The y-type polarised kinetic Sunyaev-Zeldovich effect - Pairwise & cross-pairwise estimator

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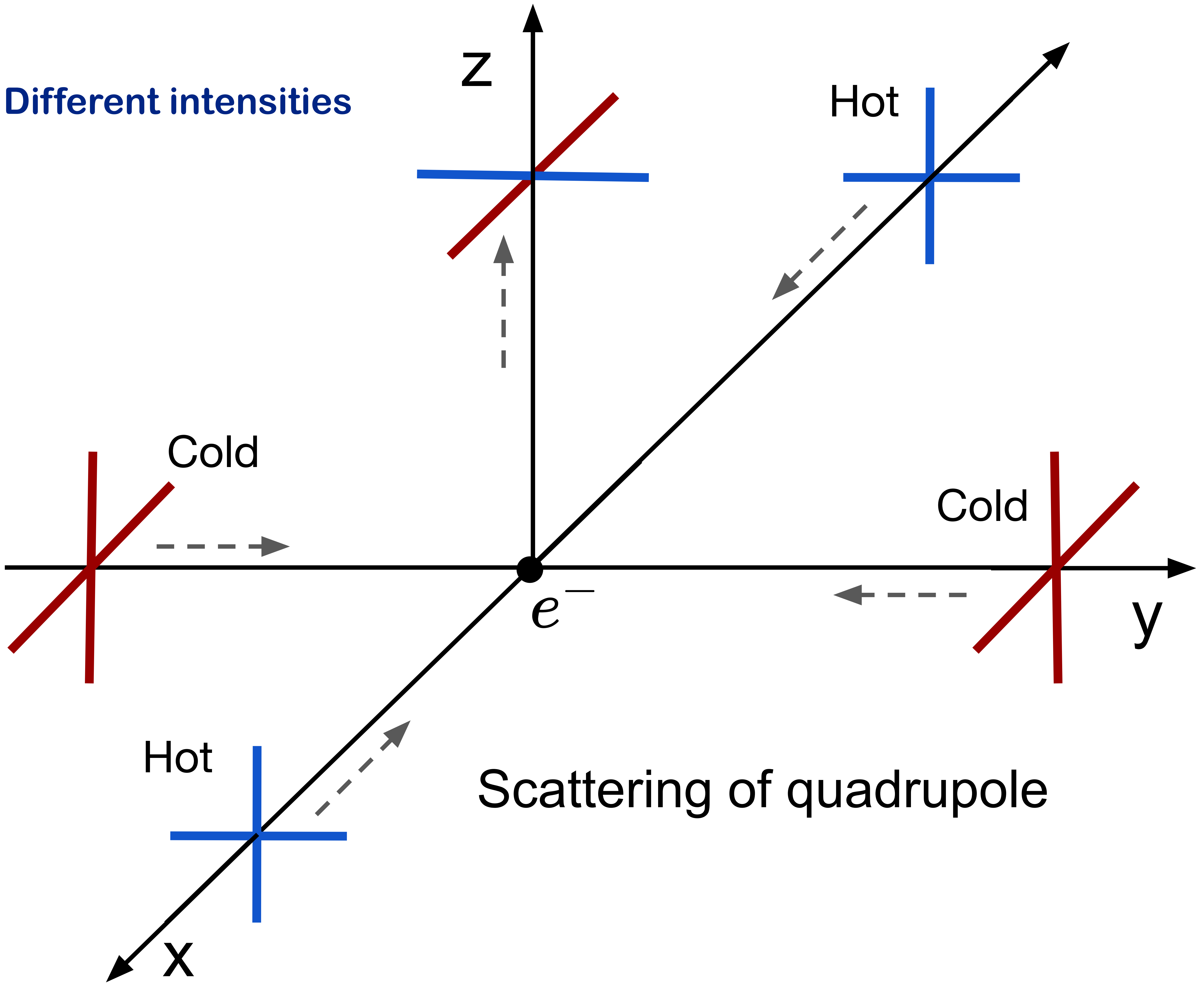
September 2023

Polarisation in the CMB: Quadrupolar anisotropy

- Thomson scattering of CMB quadrupole by the free electrons produces linear polarisation.

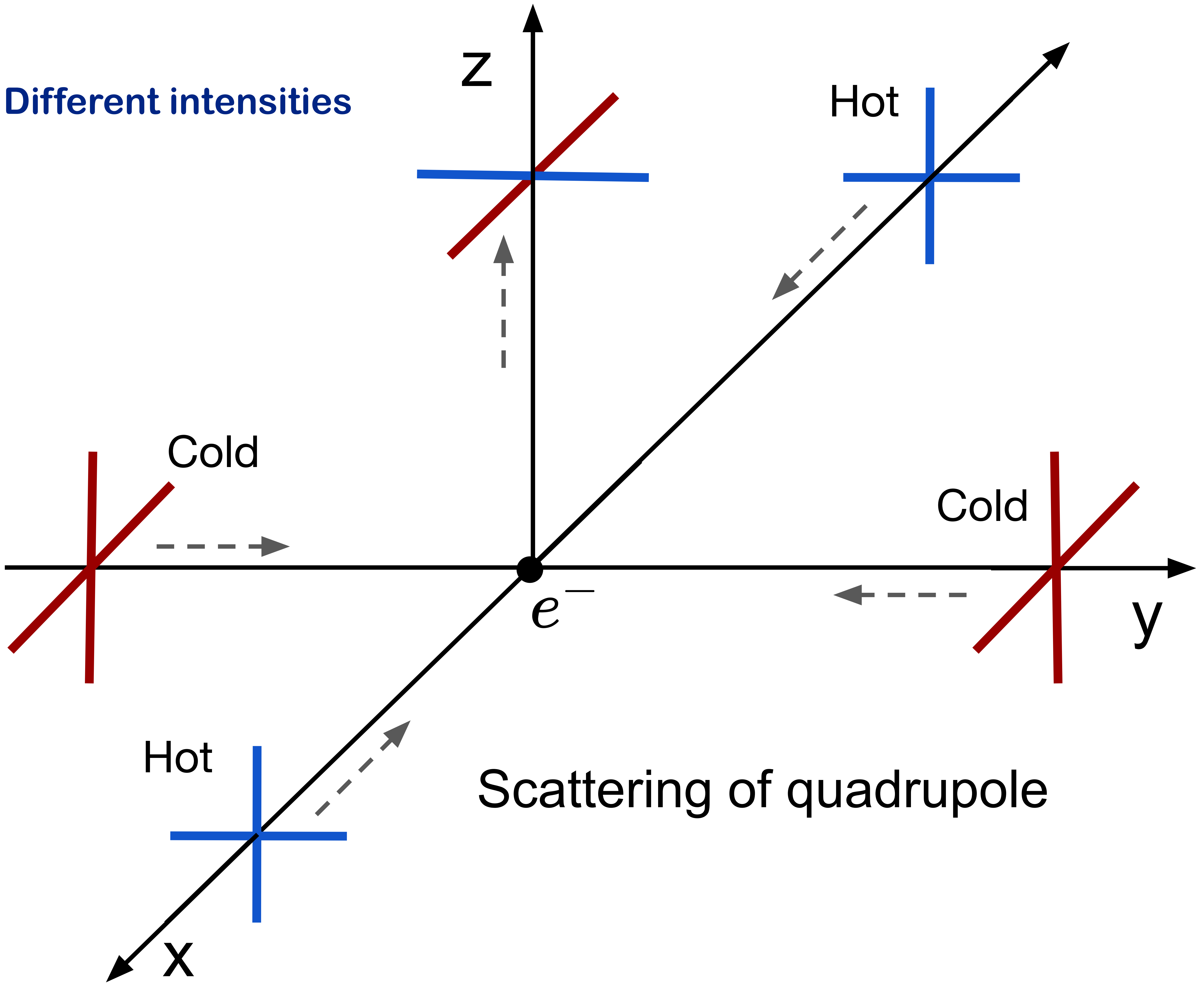


Scattering of quadrupole

Polarisation in the CMB: Quadrupolar anisotropy

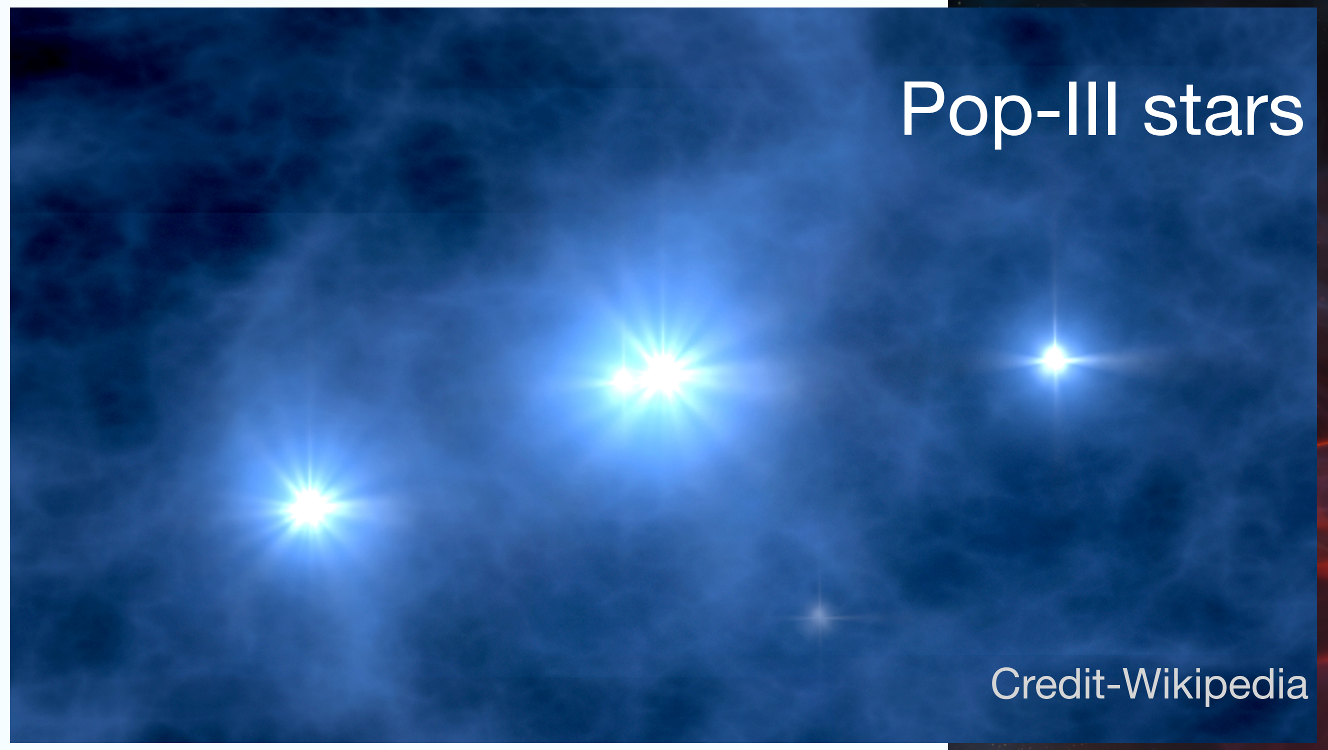
- Thomson scattering of CMB quadrupole by the free electrons produces linear polarisation.

- The source of the quadrupole can be due to different physical processes.
  - Inflationary tensor perturbation
  - Free streaming of photons
  - Peculiar velocity of free electrons



Production of free electrons: Reionization

- Free electrons are produced during reionization era when the radiation from the first collapsed objects ionises the surrounding neutral hydrogen medium.



Credit-Wikipedia

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Production of free electrons: Reionization

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- Post-reionisation these electrons are present near the galaxies, in filaments, and in the intra cluster medium (ICM) of galaxy clusters.

