Primordial black holes from inflation: dark matter, gravitational waves and imprints from evaporation

Rajeev Kumar Jain

Theoretical Cosmology group

Department of Physics, IISc Bangalore

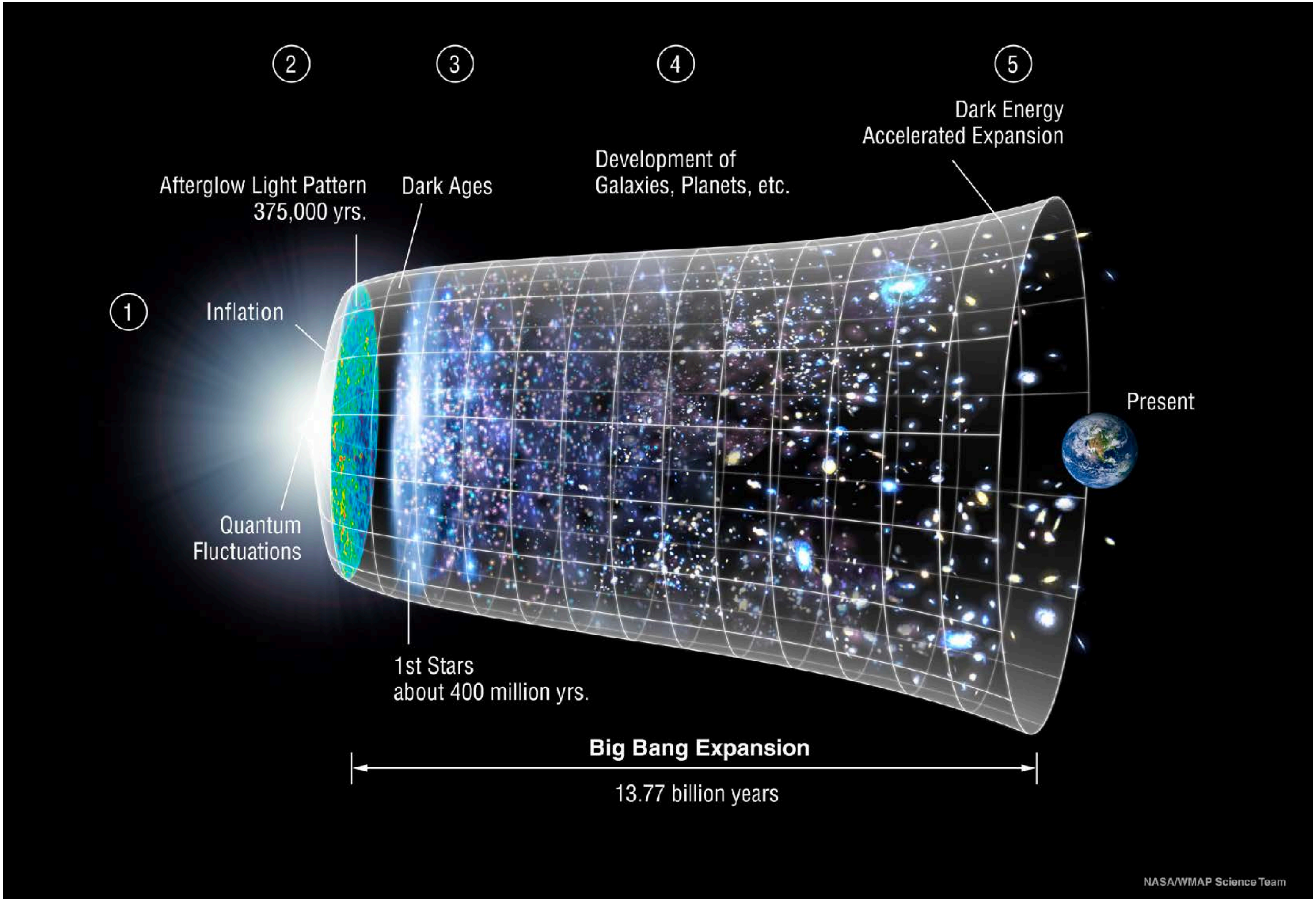
Dept. of Physics @ IIT Madras

Aug. 10, 2023

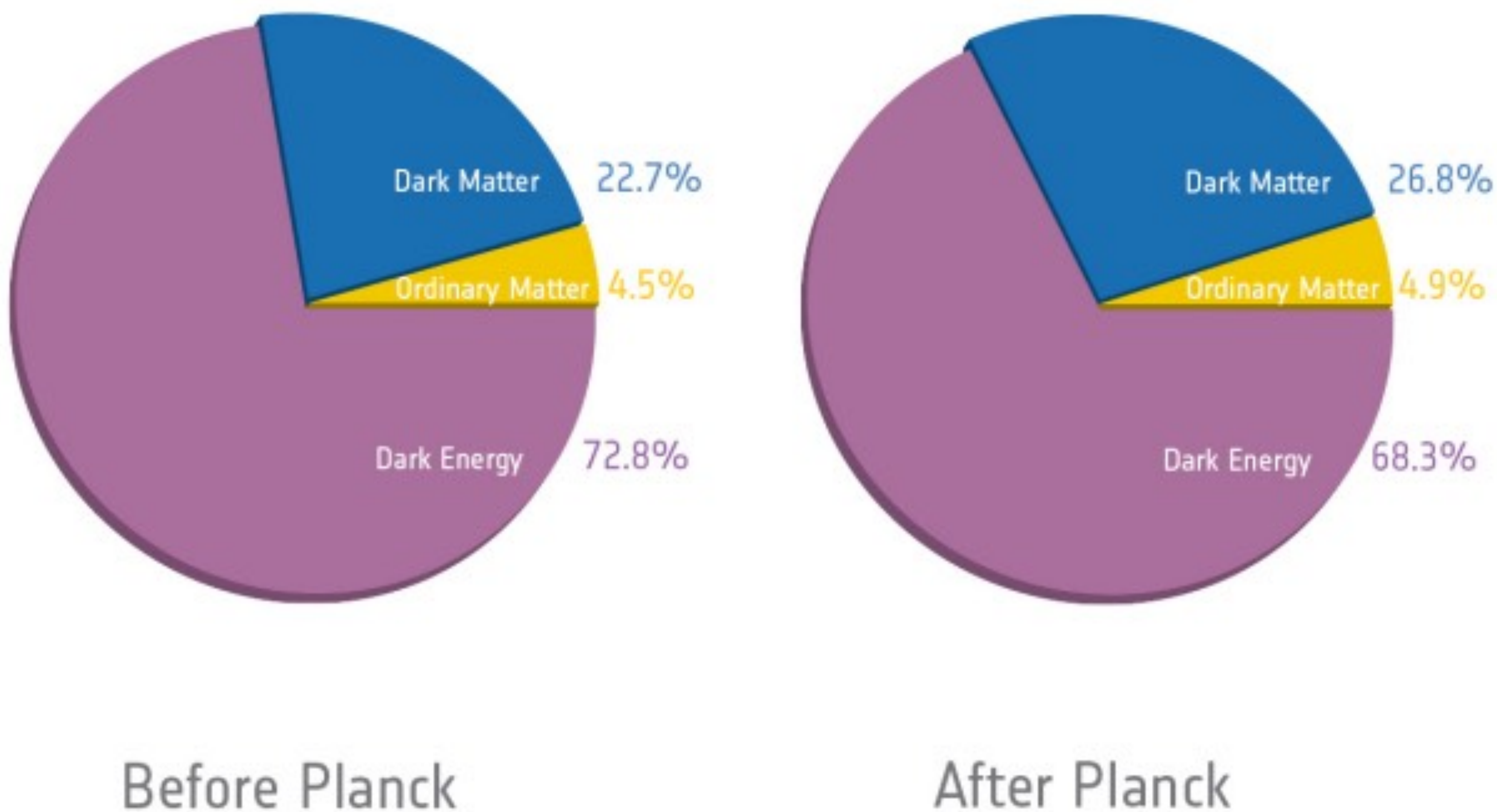
Outline of the talk

- Why Primordial Black Holes (PBH) ?
- PBH generation mechanisms — single field inflation — inflection point models
- Primordial scalar power spectra and the PBH mass fraction
- Observational imprints of PBHs
  - Induced secondary GWs
  - Imprints from Hawking evaporation
- Conclusions

