Geometrical and topological properties of CMB Polarization

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Cosmic Microwave Background

ORIGIN:

- Primordial plasma in the recombination epoch

PROPERTY:

- 2.73K black body radiation
- Highly isotropic and homogeneous
- Linear polarization

CMB fluctuations

ORIGIN OF FLUCTUATIONS:

- Primordial fluctuation in the Inflationary phase of very early Universe

PROPERTY:

- Nearly Gaussian distributed fluctuations with small deviation
- Nearly scale invariant power spectrum

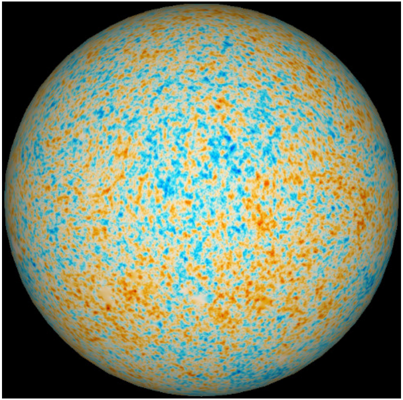


Image: thecmb.org

CMB fields

Each line of sight possess a temperature and polarization value

Temperature:

$$\frac{\Delta T}{T_0}(\theta, \varphi) = \frac{T(\theta, \varphi) - T_0}{T_0}, \text{ where } T_0 = \langle T(\theta, \varphi) \rangle$$

Polarization:

- Stokes parameters Q, U
- Transforms under rotation about line of sight
- Re-expressed in terms of E mode and B mode
- E mode and B mode are invariant under such transfomations

Polarization direction pattern

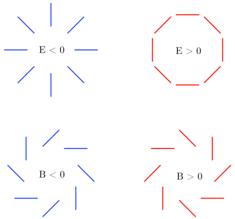


Image: https://astrobites.org/2013/07/24/lensing-b-modes-in-the-cosmic-microwave-background-polarization/

Excursion set of a field

For a fluctuating field $g(\theta, \varphi)$ on a sphere:

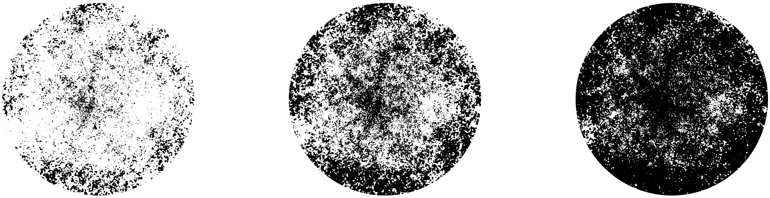
A constant field on a sphere with value $\nu\sigma$

UNIVERSAL SET: All points on the sphere

EXCURSION SET: Set of points with field value above the constant field
(white points $\equiv$ excursion set)

Systematic variation with the threshold value ν:

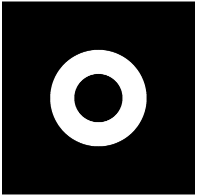
$-\sigma$ | 0 | $+\sigma$



Minkowski Functionals

Scalar Minkowski Functionals:
Area fraction, contour length and genus

For example: a ring



- Area fraction $= \frac{\pi r_{\text{out}}^2 - \pi r_{\text{in}}^2}{\text{Total area}}$
- Contour length $= 2\pi r_{\text{in}} + 2\pi r_{\text{out}}$
- Genus = number of hotspots - number of coldspots = 1 - 1 = 0

Tensor Minkowski Functionals

- Definition of $a+b$ rank tensor:
  $W_0^{a,0} = \int_S \vec{r}^{\ a}\ ds, W_j^{a,b} = \frac{1}{2}\int_C \vec{r}^{\ a} \otimes \hat{n}^b G_j d\ell,$
  for $j = 1, 2$ with $G_1 = 1$ and $G_2 = \kappa$
- 3 rank 0 scalars, 3 rank 1 vectors, and 7 rank 2 tensors
- The rank 2 tensors capture more information than the scalars
- Minkowski Functionals

$$\Downarrow$$

Motion-invariant $\left\{\text{scalar}\right\}$

Motion-covariant $\left\{\text{vector,tensor}\right\}$

$$\Downarrow$$

Translation-invariant $\left\{W_1^{1,1}, W_1^{0,2}, W_2^{1,1}, W_2^{0,2}\right\}$

Translation-covariant $\left\{\text{vector}, W_0^{2,0}, W_1^{2,0}, W_2^{2,0}\right\}$

