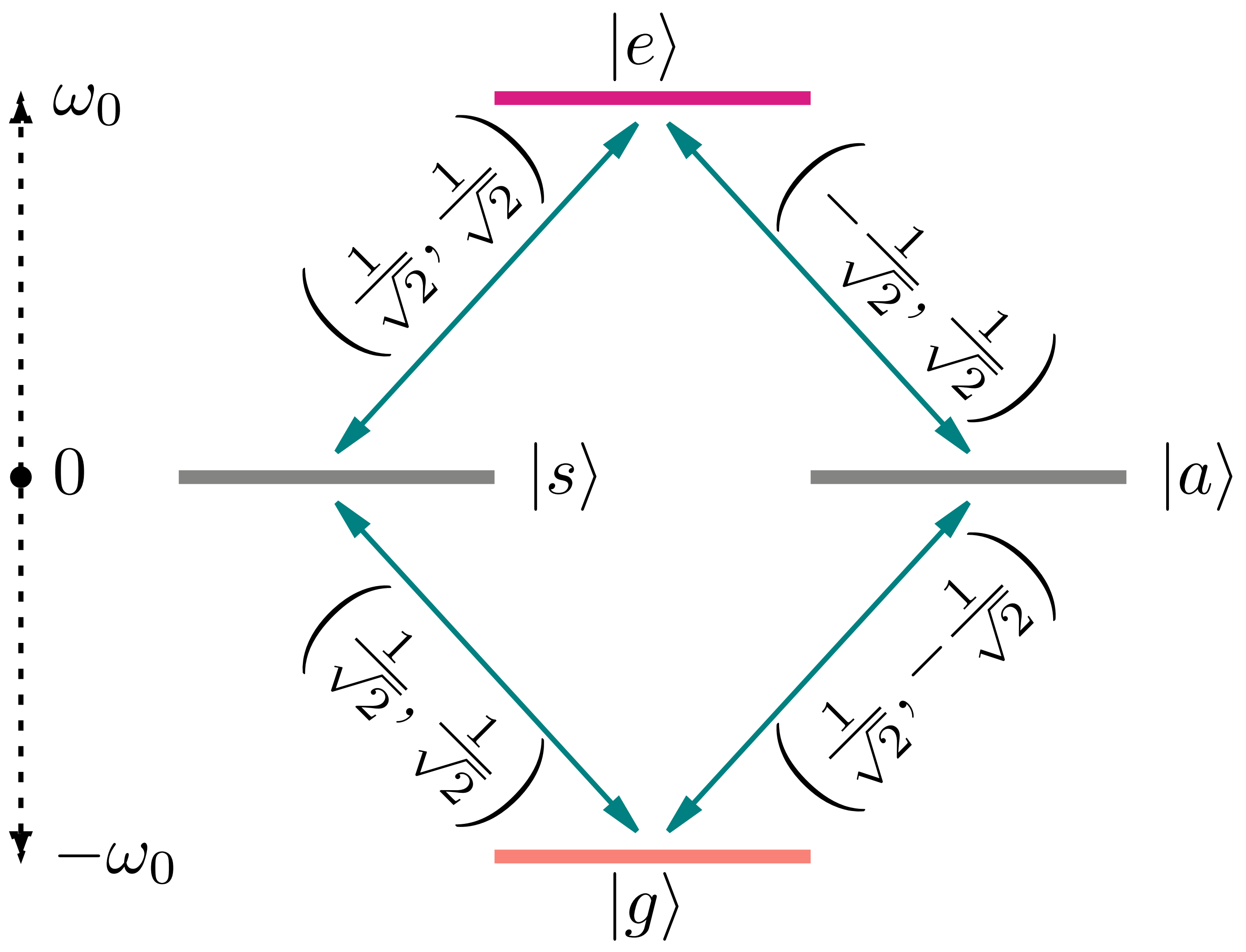
- In (1+1) dimensions one has $\mathcal{F}_{jl}\left(\Delta E,\omega_k\right) = \dfrac{Re\left[\mathcal{C}_1(b_j,b_l)\right]}{2\pi\omega_k b_j b_l} \times \left[\dfrac{e^{\frac{\pi\Delta E}{2}\left(\frac{1}{b_j}+\frac{1}{b_l}\right)}}{e^{\beta\omega_k}-1} + \dfrac{e^{-\frac{\pi\Delta E}{2}\left(\frac{1}{b_j}+\frac{1}{b_l}\right)}}{1-e^{-\beta\omega_k}}\right]$,

  where $\mathcal{C}_1(b_j,b_l) = \left(\dfrac{\omega_k}{b_j}\right)^{-\frac{i\Delta E}{b_j}}\left(\dfrac{\omega_k}{b_l}\right)^{\frac{i\Delta E}{b_l}}\Gamma\left(\dfrac{i\Delta E}{b_j}\right)\Gamma\left(-\dfrac{i\Delta E}{b_l}\right)$.

- In(1+3) dimensions

$$\mathcal{F}_{jl}\left(\Delta E,\omega_k\right) = \int_0^{\pi}\sin\theta\, d\theta\, \frac{\omega_k}{2\pi^2 b_j b_l}\,\mathcal{C}_2(\theta,b_j,b_l)\left[\frac{e^{\frac{\pi}{2}\left(\frac{\Delta E}{b_j}+\frac{\Delta E}{b_l}\right)}}{e^{\beta\omega_k}-1}\left(\frac{\delta_1}{\delta_2}\right)^{\frac{i\Delta E}{2}\left(\frac{1}{b_j}-\frac{1}{b_l}\right)} + \frac{e^{-\frac{\pi}{2}\left(\frac{\Delta E}{b_j}+\frac{\Delta E}{b_l}\right)}}{1-e^{-\beta\omega_k}}\left(\frac{\delta_1}{\delta_2}\right)^{-\frac{i\Delta E}{2}\left(\frac{1}{b_j}-\frac{1}{b_l}\right)}\right]$$

where, $\mathcal{C}_2(\theta,b_j,b_l) = \mathcal{K}\left[\dfrac{i\Delta E}{b_j},\dfrac{\omega_k\sqrt{\delta_1\delta_2}}{b_j}\right]\left(\mathcal{K}\left[\dfrac{i\Delta E}{b_l},\dfrac{\omega_k\sqrt{\delta_1\delta_2}}{b_l}\right]\right)^*$.



$$E_e = \omega_0, \quad |e\rangle = |e_1\rangle\,|e_2\rangle,$$
$$E_s = 0, \quad |s\rangle = \frac{1}{\sqrt{2}}\left(|e_1\rangle\,|g_2\rangle + |g_1\rangle\,|e_2\rangle\right),$$
$$E_a = 0, \quad |a\rangle = \frac{1}{\sqrt{2}}\left(|e_1\rangle\,|g_2\rangle - |g_1\rangle\,|e_2\rangle\right),$$
$$E_g = -\,\omega_0, \quad |g\rangle = |g_1\rangle|g_2\rangle\,.$$

FIG. 1: The energy levels and $m^j_{\Omega\omega} = \langle\Omega\,|\,m^j(0)\,|\,\omega\rangle$ with $m^j(0) = |\,e_j\rangle\langle g_j\,| + |\,g_j\rangle\langle e_j\,|\,.$

- The transition probability from the symmetric entangled state $|\,s\rangle$ to the collective excited state $|\,e\rangle$ is $\Gamma_{se} = \int_0^\infty d\omega_k\;\gamma_{se}$, where the expression of γ_{se} is given by,

$$\gamma_{se} = \frac{\mu^2}{2}\left[\{\mathcal{F}_{11}(\omega_0, \omega_k) + \mathcal{F}_{22}(\omega_0, \omega_k)\} + \{\mathcal{F}_{12}(\omega_0, \omega_k) + \mathcal{F}_{21}(\omega_0), \omega_k\}\right].$$

- Whereas the same between the anti-symmetric state $|\,a\rangle$ to the excited state $|\,e\rangle$ is provided by

$$\gamma_{ae} = \frac{\mu^2}{2}\left[\{\mathcal{F}_{11}(\omega_0, \omega_k) + \mathcal{F}_{22}(\omega_0, \omega_k)\} - \{\mathcal{F}_{12}(\omega_0, \omega_k) + \mathcal{F}_{21}(\omega_0, \omega_k)\}\right].$$

