Searching for New Physics in the Sky --the case for Inflaton

Arindam Chatterjee Indian Statistical Institute, Kolkata, India

(Based on arXiv:1103.5758, 1409.4442, 1708.07293, with Anupam Mazumdar)

Outline

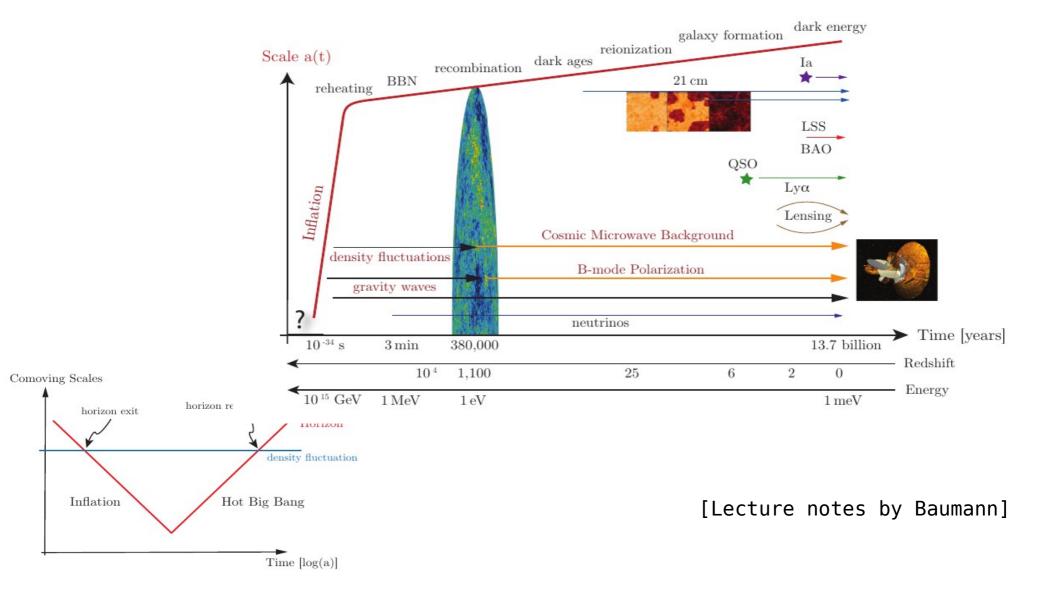
Motivation for Inflation

 Inflation within SUSY models : the case for Higgs Inflation

Probing small scale power spectrum with gravity waves (and PBH)

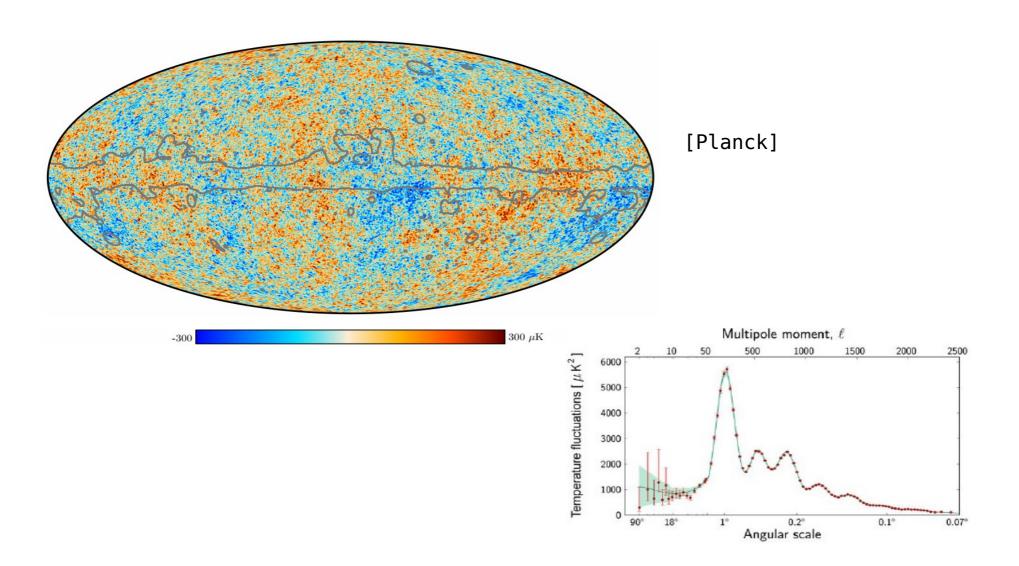
Conclusion and Outlook

History of the Universe



Introduction

Inflation: Addresses Horizon and flatness problems (also resolves monopole problems, generates entropy in the visible sector ...)



Inflation: Generalities

- "New" inflation scenarios
 Driven by a scalar field with uniform ev across Hubble patch
 [Linde; Albrecht, Steinhardt]
- Energy density dominated by the potential. Needs to be sufficiently flat
- Exponential expansion of about 45-60 efoldings
- Within SM, only scalar field is Higgs doublet, requires large non-minimal coupling with gravity! Unitarity issue..