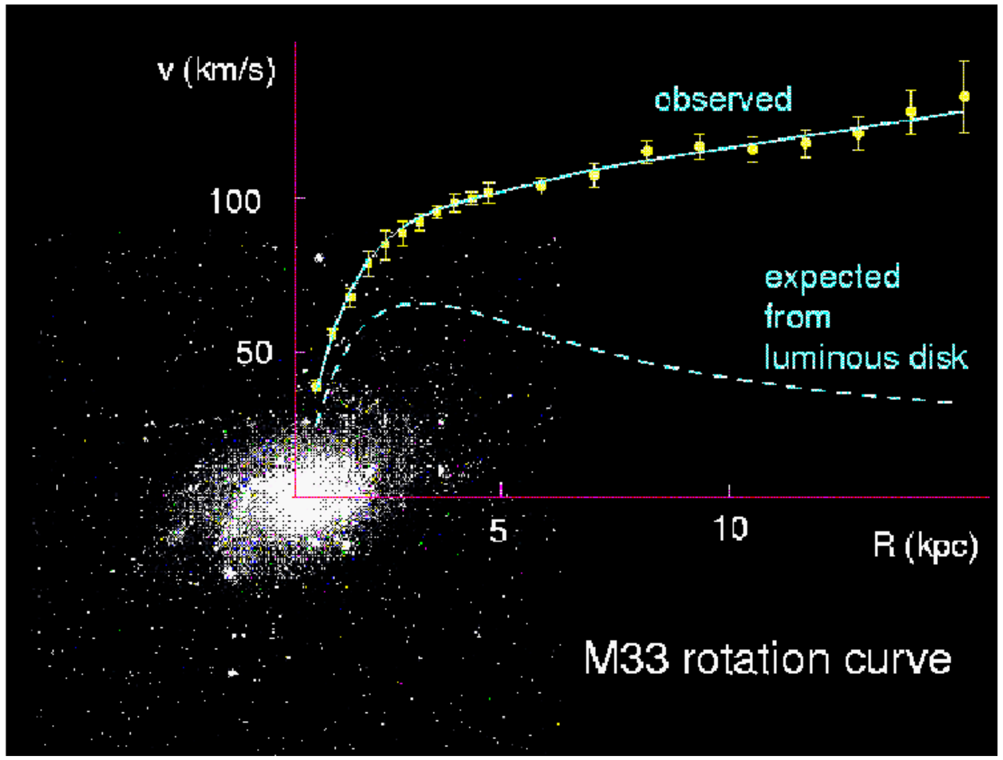
The evolution of the universe

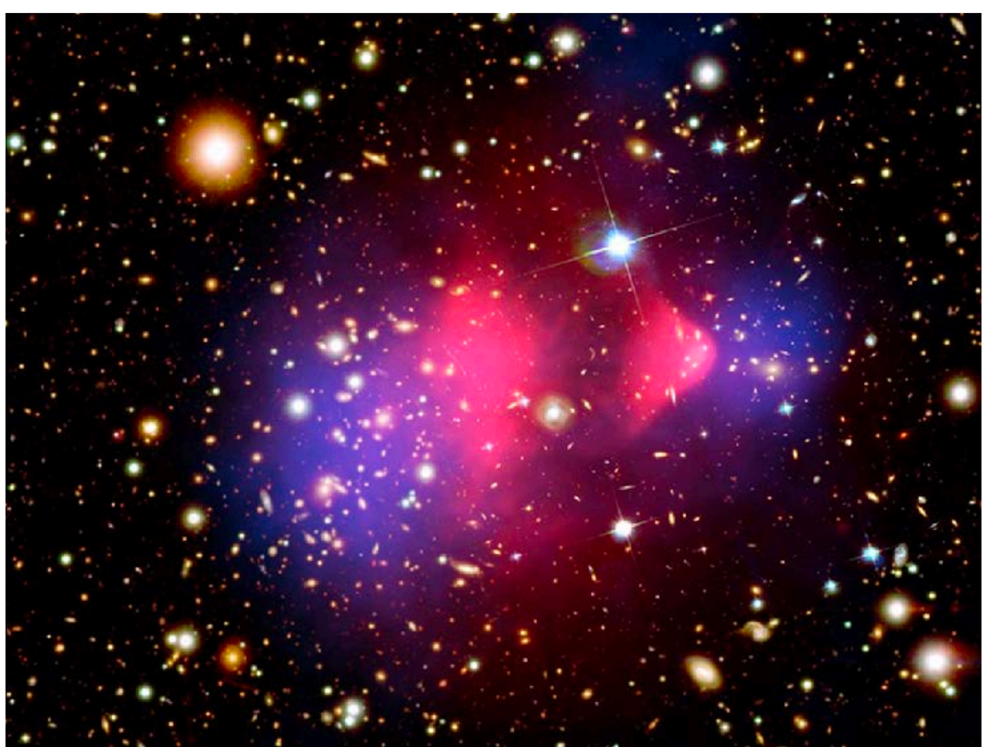


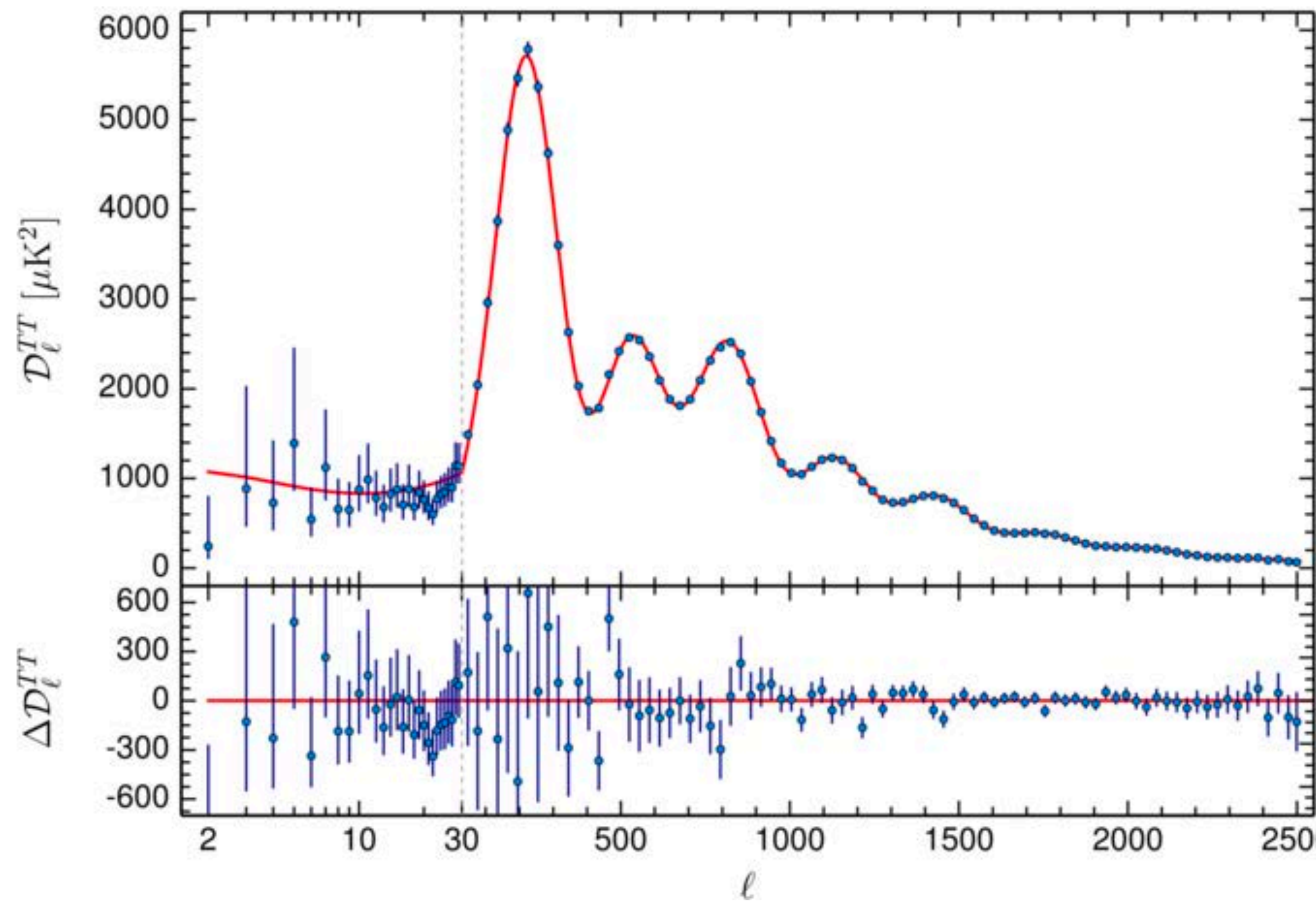
ΛCDM *and the 'Cosmic' cake*



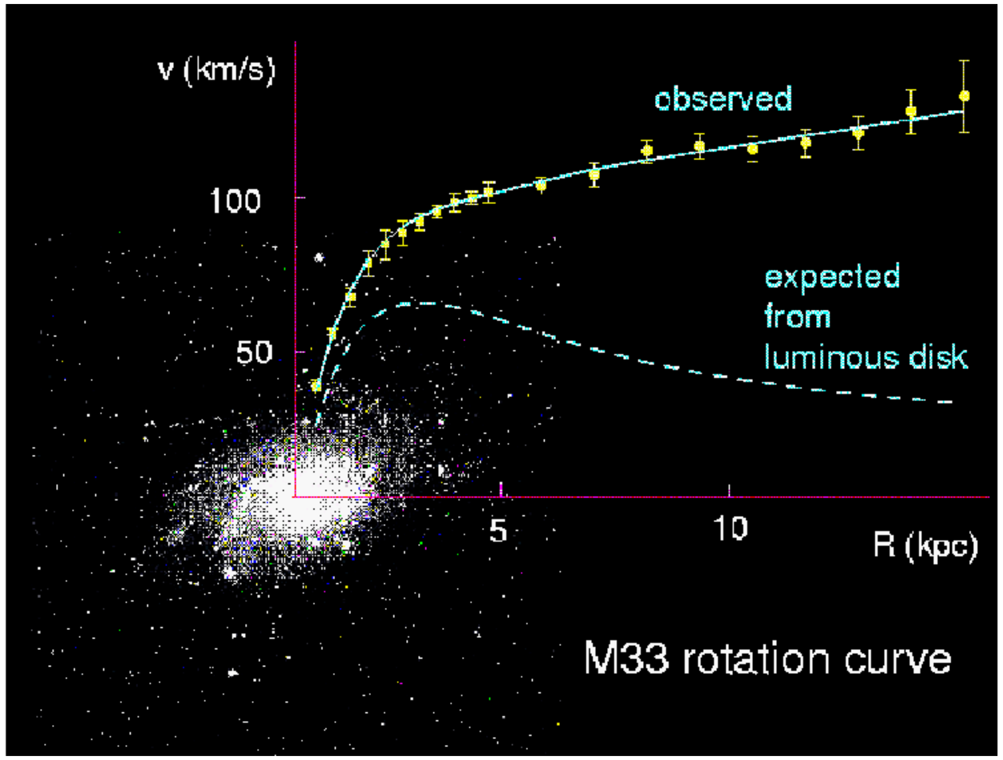
Dark matter — observational evidence

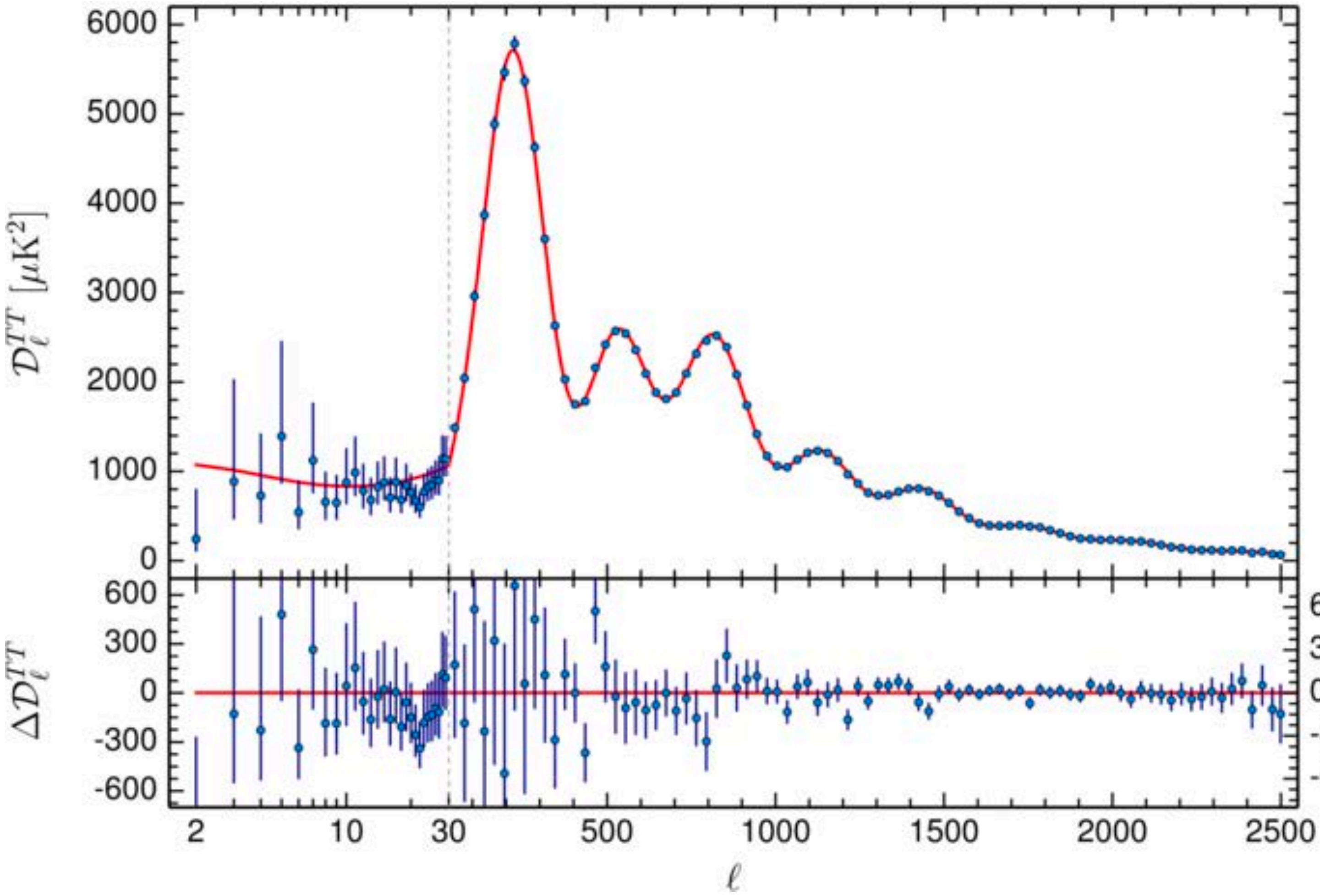






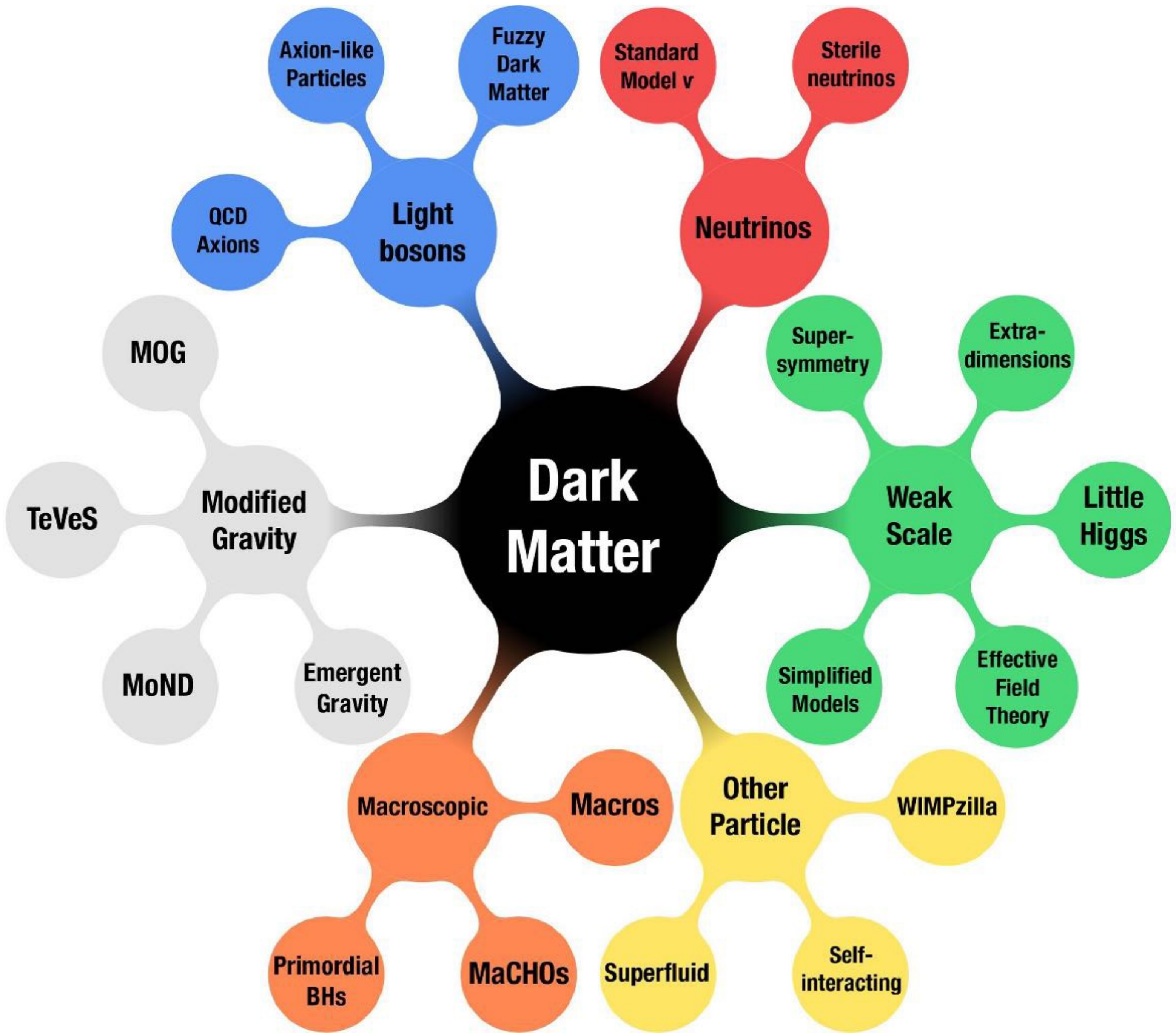
Dark matter — observational evidence





DM: A known *Unknown* !