- Using the Minkowski modes with Rindler coordinate transformation one has

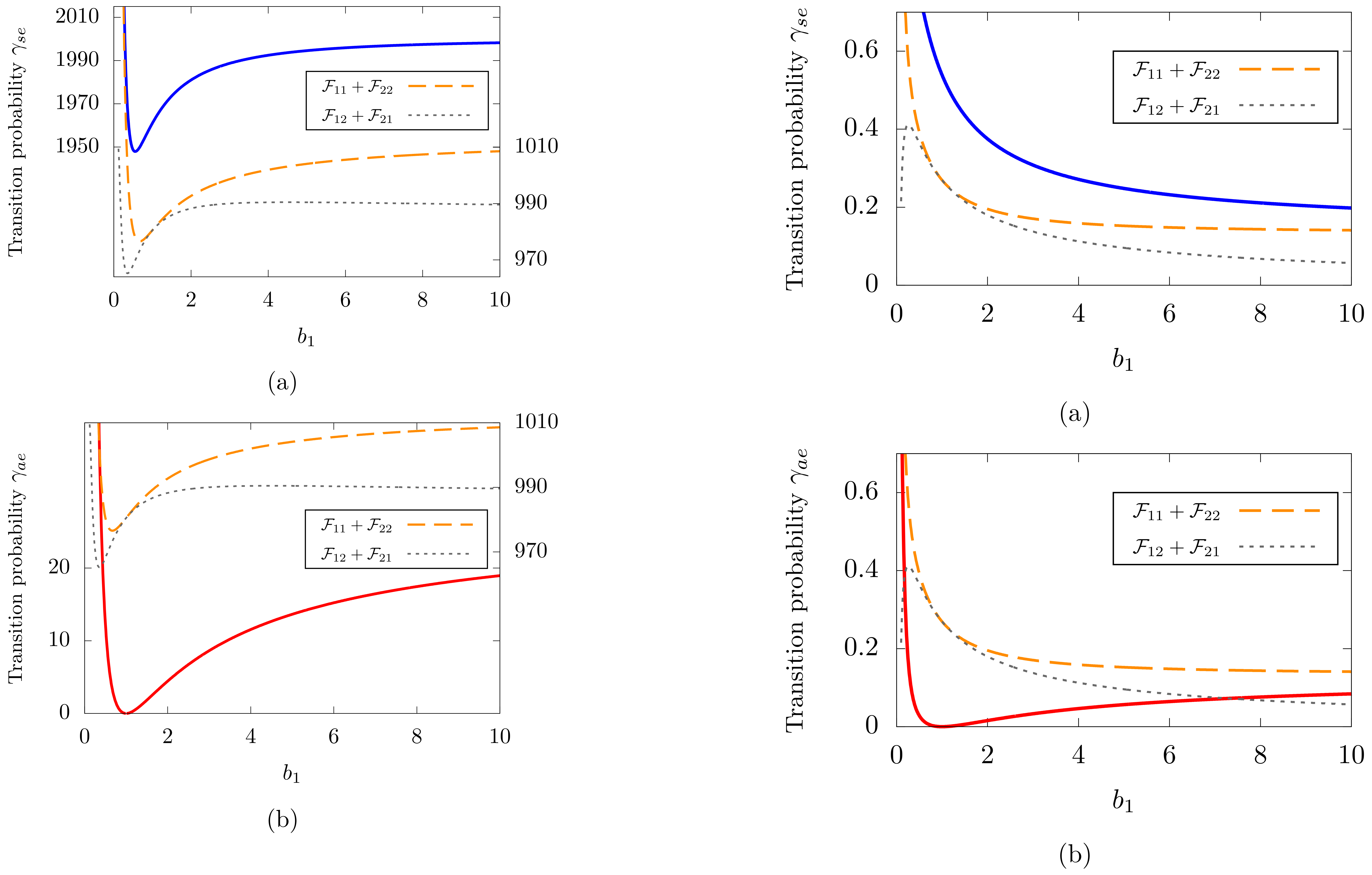


FIG. 2: The transition probabilities in (1+1) and (1+3) dimensions for $b_2 = 1,\ \Delta E = 0.1,\ \omega_k = 0.1,$ and $\beta = 2\pi$.

Radiative process using Rindler modes with the Unruh operators:

- In (1+1) dimensions scalar field EOM $\Box\Phi = e^{-2a\xi}(-\partial_\eta^2\Phi + \partial_\xi^2\Phi) = 0,$ in terms of the Rindler coordinates, has solutions ${}^{R}u_k = \frac{1}{\sqrt{4\pi\omega}} e^{ik\xi - i\omega\eta}, \quad in\ RRW;$ ${}^{L}u_k = \frac{1}{\sqrt{4\pi\omega}} e^{ik\xi + i\omega\eta}, \quad in\ LRW;$
  $= 0, \quad in\ LRW;$ $= 0, \quad in\ RRW.$

- In terms of Rindler modes $\Phi(X) = \sum_{k=-\infty}^{\infty} \left[b_k^R\ {}^{R}u_k + b_k^{R^\dagger}\ {}^{R}u_k^* + b_k^L\ {}^{L}u_k + b_k^{L^\dagger}\ {}^{L}u_k^* \right]$, where $b_k^R|0_R\rangle = 0 = b_k^L|0_R\rangle$.

- In RRW the mode ${}^{L}u_k$ vanishes and one may express $\Phi^R(X) = \sum_{k=-\infty}^{\infty} \left[b_k^R\ {}^{R}u_k + b_k^{R^\dagger}\ {}^{R}u_k^* \right]$.

- One may express, $\Phi(X) = \sum_{k=-\infty}^{\infty} \frac{1}{\sqrt{2\sinh\frac{\pi\omega}{a}}} \left[d_k^1 \left(e^{\frac{\pi\omega}{2a}}\ {}^{R}u_k + e^{-\frac{\pi\omega}{2a}}\ {}^{L}u_{-k}^* \right) + d_k^2 \left(e^{-\frac{\pi\omega}{2a}}\ {}^{R}u_{-k}^* + e^{\frac{\pi\omega}{2a}}\ {}^{L}u_k \right) \right] + h.c.$.

- The modes ${}^{R}u_k + e^{-\pi\omega/a}\ {}^{L}u_{-k}^*$ and ${}^{R}u_{-k}^* + e^{\pi\omega/a}\ {}^{L}u_k$ have the positive frequency analyticity property corresponding to the Minkowski time and the operators satisfy $d_k^1|0_M\rangle = d_k^2|0_M\rangle = 0$.

Radiative process using Rindler modes with the Unruh operators:

- Unruh and the Rindler operators are related by

$$b_k^L = \frac{1}{\sqrt{2\sinh\frac{\pi\omega}{a}}}\left[e^{\frac{\pi\omega}{2a}}d_k^2 + e^{-\frac{\pi\omega}{2a}}d_{-k}^{1^\dagger}\right]\ ,\ \ b_k^R = \frac{1}{\sqrt{2\sinh\frac{\pi\omega}{a}}}\left[e^{\frac{\pi\omega}{2a}}d_k^1 + e^{-\frac{\pi\omega}{2a}}d_{-k}^{2^\dagger}\right]\ .$$

- One may perceive $\langle_M 0\,|\,b_k^{L,R\dagger}\,b_k^{L,R}\,|\,0_M\rangle = (e^{2\pi\omega/a} - 1)^{-1}$.

- Then in RRW one may express $\Phi^R(X) = \sum_{k=-\infty}^{\infty}\frac{1}{\sqrt{2\sinh\frac{\pi\omega}{a}}}\left[d_k^1\ e^{\frac{\pi\omega}{2a}}\ Ru_k\ \ +\ d_k^2\ e^{-\frac{\pi\omega}{2a}}\ Ru_{-k}^*\right] + h.c..$

- Considering $H_k = (d_k^{1^\dagger}d_k^1 + d_k^{2^\dagger}d_k^2)\omega_k$ for the k^{th} excitation one can obtain $G_{\beta_R}^+(X_{j,2}, X_{l,1}) = \langle\Phi^R(X_{j,2})\Phi^R(X_{l,1})\rangle_\beta$,

$$G_{\beta_R}^+\left(\Delta\xi_{jl}, \Delta\eta_{jl}\right) = \int_{-\infty}^{\infty}\frac{dk}{8\pi\omega_k\sqrt{\sinh\frac{\pi\omega_k}{a_j}\sinh\frac{\pi\omega_k}{a_l}}}\left[\frac{1}{1-e^{-\beta\omega_k}}\left\{e^{ik\Delta\xi_{jl}-i\omega_k\Delta\eta_{jl}}\ e^{\frac{\pi\omega_k}{2}\left(\frac{1}{a_j}+\frac{1}{a_l}\right)} + e^{ik\Delta\xi_{jl}+i\omega_k\Delta\eta_{jl}}\ e^{-\frac{\pi\omega_k}{2}\left(\frac{1}{a_j}+\frac{1}{a_l}\right)}\right\}\right.$$
$$\left. + \frac{1}{e^{\beta\omega_k}-1}\left\{e^{-ik\Delta\xi_{jl}+i\omega_k\Delta\eta_{jl}}\ e^{\frac{\pi\omega_k}{2}\left(\frac{1}{a_j}+\frac{1}{a_l}\right)} + e^{-ik\Delta\xi_{jl}-i\omega_k\Delta\eta_{jl}}\ e^{-\frac{\pi\omega_k}{2}\left(\frac{1}{a_j}+\frac{1}{a_l}\right)}\right\}\right],$$

where $\Delta\xi_{jl} = \xi_{j,2} - \xi_{l,1}$ and $\Delta\eta_{jl} = \eta_{j,2} - \eta_{l,1} = \tau_{j,2} - \tau_{l,1}$ (if $\xi_j = 0 = \xi_l$).