Credit-Wikipedia

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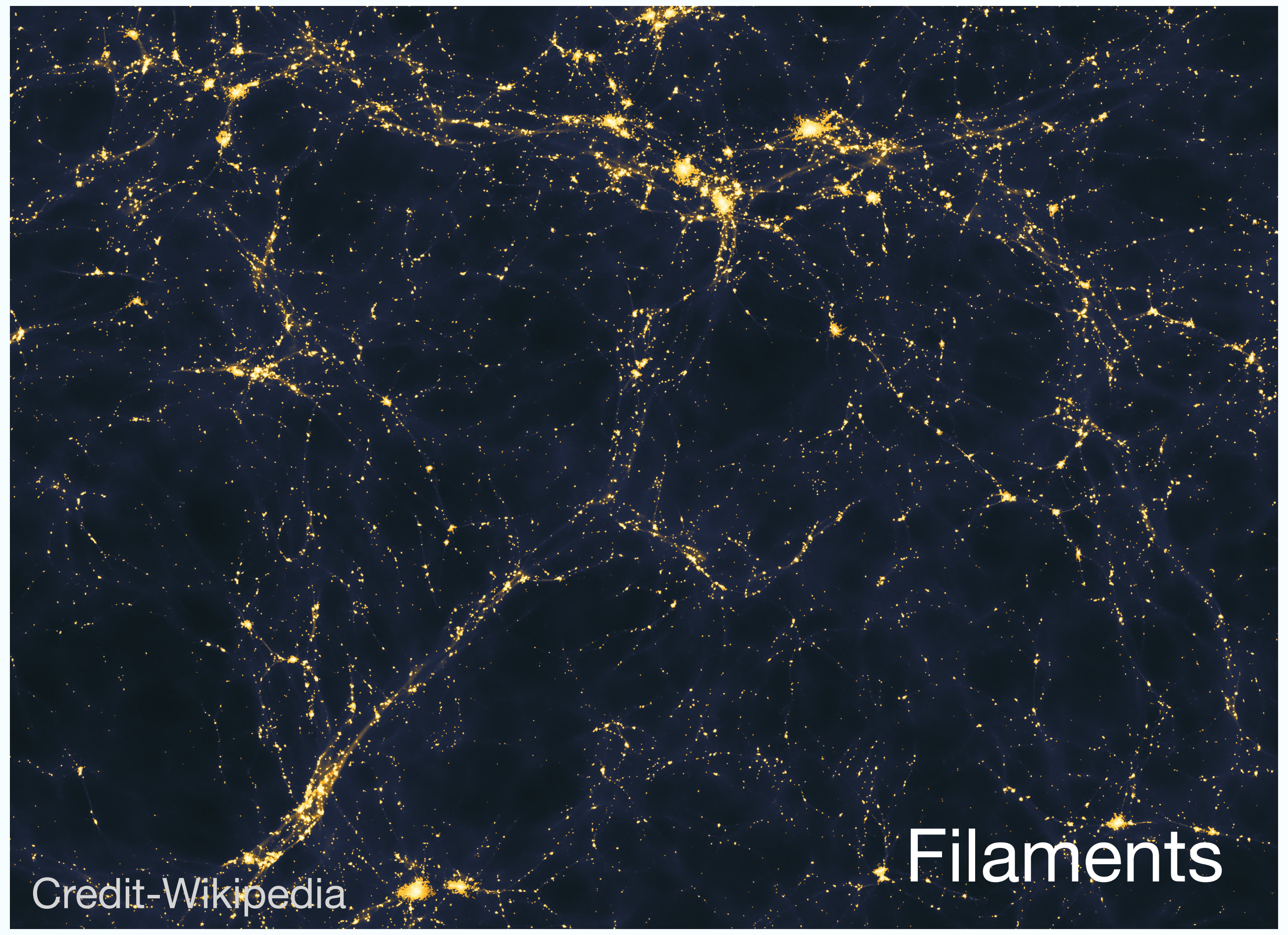
Credit-NASA



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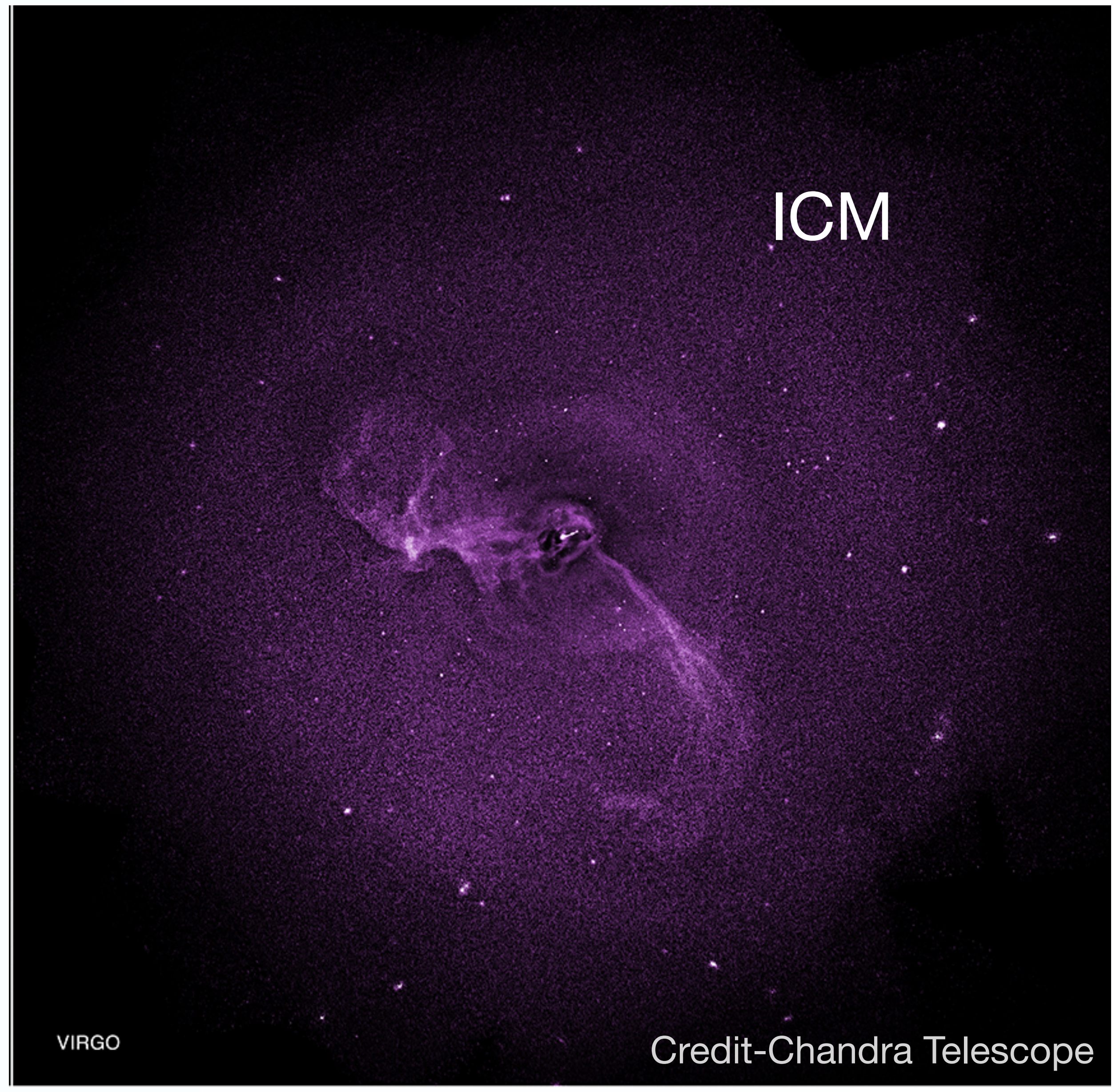
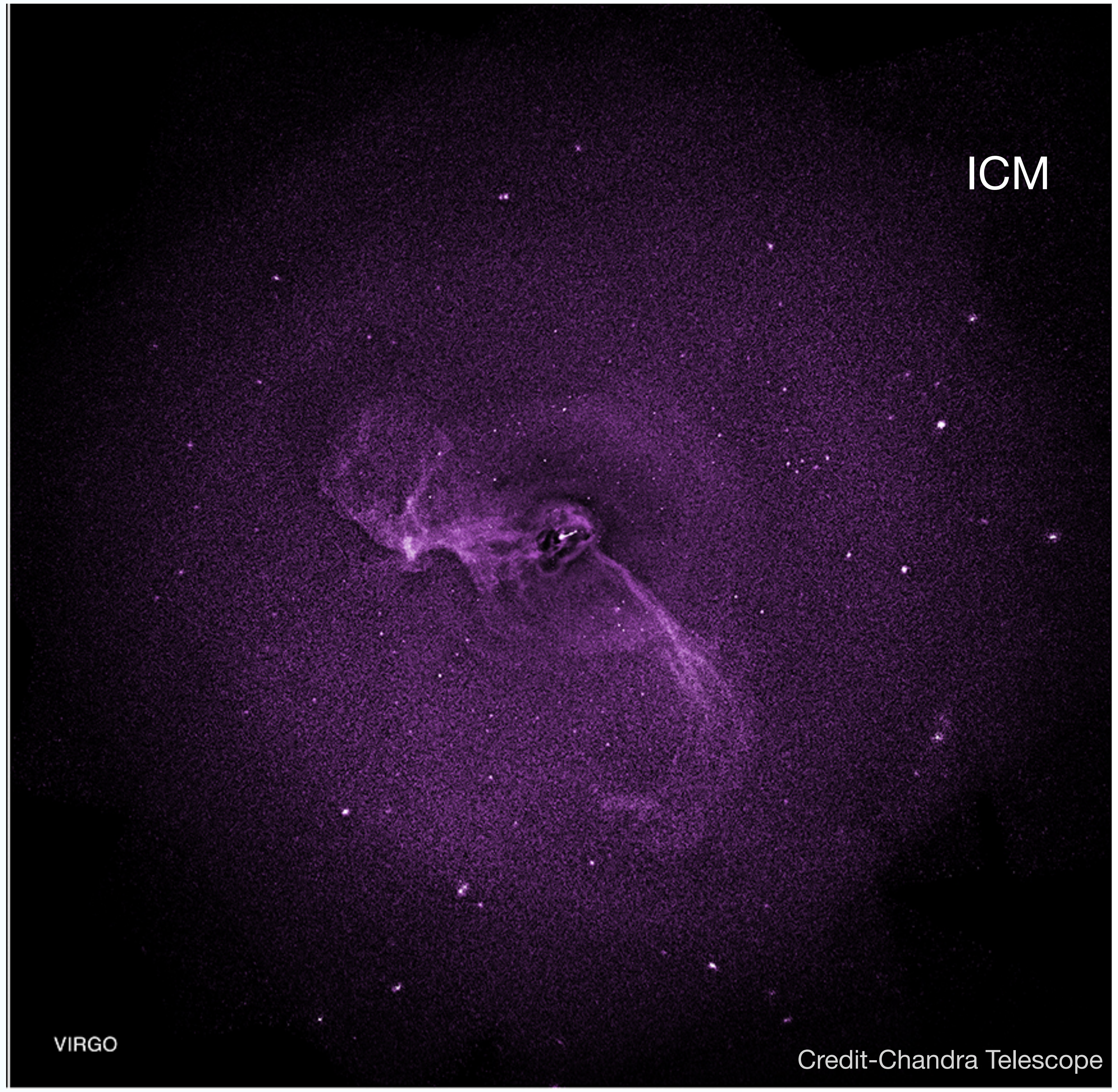


Credit-Chandra Telescope

Electron peculiar velocities at second order generate polarisation in the CMB: The pkSZ effect

- Post-reionisation, the ICM provides the highest optical depth to Thomson scattering.



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- Previous works: Renaux-Petel et al. 2013 Hotinli et al. 2022

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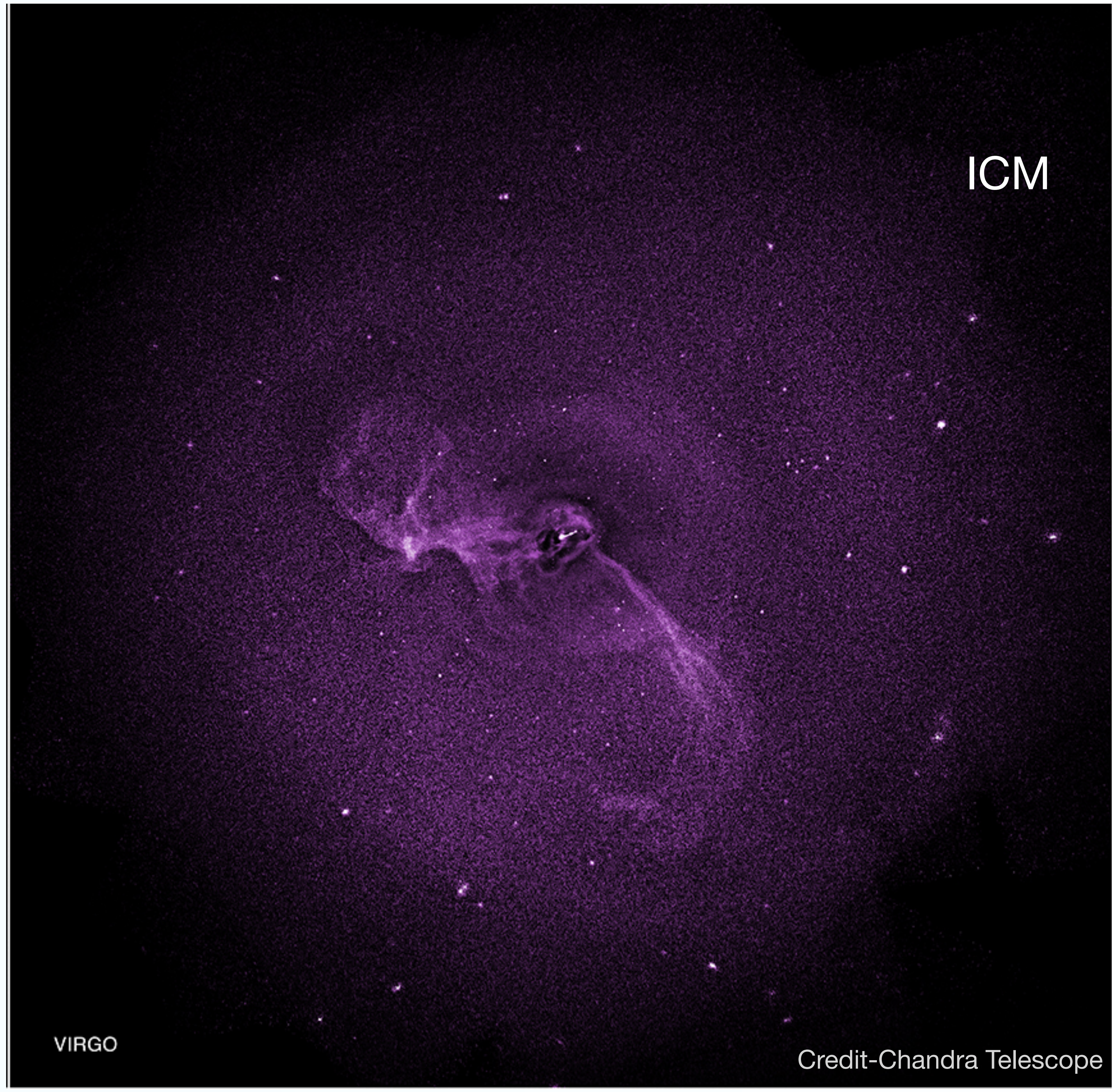