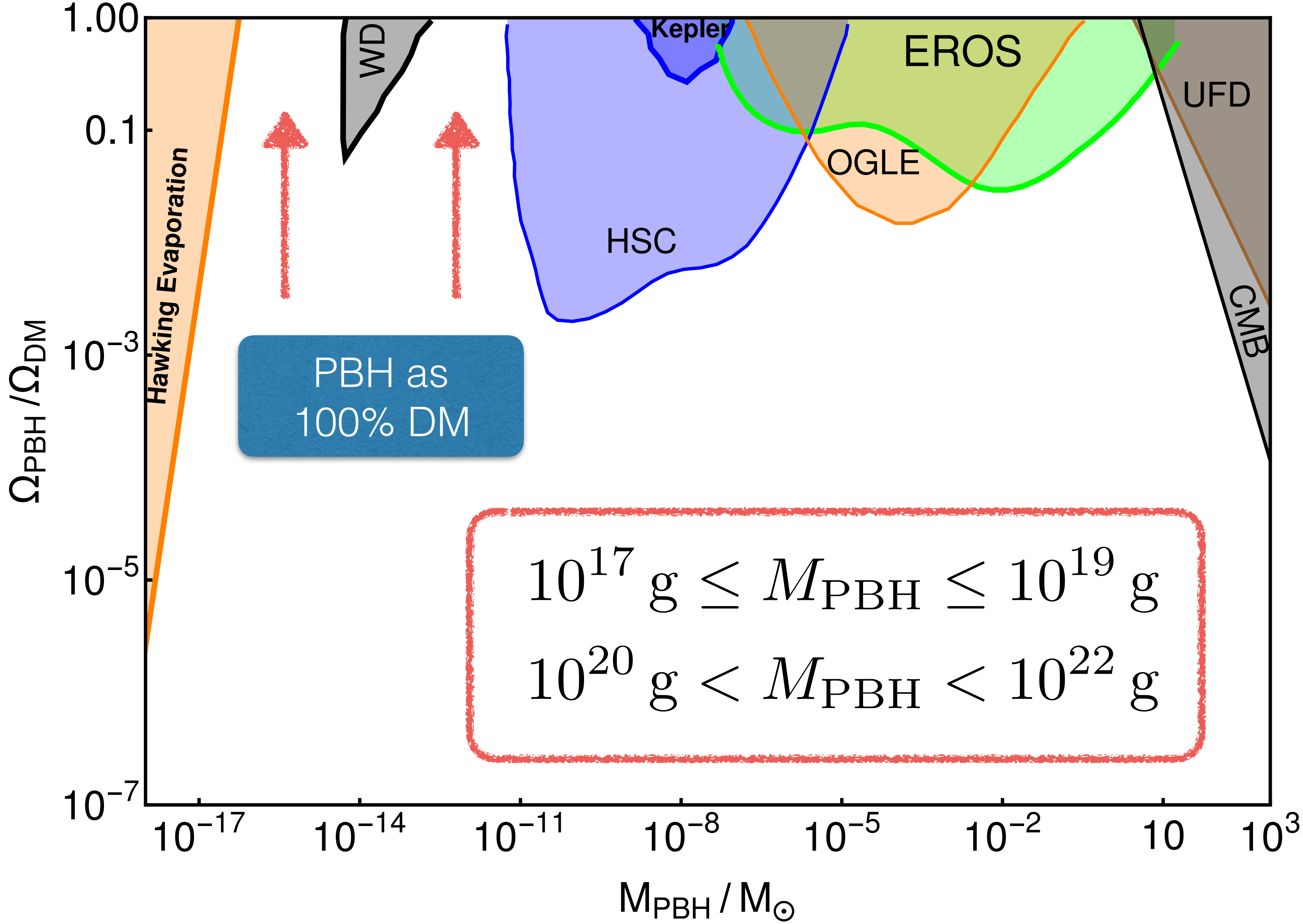
Dark matter candidates



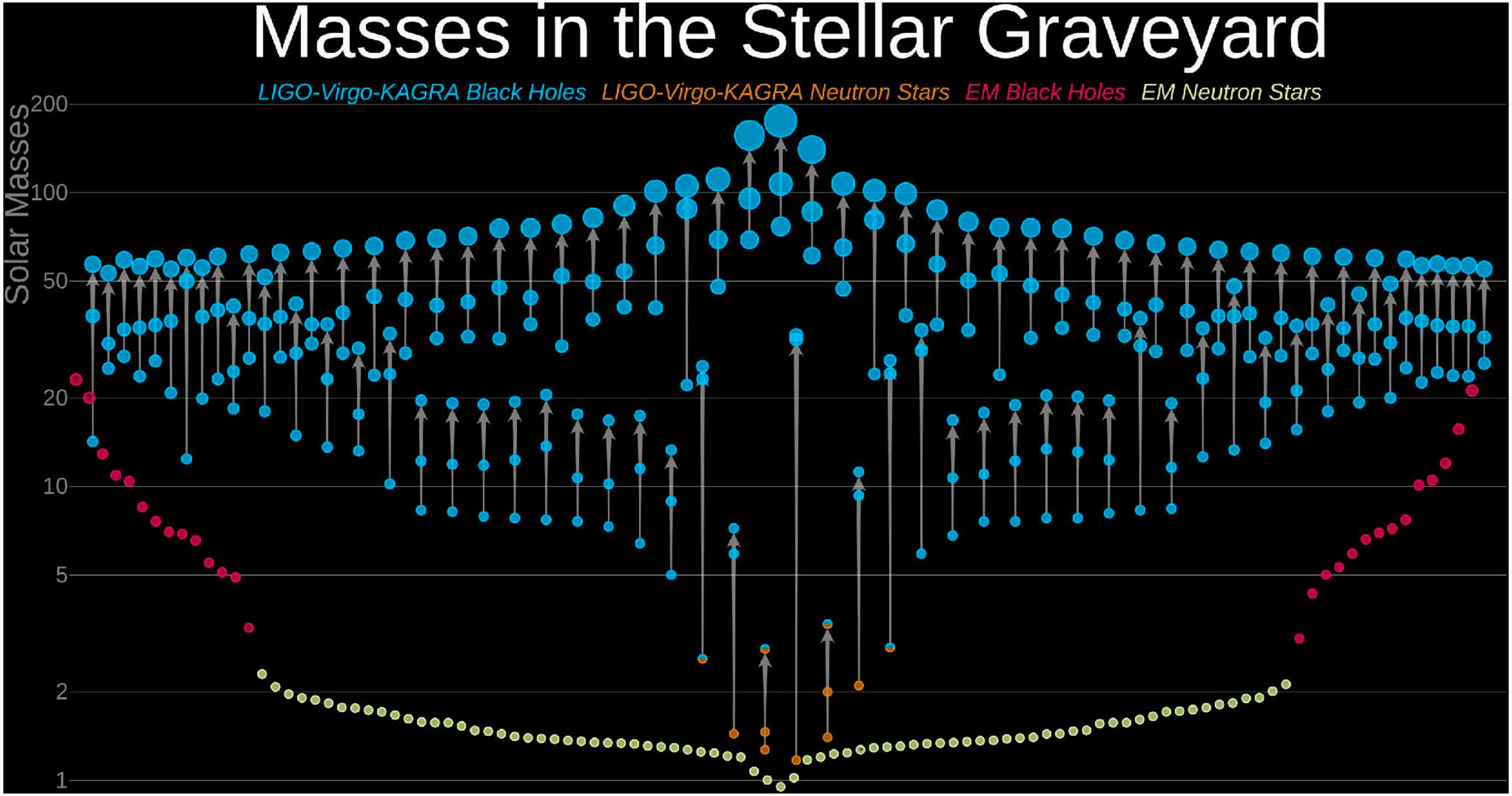
Why Primordial Black Holes (PBH) ?

- A novel and promising candidate for cold dark matter
- Inflation can generate PBH abundantly
- Non-baryonic, non-relativistic and nearly collision-less
- No new physics required !
- LIGO detection of GWs from supermassive black holes — seeds from PBHs

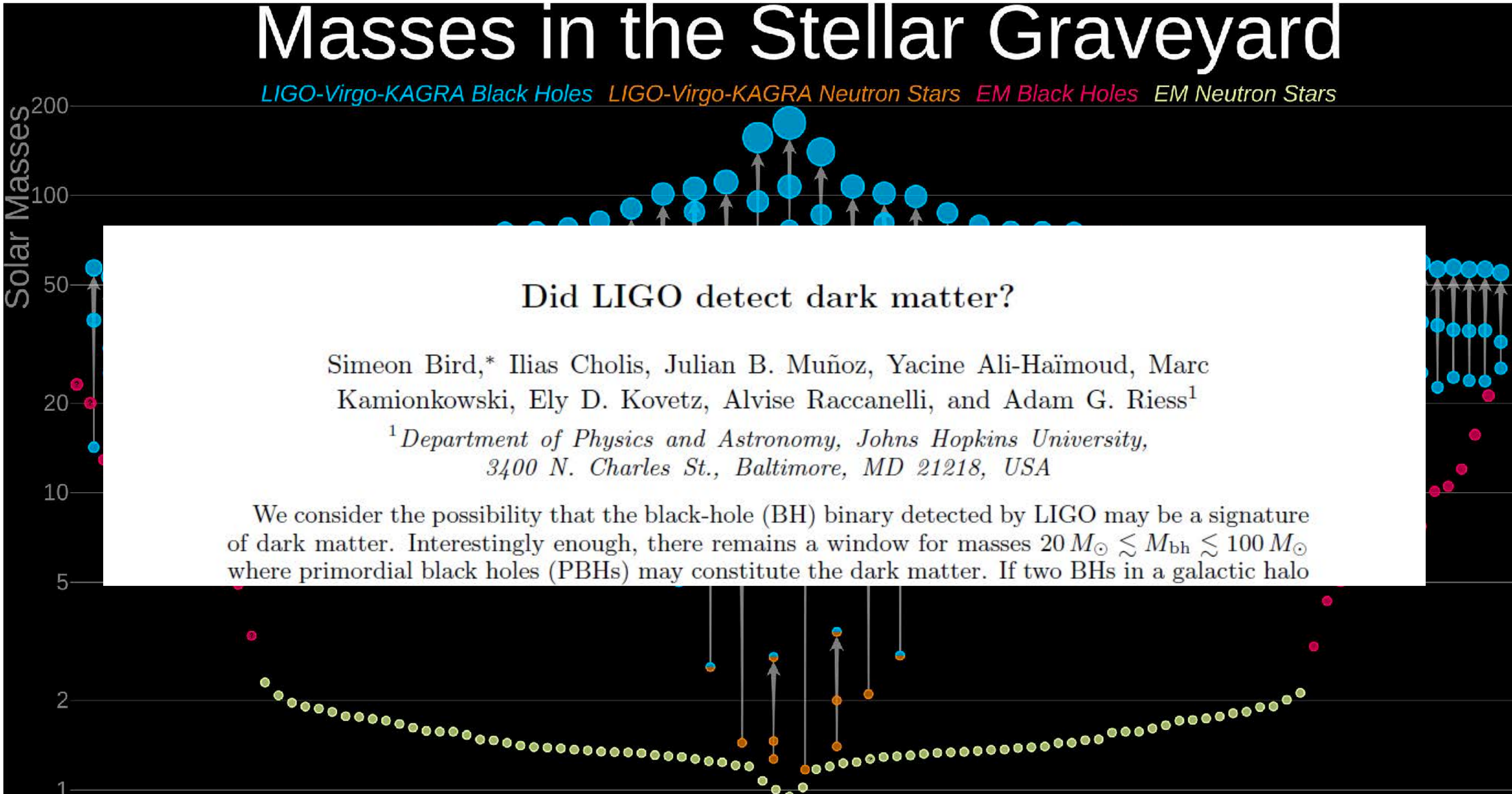
PBH as DM — Current constraints



Dark matter — LIGO GWs



Dark matter — LIGO GWs



Did LIGO detect dark matter?

Simeon Bird,* Ilias Cholis, Julian B. Muñoz, Yacine Ali-Haïmoud, Marc Kamionkowski, Ely D. Kovetz, Alvise Raccanelli, and Adam G. Riess[1]

[1] *Department of Physics and Astronomy, Johns Hopkins University, 3400 N. Charles St., Baltimore, MD 21218, USA*

We consider the possibility that the black-hole (BH) binary detected by LIGO may be a signature of dark matter. Interestingly enough, there remains a window for masses $20\,M_\odot \lesssim M_{\rm bh} \lesssim 100\,M_\odot$ where primordial black holes (PBHs) may constitute the dark matter. If two BHs in a galactic halo

Can LIGO black holes be primordial ?

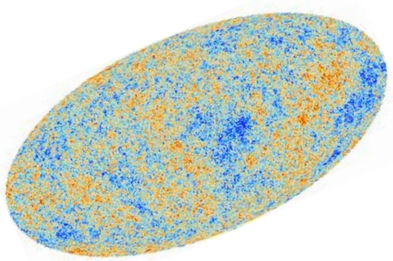
- LIGO observations consistent with no spin (hard to produce astrophysically !)

- Black holes with masses $M > 10^{-17} M_\odot \sim 10^{16}\,\mathrm{g}$ can survive the age of the universe without Hawking evaporating

$$t_{\rm ev}(M) \sim \frac{G^2 M^3}{\hbar\, c^4} \sim 10^{63} \left(\frac{M}{M_\odot}\right)^3 {\rm yr.}$$

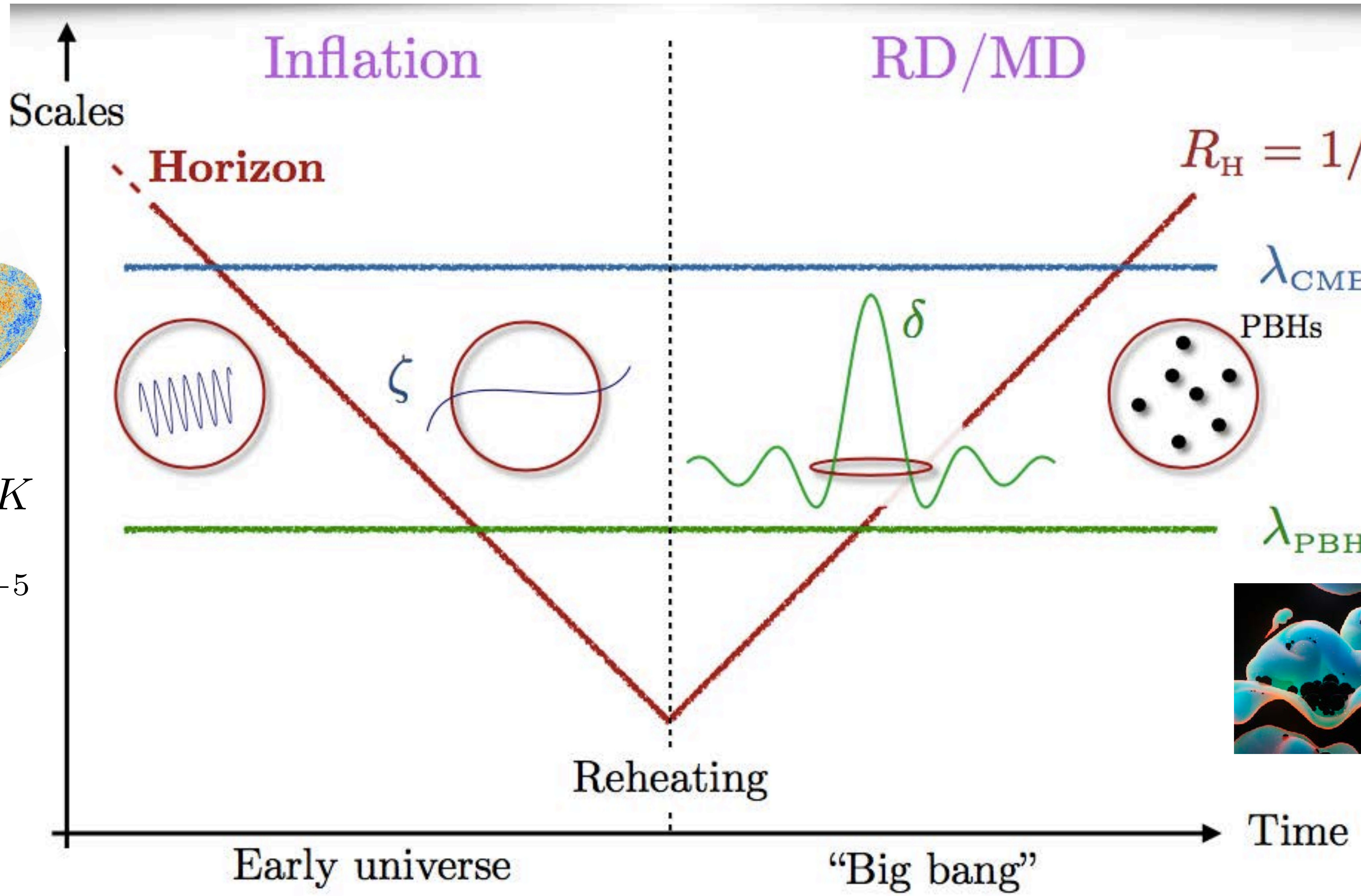
- Observations of below Chandrasekhar mass black holes would be a smoking gun proof of the existence of PBH !