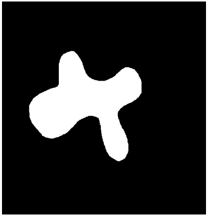
Definition of $W_2^{1,1}$

$$W_2^{1,1} = \frac{1}{2}\int_C \vec{r} \otimes \hat{n}\ \kappa\ d\ell$$

$\vec{r} \rightarrow$ position vector,

$\hat{n} \rightarrow$ unit normal vector,

$\kappa \rightarrow$ local curvature of contour C

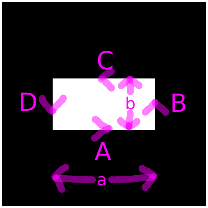


Invariant under translation operation or choice of origin

Real data

Formula for $W_2^{1,1}$ of real data [1]:

$$W_2^{1,1} = \sum_i \frac{1}{2} |\vec{e}_i|^{-1} (\vec{e}_i \otimes \vec{e}_i)$$



For example: a quadrilateral

$$W_2^{1,1} = \frac{1}{2}|\vec{e}_A|^{-1} \begin{bmatrix} e_A^x * e_A^x & e_A^x * e_A^y \\ e_A^y * e_A^x & e_A^y * e_A^y \end{bmatrix} + \frac{1}{2}|\vec{e}_B|^{-1} \begin{bmatrix} e_B^x * e_B^x & e_B^x * e_B^y \\ e_B^y * e_B^x & e_B^y * e_B^y \end{bmatrix} + \dots$$

$$= \frac{1}{2} a^{-1} \begin{bmatrix} a^2 & 0 \\ 0 & 0 \end{bmatrix} + \frac{1}{2} b^{-1} \begin{bmatrix} 0 & 0 \\ 0 & b^2 \end{bmatrix} + \frac{1}{2} a^{-1} \begin{bmatrix} a^2 & 0 \\ 0 & 0 \end{bmatrix} + \frac{1}{2} b^{-1} \begin{bmatrix} 0 & 0 \\ 0 & b^2 \end{bmatrix}$$

$$= \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$

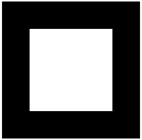
[1] G.E. Schroder-Turk *et al.*, J. Microsc. **238** 57 (2010)

Real data with many structures

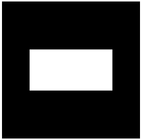
Anisotropy Measure β:

- λ_1, λ_2 are eigenvalues of $W_2^{1,1}$ with $\lambda_1 \leq \lambda_2$ for each structure
- Average $\dfrac{\lambda_1}{\lambda_2}$ over all the structures
- $\beta = \left\langle \dfrac{\lambda_1}{\lambda_2} \right\rangle$, $0 \leq \beta \leq 1$
- Encapsulates net anisotropy of the structures
- For quadrilateral: $\beta = \dfrac{b}{a}$

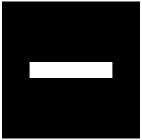
$\beta = 1$



$\beta = 0.5$



$\beta = 0.2$

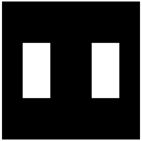


Real data with many structures

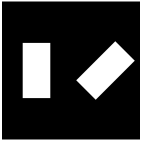
Orientation Measure α:

- Average $W_2^{1,1}$ over all the structures
- Λ_1, Λ_2 are eigenvalues of $\langle W_2^{1,1} \rangle$ with $\Lambda_1 \leq \Lambda_2$
- $\alpha = \frac{\Lambda_1}{\Lambda_2}$, $\beta \leq \alpha \leq 1$
- Encapsulates net orientation of the structures
- Completely aligned : $\alpha = \beta$
  Randomly aligned : $\alpha = 1$

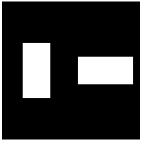
$\alpha = 0.5$



$\alpha = 0.75$



$\alpha = 1$



Numerical calculation of $W_2^{1,1}$ for any general planar field

TMFCode $\implies$ Computes α, β for an excursion set of any general planar field

`ALGORITHM:`

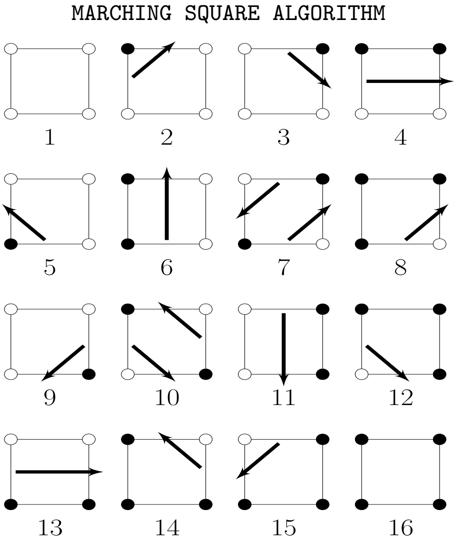
1. Scanning and tracking individual structures:
   - Outer scan : Pixel with field value above the threshold value is found
   - Inner scan : All connected pixels are found and labelled as a single structure

Numerical calculation of $W_2^{1,1}$ for any general planar field

2. Defining the boundaries for structures:
   - Planar field is divided into area segments with pixel centers as its vertices
   - Line segments are defined based on the surrounding four pixel centers configuration
3. Computation of α and β:
   - $W_2^{1,1}$ is calculated for individual structures
   - α and β is then computed