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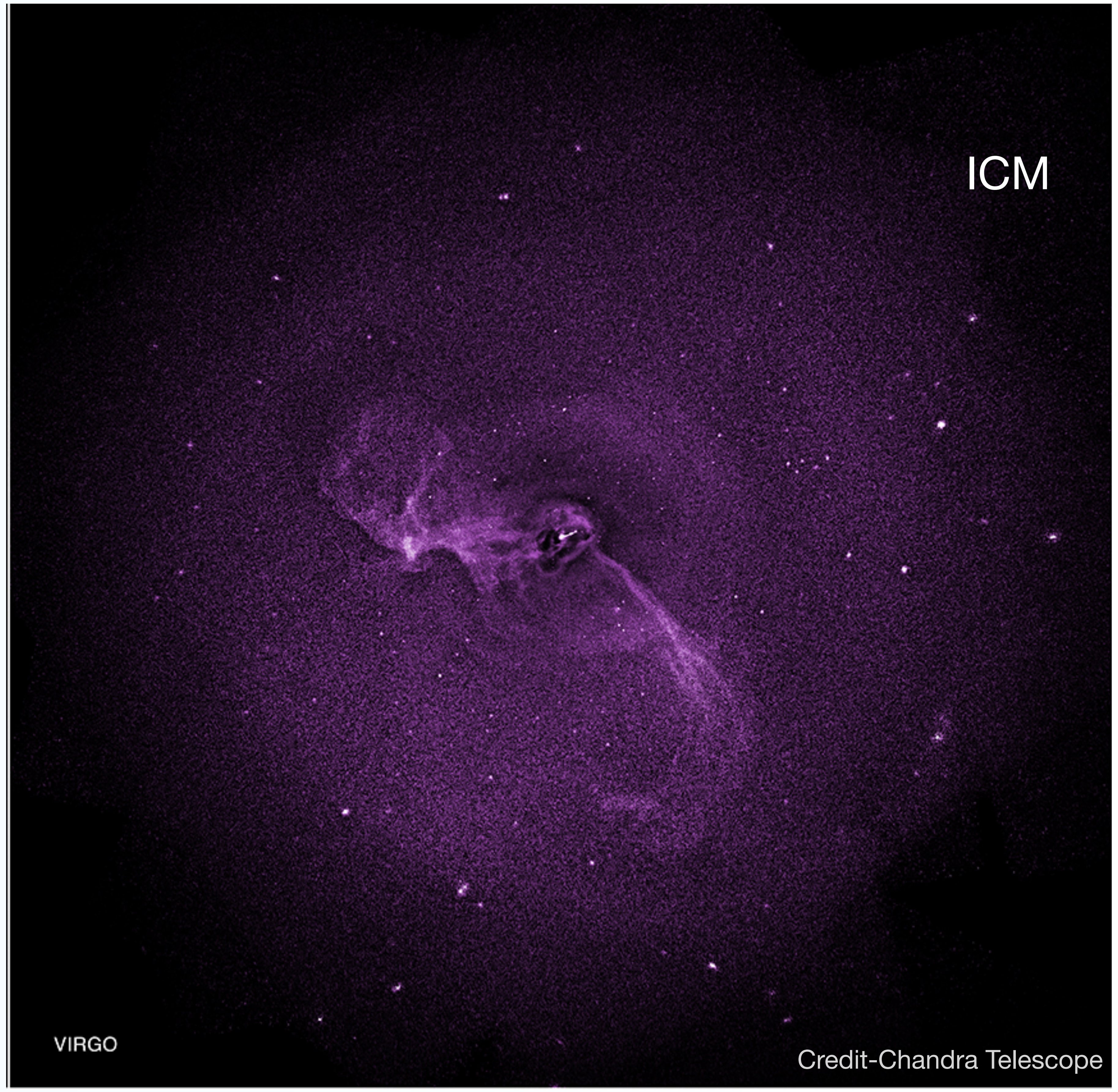
- Clusters have peculiar velocities; CMB is not isotropic in electron's rest frame - In particular a quadrupole is present.
  - Non-linear nature of Relativistic Doppler shift.
  - A non-linear relation between temperature and intensity in the Planck spectrum



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Beating the cosmic variance with pkSZ effect

- Photons from different blackbody spectra with different temperatures mix.

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$$\delta n_\nu = \frac{1}{2h\nu^3}\delta I_\nu = \left(\theta + \theta^2\right)\left(T\frac{\partial n_{pl}}{\partial T}\right)\Bigg|_{T_0} + \frac{\theta^2}{2}\left(T^4\frac{\partial}{\partial T}\left(\frac{1}{T^2}\frac{\partial n_{pl}}{\partial T}\right)\right)\Bigg|_{T_0} + \mathcal{O}(\theta^3) \ \cdots$$

$$x = \frac{h\nu}{k_B T_0}$$

$$\left(\frac{\delta I}{I}\right)\Bigg|_{\text{(quadrupolar)}} = 2\left(\mathbf{v}\cdot\hat{\mathbf{n}}'\right)^2 g(x) + \frac{1}{2}y(x)\left(\mathbf{v}\cdot\hat{\mathbf{n}}'\right)^2$$

$$y(x) = \frac{xe^x}{(e^x-1)}\left(x\frac{e^x+1}{e^x-1} - 4\right)$$

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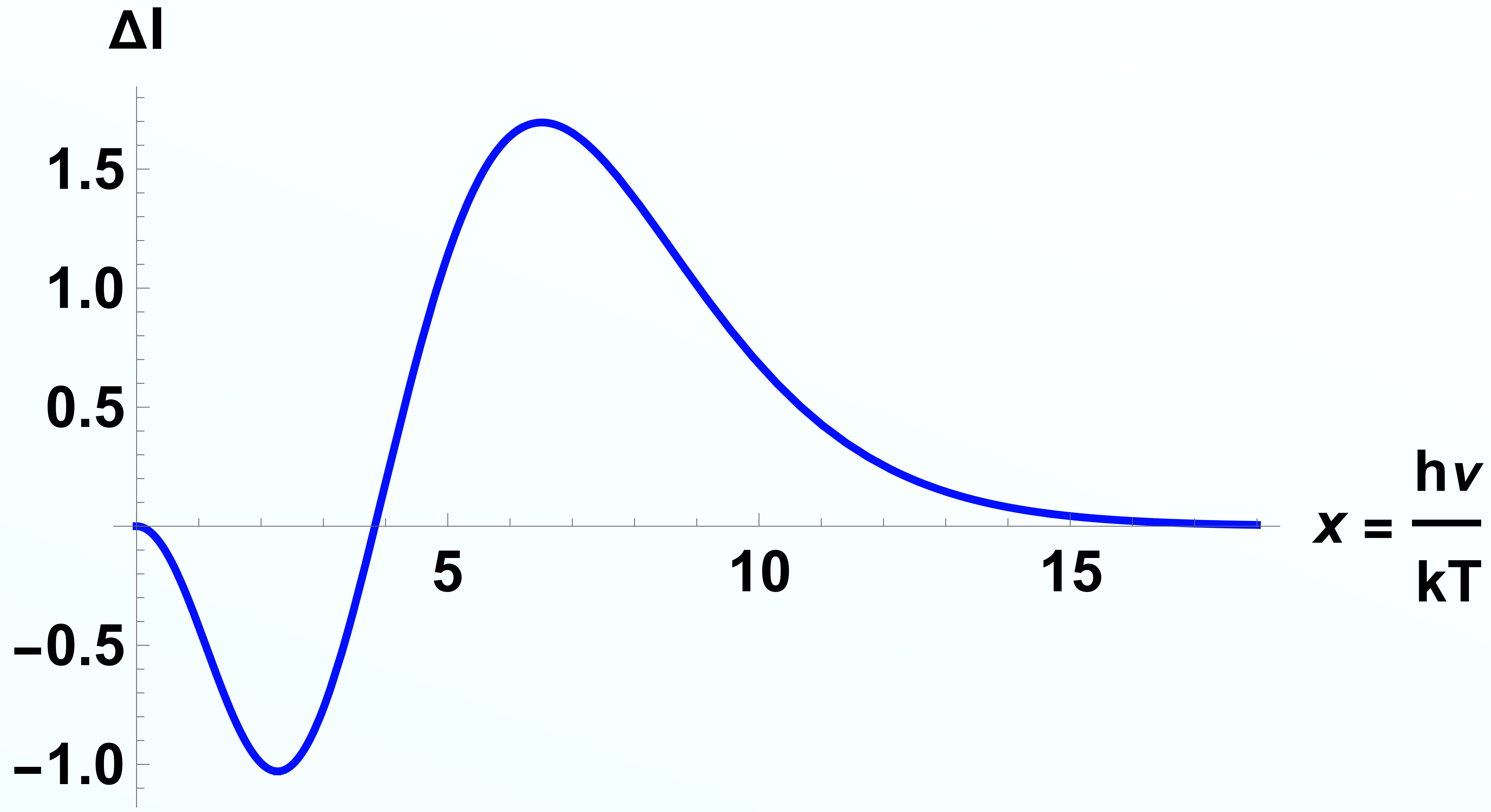