Radiative process using Rindler modes with the Unruh operators:

- In both (1+1) and (1+3) dimensions the Green's function corresponding to an accelerated observer in thermal bath using the Rindler modes are time translational invariant.

- With $u_{jl} = \tau_{j,2} - \tau_{l,1}$ and Rindler modes one may define the transition rates

$$R_{jl}(\Delta E) = \int_{-\infty}^{\infty} du_{jl}\, e^{-iu_{jl}\Delta E}\, G^{+}_{\beta_R}(u_{jl})\,.$$

- Using the Rindler modes in (1+1) dimensions

$$R_{jl}(\Delta E) = \frac{1}{4\Delta E\sqrt{\sinh\frac{\pi\Delta E}{a_j}\sinh\frac{\pi\Delta E}{a_l}}}\left[\frac{e^{-\frac{\pi\Delta E}{2}\left(\frac{1}{a_j}+\frac{1}{a_l}\right)}}{1-e^{-\beta\Delta E}} + \frac{e^{\frac{\pi\Delta E}{2}\left(\frac{1}{a_j}+\frac{1}{a_l}\right)}}{e^{\beta\Delta E}-1}\right].$$

- In (1+3) dimensions

$$R_{jl}(\Delta E) = \frac{2}{(2\pi)^2\sqrt{a_j a_l}}\left[\frac{e^{-\frac{\pi\Delta E}{2}\left(\frac{1}{a_j}+\frac{1}{a_l}\right)}}{1-e^{-\beta\Delta E}} + \frac{e^{\frac{\pi\Delta E}{2}\left(\frac{1}{a_j}+\frac{1}{a_l}\right)}}{e^{\beta\Delta E}-1}\right]\int_0^{\infty}\kappa_p d\kappa_p\, \mathscr{K}\left[\frac{i\Delta E}{a_j},\frac{|\kappa_p|}{a_j}\right]\mathscr{K}\left[\frac{i\Delta E}{a_l},\frac{|\kappa_p|}{a_l}\right].$$

- The transition probability rates are $\gamma^R_{se} = \left[\{R_{11}(\omega_0,\omega_k)+R_{22}(\omega_0,\omega_k)\} + \{R_{12}(\omega_0,\omega_k)+R_{21}(\omega_0),\omega_k\}\right]$, $\gamma^R_{ae} = \left[\{R_{11}(\omega_0,\omega_k)+R_{22}(\omega_0,\omega_k)\} - \{R_{12}(\omega_0,\omega_k)+R_{21}(\omega_0,\omega_k)\}\right].$

- Using the Rindler modes with the Unruh operators one has

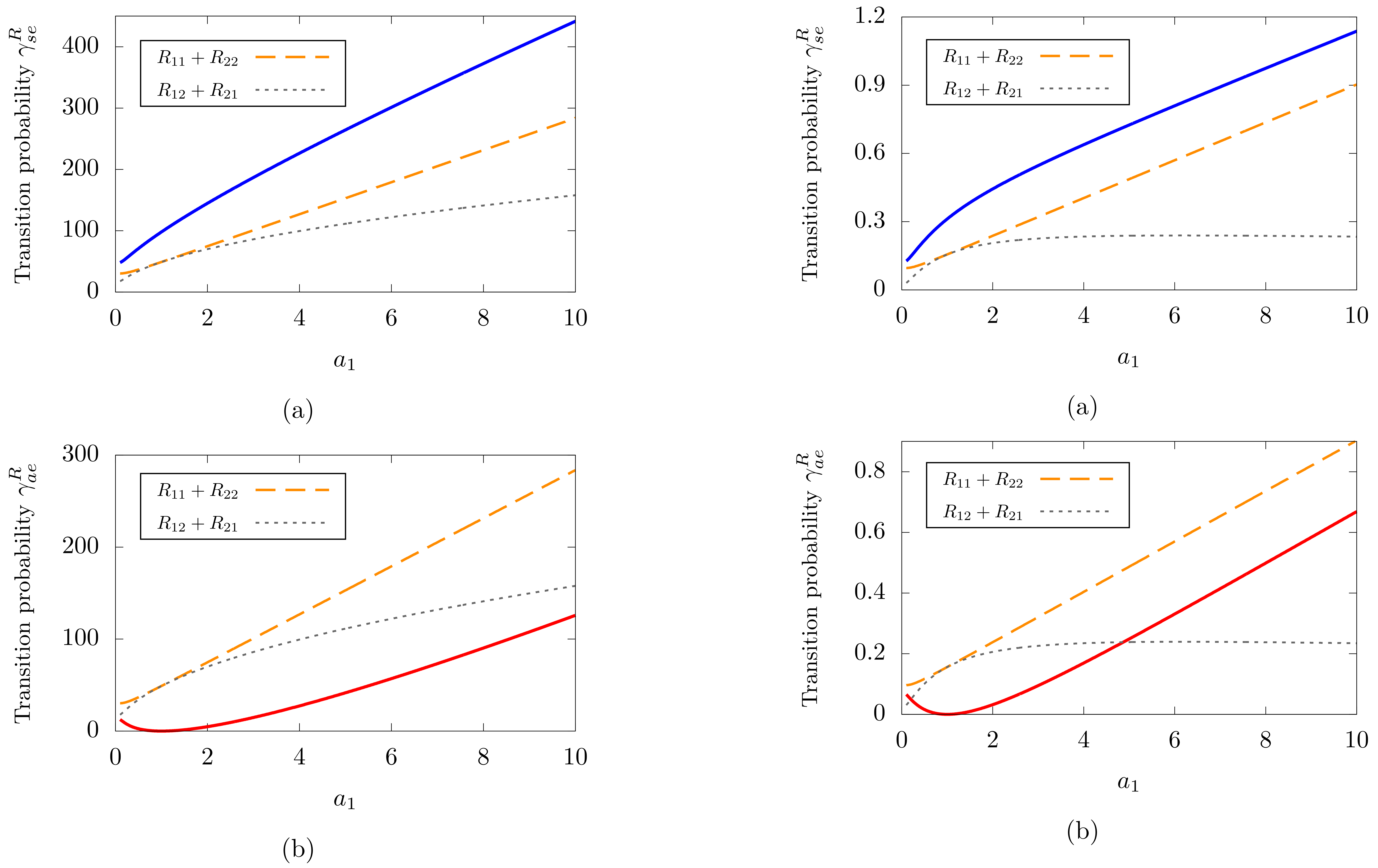


FIG. 3: The transition probabilities in (1+1) and (1+3) dimensions.

The anti-Unruh phenomena:

- The weak anti-Unruh effect is defined as $\partial_{b_1}\mathcal{F}_{jl} < 0$; $\partial_{b_1}\gamma_{\omega\Omega} < 0$; $\partial_{a_1}R_{jl} < 0$.

- To specify the strong anti-Unruh effect one needs to first define the excitation to de-excitation ratios (EDR), such as

$$\mathcal{R}_{\mathcal{F}}(\Delta E) = \frac{\mathcal{F}_{jl}(\Delta E)}{\mathcal{F}_{jl}(-\Delta E)} \; ; \quad \mathcal{R}_{\gamma}(\Delta E) = \frac{\gamma_{\Omega\omega}(\Delta E)}{\gamma_{\Omega\omega}(-\Delta E)} \; ; \quad \mathcal{R}_{R}(\Delta E) = \frac{R_{jl}(\Delta E)}{R_{jl}(-\Delta E)} \; .$$

- Then EDR inverse temperature is $\mathcal{B}_{EDR} = -(1/\Delta E)\,\ln(\mathcal{R})$, and the condition for strong anti-Unruh effect is $\partial_{b_1}\mathcal{B}_{EDR}(\Delta E, b_2, \omega_k) > 0$.