`NOTE:` We use stereo-graphic projection to map CMB field onto a plane

Boundary for pixel center configurations



Pixelization error

Analytical formula of $W_2^{1,1}$ for an ellipse with major axis p and minor axis q:

$$W_2^{1,1} = \begin{bmatrix} f_2^{1,1}(p,q) & 0 \\ 0 & f_2^{1,1}(q,p), \end{bmatrix}, \quad f_2^{1,1}(p,q) = \frac{1}{2} p^2 q^2 \int_0^{2\pi} d\varphi \frac{cos^2\varphi}{[p^2 - (p^2 - q^2) cos^2\varphi]^{3/2}}$$

Single ellipse on a plane

q/p	β from analytical formula	β from TMFCode 3000^2 pixels	% error
1.0000	1.0000	1.0000	0.0
0.8000	0.7154	0.7641	6.8
0.6000	0.4638	0.5418	16.8
0.5000	0.3518	0.4370	24.2
0.3000	0.1602	0.2432	51.8
0.1000	0.0274	0.0741	170.4

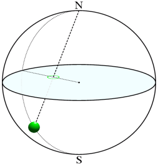
- Interpolating the % error at $\beta = 0.68 \rightarrow 9.68$ %
- Corrected $\beta = 0.62$

Double ellipse on a plane

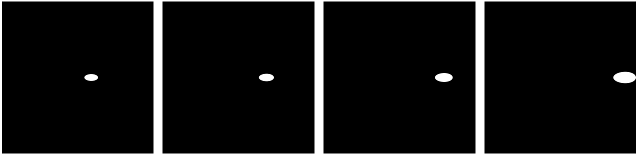
Angle between major axis of the ellipses	α from analytical formula	α from TMFCode 3000^2 pixels	% error
0°	0.3518	0.4369	24.2
20°	0.3787	0.4674	23.4
45°	0.4936	0.5661	14.7
60°	0.6132	0.6727	9.7
90°	1.0000	1.0000	0.0

- % error for $\alpha \sim 1$ is negligible
- No correction required

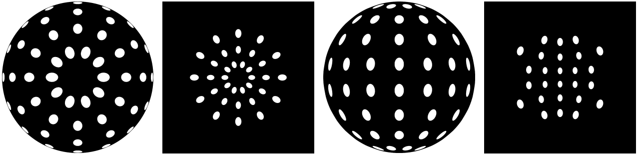
Stereographic projection effects



Structures as it gets closer to the equator:



Orientation measure for different projection planes:



What Standard model predicts for α and β?

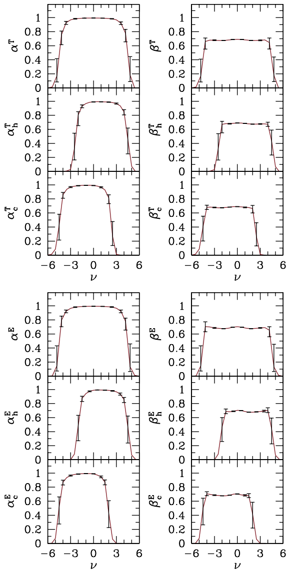
Threshold: $|\nu| = 1$

Prediction for Gaussian and isotropic CMB fields:

Temperature $\Longrightarrow$ $\alpha = 1$, $\beta = 0.62$

E mode $\Longrightarrow$ $\alpha = 1$, $\beta = 0.63$

Implications:
Statistical isotropy and *Intrinsic anisotropy*



Analysis of PLANCK data

- Foreground separated maps: SMICA, COMMANDER, NILC, and SEVEM
- Frequency simulation maps of 44GHz with instrumental noise effects
- An excursion set contains a group of zero or one or more structures. Different phenomenon may induce different characteristic pattern in these structures and their variation with the threshold value
- Anisotropy measure (β) of PLANCK temperature and E mode field are consistent within $2-\sigma$
- Orientation measure of PLANCK temperature field is consistent within $1.2-\sigma$
- Orientation measure of PLANCK E mode field shows deviations $3-\sigma$ to $5.3-\sigma$ from the standard model

Aspects of CMB polarization fields

`Study of non-Gaussian features in CMB polarization fields:`

- E mode field can provide independent and equally strong constraint on f_{NL}
- PLANCK results confirmed these theoretical expectations but it was not as significant as expected due to the presence of instrumental effects
- Polarization intensity, Q and U fields are not capable of providing an independent constraint on f_{NL}
- The CMB fields can be used in conjunction to distinguish primordial non-Gaussianity from other sources

`Study of the presence of tensor perturbation in CMB polarization fields:`

- SMFs of Q, U and I_P are sensitive to the presence of tensor perturbation and their amplitude decreases with r.
- Number density of singularities in I_P is also sensitive and it decreases with r.
- These findings will be useful for the searches of B mode polarization in the future experiments.

THANK YOU