PBH formation from inflation — in a nutshell



$T_0 = 2.73K$

$\frac{\Delta T}{T_0} \sim 10^{-5}$



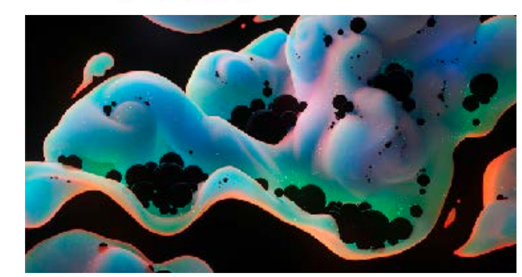
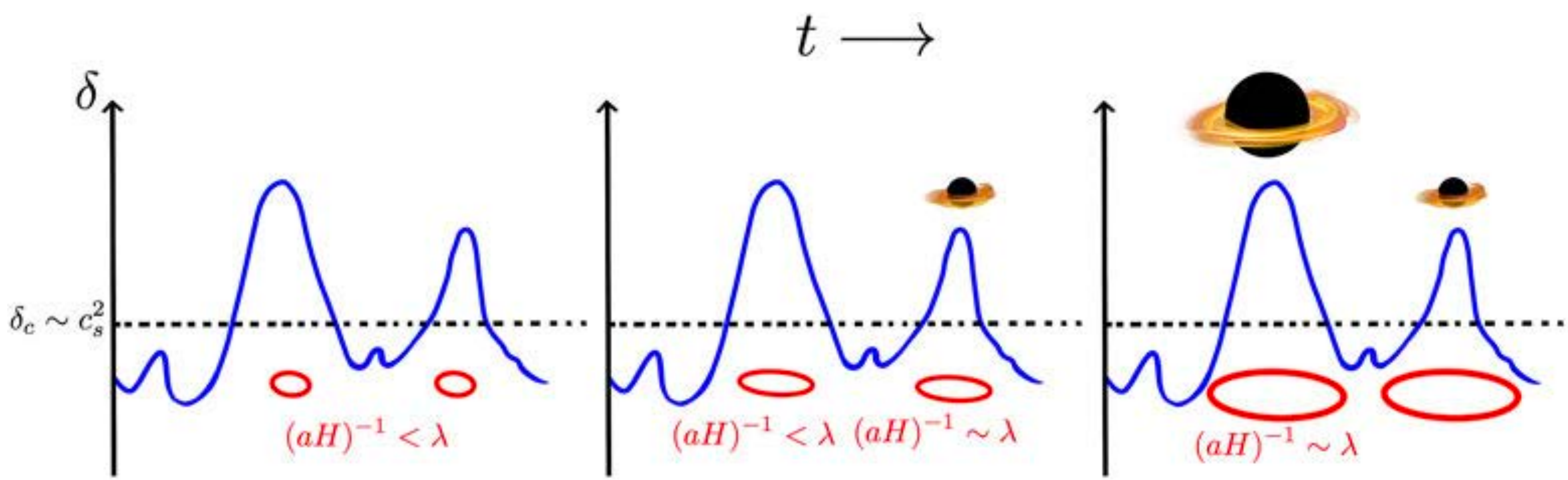


Fig. credit: G. Franciolini

PBH formation from inflation — in a nutshell



Collapse of overdense fluctuations during radiation domination

Fig. credit: Front. Astron. Space Sci., 2021

PBH formation from inflation — in a nutshell

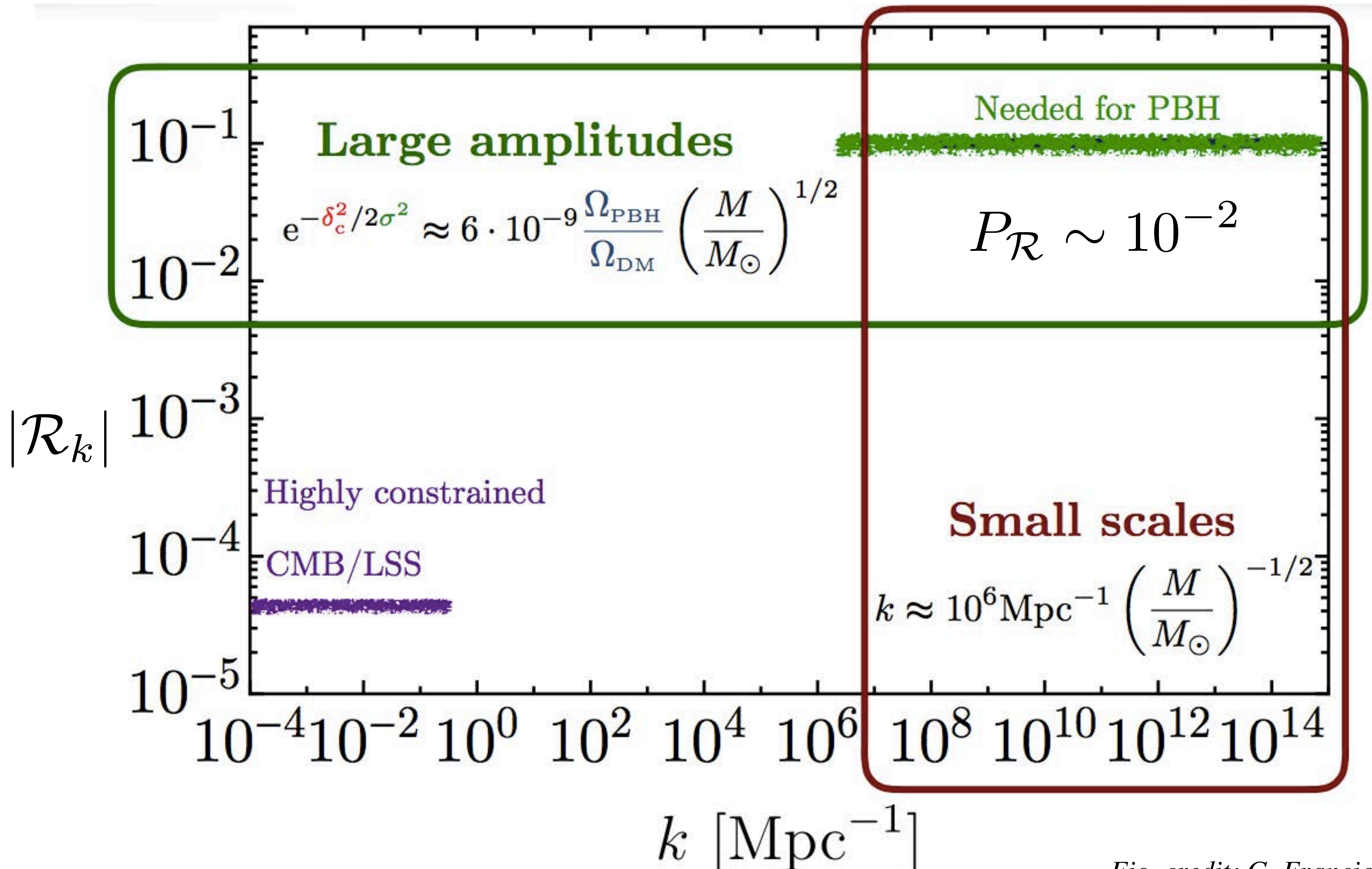
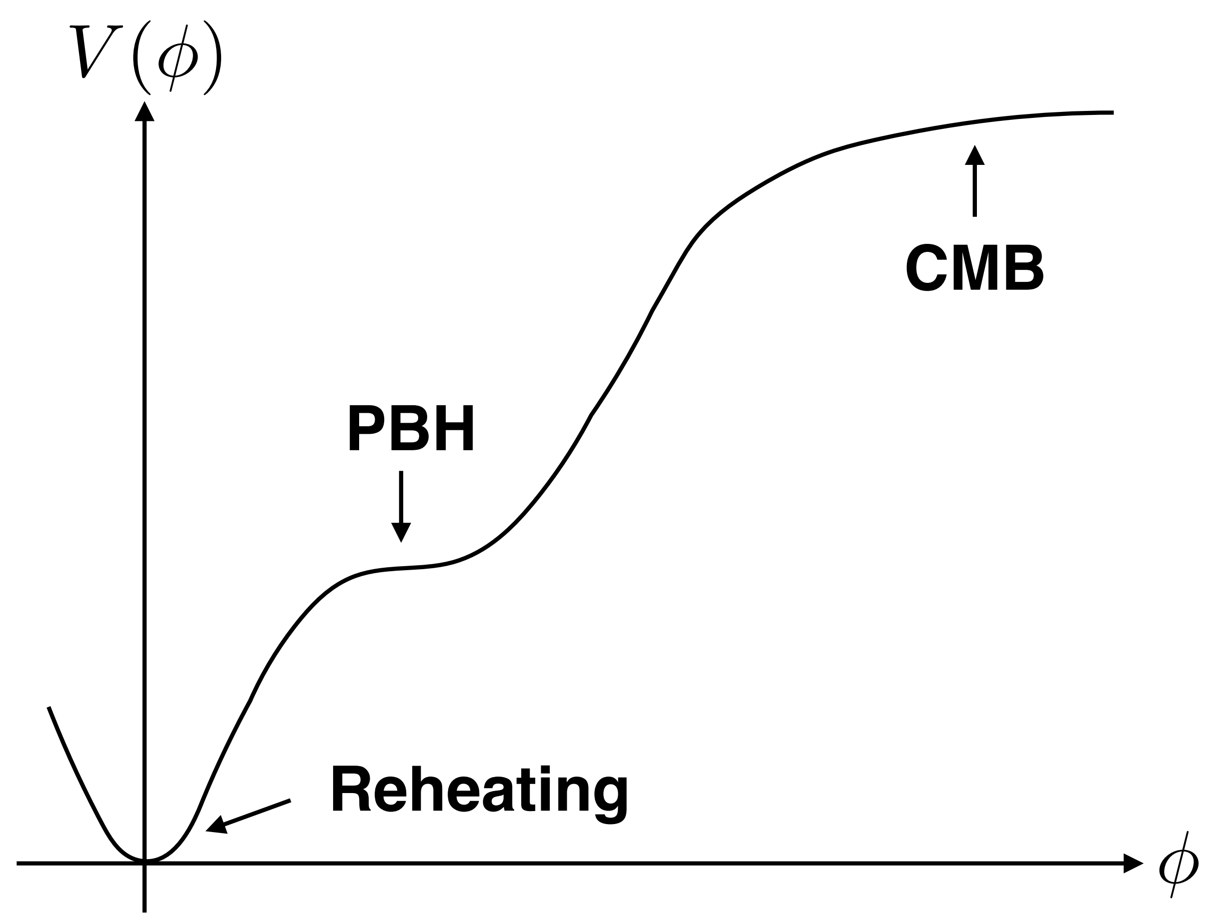


Fig. credit: G. Franciolini

PBH formation from inflation

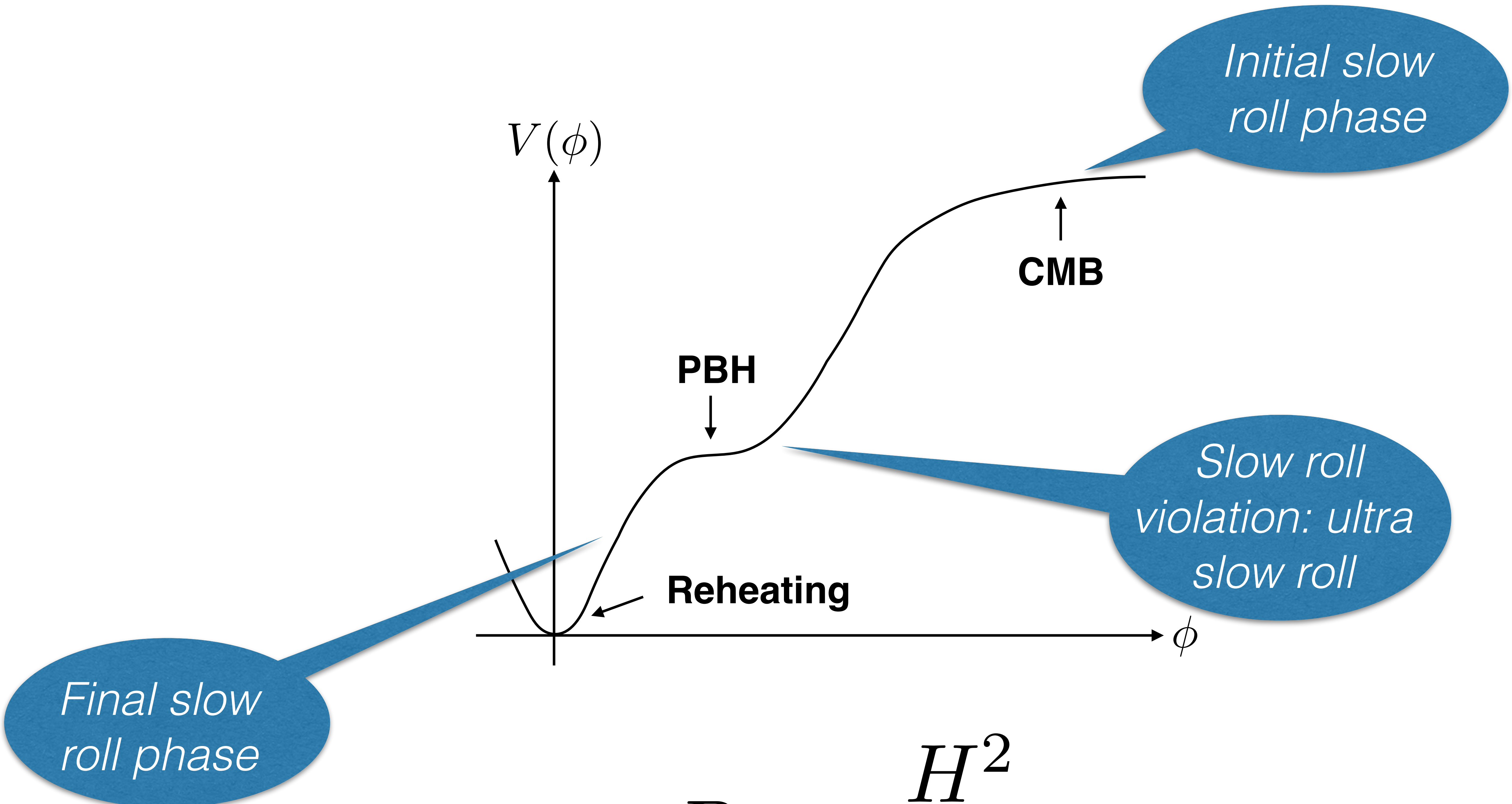
- Single field inflation with polynomial inflection point models
- Inflation with running spectral index — but running is small !
- Preheating after inflation
- Hybrid inflation
- Inflating/axionlike curvaton
- Particle production during inflation
- Critical Higgs inflation, string inflation, thermal inflation….