- The strong anti-Unruh effect always guarantees the satisfaction of the weak anti-Unruh effect unless $\partial_{b_1}\mathbb{F}(-\Delta E) > 0$, and $\partial_{b_1}\mathbb{F}(\Delta E) > 0$

  where $\mathbb{F}$ are generalisations for $\mathcal{F}_{jl}$, $\gamma_{\omega\Omega}$ or R_{jl}.

- One can compare the results with the situation $\beta \to \infty$,

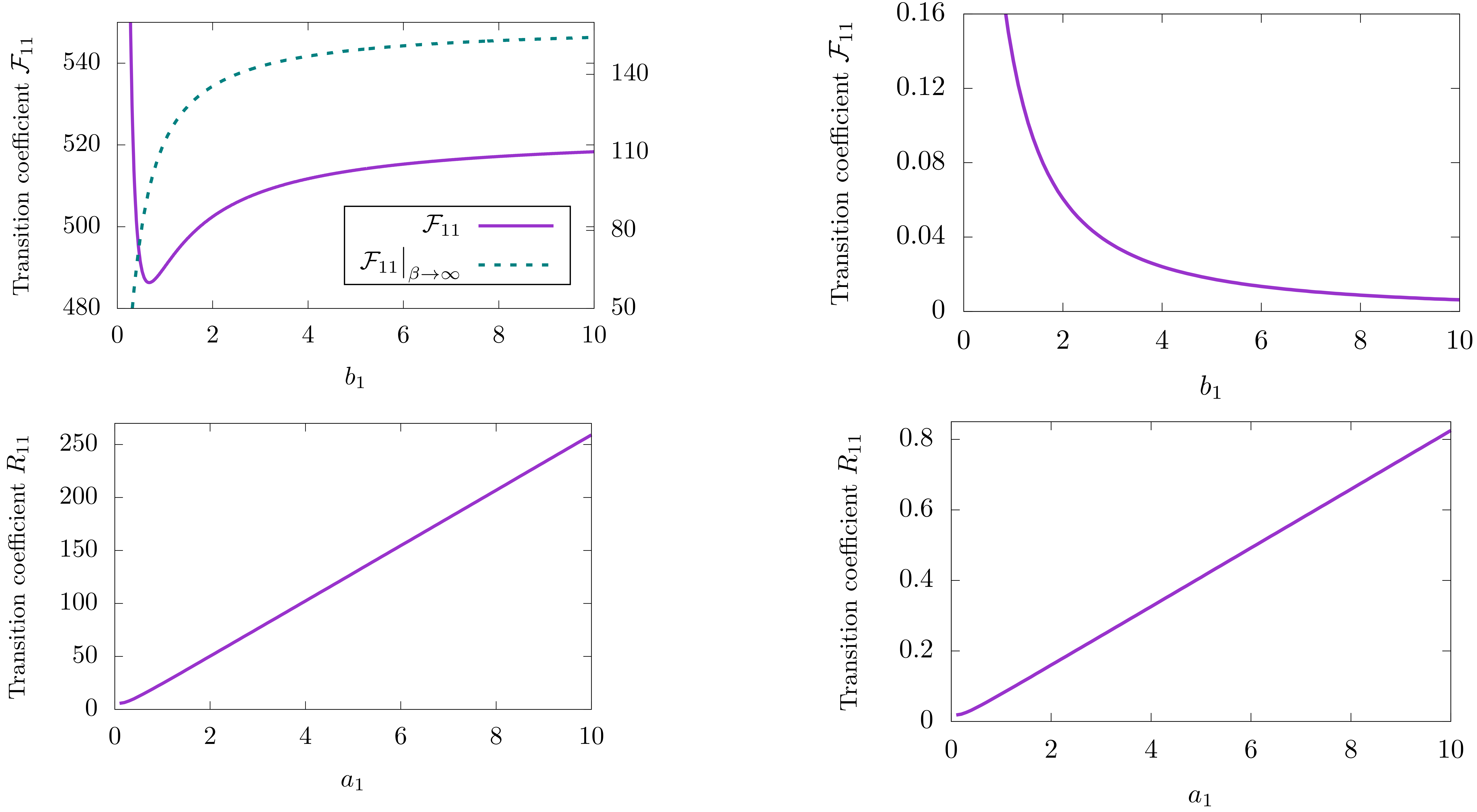


FIG. 4: The transition probabilities $\mathscr{F}_{11}$ and $\mathscr{R}_{11}$ in (1+1) and (1+3) dimensions.

TABLE. 1: The case with the Minkowski modes

	Transitions	Anti-Unruh-like effect	Nature
(1+1) dimensions	γ_{se}	Yes	Entirely weak
	γ_{ae}	Yes	Entirely weak
	$\mathscr{F}_{11}$	Yes	Entirely weak
(1+3) dimensions	γ_{se}	Yes	Mostly weak, strong in some region
	γ_{ae}	Yes	Mostly weak, strong in some region
	$\mathscr{F}_{11}$	Yes	Entirely weak

TABLE. 2: The case with the Rindler modes

	Transitions	Anti-Unruh-like effect	Nature
(1+1) dimensions	γ_{se}^{R}	No	-
	γ_{ae}^{R}	Yes	Both strong and weak
	$\mathscr{R}_{11}$	No	-
(1+3) dimensions	γ_{se}^{R}	No	-
	γ_{ae}^{R}	Yes	Entirely weak
	$\mathscr{R}_{11}$	No	-

Entanglement harvesting

Model set-up:

- Here the initial state is $|in\rangle = |0\rangle|E_0^A\rangle|E_0^B\rangle$ and final one is $|out\rangle = U|in\rangle$.
- One can get the final detector density matrix as

$$\rho_{AB} = \begin{bmatrix} 0 & 0 & 0 & c_a c_b \varepsilon \\ 0 & c_a^2 P_A & c_a c_b P_{AB} & c_a^2 W_A^{(N)} + c_a c_b W_A^{(S)} \\ 0 & c_a c_b P_{AB}^* & c_b^2 P_B & c_b^2 W_B^{(N)} + c_a c_b W_B^{(S)} \\ c_a c_b \varepsilon^* & c_a^2 W_A^{(N)*} + c_a c_b W_A^{(S)*} & c_b^2 W_B^{(N)*} + c_a c_b W_B^{(S)*} & 1 - (c_a^2 P_A + c_b^2 P_B) \end{bmatrix} + \mathcal{O}(c^4) \ ,$$

Where, $P_j = |\langle E_1^j | m_j(0) | E_0^j \rangle|^2 \ \mathcal{I}_j, \varepsilon = \langle E_1^B | m_B(0) | E_0^B \rangle \langle E_1^A | m_A(0) | E_0^A \rangle \mathcal{I}_\varepsilon,$
$P_{AB} = \langle E_1^A | m_A(0) | E_0^A \rangle \langle E_1^B | m_B(0) | E_0^B \rangle^\dagger \mathcal{I}_{AB}.$

- Here the quantities $\mathcal{I}_j = \int_{-\infty}^{\infty} d\tau_j' \int_{-\infty}^{\infty} d\tau_j \ e^{-i\Delta E^j(\tau_j' - \tau_j)} \ G_W(X_j', X_j),$

$$\mathcal{I}_\varepsilon = -i \int_{-\infty}^{\infty} d\tau_B' \int_{-\infty}^{\infty} d\tau_A \ e^{i(\Delta E^B \tau_B' + \Delta E^A \tau_A)} \ G_F(X_B', X_A), \ \mathcal{I}_{AB} = \int_{-\infty}^{\infty} d\tau_B' \int_{-\infty}^{\infty} d\tau_A \ e^{i(\Delta E^A \tau_A - \Delta E^B \tau_B')} \ G_W(X_B', X_A) \, .$$

Entanglement harvesting condition:

- Entanglement harvesting is possible only when the partial transposition of the reduced density matrix has negative eigenvalues[5].

- The entanglement harvesting condition is $P_A P_B < |\varepsilon|^2$ which turns into $\mathcal{I}_A \mathcal{I}_B < |\mathcal{I}_\varepsilon|^2$.