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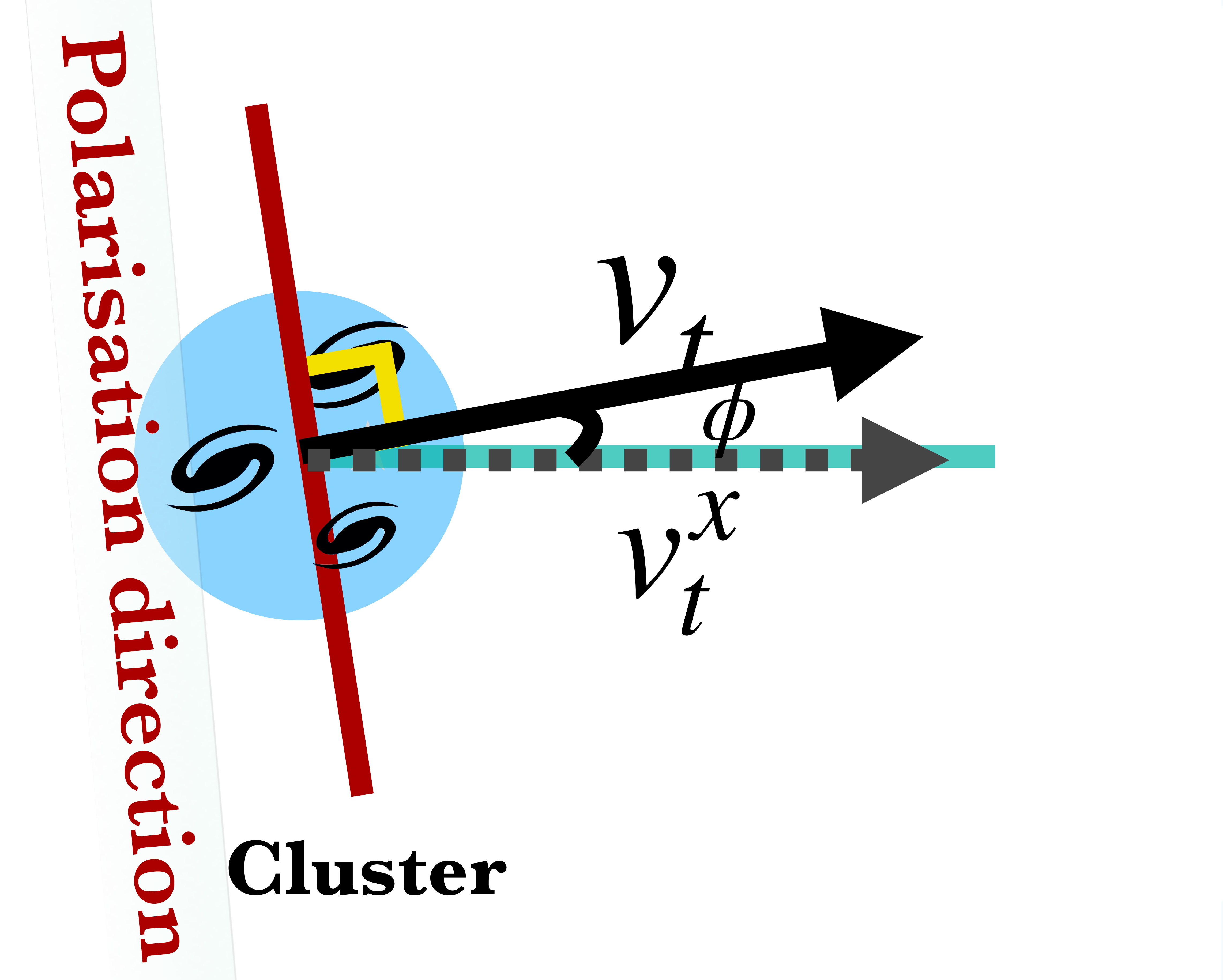
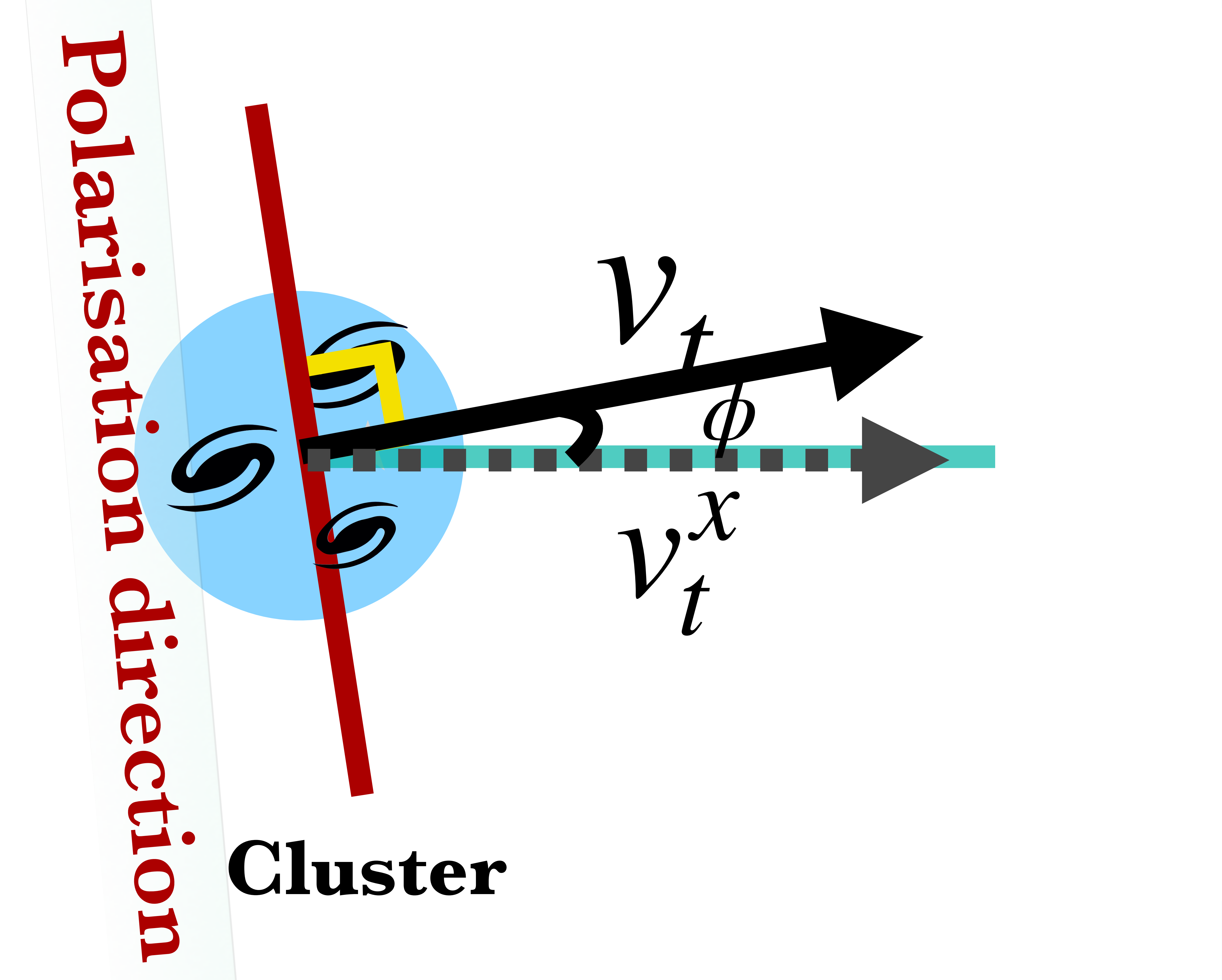
Polarisation direction is always perpendicular to the transverse velocity direction



✱ **The polarisation field :** $\left(\mathcal{Q} \pm i\mathcal{U}\right)\left(\hat{\mathbf{n}}\right) \equiv P_{\pm}\left(\hat{\mathbf{n}}\right)$

$$P_+\left(\hat{\mathbf{n}} \equiv \hat{\mathbf{z}}\right) = -\frac{1}{10}\tau_{\rm eff}\, v_t^2(\mathbf{x})\, e^{-2i\phi}$$

Signal too small to observe directly !



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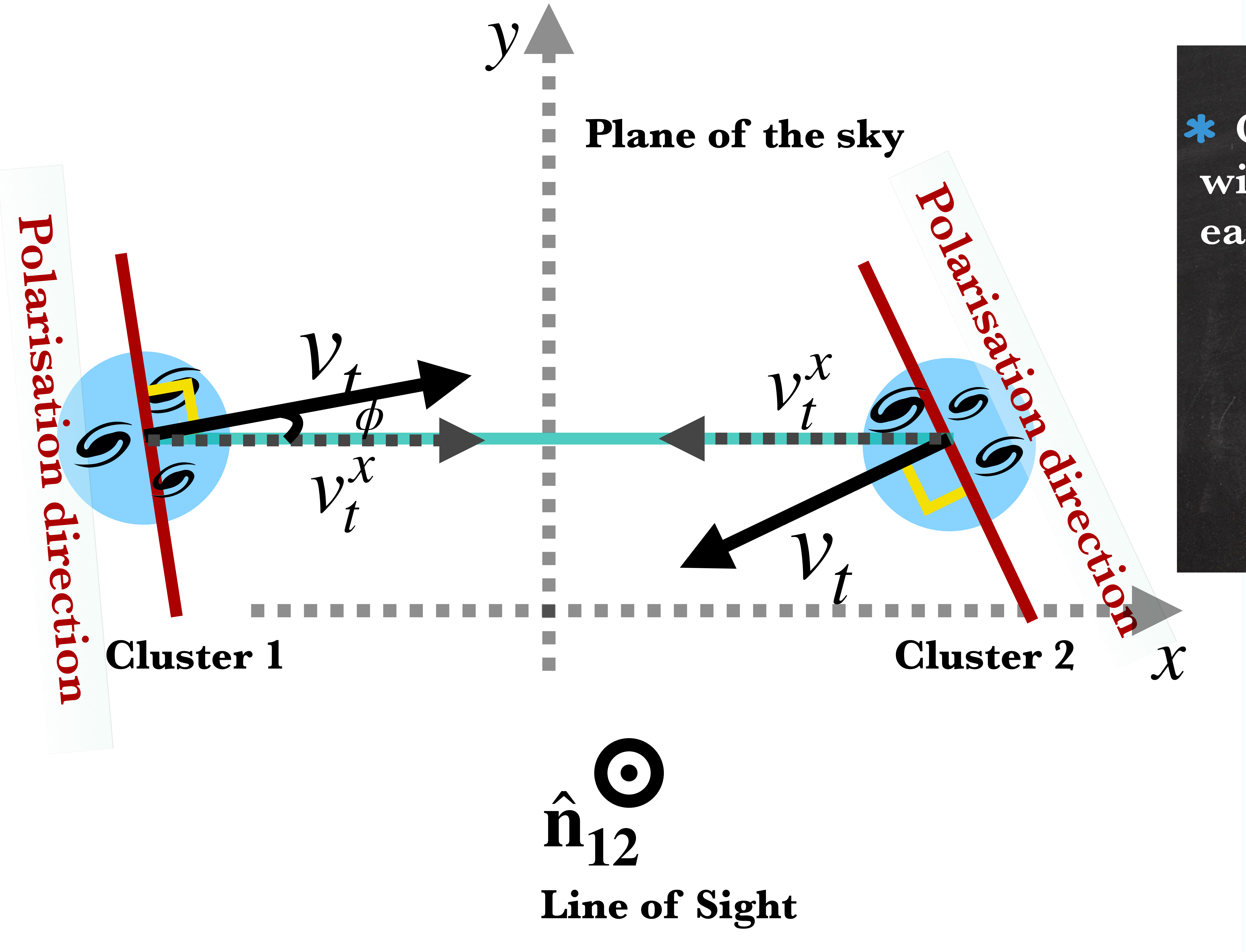
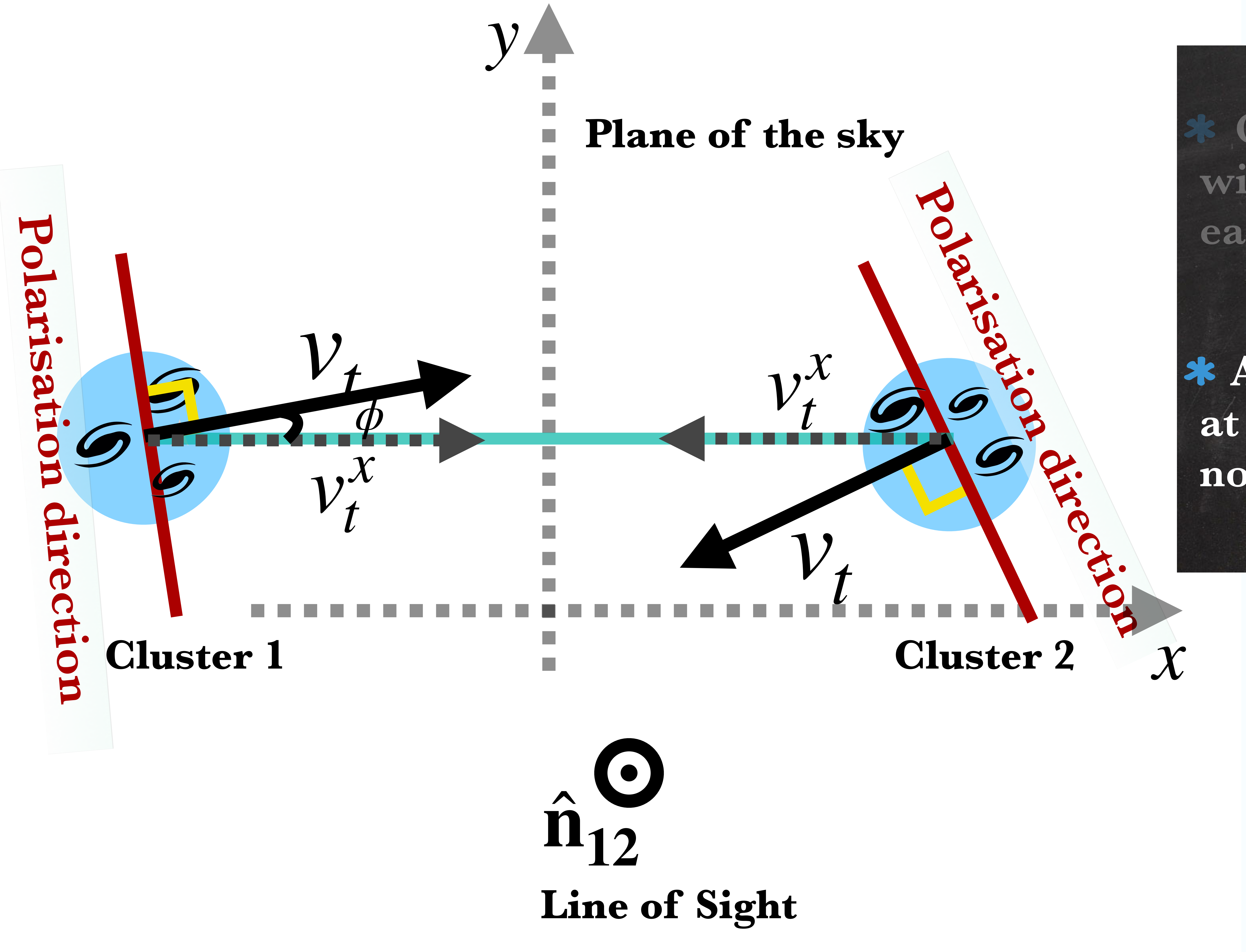
$$\tau_{\mathrm{eff}} \sim 10^{-3} \qquad v_t^2 \sim 10^{-6}$$