Inflation — inflection point models



$$P_{\mathcal{R}} \sim \frac{H^2}{\epsilon}$$

Inflation — inflection point models



$$P_{\mathcal{R}} \sim \frac{H^2}{\epsilon}$$

An inflection point scenario

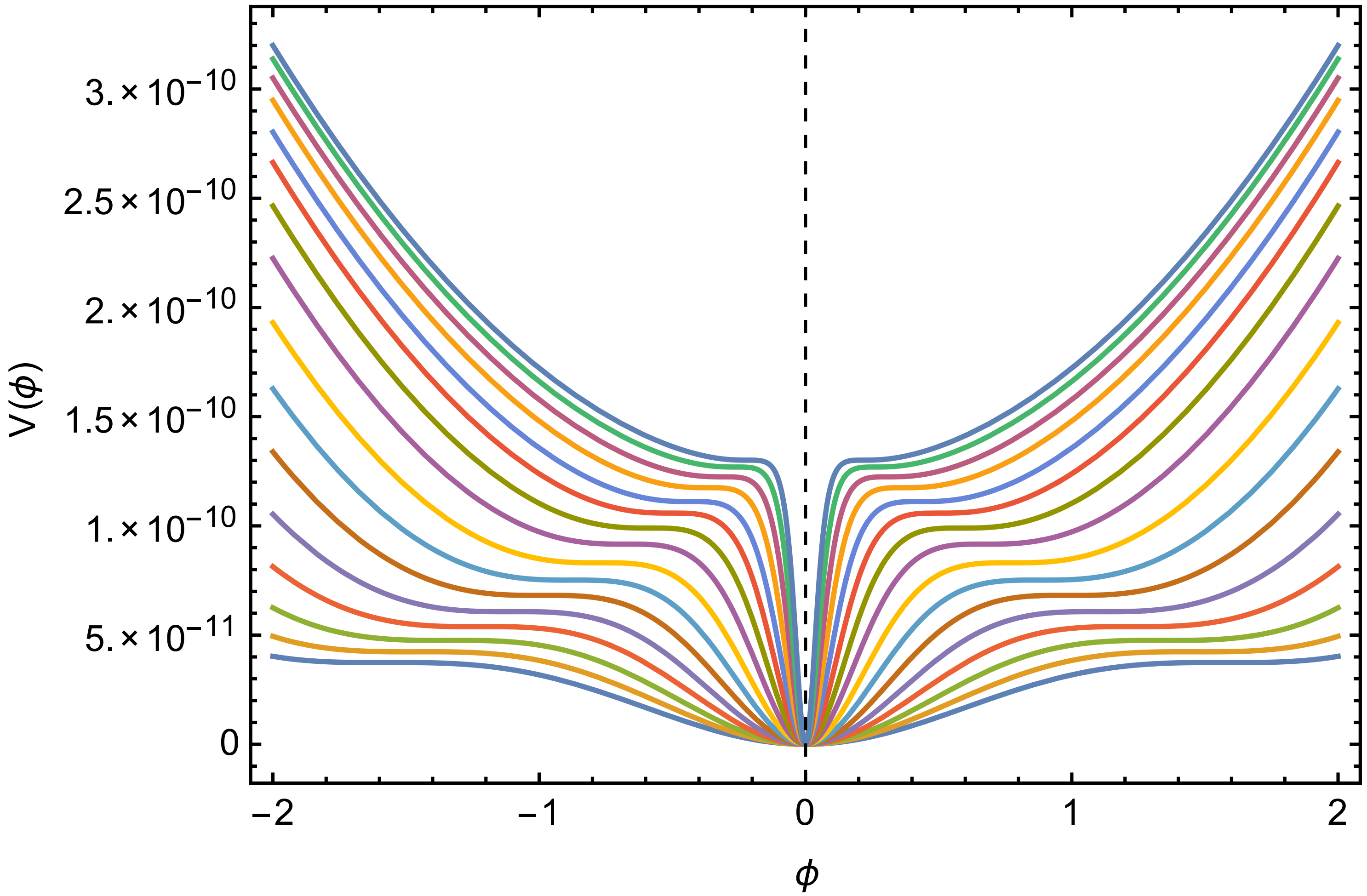
$$V(x) = V_0 \frac{ax^2 + bx^4 + cx^6}{(1+dx^2)^2}, \qquad x = \phi/v$$

$$x \gg 1 : V(x) \simeq \frac{V_0 c}{d^2} x^2$$

$$x \ll 1 : V(x) \simeq V_0 a x^2$$

Quadratic for both large & small field values

$$r \sim 0.05$$



N. Bhaumik & **RKJ**, JCAP 01, 037 (2020)

Slow roll, ultra slow roll and all that…

Background evolution

$$H^2 = \frac{V(\phi)}{M_{\rm Pl}^2(3-\epsilon)},$$

$$\frac{d^2\phi}{dN^2} + (3-\epsilon)\frac{d\phi}{dN} + \frac{1}{H^2}V'(\phi) = 0,$$

SR: $$\frac{d\phi}{dN} + \frac{1}{3H^2}V'(\phi) \simeq 0,$$

USR: $$\frac{d^2\phi}{dN^2} + 3\frac{d\phi}{dN} \simeq 0,$$

$$\epsilon \sim \exp\left[-6(N-N_i)\right]$$

$$\eta \simeq \epsilon + (3-\epsilon) \sim 3$$

Curvature perturbations

$$\mathcal{R}_k'' + 2\left(\frac{z'}{z}\right)\mathcal{R}_k' + k^2\mathcal{R}_k = 0.$$

$$\frac{z'}{z} = aH(1+\epsilon-\eta)$$

$$\mathcal{R}_k(\tau) \simeq C_1 + C_2\int\frac{d\tau}{z^2}$$

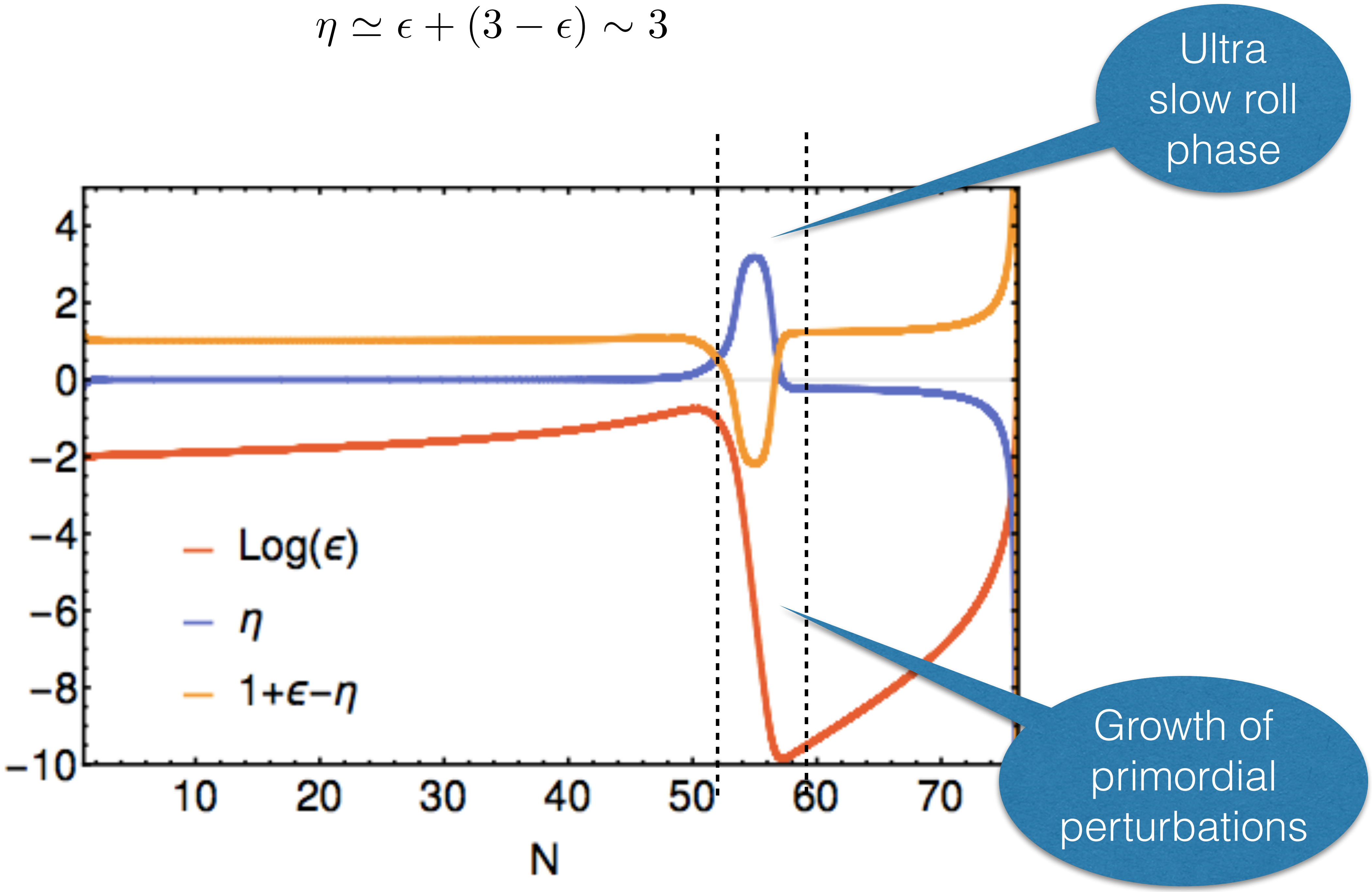
$$P_{\mathcal{R}} = \frac{k^3}{2\pi^2}|\mathcal{R}_k|^2$$

$$P_{\mathcal{R}} \sim \frac{H^2}{\epsilon}$$

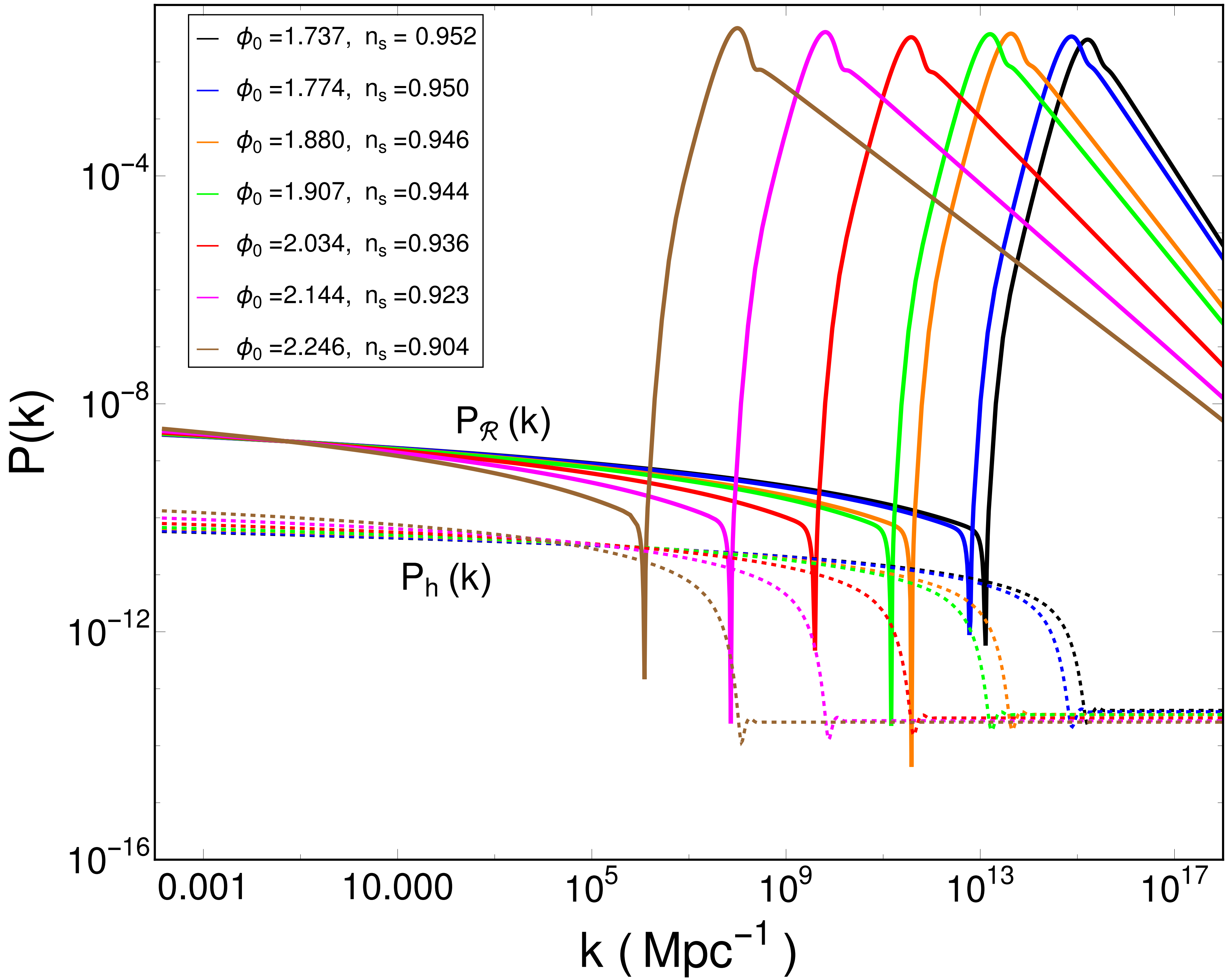
Slow roll, ultra slow roll and all that…