- Concurrence is an entanglement measure defined as $\mathscr{C}(\rho_{AB}) = max\left[0,\ 2c^2\left(|\varepsilon| - \sqrt{P_A P_B}\right) + \mathcal{O}(c^4)\right]$.

- We consider a simplified form of it, $\mathscr{C}_{\mathcal{I}} = \left(|\mathcal{I}_\varepsilon| - \sqrt{\mathcal{I}_A \mathcal{I}_B}\right)$.

- Total correlation is defined by Mutual information
  $\mathscr{M}(\rho_{AB}) \equiv S(\rho_A) + S(\rho_B) - S(\rho_{AB}) = c^2\left[P_+ \ln P_+ + P_- \ln P_- - P_A \ln P_A - P_B \ln P_B\right] + \mathcal{O}(c^4)$

  where, $P_\pm = \dfrac{1}{2}\left[P_A + P_B \pm \sqrt{(P_A - P_B)^2 + 4|P_{AB}|^2}\right]$.

[5] A. Peres, Phys. Rev. Lett. 77 (1996) 1413-1415.

Considered system:

- Here we shall also consider anti-parallel detectors. The relevant coordinate transformations are

$$T = \frac{e^{a\xi}}{a}\sinh a\eta,\; X = \frac{e^{a\xi}}{a}\cosh a\eta; \quad \text{in RRW}$$

$$T = -\frac{e^{a\xi'}}{a}\sinh a\eta',\; X = -\frac{e^{a\xi'}}{a}\cosh a\eta'; \quad \text{in LRW.}$$

- Both of them correspond to the same line-element $ds^2 = e^{2a\xi}\left[-d\eta^2 + d\xi^2\right] + dY^2 + dZ^2$.

- One can define the proper times, and accelerations as

$$\tau = e^{a\xi}\eta,\; b = ae^{-a\xi}, \text{ in RRW; and } \tau' = -\,e^{a\xi'}\eta',\; b' = ae^{-a\xi'}, \text{ in LRW.}$$

- In terms of these proper times and accelerations one has

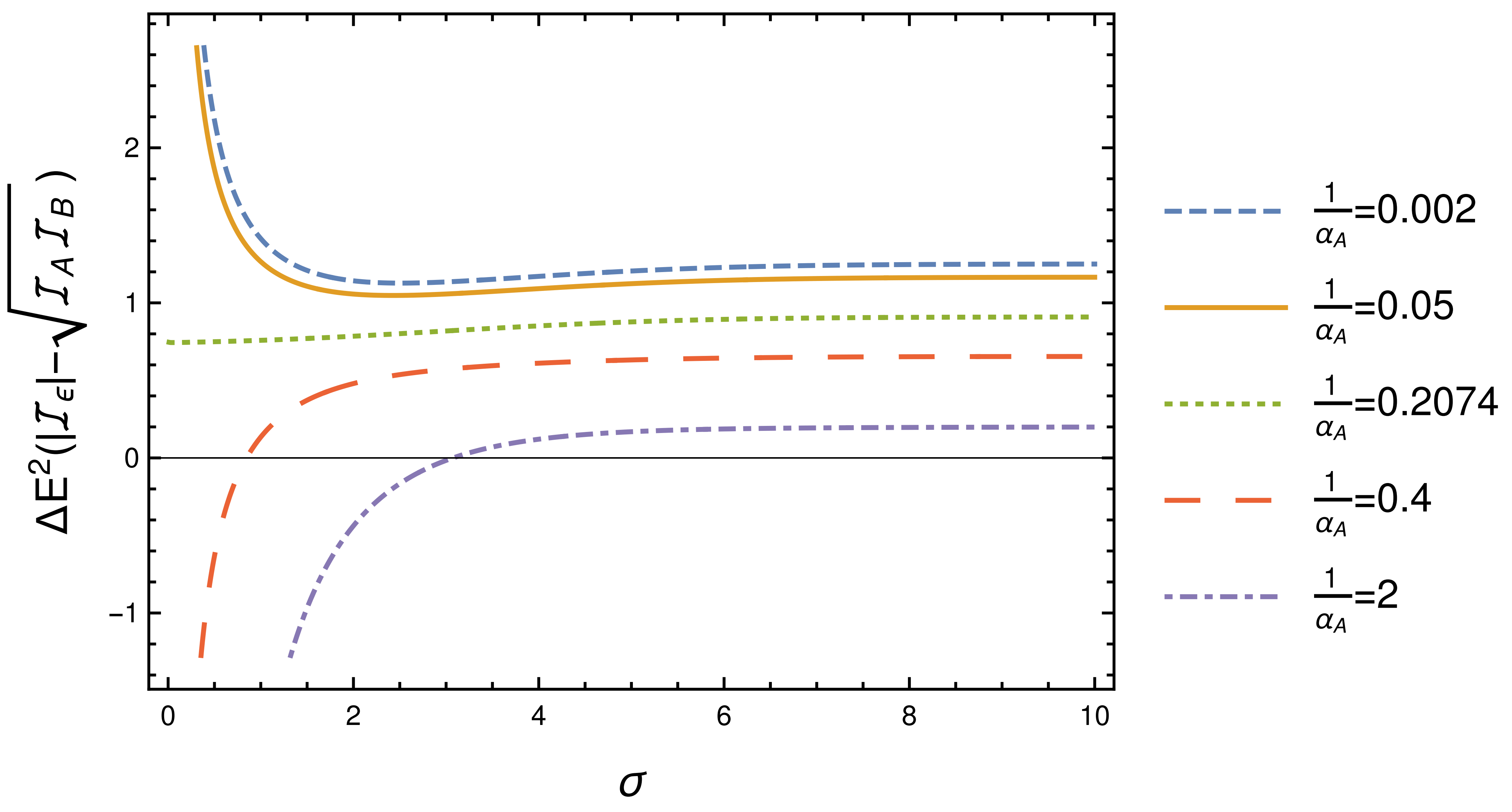
$$T = \frac{1}{b}\sinh b\tau,\; X = \frac{1}{b}\cosh b\tau, \text{ in RRW}$$

$$T = \frac{1}{b'}\sinh b'\tau',\; X = -\frac{1}{b'}\cosh b'\tau', \text{ in LRW.}$$

Results in (1+1) dimensions:

- For parallel detectors there is no entanglement harvesting.
- We consider *Alice* in right and *Bob* in the left Rindler wedge so that they are anti-parallelly accelerated in a thermal bath.
- Then in (1+1)-dimensions entanglement harvesting condition is

$$\left(\frac{e^{-\frac{\pi\Delta E}{a_A}}}{1-e^{-\beta\Delta E}}+\frac{e^{\frac{\pi\Delta E}{a_A}}}{e^{\beta\Delta E}-1}\right)\left(\frac{e^{-\frac{\pi\Delta E}{a_B}}}{1-e^{-\beta\Delta E}}+\frac{e^{\frac{\pi\Delta E}{a_B}}}{e^{\beta\Delta E}-1}\right)<4\left[\frac{e^{\frac{\pi\Delta E}{2}\left(\frac{1}{a_B}-\frac{1}{a_A}\right)}}{1-e^{-\beta\Delta E}}+\frac{e^{-\frac{\pi\Delta E}{2}\left(\frac{1}{a_B}-\frac{1}{a_A}\right)}}{e^{\beta\Delta E}-1}-\sinh\left\{\frac{\pi\Delta E}{2}\left(\frac{1}{a_B}-\frac{1}{a_A}\right)\right\}\right]^2$$