[Shaposnikov, Bezrukov, 2008; Burgess et.al., 2010]

- To generate visible sector particles inflaton must couple to these dof. The potential can be sensistive to radiative corrections. No protection for the flatness!
- Mass of a scalar field is not protected, mass term has "quadratic" dependence on cut-off scale!

The case for SUSY

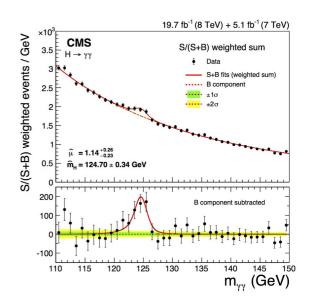
- A hypothetical symmetry relating Bosons and Fermions, has "fermionic" generators
- Solves Hierarchy problem, cencels quadratic divergencies
- Unifies gauge couplings at a high energy scale
- Haag-Lopuzansky-Sohnius theorem
- Offers protection against rad.corr. "non-renormalization" theorem valid in ALL orders of perturbation theory
- Local SUSY > SUGRA (Special Breakthrough Prize: Freedman, Nieuwenhuizen, Ferrara)

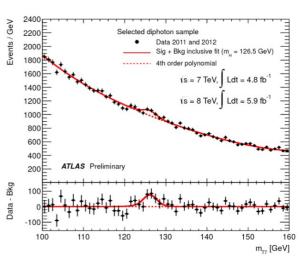
$$\tilde{\lambda}_f = -\lambda_f^2 \qquad -2N(f)\lambda_f^2\int\frac{d^4k}{(2\pi)^4}\left[\frac{1}{k^2-m_f^2} + \frac{2m_f^2}{\left(k^2-m_f^2\right)^2}\right]$$
 [Witten; Kaul, Majumdar'82]

$$\delta = \frac{3\bar{m}_t^4}{2\pi^2 v^2 \sin^2\beta} \left[\log \frac{M_S^2}{\bar{m}_t^2} + \frac{X_t^2}{2M_S^2} \left(1 - \frac{X_t^2}{6M_S^2} \right) \right]$$

$$M_h \stackrel{M_A \gg M_Z}{\to} \sqrt{M_Z^2 \cos^2 2\beta + \epsilon \sin^2 \beta} \left[1 + \frac{\epsilon M_Z^2 \cos^2 \beta}{2M_A^2 (M_Z^2 + \epsilon \sin^2 \beta)} - \frac{M_Z^2 \sin^2 \beta + \epsilon \cos^2 \beta}{2M_A^2} \right]$$

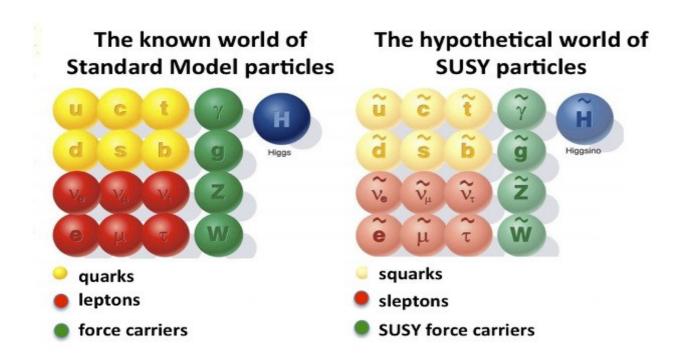
$$[Review : Djouadi'05]$$





$$\epsilon = \frac{3 \,\bar{m}_t^4}{2 \pi^2 v^2 \sin^2 \beta} \left[\log \frac{M_S^2}{\bar{m}_t^2} + \frac{X_t^2}{2 \,M_S^2} \left(1 - \frac{X_t^2}{6 \,M_S^2} \right) \right]$$

The case for SUSY ...



The Minimal SUSY (extension of the) Standard Model (MSSM)

- Has the same gauge group as SM : $SU(3)_c xSU(2)_L xU(1)_Y$
- Particle content follow from SM + extra Higgs doublet
- A discrete symmetry R-parity is assumed to presvent p decay via renormalizable terms, provides with a Dark Matter candidate
- Soft-SUSY breaking is assumed
- Neutrino mass generation mechanism requires extension

Inflation within SUSY

- For high scale inflation SUGRA needs to be invoked
- Inevitable correction from SUGRA, eta problem, need protection: Shift symmetry, Heisenberg symmetry ...

[Dine, Fischler, Nemeschansky, 1984; Coughlan, Holman, Ramond, Ross, 1984; Binetruy, Gaillard 1987, Gillard, Murayama, Olive, 1995...]

Supergravity potential : $V_F = e^K \left[D_{\Phi_i} W K_{ij^*}^{-1} D_{\Phi_j^*} W^* - 3 |W|^2
ight]$

$$D_{\Phi_i} W = \frac{\partial W}{\partial \Phi_i} + \frac{\partial K}{\partial \Phi_i} W.$$

[Review Yamaguchi 2011]

Low scale inflations do not necessarily suffer

$$V(\phi) = H^2 M_{\rm P}^2 f\left(\frac{\phi}{M_{\rm P}}\right) \qquad H_{\rm inf}^2 M_{\rm P}^2 \left(\frac{\phi}{M_{\rm P}}\right)^p \ll m_\phi^2 \phi_0^2$$

- For low scale inflation : SUGRA corrections negligible
- Within MSSM, can accommodate inflaton

[Allahverdi, Enqvist, Mazumdar, Garcia-Bellido, 2006..]