$$\epsilon \sim \exp\left[-6(N - N_i)\right]$$

$$\eta \simeq \epsilon + (3 - \epsilon) \sim 3$$



Primordial power spectra



N. Bhaumik & **RKJ**, JCAP 01, 037 (2020)

PBH mass fraction: Press-Schechter formalism

$$\delta(k,t) \simeq \frac{2(w+1)}{(3w+5)} \left(\frac{k}{aH}\right)^2 \mathcal{R}_k.$$

$$\sigma_\delta^2(R) \simeq \frac{16}{81} \int \frac{dk}{k} (kR)^4 P_{\mathcal{R}}(k) W^2(k,R).$$

$$\beta_f(M) \simeq \sqrt{\frac{1}{2\pi}} \frac{\sigma_\delta(M(R))}{\delta_c} \exp\left(-\frac{\delta_c^2}{2\sigma_\delta^2(M(R))}\right)$$

$$\beta(M) \equiv \frac{\rho_{\rm PBH}(M)}{\rho_{\rm tot}}.$$

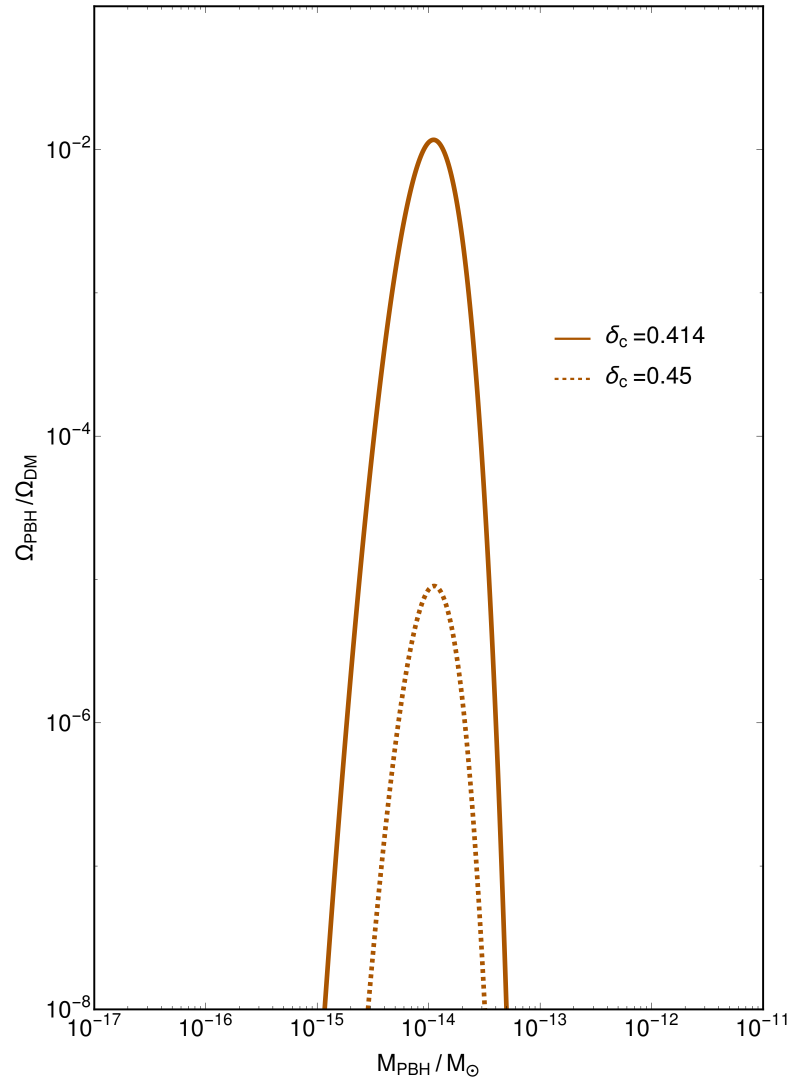
$$\beta_{\rm eq}(M) \simeq \beta_f(M) \left(\frac{a_{\rm eq}}{a_f}\right)$$

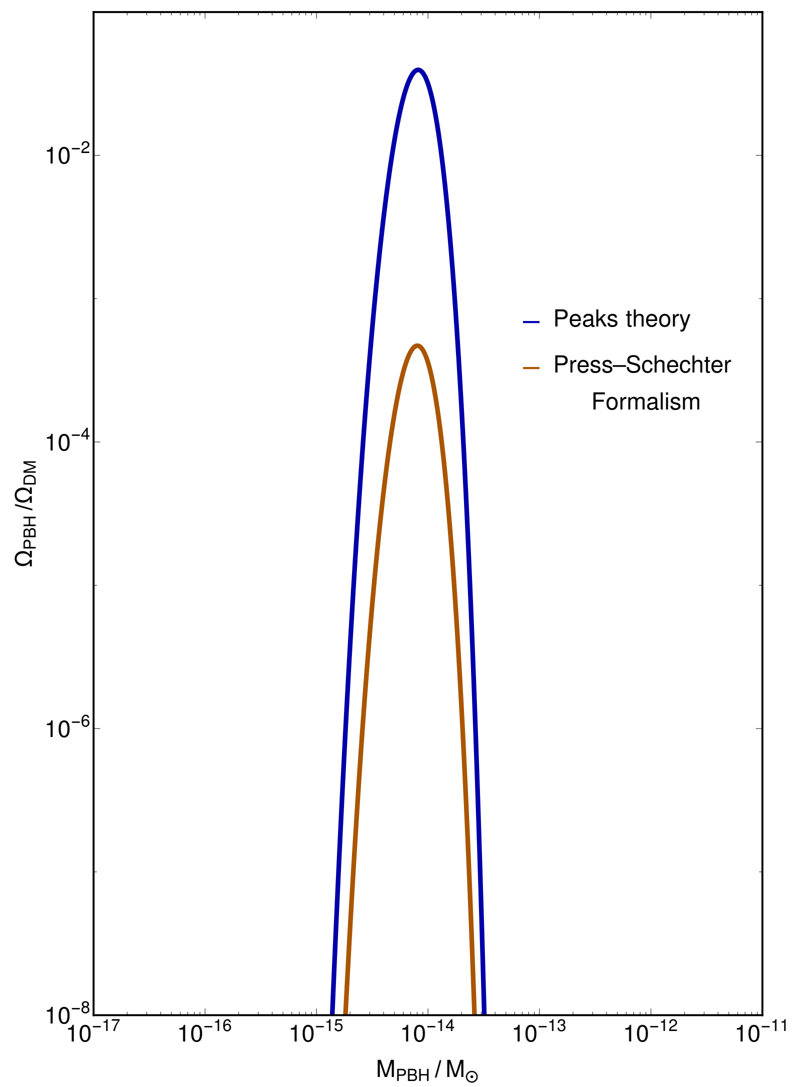
$$f_{\rm PBH}(M) \equiv \frac{\Omega_{\rm PBH}(M)}{\Omega_{\rm DM}} = \frac{\beta(M)}{8\times 10^{-16}} \left(\frac{\gamma}{0.2}\right)^{3/2} \left(\frac{g_*}{106.75}\right)^{-1/4} \left(\frac{M}{10^{18}\,{\rm g}}\right)^{-1/2}$$

PBH Recipe

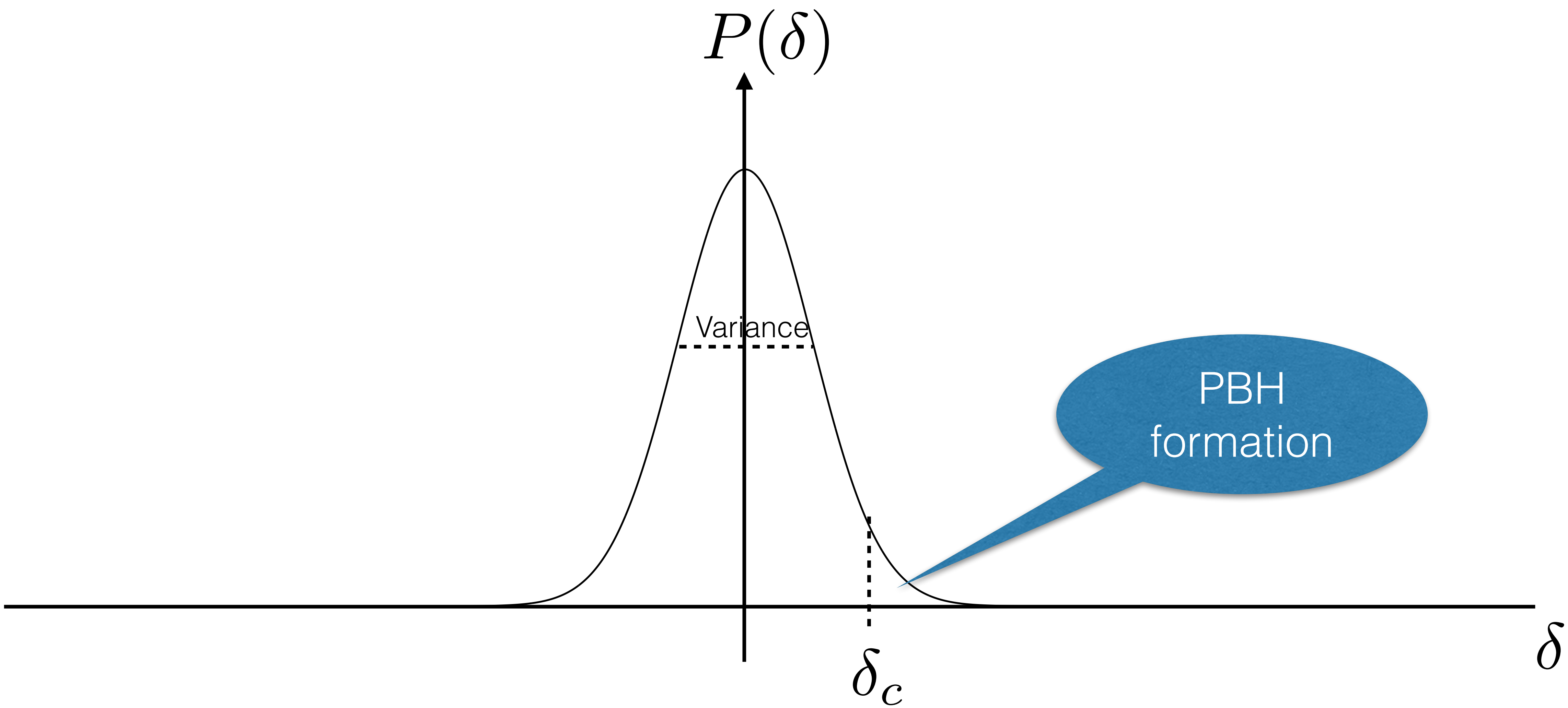
PBH mass fraction: Some uncertainties

- Peaks theory vs. Press-Schechter
- Choice of the window function
- Value of the critical density contrast





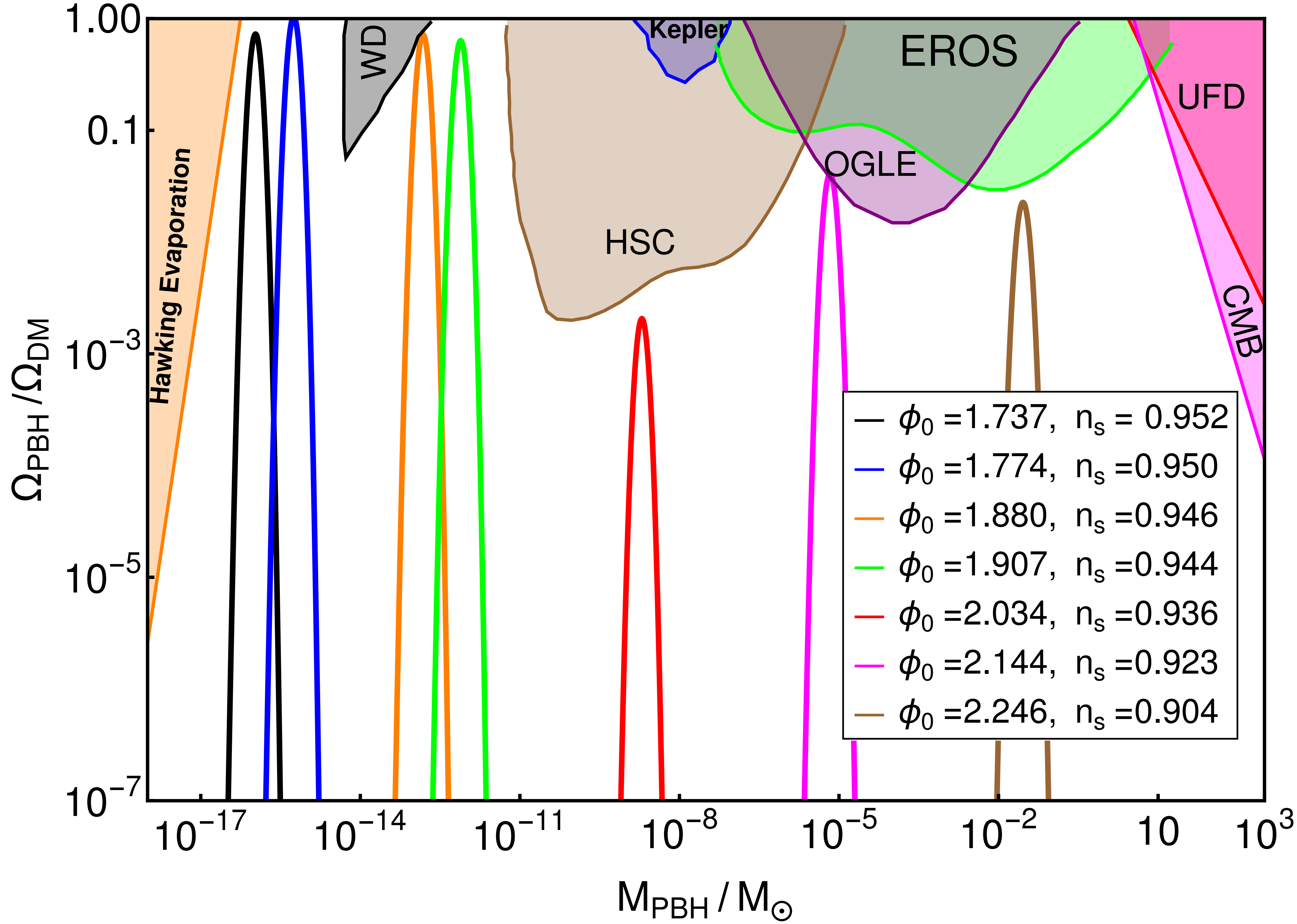
PBH mass fraction: Press-Schechter formalism



$$\beta_f(M) = \frac{1}{\sqrt{2\pi\sigma_\delta^2(M(R))}} \int_{\delta_c}^{\infty} d\delta \exp\left(-\frac{\delta^2}{2\sigma_\delta^2(M(R))}\right)$$