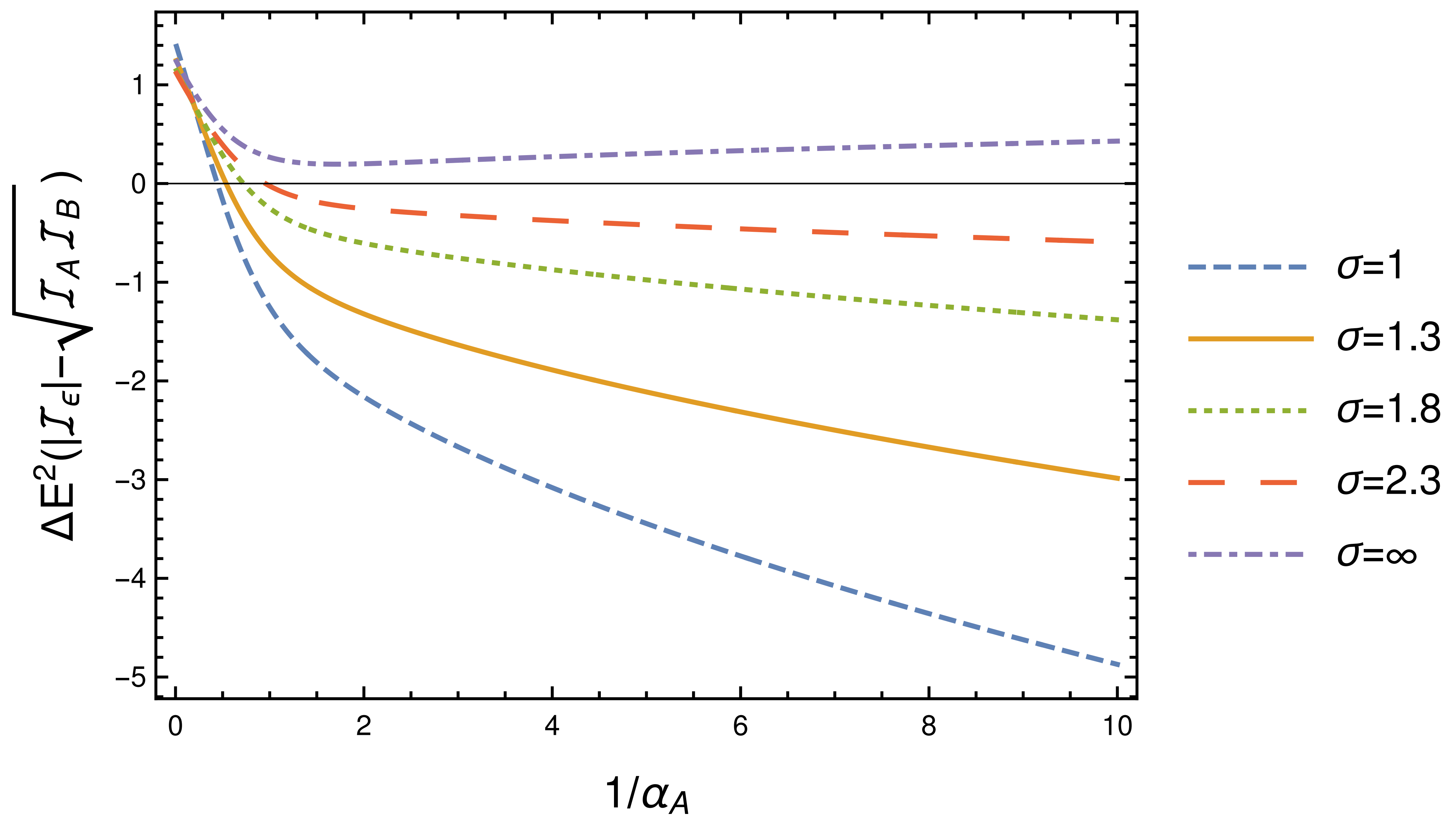


FIG. 5: In (1+1) dimensions the quantity $\Delta E^2\left(|\mathcal{I}_\varepsilon|-\sqrt{\mathcal{I}_A\mathcal{I}_B}\right)$ w.r.t. $\sigma=\beta\Delta E$ and $\alpha_A=a_A/\Delta E$ for $\alpha_B=a_B/\Delta E=1$.

- There is transition in the nature of $\Delta E^2\left(|\mathcal{J}_\varepsilon| - \sqrt{\mathcal{J}_A\mathcal{J}_B}\right)$ after a certain value of α_A.

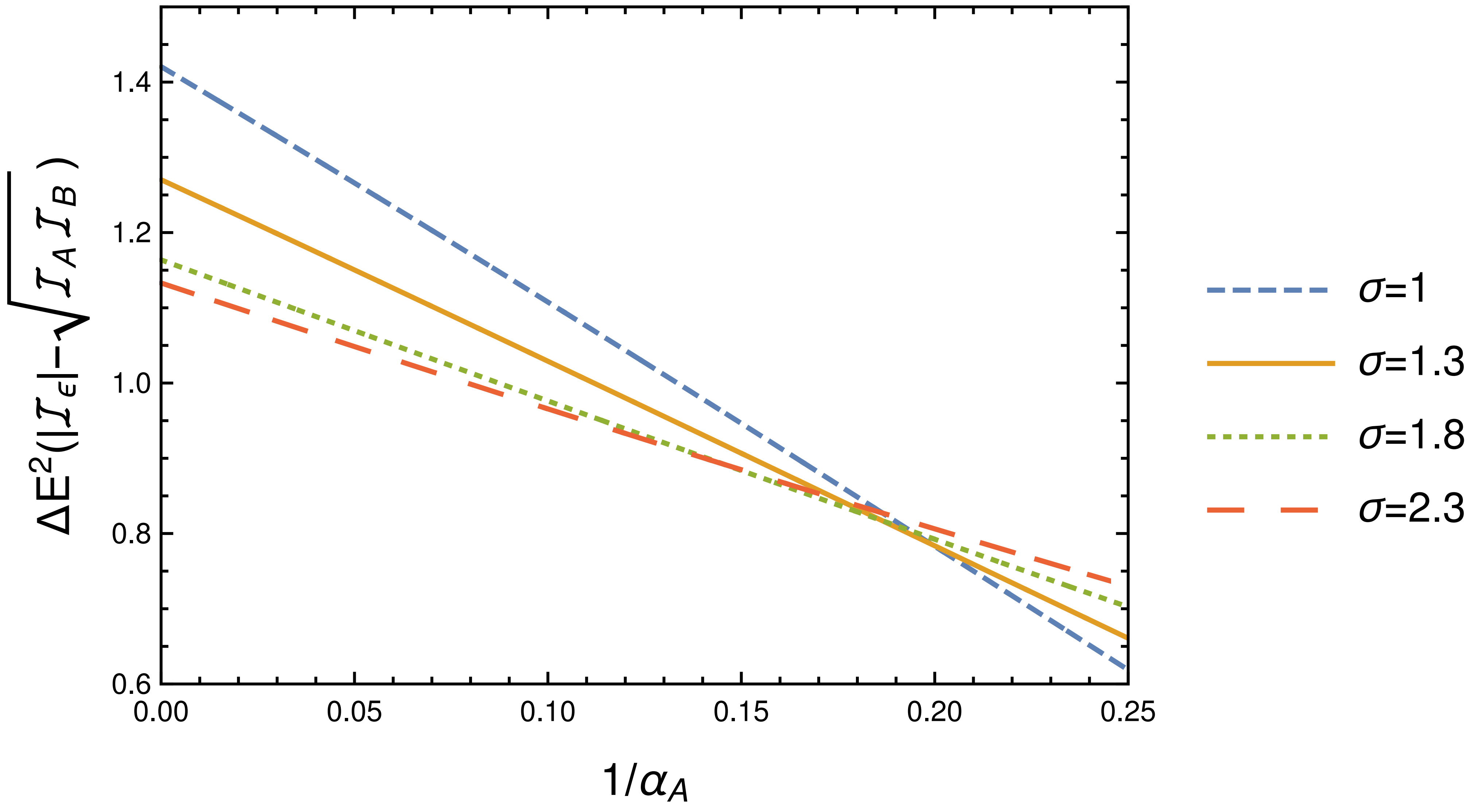


FIG. 6: In (1+1) dimensions the quantity $\Delta E^2\left(|\mathcal{J}_\varepsilon| - \sqrt{\mathcal{J}_A\mathcal{J}_B}\right)$ is plotted w.r.t. $\alpha_A = a_A/\Delta E$ for different $\sigma = \beta\Delta E$ and $\alpha_B = a_B/\Delta E = 1$.