- Several flat directions : UDD, LLE ...
- ullet How about the Higgs Bosons : H_1 H_2
- D-flat direction, F-flatness lifted by mu term,
 also any number of Planck suppressed terms may be present

$$\mathcal{W} = \mu \mathbf{H_1}.\mathbf{H_2} + \frac{\lambda_k}{k} \frac{(\mathbf{H_1}.\mathbf{H_2})^k}{M_P^{2k-3}}$$

• Mu term is >0(100) GeV [LHC constraints!]

[ATLAS, CMS electroweak-ino searches]

• The soft-SUSY breaking terms :

$$V_{H,Soft} = m_1^2 |H_1|^2 + m_2^2 |H_2|^2 + (B\mu H_1 \cdot H_2 + h.c.)$$

• The scalar potential :

$$\begin{split} \tilde{V}(\varphi,\theta) &= \frac{1}{2} m^2(\theta) \varphi^2 + (-1)^{(k-1)} 2\lambda_k' \mu \cos((2k-2)\theta) \varphi^{2k} + 2\lambda_k'^2 \varphi^{2(2k-1)} \\ \phi &= |\phi| e^{i\theta}, \ \varphi = \sqrt{2} |\phi| \end{split}$$

Some details :

$$H_1 = \frac{1}{\sqrt{2}} (\phi, 0)^T,$$

$$H_2 = \frac{1}{\sqrt{2}} (0, \phi)^T,$$

$$m^2(\theta) = \frac{1}{2} (m_1^2 + m_2^2 + 2\mu^2 - 2B\mu \cos 2\theta)$$

$$\lambda'_k = \frac{\lambda_k}{2^{(2k-1)} M_P^{2k-3}}.$$

 Assume B and mu to be real, experimental constraint on complex phases, 0 phase is well-motivated

- The scalar potential can have a local minima in the angular direction for
- And a suitable inflection point for :

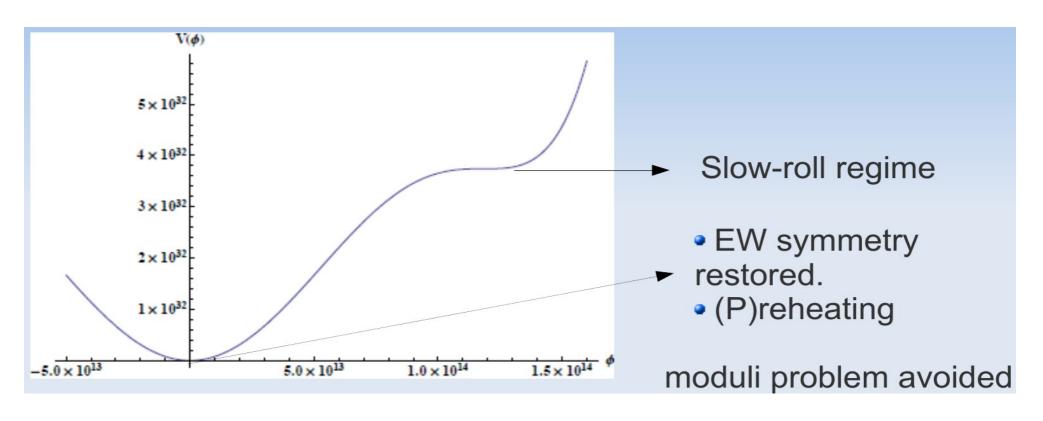
$$m_0^2 = \frac{k^2 \mu^2}{(2k-1)} + \tilde{\lambda}^2$$

$$\varphi_0 = \left(\frac{k|\mu||\lambda_k'|^{-1}}{2(2k-1)}\right)^{1/(2k-2)} (1-\lambda^2) \qquad \lambda^2 = \frac{\tilde{\lambda}^2 m_0^{-2}}{8(k-1)^2}$$

The potential is :

$$V_0 = \frac{(k-1)^2 m_0^2}{k(2k-1)} \varphi_0^2 + \mathcal{O}(\lambda^2)$$

- Sub-Planckian ev, scale of inflation low, no eta problem
- Inflation ends when $|\eta|\simeq 1$ $\frac{|\varphi_0-\varphi|}{\varphi_0}\sim \left(\frac{\varphi_0}{8k(2k-1)M_P}\right)^2\sim 10^{-8}$
- Can get sufficient e-foldings



$$\varphi_0 \sim 10^{14} \text{ GeV}$$
 $V(\varphi_0) = V_0 \simeq 10^{32} \text{ GeV}^4$

$$H_{\text{inf}} \simeq \sqrt{\frac{V_0}{3M_P^2}} = \frac{k-1}{\sqrt{3k(2k-1)}} \frac{m_0 \varphi_0}{M_P} \sim 10^{-1} \text{ GeV}$$