$$P_+ \sim 10^{-4}\,\mu K$$

Sensitivity of current generation experiments: $\sim \mu K$



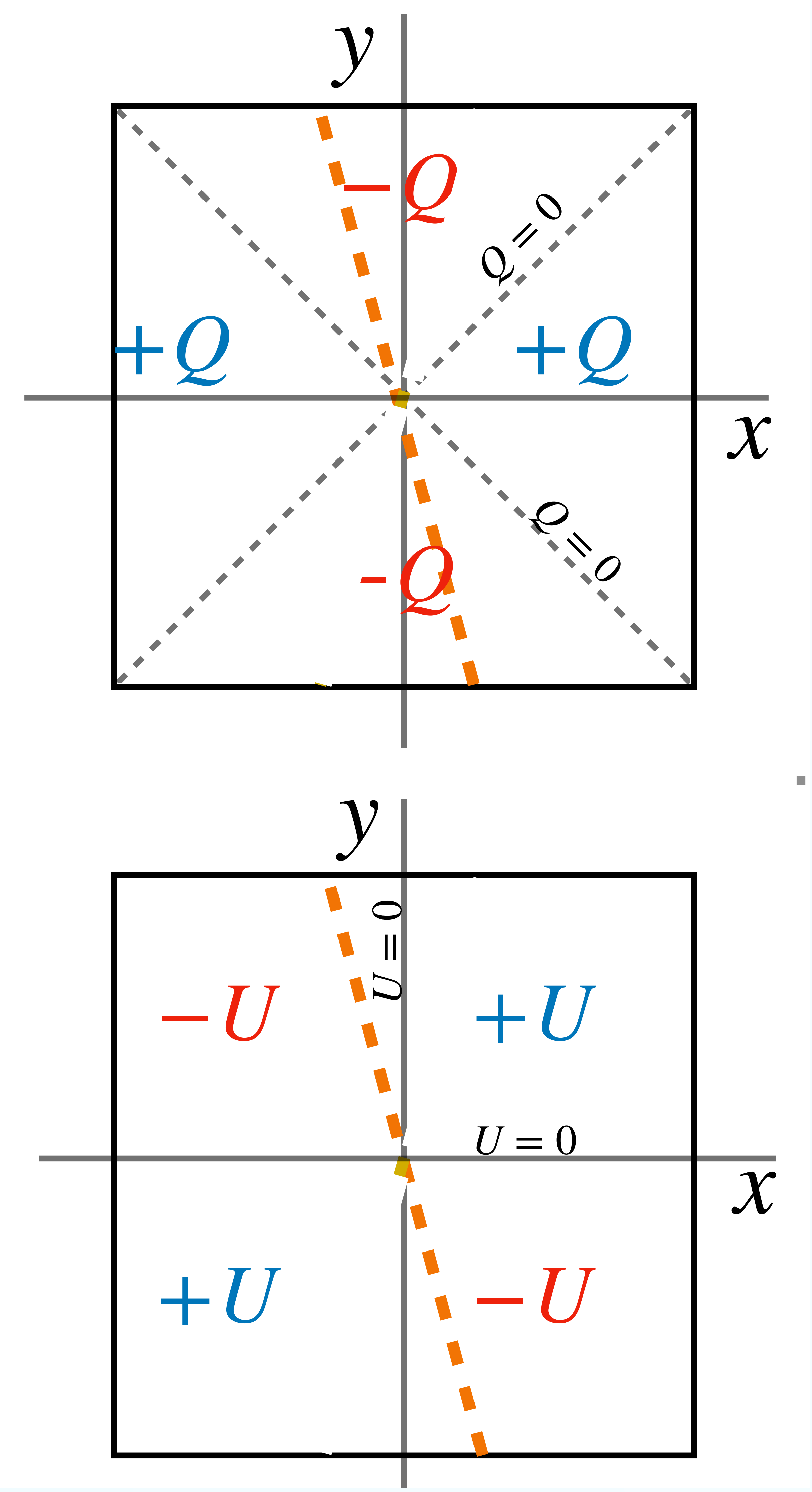
- Clusters which are close to each other will have peculiar velocities ~ towards each other

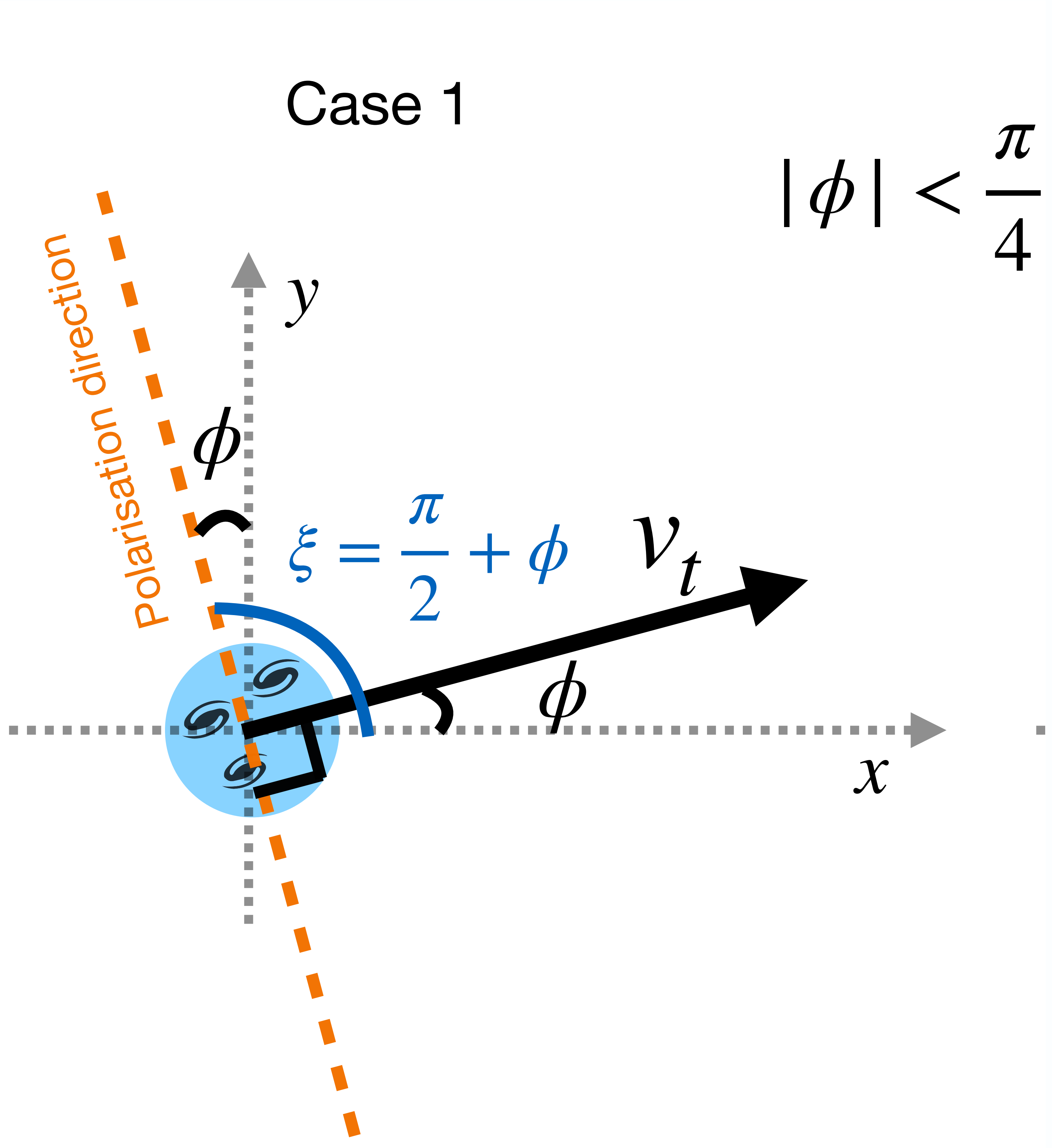


- Clusters which are close to each other will have peculiar velocities ~ towards each other

- Averaging over many clusters which are at a fixed separation will generate a net non-zero polarisation signal.

Coherent addition of the Q parameter gives a net non-zero polarisation signal.





$$Q \propto \cos(2\xi) < 0$$

$$U \propto \sin(2\xi) < 0$$

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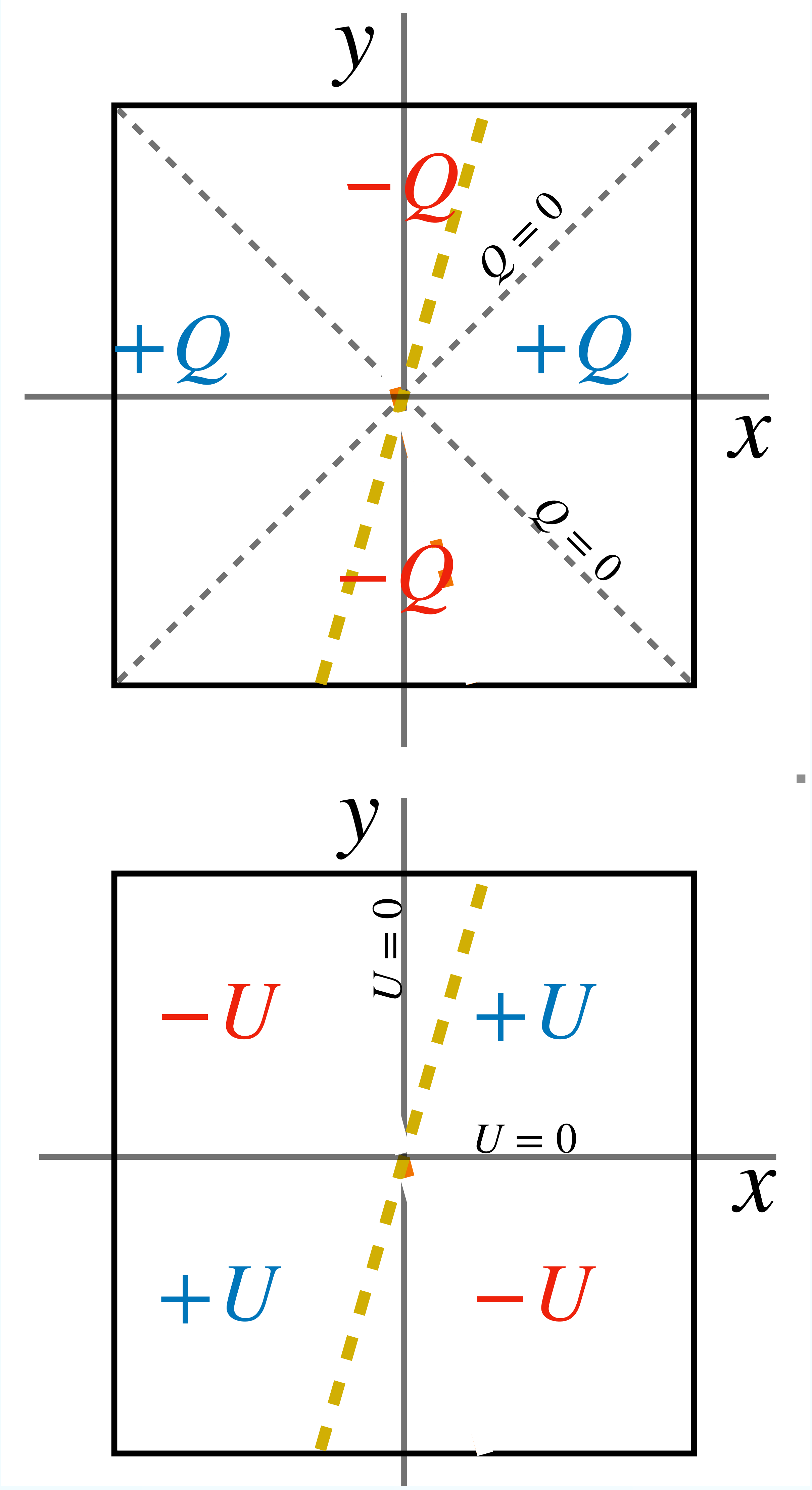

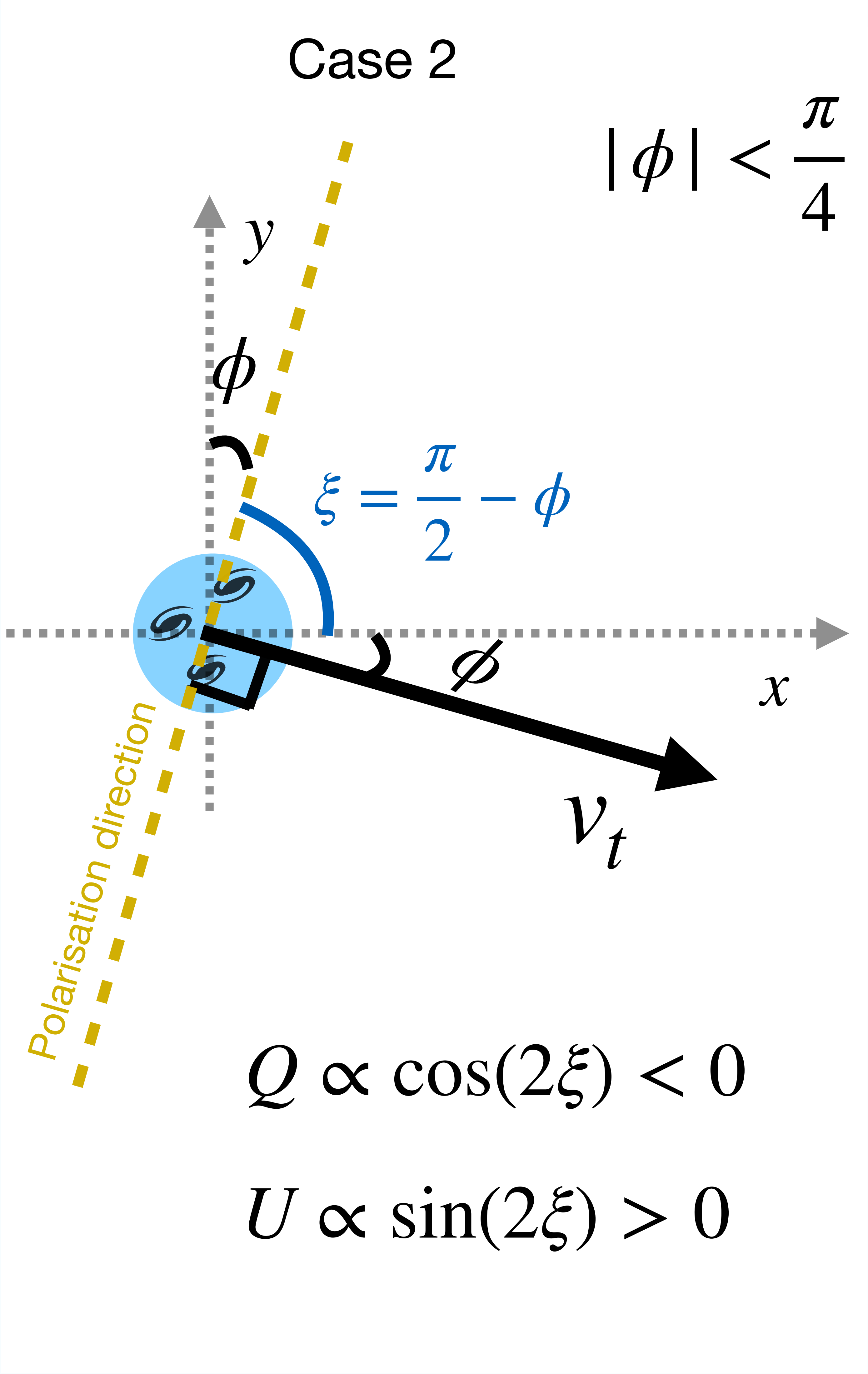

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Theoretical formalism of the pairwise estimator