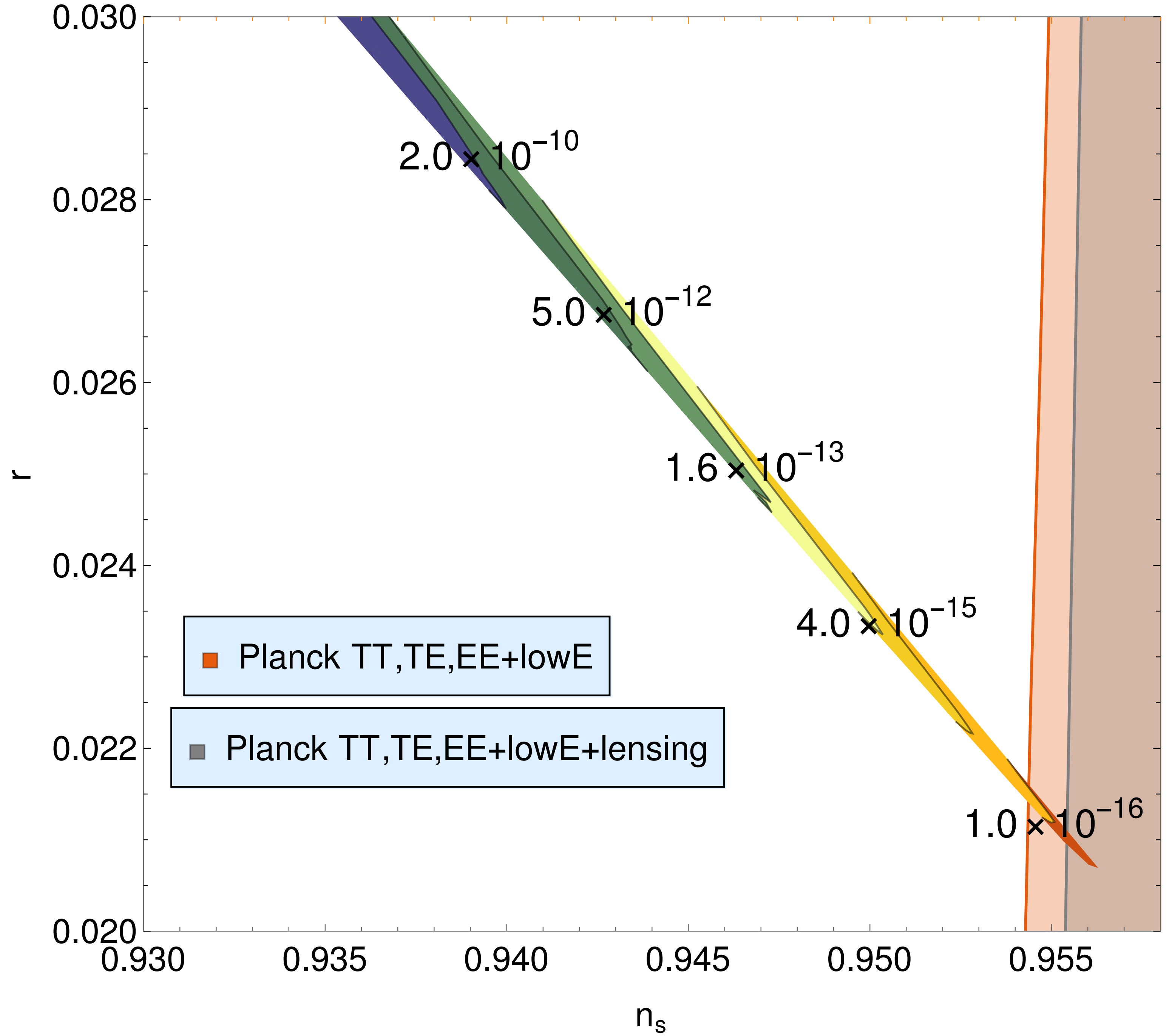
Non-gaussianities play very important role in PBH formation !

PBH mass fraction



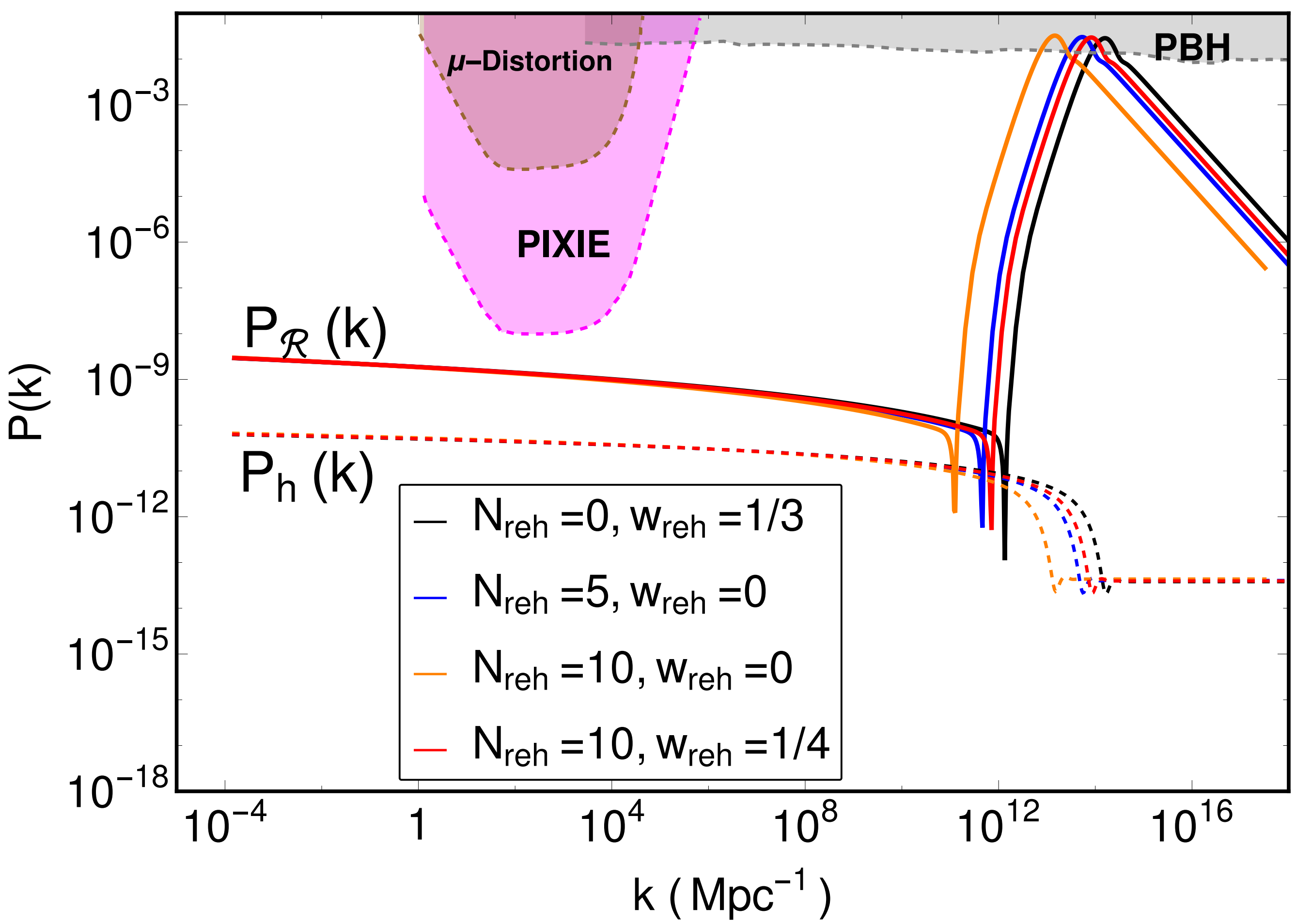
N. Bhaumik & **RKJ**, JCAP 01, 037 (2020)