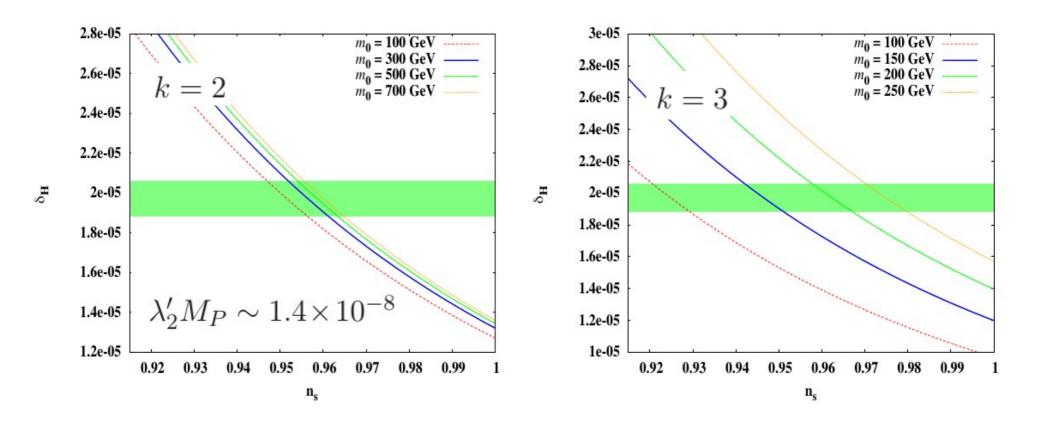
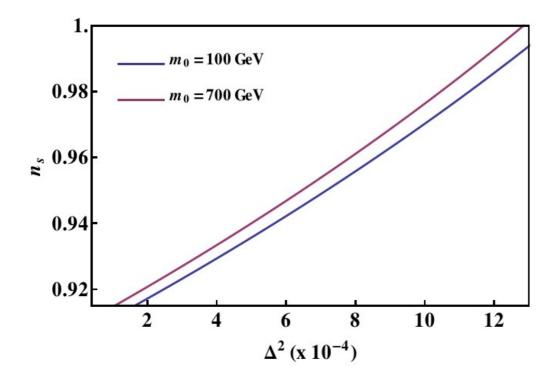


Figure 1. δ_H and n_s have been plotted for different values of m_0 and at the inflection point VEV, $\varphi_0 \sim 3 \times 10^{14} \,\text{GeV}$. In frame (a) we have used k=2 in the superpotential, we have taken $\lambda_2' M_P = 1.4 \times 10^{-8}$, see eq. (2.6). Although the splitting between these curves are not so sensitive to the inflaton mass, varying λ_2' it is possible to span the complete range in the n_s - δ_H plane. In frame (b) we have used k=3 in the superpotential, and have used $\lambda_3' M_P^3 = -0.71$. The green bands denote 2σ allowed region of δ_H [1].

Fine tuning required :

$$n_s = 1 - 4\sqrt{\Delta^2} \cot[\mathcal{N}_{\text{COBE}}\sqrt{\Delta^2}]$$

$$\delta_H \simeq \frac{1}{5\pi} \sqrt{\frac{2}{3} 2k(2k-1)} (2k-2) \left(\frac{m_0 M_P}{\varphi_0^2}\right) \frac{1}{\Delta^2} \sin^2[\mathcal{N}_{\text{COBE}} \sqrt{\Delta^2}]$$



$$\Delta^2 \sim 10^{-3} \implies \lambda \sim 10^{-11}$$

$$m_0^2 = \frac{k^2 \mu^2}{(2k-1)} + \tilde{\lambda}^2$$

Figure 2. n_s has been plotted against vs Δ^2 for different values of m_0 for k=2 case with $\lambda_2' M_P = 1.4 \times 10^{-8}$.

Inflation within SUSY: EWSB ...

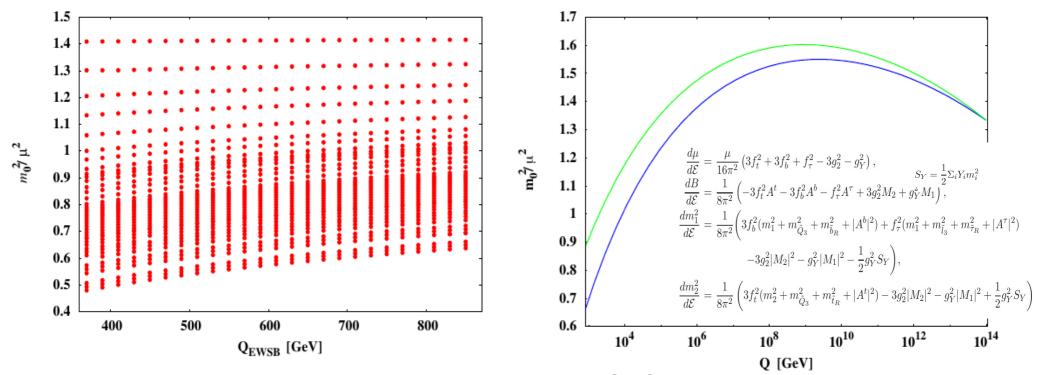
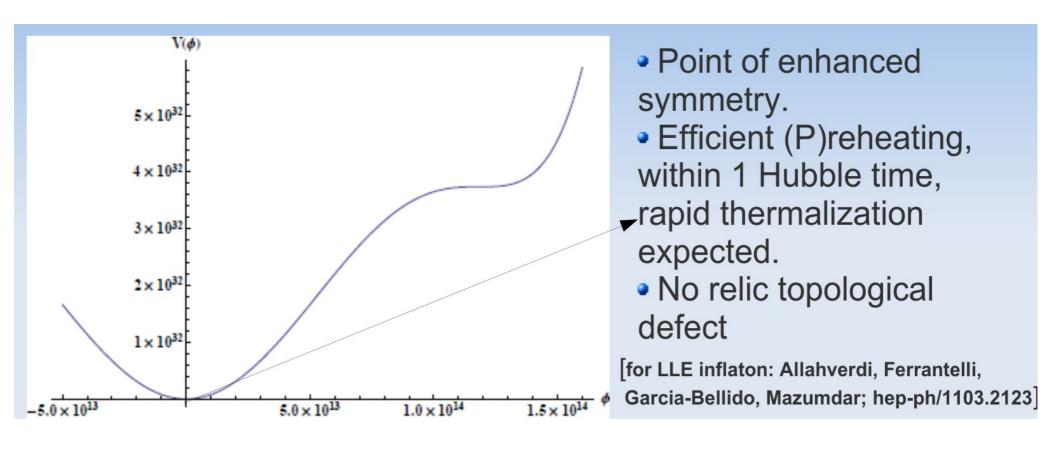