$$\hat{P}_{\text{pairwise}}(x) = \sum_{\text{i}} w_i \left(P_{i1+} + P_{i2+}\right) \bigg|_{\substack{\text{separation along} \\ \text{x} - \text{axis}}}$$

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Pairwise signal - dependent cosmological and astrophysical parameters

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Thomson Optical depth of clusters

$$P_{\text{pairwise}}(\mathbf{x}, \hat{\mathbf{n}}_{\mathbf{12}} \,|\, m, \chi) = \Bigg[\frac{\sqrt{\pi}}{10\pi^5} D^4 (Hfa)^2 \, \tau_{\text{eff}} \, Y_{2-2}(\hat{\mathbf{x}}; \hat{\mathbf{n}}_{\mathbf{12}}) \sum_{L_1, L_2} \sum_{q=0}^{2} \sum_{l} i^{(L_1+L_2)} (-1)^{(L_1+1)} (2L_1+1)(2L_2+1)(2l+1)$$

$$\frac{q!}{(q-l)!!(q+l+1)!!} \begin{pmatrix} 1 & L_1 & l \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & L_2 & l \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} L_1 & L_2 & 2 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 2 & L_2 & L_1 \\ l & 1 & 1 \end{pmatrix}$$

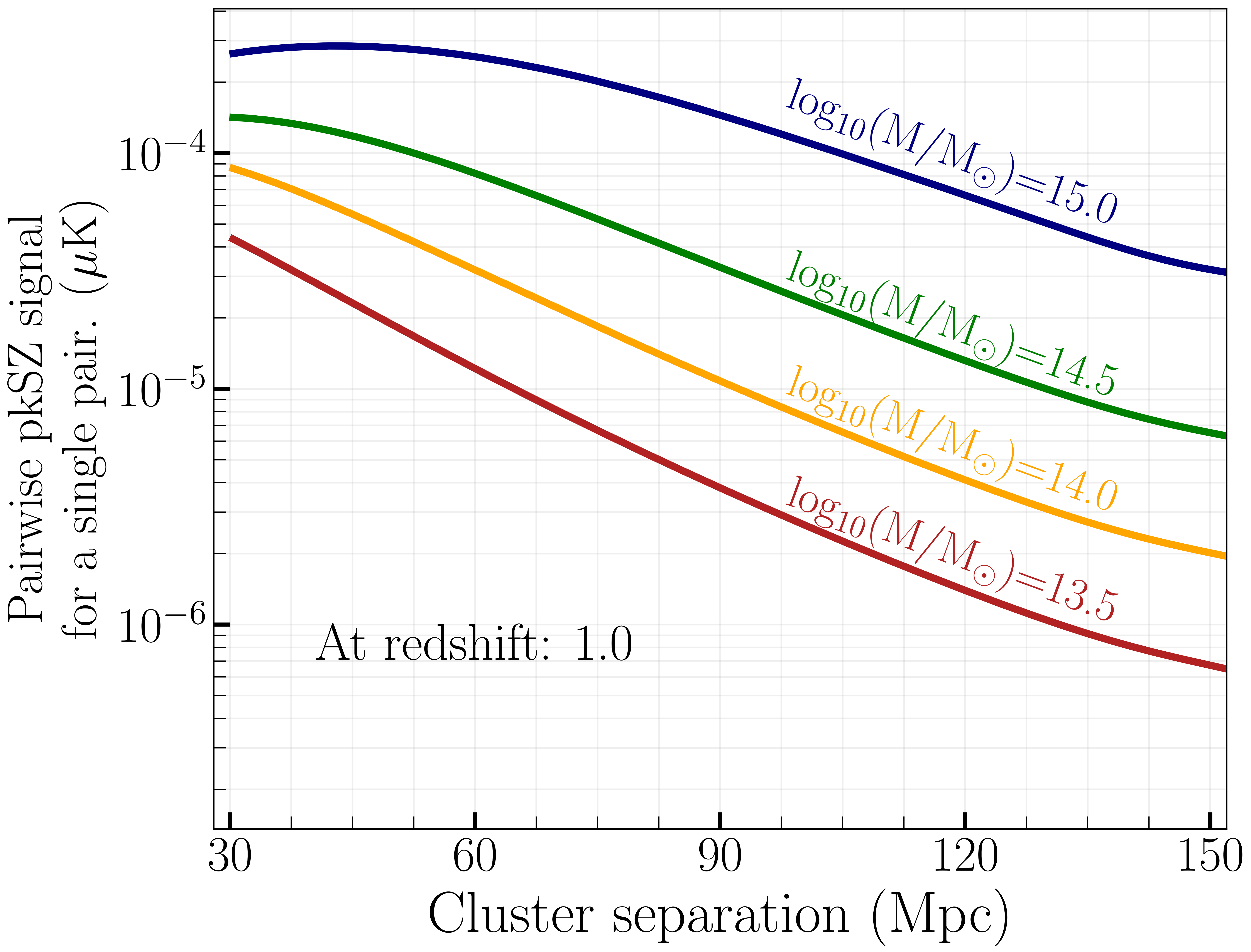
$$\int dk_1 dk_2 \, k_1^2 k_2^2 \, G_q(k_1, k_2, b_1, b_2) j_{L_1}(k_1 x) \, j_{L_2}(k_2 x) P(k_1) P(k_2) \Bigg] \left[1 + \frac{D^2 \, b_1^2}{2\pi^2} \int dk \, k^2 j_0(kx) P(k) \right]^{-1}$$

Cosmological parameters

Cluster bias factors

Linear matter power spectrum

Smaller mass cluster have lower optical depth, thus lower polarisation signal.



Choosing proper weights to optimise the estimator

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Denoting the orientation of the cluster pair wrt LOS

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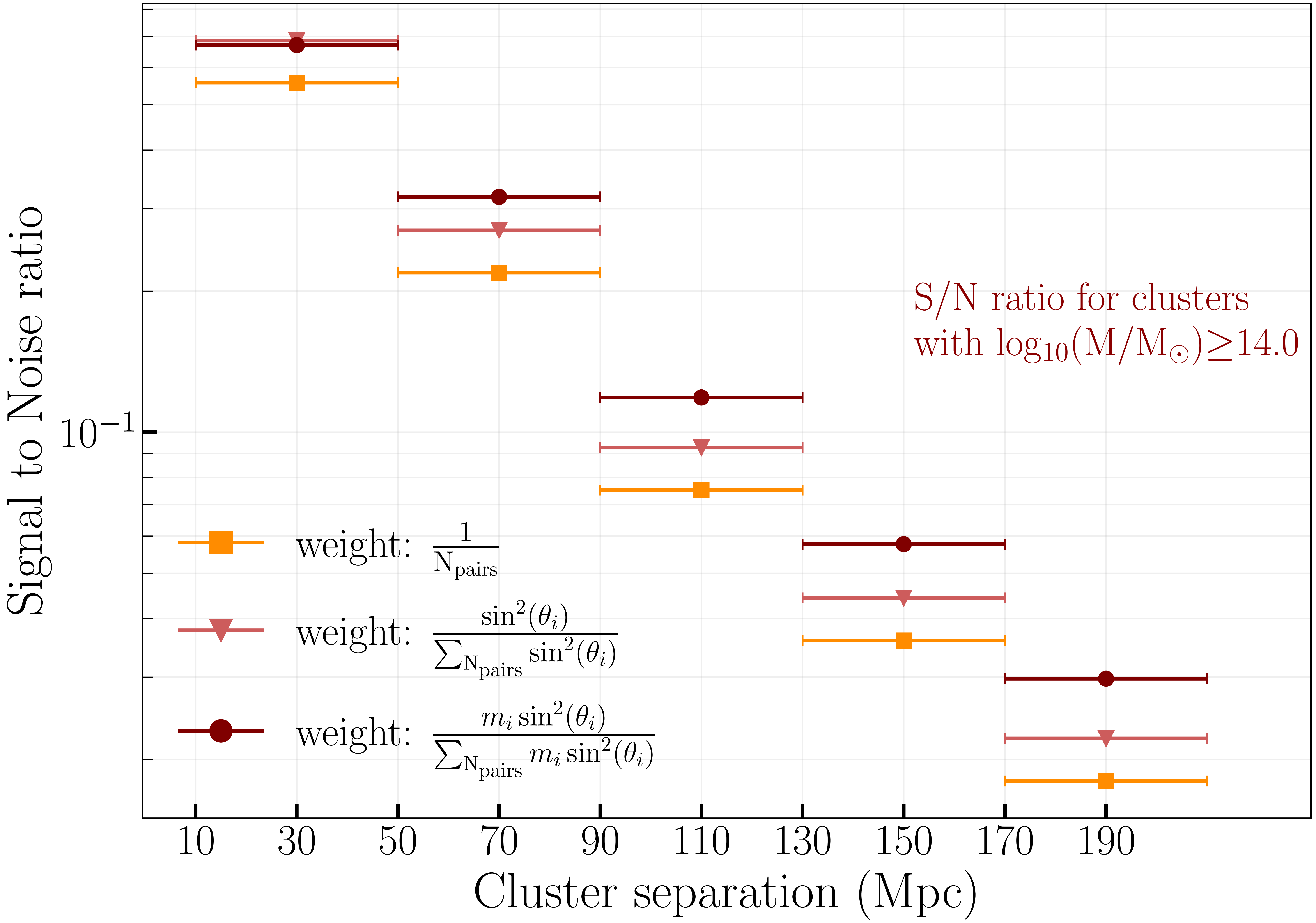
Denoting the orientation of the cluster pair wrt LOS

- Give more weights to pair which are more aligned to the plane of the sky
- Give more weights to more massive clusters

$$\hat{P}_{\text{pairwise}}(x) = \sum_{\text{i}} w_i \,(P_{i1+} + P_{i2+})\Big|_{\substack{\text{separation along}\\ \text{x} - \text{axis}}}$$