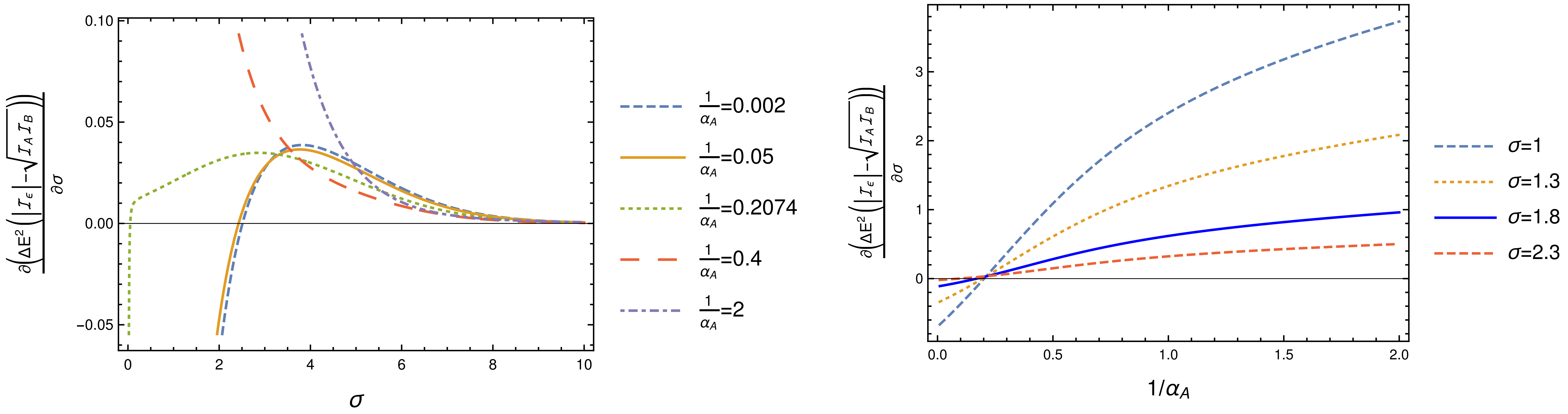


FIG. 7: In (1+1) dimensions the derivative of $\Delta E^2\left(|\mathcal{J}_\varepsilon| - \sqrt{\mathcal{J}_A\mathcal{J}_B}\right)$ is plotted w.r.t. $\sigma = \beta\Delta E$ and $\alpha_A = a_A/\Delta E$ for $\alpha_B = a_B/\Delta E = 1$.

- The feature of transition in the nature of $\Delta E^2\left(|\mathcal{I}_\varepsilon| - \sqrt{\mathcal{I}_A\mathcal{I}_B}\right)$ is easily visualized when $\alpha_A = \alpha_B = \alpha$.

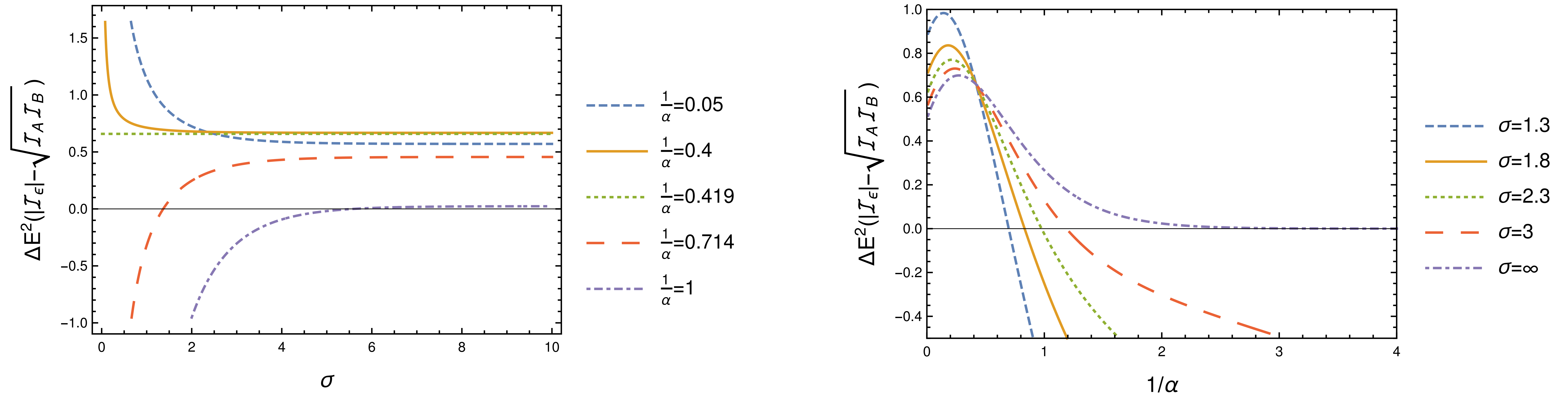


FIG. 8: In (1+1) dimensions the quantity $\Delta E^2\left(|\mathcal{I}_\varepsilon| - \sqrt{\mathcal{I}_A\mathcal{I}_B}\right)$ is plotted w.r.t. $\sigma = \beta\Delta E$ and α where $\alpha = \alpha_A = \alpha_B$.

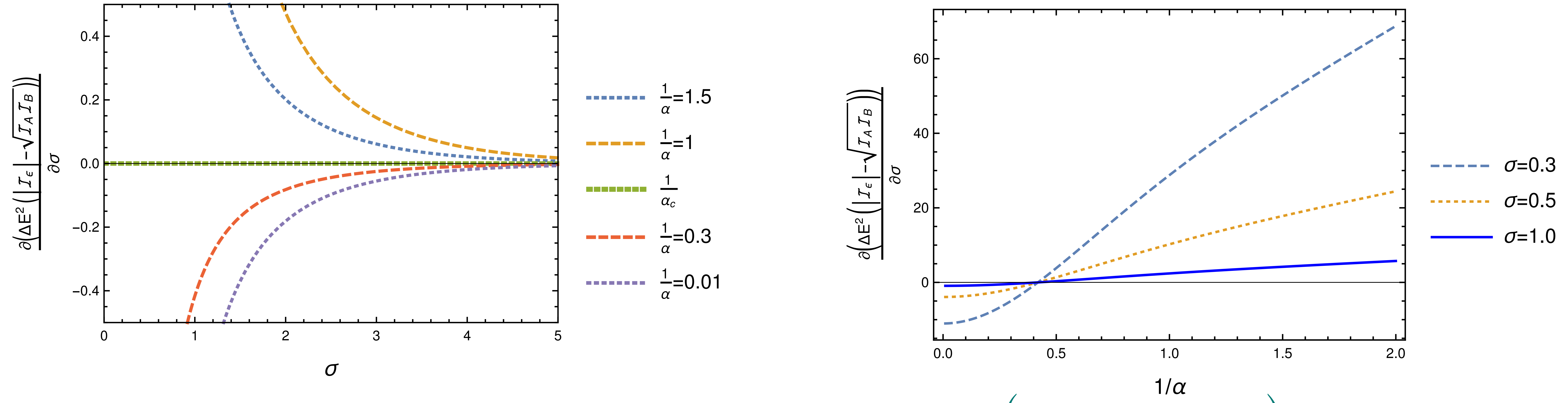


FIG. 9: In (1+1) dimensions the derivative of $\Delta E^2\left(|\mathcal{I}_\varepsilon| - \sqrt{\mathcal{I}_A\mathcal{I}_B}\right)$ is plotted w.r.t. $\sigma = \beta\Delta E$ and α where $\alpha = \alpha_A = \alpha_B$.

Results in (1+3) dimensions:

- Then in (1+3)-dimensions entanglement harvesting condition is

$$\left(\frac{e^{-\frac{\pi\Delta E}{a_A}}}{1-e^{-\beta\Delta E}}+\frac{e^{\frac{\pi\Delta E}{a_A}}}{e^{\beta\Delta E}-1}\right)\left(\frac{e^{-\frac{\pi\Delta E}{a_B}}}{1-e^{-\beta\Delta E}}+\frac{e^{\frac{\pi\Delta E}{a_B}}}{e^{\beta\Delta E}-1}\right)\Upsilon\left(\Delta E,a_A,a_A\right)\Upsilon\left(\Delta E,a_B,a_B\right)<4\left[\frac{e^{\frac{\pi\Delta E}{2}\left(\frac{1}{a_B}-\frac{1}{a_A}\right)}}{1-e^{-\beta\Delta E}}+\frac{e^{-\frac{\pi\Delta E}{2}\left(\frac{1}{a_B}-\frac{1}{a_A}\right)}}{e^{\beta\Delta E}-1}-\sinh\left\{\frac{\pi\Delta E}{2}\left(\frac{1}{a_B}-\frac{1}{a_A}\right)\right\}\right]^2\Upsilon\left(\Delta E,a_A,a_B\right)^2,$$

where, $\Upsilon\left(\bar{\varepsilon},a_j,a_l\right)=\int_0^\infty \kappa_p\,d\kappa_p\,\mathscr{K}\left[\frac{i\bar{\varepsilon}}{a_j},\frac{\kappa_p}{a_j}\right]\mathscr{K}\left[\frac{i\bar{\varepsilon}}{a_l},\frac{\kappa_p}{a_l}\right].$