Figure 3. Frame (a): A sample plot where the ratio m_0^2/μ^2 , see eq. (3.1), for k=2, has been evaluated at the EWSB scale. The corresponding value at a high scale, $\varphi_0 \sim 10^{14} \,\text{GeV}$, is set to 4/3, see eq. (3.1), with an accuracy of 0.1%. Frame (b): The ratio m_0^2/μ^2 for k=2, has been evolved from $10^{14} \,\text{GeV}$ to the EWSB scale (chosen to be 850 GeV). The green line and the blue line correspond to $m_0 = 323.4 \,\text{GeV}$ and $m_0 = 354 \,\text{GeV}$ at $10^{14} \,\text{GeV}$ respectively. The ratio at the high scale ($10^{14} \,\text{GeV}$) is set to 4/3, see eq. (3.1), with an accuracy of 0.1%. The RGE accuracy in SuSpect [27] is about 0.01%.

 No UV fixed point. Parameter space consistent with Higgs mass of 125 GeV

Inflation within SUSY: (P)reheating...



• The final reheat temperature :

$$g_* = 228.75$$

$$\rho_0 = (4/15)m_0^2 \phi_0^2$$
 $T_{\rm rh} \simeq \left(\frac{30}{\pi^2 g_*}\right)^{1/4} \rho_0^{1/4} \simeq 2 \times 10^8 \text{ GeV}$

Questions

- Can we generate sizable r with sub-Planckian excursion? Lyth Bound assumes constant (or monotonic variation) in slow-roll parameter $\frac{\Delta \phi}{M_{\rm Pl}} = \mathscr{O}(1) \sqrt{\frac{r}{0.01}} \quad \text{[Lyth,1996;Lyth,Boubekeur,2006]}$
- How to probe the full inflationary potential in such cases? Any possibility to produce adequate PBH DM?

A "Phenomenological" Potential for Inflation

Motivated by SUSY/SUGRA

$$V(\phi) = V_0 + A\phi^2 - B\phi^3 + C\phi^4$$

• The scalar potential :
With sizable SUGRA contribution :

$$V_0 + c_H H^2 \phi^2 - a_H H \lambda_n \phi^n + \lambda_n^2 \phi^{2n-2}$$

• N=3,4,6
$$\phi = \frac{\widetilde{N} + H_u + \widetilde{L}}{\sqrt{3}} \qquad \phi = \frac{H_u + H_d}{\sqrt{2}}$$

$$\phi = \frac{\widetilde{u} + \widetilde{d} + \widetilde{d}}{\sqrt{3}} \qquad \phi = \frac{\widetilde{L} + \widetilde{L} + \widetilde{e}}{\sqrt{3}}$$

[See e.g. review Mazumdar, Rocher, 2010...]

Ways to generate V0 : String theory "Landscape", Hidden sector with a heavy superfield, hybrid scenarios

[Allahverdi,Frey,Mazumdar; Enqvist,
Mether,Nurmi; Lalak,Turzynski]

Reconstructing the Parameters

Some relevant "observables":

$$\begin{split} \mathcal{P}_{s}(k) &= \frac{1}{8\pi^{2}} \frac{H^{2}}{\varepsilon_{V}} \Big|_{k=aH} \\ &= A_{s} \left(\frac{k}{k_{*}} \right)^{n_{s} - 1 + \frac{1}{2} \operatorname{d}n_{s} / \operatorname{d} \ln k \ln(k/k_{*}) + \frac{1}{6} \operatorname{d}^{2}n_{s} / \operatorname{d} \ln k^{2} (\ln(k/k_{*}))^{2} + \dots} \\ \mathcal{P}_{t}(k) &= \frac{2H^{2}}{\pi^{2}} \Big|_{k=aH} \\ &= A_{t} \left(\frac{k}{k_{*}} \right)^{n_{t} + \frac{1}{2} \operatorname{d}n_{t} / \operatorname{d} \ln k \ln(k/k_{*}) + \dots} , \end{split}$$