Optimal estimator

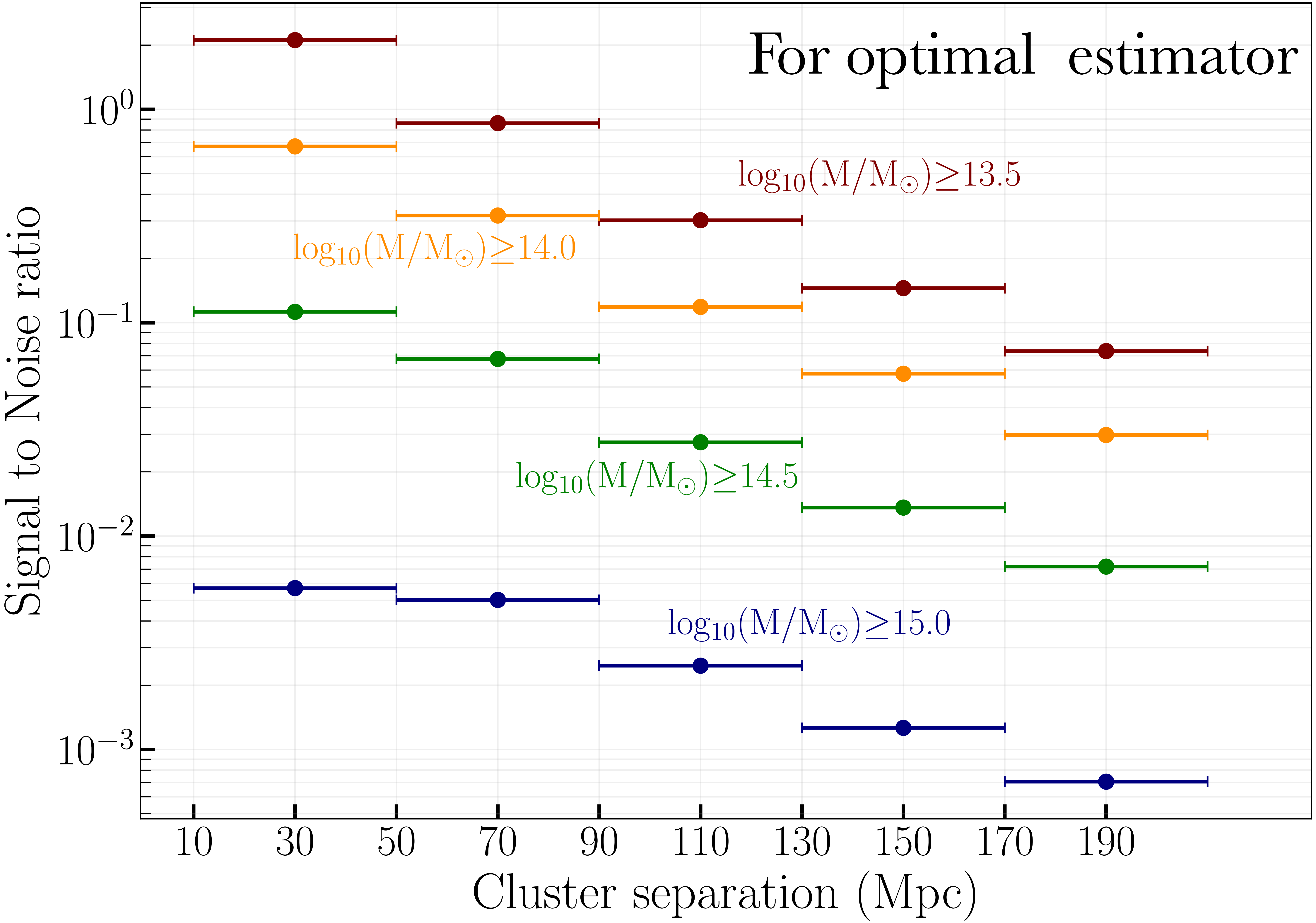
$$w_i = \frac{1}{\sum_i m_i \sin^2\theta_i}\, m_i \sin^2\theta_i$$



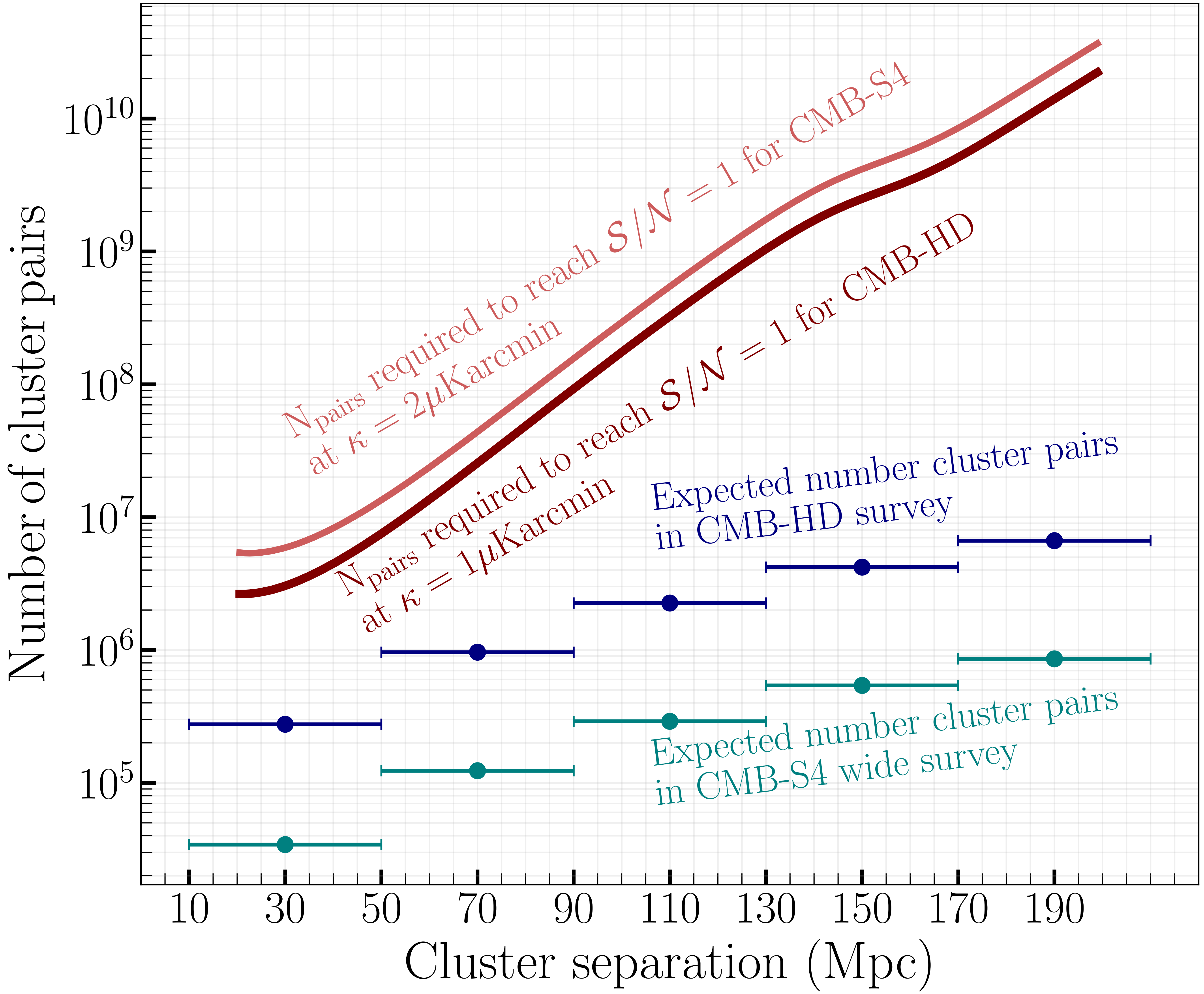
Detecting all clusters with

$\log_{10}\left(M_{\rm min}^{500c}/M_{\odot}\right) \geq 13.5$

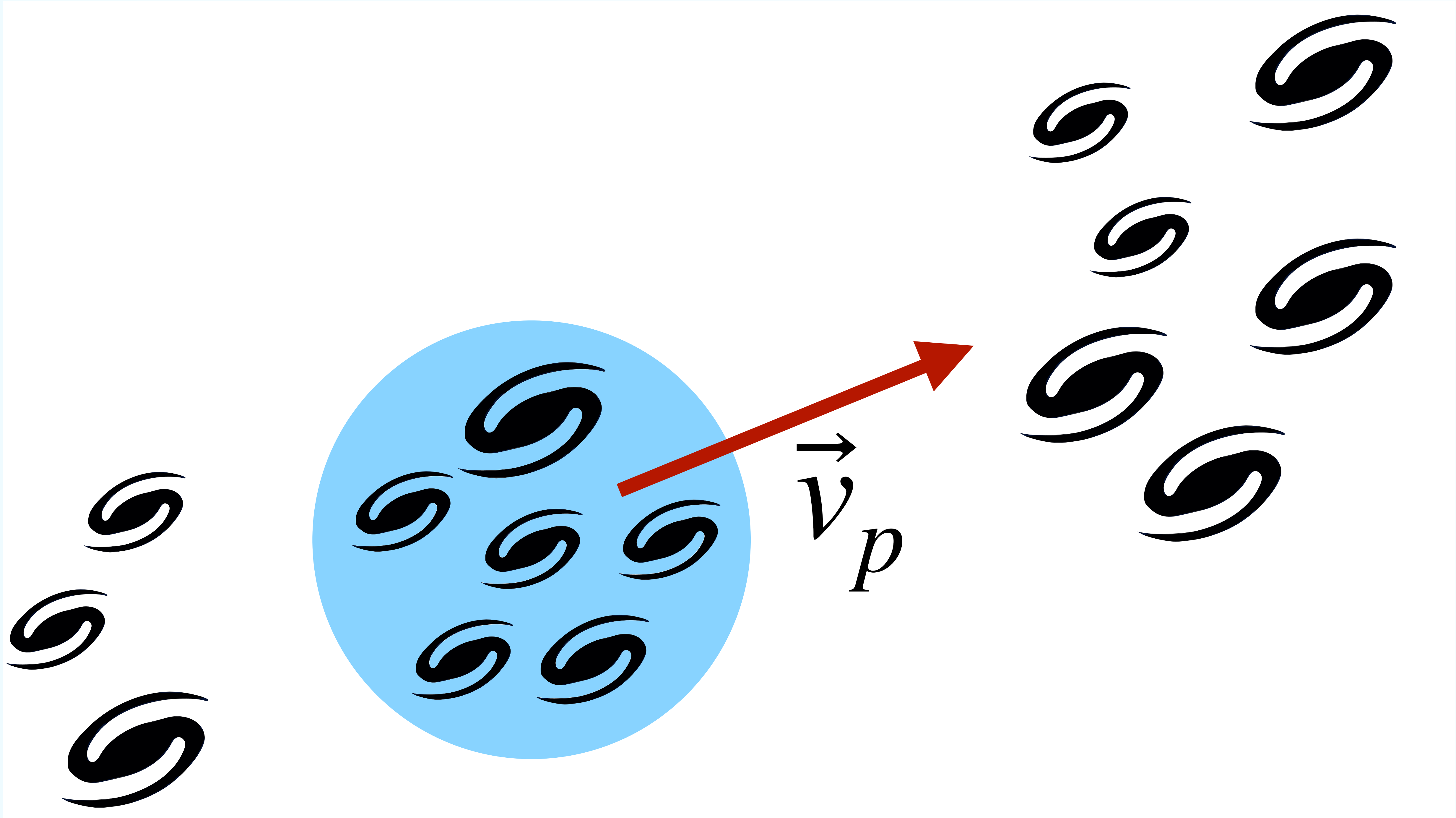
will enable a detection of the pairwise pkSZ effect .



Forecast for CMB-S4 and CMB-HD



Cross-pairwise pkSZ effect to the rescue!



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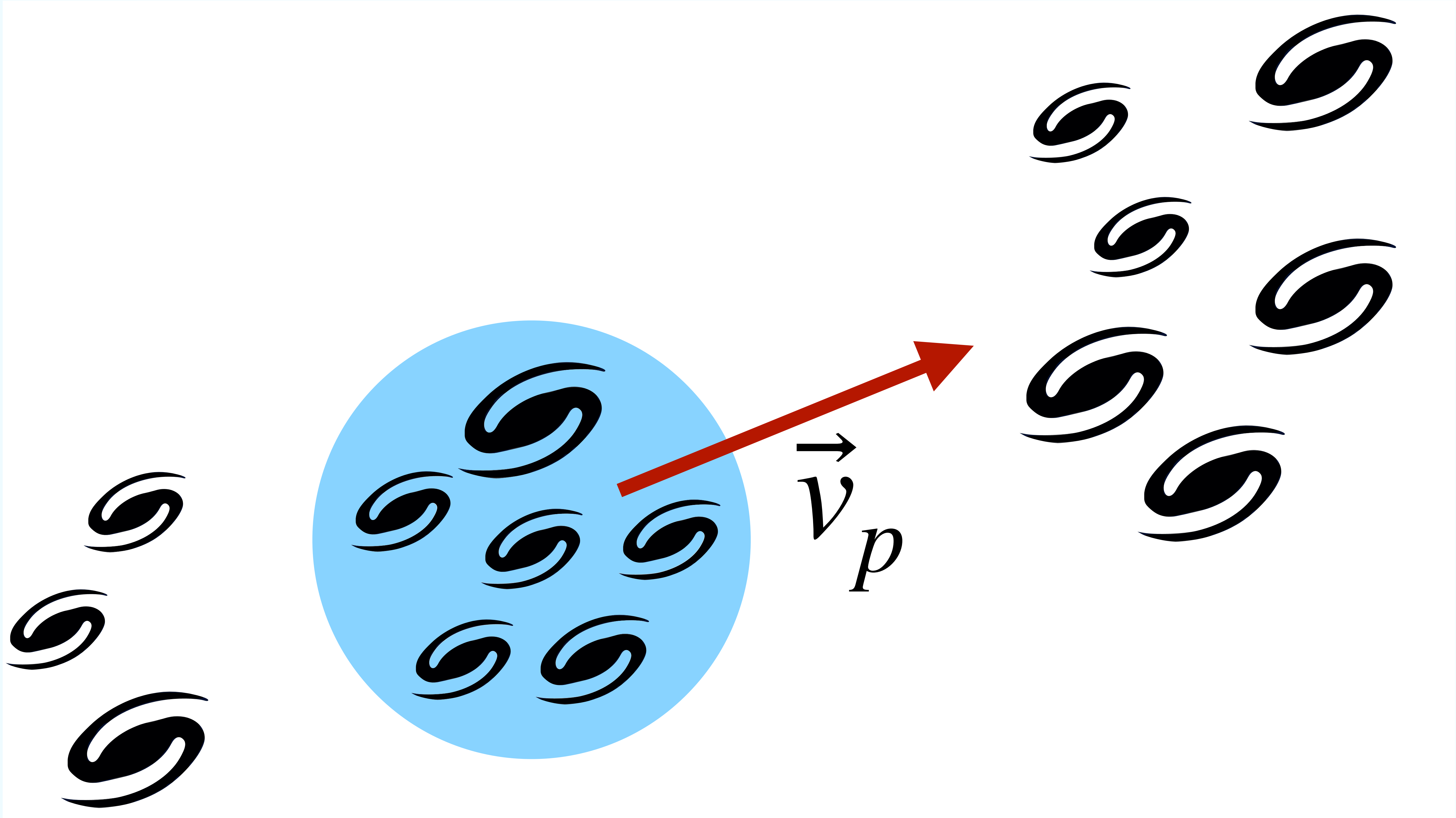