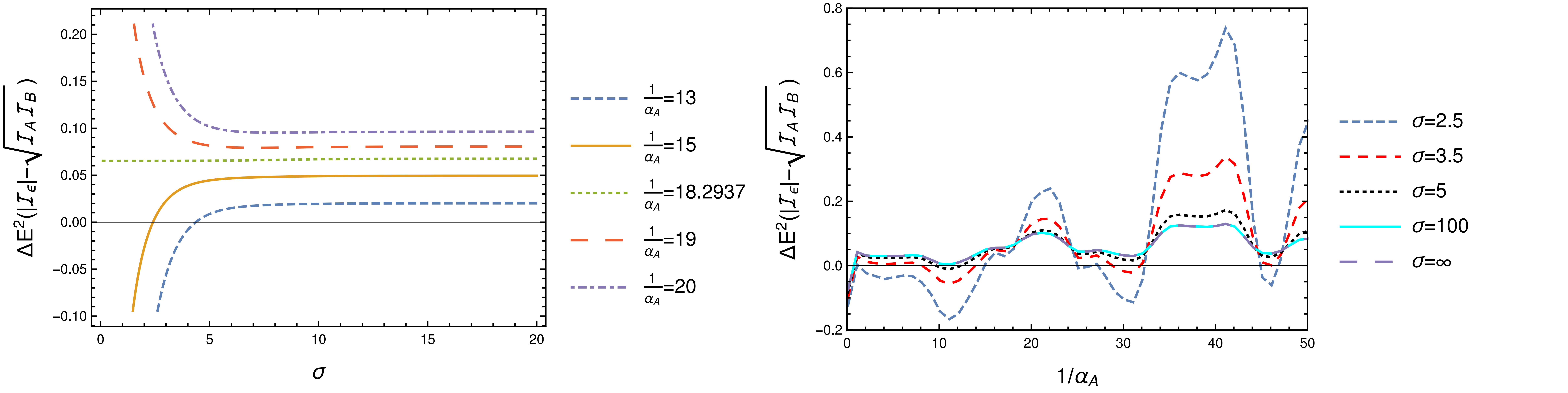


FIG. 10: In (1+3) dimensions the quantity $\Delta E^2\left(|\mathcal{J}_\varepsilon|-\sqrt{\mathcal{J}_A\mathcal{J}_B}\right)$ w.r.t. $\sigma=\beta\Delta E$ and $\alpha_A=a_A/\Delta E$ for $\alpha_B=a_B/\Delta E=1$.

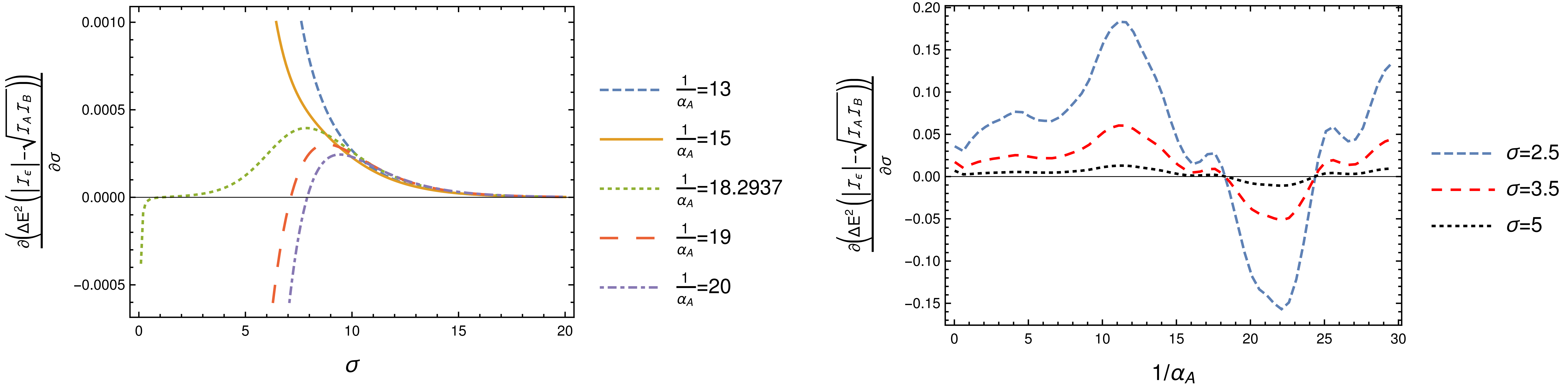


FIG. 11: In (1+3) dimensions the derivative of $\Delta E^2 \left(|\mathcal{I}_\varepsilon| - \sqrt{\mathcal{I}_A \mathcal{I}_B} \right)$ is plotted w.r.t. $\sigma = \beta \Delta E$ and $\alpha_A = a_A/\Delta E$ for $\alpha_B = a_B/\Delta E = 1$.

- In (1+3)- dimensions multiple such transition points exist.
- However, when $\alpha_A = \alpha_B$ the plots are the same as (1+1)-dimensional ones.

Mutual information in (1+1) and (1+3) dimensions:

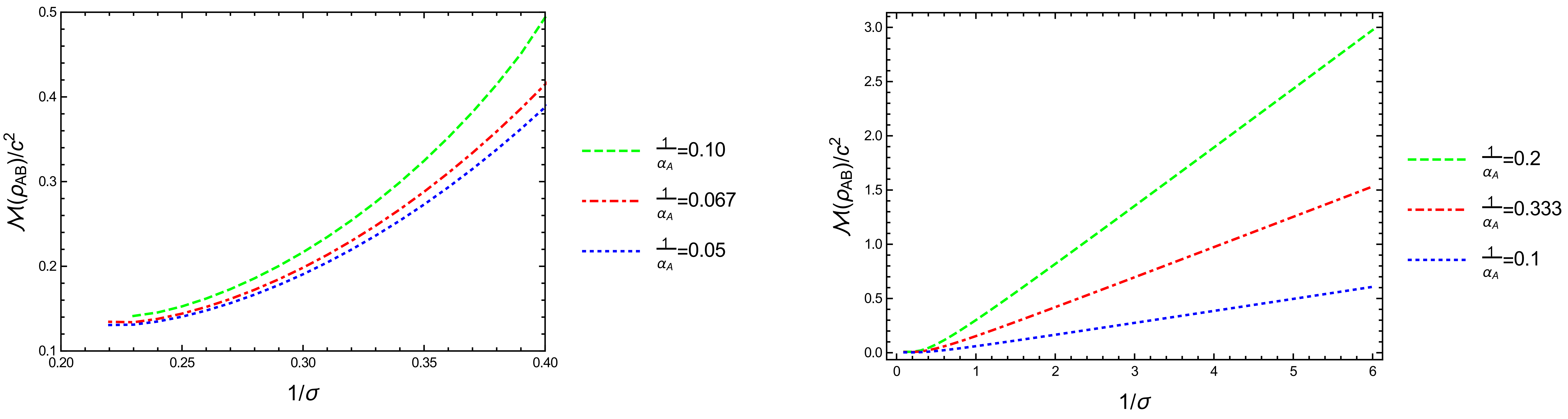


FIG. 12: The mutual information in (1+1) and (1+3) dimensions plotted w.r.t. $\sigma = \beta\Delta E$, for $\alpha_A = a_A/\Delta E$, and $\alpha_B = a_B/\Delta E = 1$.

- Mutual information is zero for anti-parallel accelerated detectors.
- Mutual information is non-zero for parallel accelerated detectors.

Conclusion

Radiative process of entangled atoms:

- With Minkowski modes we have observed that there are anti-Unruh effect in both (1+1) and (1+3) dimensions for both transitions from the symmetric and anti-symmetric states to the excited state.

- Here it is observed that thermal bath plays a role in the occurrence of the anti-Unruh effect.

- With Rindler modes it is observed that only for the transition from the anti-symmetric states to the excited state there is anti-Unruh effect and entanglement must have contributed to it.

Entanglement harvesting:

- At higher temperature greater acceleration is needed to start entanglement harvesting.

- Below a certain acceleration temperature has diminishing effect on harvesting while above that the characteristic is opposite in (1+1) dimensions.

- In (1+3) dimensions there are multiple such transition points for different accelerations of the detectors.

- For equal accelerations of the detectors the (1+1) and (1+3) dimensional results are the same.

THANK YOU FOR LISTENING!