$$A_{
m s} pprox rac{v}{24\pi^2 M_{
m pl}^4 arepsilon_V}, \ n_{
m s} - 1 pprox 2\eta_V - 6arepsilon_V, \ {
m d}n_{
m s}/{
m d} \ln k pprox 16arepsilon_V \eta_V - 24arepsilon_V^2 - 2\xi_V^2, \ {
m d}^2 n_{
m s}/{
m d} \ln k^2 pprox -192arepsilon_V^3 + 192arepsilon_V^2 \eta_V - 32arepsilon_V \eta_V^2 \ - 24arepsilon_V \xi_V^2 + 2\eta_V \xi_V^2 + 2\sigma_V^3, \ A_{
m t} pprox rac{2V}{3\pi^2 M_{
m pl}^4}, \ n_{
m t} pprox -2arepsilon_V, \ {
m d}n_{
m t}/{
m d} \ln k pprox 4arepsilon_V - 8arepsilon_V^2.$$

ullet Reconstructing the potential at pivot scale $\phi_{
m CMB}=1~M_P$

$$\begin{pmatrix} \phi_{\text{CMB}}^{2} & -\phi_{\text{CMB}}^{n} & \phi_{\text{CMB}}^{2(n-1)} \\ 2\phi_{\text{CMB}} & -n\phi_{\text{CMB}}^{(n-1)} & 2(n-1)\phi_{\text{CMB}}^{2n-3} \\ 2 & -n(n-1)\phi_{\text{CMB}}^{(n-2)} & 2(n-1)(2n-3)\phi_{\text{CMB}}^{2(n-2)} \end{pmatrix} \begin{pmatrix} A \\ B \\ C \end{pmatrix} = \begin{pmatrix} V(\phi_{\text{CMB}}) - V_{0} \\ V'(\phi_{\text{CMB}}) \\ V''(\phi_{\text{CMB}}) \end{pmatrix}$$

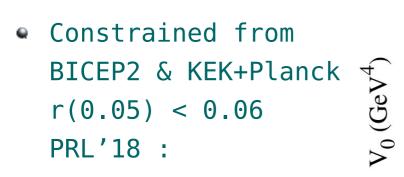
[Hotchkiss, Mazumdar, Nadathur'11]

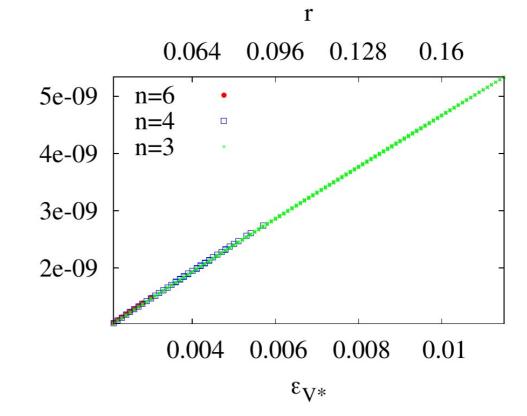
Using:
$$V(\phi_{\rm CMB}) = \frac{3}{2} A_s r \pi^2,$$
 $V'(\phi_{\rm CMB}) = \frac{3}{2} \sqrt{\frac{r}{8}} \left(A_s r \pi^2 \right),$ $A_s \approx \frac{V}{24 \pi^2 M_{\rm pl}^4 \varepsilon_V} \approx 2.2 \times 10^{-9}$ $V''(\phi_{\rm CMB}) = \frac{3}{4} \left(\frac{3r}{8} + n_s - 1 \right) \left(A_s r \pi^2 \right)$ $n_s \approx 1 + 2 \eta_V - 6 \varepsilon_V \approx 0.96$

Reconstructing the Parameters

Some constraints from Planck:

Model	Parameter	Planck+WP+lensing Planck+WP+hi		
	$n_{ m s}$	$0.9573_{-0.079}^{+0.077}$	$0.9476^{+0.086}_{-0.088}$	
$\Lambda {\rm CDM} + {\rm d}n_{\rm s}/{\rm d}\ln k$	$\mathrm{d}n_\mathrm{s}/\mathrm{d}\ln k$	$0.006^{+0.015}_{-0.014}$	$0.001^{+0.013}_{-0.014}$	
$+ d^2 n_{\rm s}/d \ln k^2$	$\mathrm{d}^2 n_\mathrm{s} / \mathrm{d} \ln k^2$	$0.019^{+0.018}_{-0.014}$	$0.022^{+0.016}_{-0.013}$	
	$n_{ m s}$	0.9633 ± 0.0072	0.9570 ± 0.0075	
	r	< 0.26	< 0.23	
$\Lambda \text{CDM} + r + dn_{\text{s}}/d\ln k$	$\mathrm{d}n_\mathrm{s}/\mathrm{d}\ln k$	-0.017 ± 0.012	$-0.022^{+0.011}_{-0.010}$	





Results: Benchmarks

TABLE I. We have used $n_s = 0.96$, $A_s = 2.2 \times 10^{-9}$, $\phi_{\text{CMB}} = 1$ in the Planck units for all the benchmarks evaluated at $k_* = 0.05 \text{ Mpc}^{-1}$. The three benchmark points match the current CMBR data, i.e. the central values used in Eqs. (2), (3).