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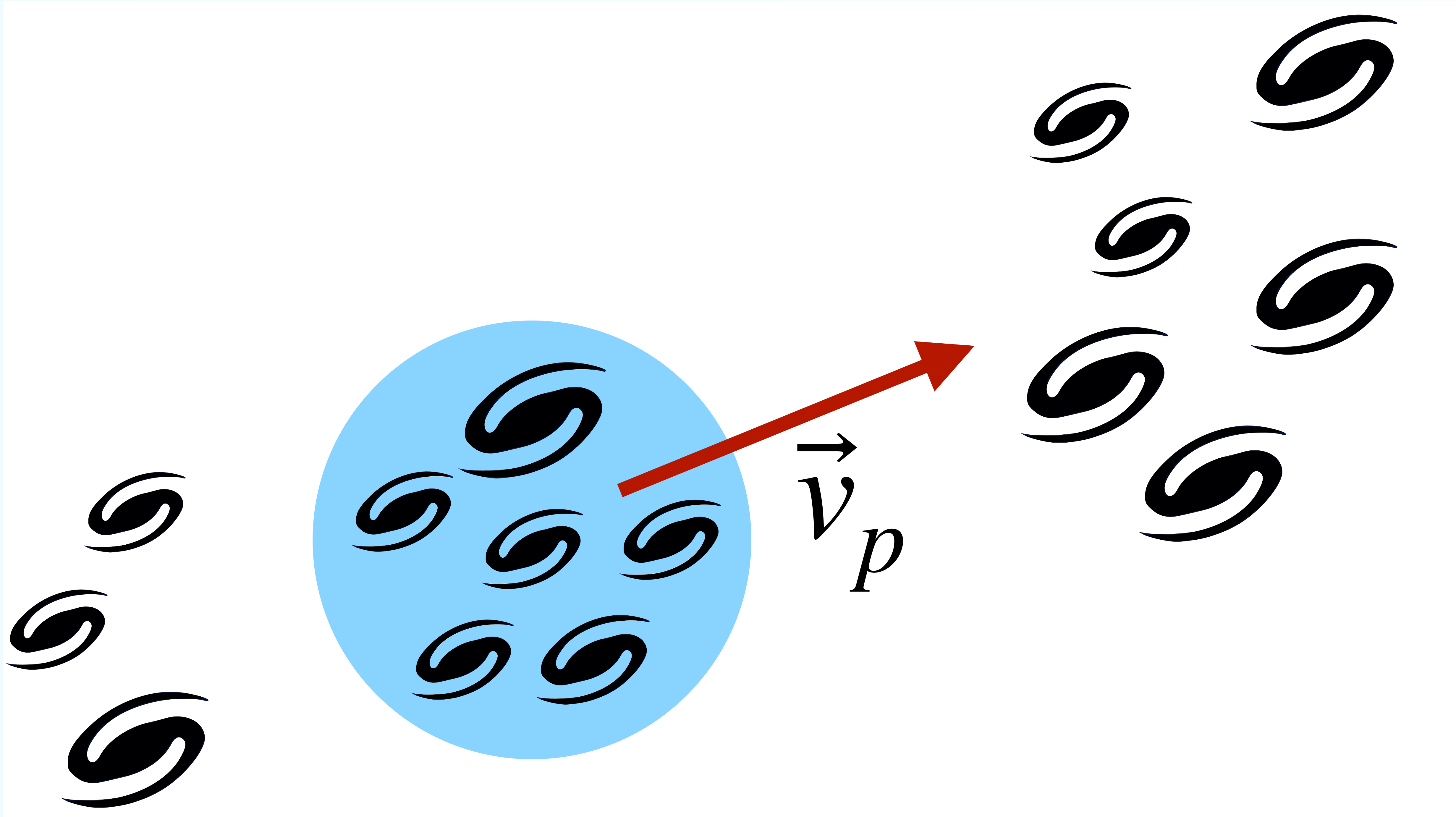

- Clusters being rare - noisy tracers, better to use galaxies.
- In general, we can pair up a cluster with any other indicator of the large-scale gravitational potential around the cluster.
- **Polarisation signal from galaxies is negligible, but they are much more in number !**

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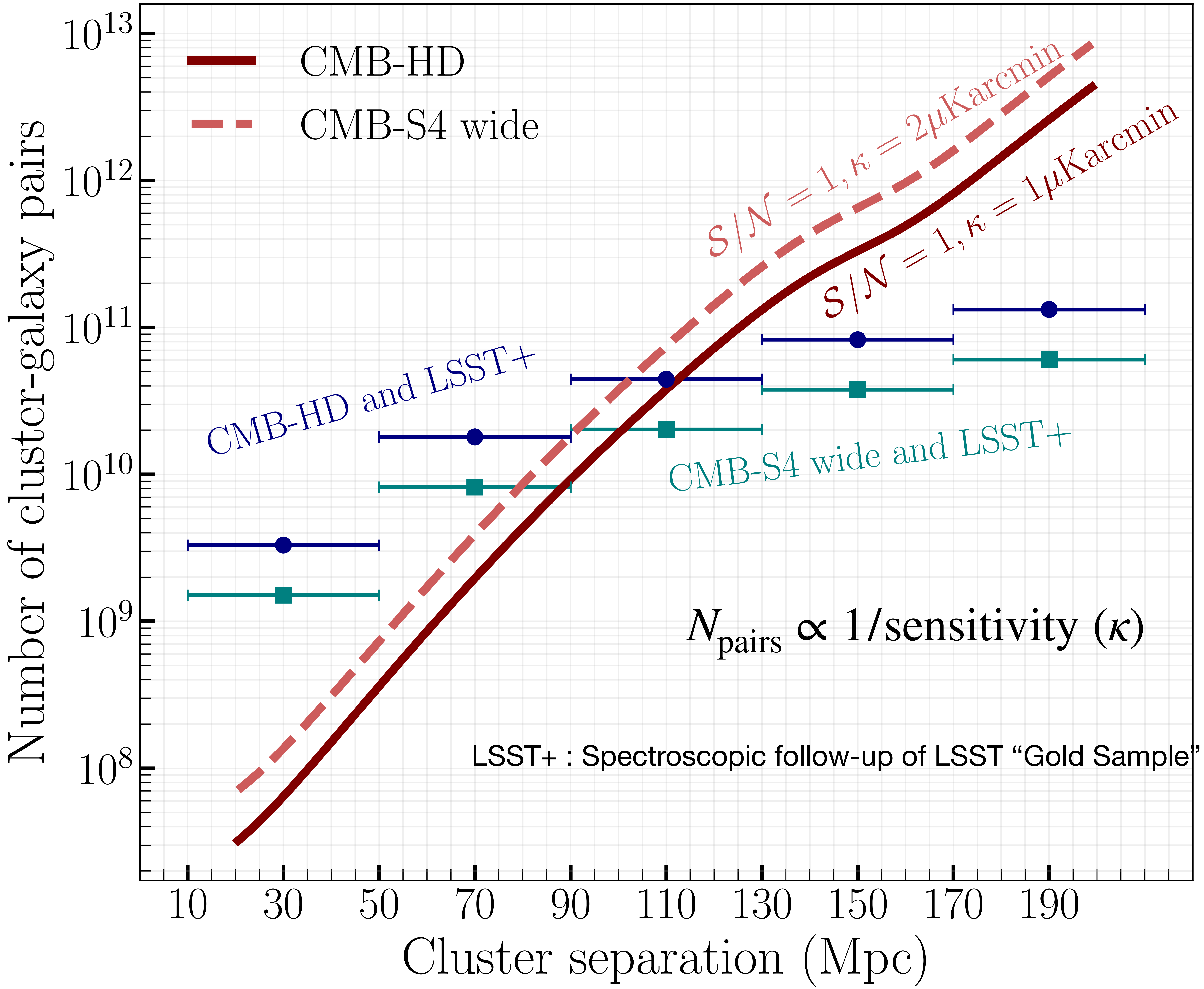
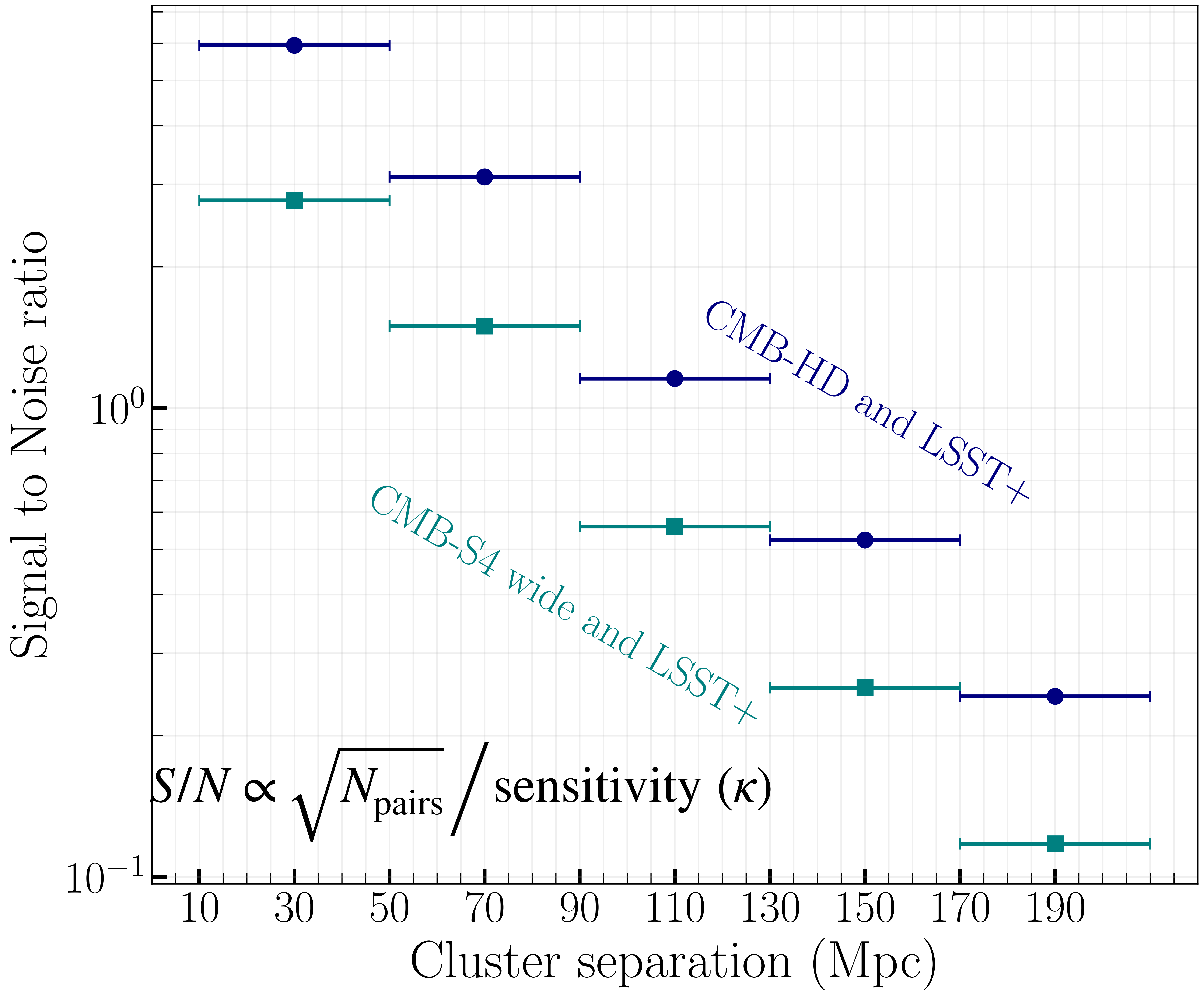
Forecast for Cross-pairwise pkSZ effect



LSST+ : Spectroscopic follow-up of LSST "Gold Sample"

Forecast for Cross-pairwise pkSZ effect



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- The pkSZ effect is sensitive to the transverse component, knowing the full velocity field can act as a consistency check of the underlying matter density field
- It is sensitive to cosmological parameters, probe of large scale structure of the universe

Concluding Remarks

- This **polarisation signal** with **y-type distortion** exists within the **Standard Cosmological model** of the Universe

- The **cross-pairing** clusters from CMB-S4 with galaxies from large overlapping spectroscopic survey can provide a way to **detect the signal.**

- **Free** from the **cosmic variance** of the primary CMB polarisation signal and lensing B modes.

Thank You