Benchmark Points (BP)	n	$V_0(k_*)$	$A(k_*)$	$B(k_*)$	$C(k_*)$	$rac{dn_s}{d\ln k}(k_*)$	$rac{d^2n_s}{d\ln k^2}(k_*)$	$r(k_*)$
1	3	7.44×10^{-10}	0.868×10^{-10}	0.689×10^{-10}	0.190×10^{-10}	-0.006	0.003	0.024
2	3	1.506×10^{-10}	0.2046×10^{-10}	0.2246×10^{-10}	0.0757×10^{-10}	-0.0148	0.001	0.005
3	4	14.245×10^{-10}	1.240×10^{-10}	0.500×10^{-10}	0.112×10^{-10}	-0.0148	0.021	0.046

The slow-roll parameters (non-monotonic):

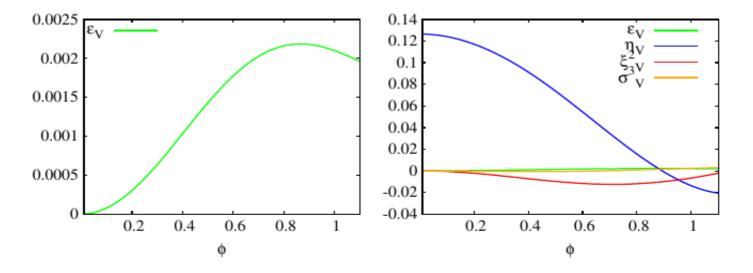
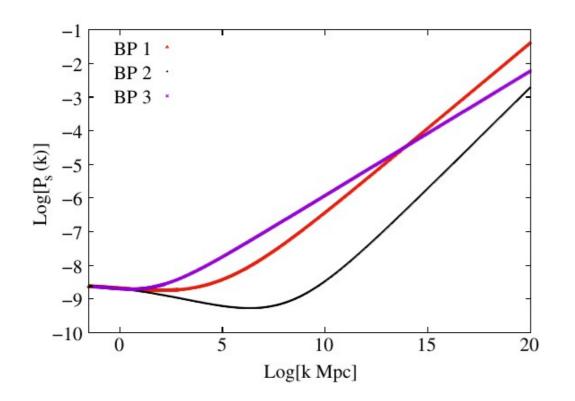
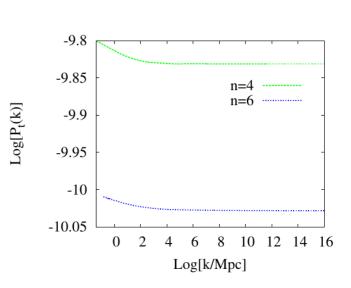


FIG. 2. The slow-roll parameters have been shown for the benchmark scenario BP-3 in Table I.

Results: Power Spectra

- The scalar power spectrum : (Solving full Mukhanov-Sasaki equation)
- Large power at small scales, slope dictated by A/VO





Results: Gravitational Wave at Small Scales

The metric

$$ds^{2} = -a(\eta)^{2} \left[(1+2\Phi)d\eta^{2} + \left\{ (1-2\Phi)\delta_{ij} + \frac{1}{2}h_{ij} \right\} dx^{i} dx^{j} \right]$$

The transverse and traceless tensor perturbation

$$h_{ij}(\mathbf{x},\eta) = \frac{1}{(2\pi)^{3/2}} \int d^3\mathbf{k} \, e^{i\mathbf{k}.\mathbf{x}} [h_{\mathbf{k}}(\eta) e_{ij}(\mathbf{k}) + \bar{h}_{\mathbf{k}}(\eta) \tilde{e}_{ij}(\mathbf{k})] \qquad e^{ij} e_{ij} = 1 = \tilde{e}^{ij} \tilde{e}_{ij}, e^{ij} \tilde{e}_{ij} = 0$$

• The equation in the Fourier space (for + polarization):

$$h_{\mathbf{k}}^{"} + 2\mathcal{H}h_{\mathbf{k}}^{'} + k^{2}h_{\mathbf{k}} = \mathcal{S}(\mathbf{k}, \eta)$$

Note that second order scalar perturbation can source tensor perturbation

[Ananda, Clarkson, Wands; Baumann, Steinhardt, Takahashi, Ichiki, 2007]

Results: Gravitational Wave at Small Scales

• The source term :

$$\begin{split} \mathcal{S}(\mathbf{k},\eta) &= -4e^{lm}(\mathbf{k})\mathcal{S}_{lm}(\mathbf{k}) \\ &= \int \frac{d^3\mathbf{q}}{(2\pi)^{3/2}}e^{lm}(\mathbf{k})q_lq_m\mathcal{F}(\mathbf{k},\mathbf{q},\eta) \\ &+ \frac{4}{\mathcal{H}^2}\Phi'(q,\eta)\Phi(|\mathbf{k}-\mathbf{q}|,\eta). \end{split}$$

• The second order source term is significant for

$$k \gg k_{\rm eq} \sim 0.01 \ {\rm Mpc^{-1}}$$

- The corresponding modes enter during radiation domination
- Bardeen potential satisfies : $\Phi'' + \frac{6(1+w)}{(1+3w)n}\Phi' + wk^2\Phi = 0$, w = 1/3
- Primordial and transfer fn

$$\Phi(k,\eta) = \frac{c(k)}{(k\eta)^3} \left[\frac{k\eta}{\sqrt{3}} \cos\left(\frac{k\eta}{\sqrt{3}}\right) - \sin\left(\frac{k\eta}{\sqrt{3}}\right) \right]$$

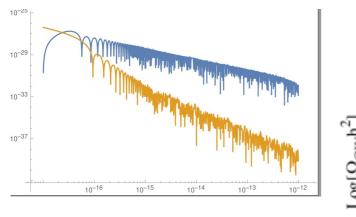
 $\mathcal{F}(\mathbf{k}, \mathbf{q}, \eta) = 12\Phi(q, \eta)\Phi(|\mathbf{k} - \mathbf{q}|, \eta)$

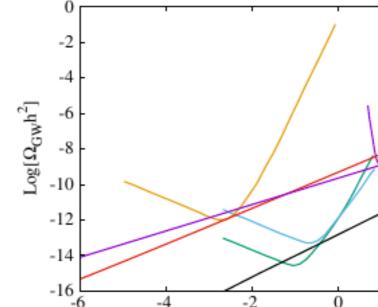
$$\Phi(k,\eta) = \Phi(k\eta)\phi_{\mathbf{k}} \qquad c(k)^2 \simeq (9\sqrt{3})^2 \frac{4}{9} \frac{2\pi^2}{k^3} P_s(k) = \frac{216\pi^2}{k^3} P_s(k)$$

PBH constraints respected, BBN+CMB constraints respected

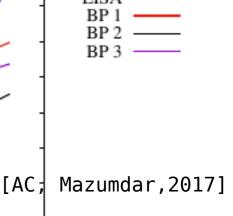
$$\Omega_{\rm GW} \lesssim 10^{-5}$$

Primordial Gravitational Wave at Small Scales





Log[f (Hz)]



$$\mathcal{P}_h(k,\eta) = \frac{k^3}{2\pi^2} (|h_{\mathbf{k}}(\eta)|^2 + |\bar{h}_{\mathbf{k}}(\eta)|^2)$$

2

$$\rho_{\text{GW}}(k,\eta) = \frac{\langle \dot{h}_{ij}\dot{h}^{ij}\rangle}{32\pi G} = \frac{1}{32\pi G} \frac{k^2}{a(\eta)^2} \mathcal{P}_h(k,\eta)$$

[Maggiore, 2000]

$$\Omega_{\mathrm{GW}}^0(k)h^2 = \frac{\Omega_{\mathrm{rad}}^0h^2}{2\Omega_{\mathrm{rad}}^{\mathrm{eq}}} \left(\frac{g_{*\mathrm{eq}}}{g_{*i}}\right)^{1/3} \frac{k^2\mathcal{P}_h(k,\eta_i)}{12a(\eta_i)^2H(\eta_i)^2}$$

Conclusion and Outlook

- Inflation can be embedded within MSSM, with fine-tuning Low-scale inflation, negligible SUGRA corrections, no moduli problem
- Large V0 can reduce the fine-tuning. Can obtain large r in presence of V0, with sub-Planckian excursion of the inflaton. Possible to obtain such potentials from SUGRA.
- PBHs (as DM) are difficult to produce in such simple (polynomial) inflationary models.
- Possible to probe (such large) scalar power at small scales with gravitational wave.

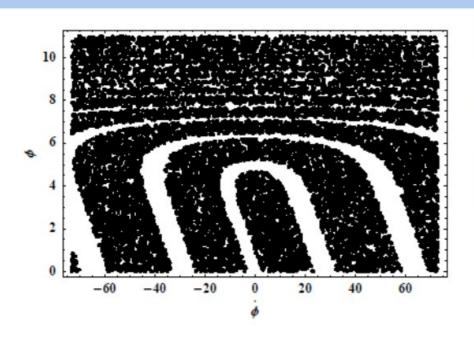


FIG. 4: We plot initial values of ϕ versus $\dot{\phi}$ for $H_{\rm false} = 10^2 m_{\phi}$. The dots show the initial values for which ϕ settles to $\pm \phi_0$ and the white bands \cap show the critically damped regions where ϕ settles to zero.

Attraction towards the inflection Point:

- ★ MSSM has metastable vacua
- * Need inflation in the flase vacua
- ★ If m<H_{false} , scalar fields can get displaced
- ★ If started at a small VEV, MSSM inflaton can obtain quantum jumps of the order H_{false} /2 pi
- ★ Need the rms value greater than the VEV at the inflection point : need ~10⁶ efoldings of false vacuum inflation.

[Allahverdi, Dutta, Mazumdar, hep-ph/08064557]

★ In the string landscape favorable conditions exist [Allahverdi, Frey, Mazumdar;hep-th/0701233]