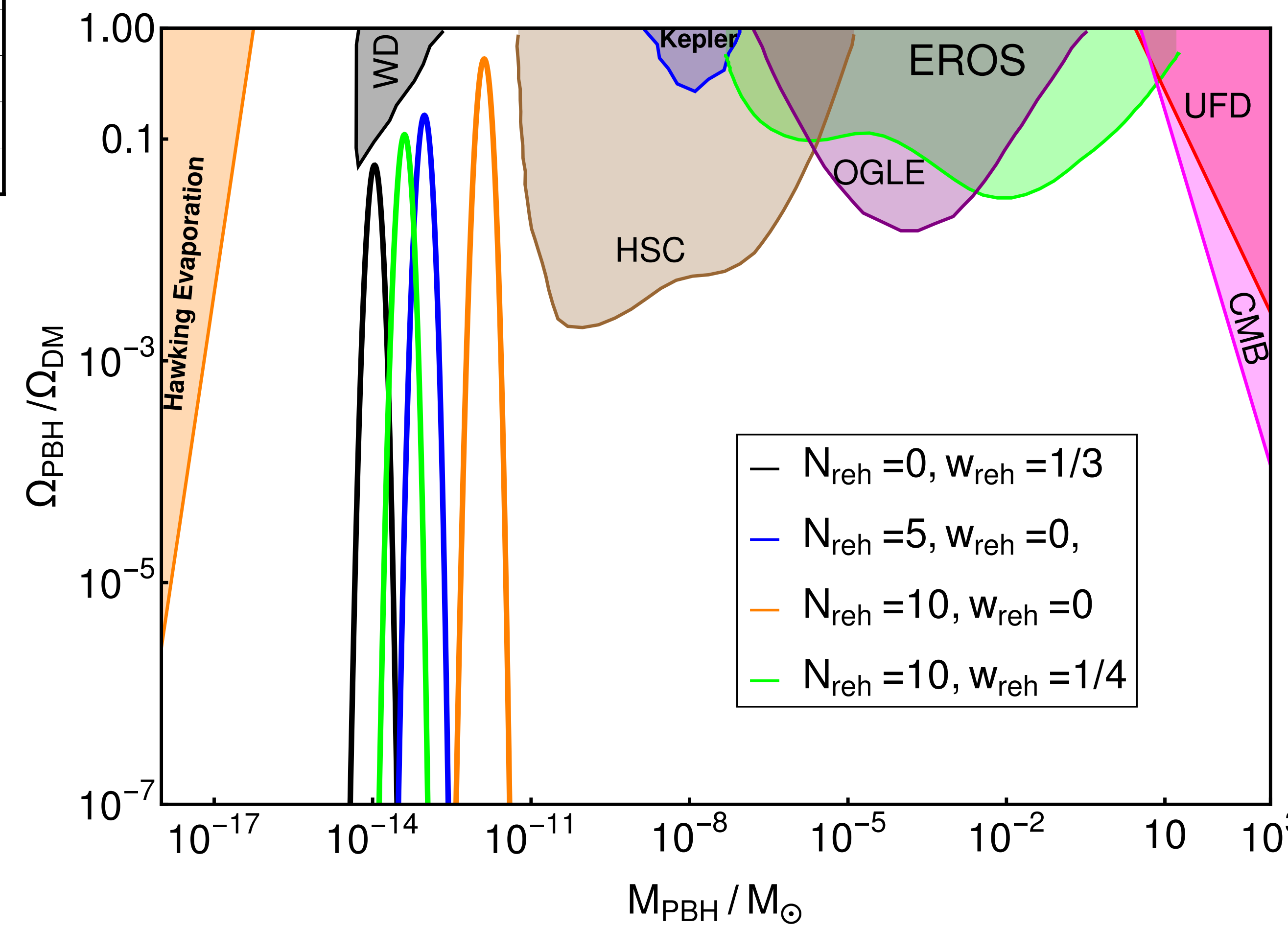
PBH mass contours: n_s vs. r



N. Bhaumik & **RKJ**, JCAP 01, 037 (2020)

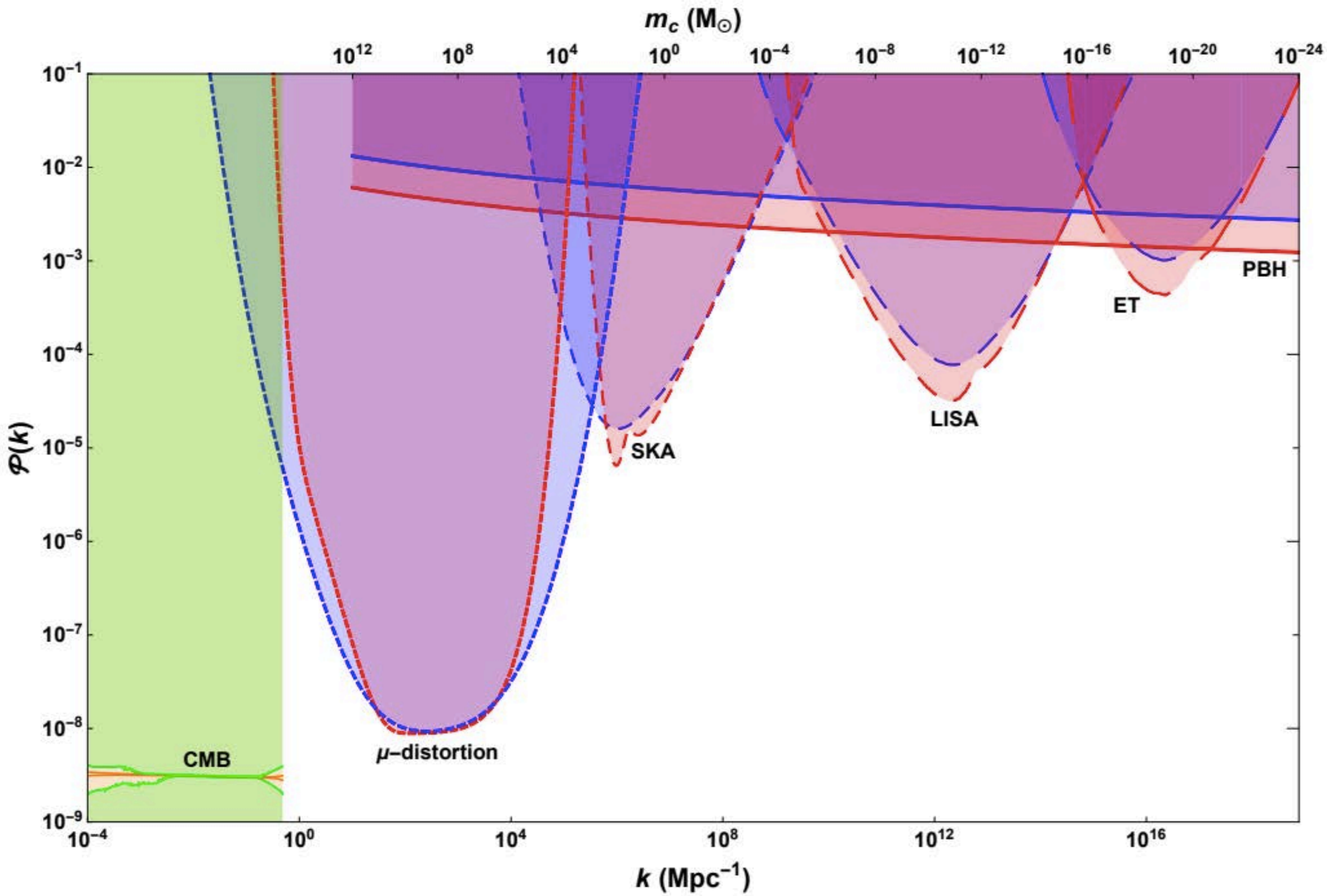
Effects of reheating





N. Bhaumik & **RKJ**, JCAP 01, 037 (2020)

(future) constraints on small scales



Gow et.al., 2020

PBH observational imprints: Induced secondary GWs

Induced secondary GWs

Tensor modes sourced by scalar perturbations

$$h''_{\mathbf{k}}(\tau) + 2\mathcal{H}h'_{\mathbf{k}}(\tau) + k^2 h_{\mathbf{k}}(\tau) = 4S_{\mathbf{k}}(\tau),$$

$$S_{\mathbf{k}} = \int \frac{\mathrm{d}^3 q}{(2\pi)^{3/2}} e^{\lambda}_{ij}(\mathbf{k}) q_i q_j \left[2\Phi_{\mathbf{q}}\Phi_{\mathbf{k}-\mathbf{q}} + \left(\mathcal{H}^{-1}\Phi'_{\mathbf{q}} + \Phi_{\mathbf{q}}\right)\left(\mathcal{H}^{-1}\Phi'_{\mathbf{k}-\mathbf{q}} + \Phi_{\mathbf{k}-\mathbf{q}}\right)\right].$$

$$\Phi_{\mathbf{k}}(\tau) = \frac{2}{3}\mathcal{T}(k\tau)\mathcal{R}(\mathbf{k}). \qquad \mathcal{T}(k\tau) = \frac{9}{(k\tau)^2}\left[\frac{\sqrt{3}}{k\tau}\sin\left(\frac{k\tau}{\sqrt{3}}\right) - \cos\left(\frac{k\tau}{\sqrt{3}}\right)\right].$$

$$\frac{k^3}{2\pi^2}\left\langle h^{\lambda}_{\mathbf{k}}(\tau)\, h^{\lambda'}_{\mathbf{k}'}(\tau)\right\rangle = \delta_{\lambda\lambda'}\delta^3(\mathbf{k}+\mathbf{k}')\mathcal{P}_h(\tau, k),$$

$$\Omega_{\mathrm{GW}}(\tau, k) \equiv \frac{1}{\rho_c}\frac{\mathrm{d}\,\rho_{\mathrm{GW}}}{\mathrm{d}\ln k} = \frac{\rho_{\mathrm{GW}}(\tau, k)}{\rho_{\mathrm{tot}}(\tau)} = \frac{1}{24}\left(\frac{k}{\mathcal{H}}\right)^2 \overline{\mathcal{P}_h(\tau, k)},$$

Induced secondary GWs

$$\Omega_{\rm GW}(\tau_0,k)\,h^2 \simeq 2.4\times10^{-5}\left(\frac{\Omega_{r,0}\,h^2}{4.0\times10^{-5}}\right)\left(\frac{k}{\mathcal{H}(\tau_f)}\right)^2\int_{-\frac{1}{\sqrt{3}}}^{\frac{1}{\sqrt{3}}}{\rm d}d\int_{\frac{1}{\sqrt{3}}}^{\infty}{\rm d}s\left[\frac{(d^2-1/3)(s^2-1/3)}{s^2-d^2}\right]^2$$

$$\times\,\mathcal{P}_{\mathcal{R}}\left(\frac{k\sqrt{3}}{2}(s+d)\right)\mathcal{P}_{\mathcal{R}}\left(\frac{k\sqrt{3}}{2}(s-d)\right)\left[\mathcal{I}_c^2(d,s)+\mathcal{I}_s^2(d,s)\right].$$

$$\mathcal{I}_c(d,s) = -36\pi\frac{(s^2+d^2-2)^2}{(s^2-d^2)^3}\theta(s-1),$$

$$\mathcal{I}_s(d,s) = -36\frac{(s^2+d^2-2)}{(s^2-d^2)^2}\left[\frac{(s^2+d^2-2)}{(s^2-d^2)}\log\frac{(1-d^2)}{|s^2-1|}+2\right]$$

$$d=(u-v)/\sqrt{3}$$

$$s=(u+v)/\sqrt{3},$$

$$v=p/k,\; u=|\mathbf{k}-\mathbf{p}|/k$$

The ‘three’ peaks

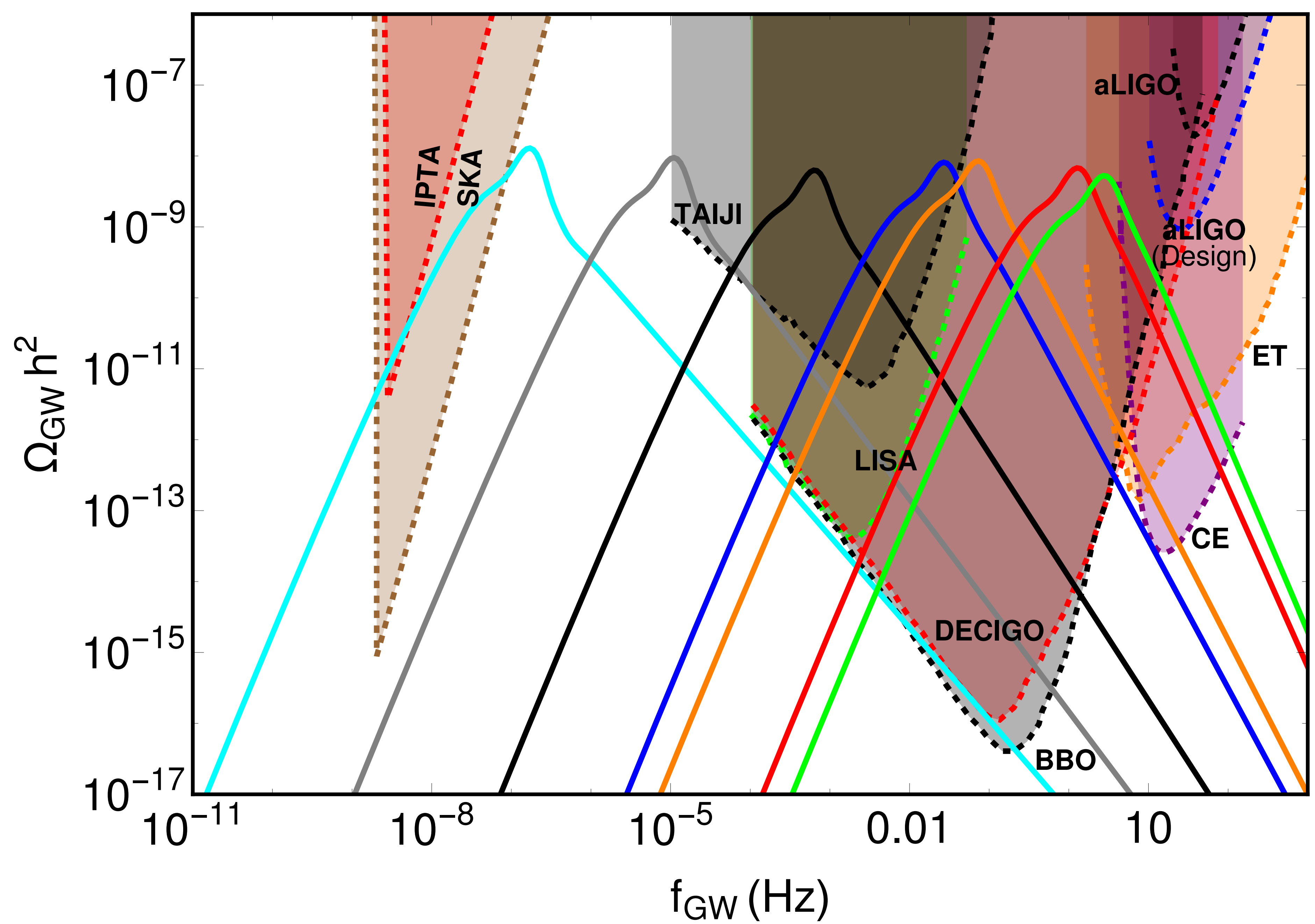
An interesting and useful relation between the ‘three’ peaks

$$\left(\frac{M_{\rm PBH}}{10^{17}\,{\rm g}}\right)^{-1/2} \simeq \frac{k}{2\times 10^{14}\,{\rm Mpc}^{-1}} = \frac{f}{0.3\,{\rm Hz}},$$

↑ $f_{\rm PBH}$ ↑ $P_{\mathcal{R}}$ ↑ $\Omega_{\rm GW}$

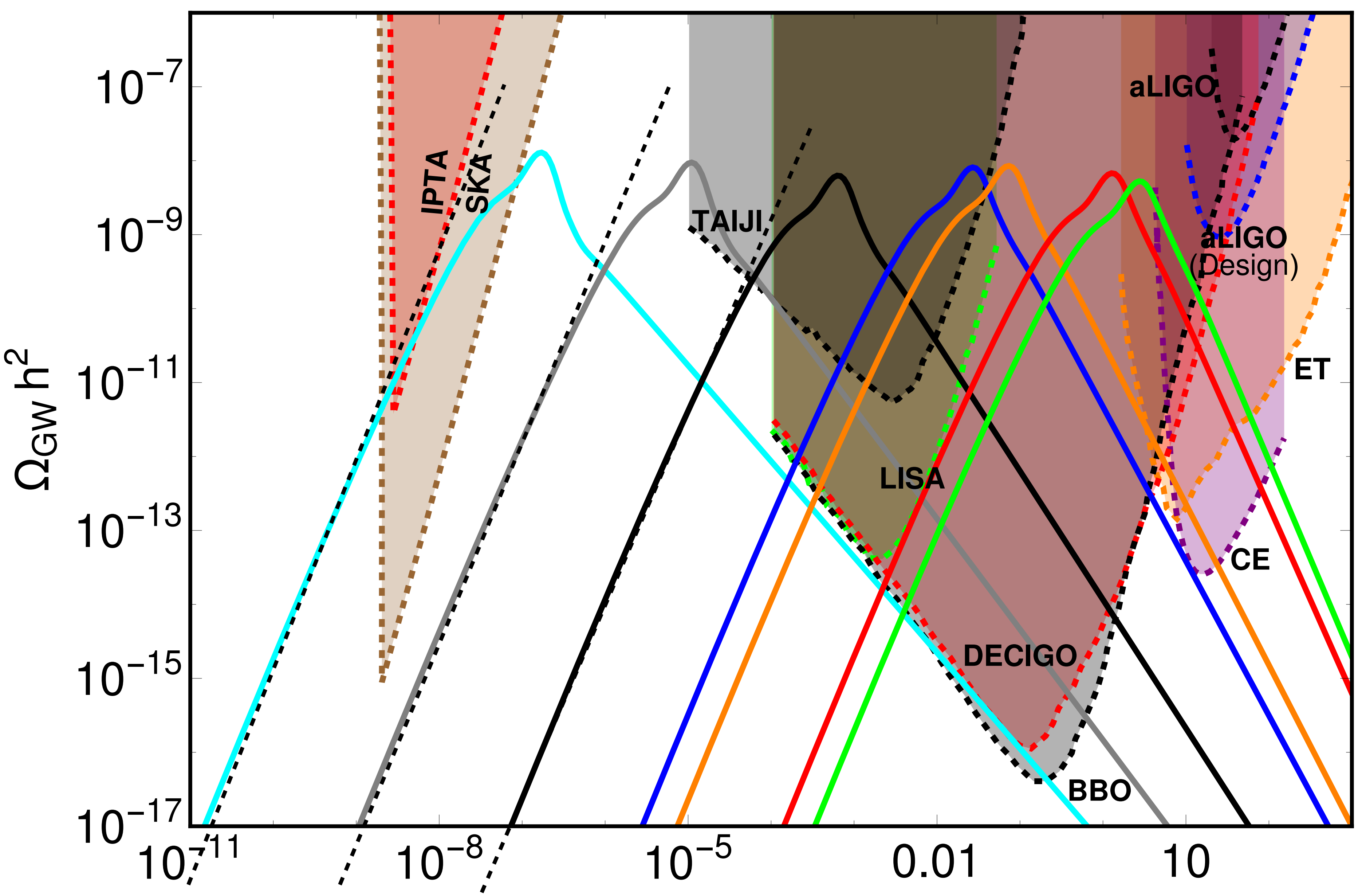
- LISA: f ~ mHz → $k \sim 10^{12}\,{\rm Mpc}^{-1}$ → $M_{\rm PBH} \sim 10^{-12} M_\odot$
- Advanced LIGO: f ~ 30 Hz → $M_{\rm PBH} \sim 10^{13}\,{\rm g} \sim 10^{-20} M_\odot$

Induced secondary GWs



N. Bhaumik & **RKJ**, Phys. Rev. D 104, 023531 (2021)

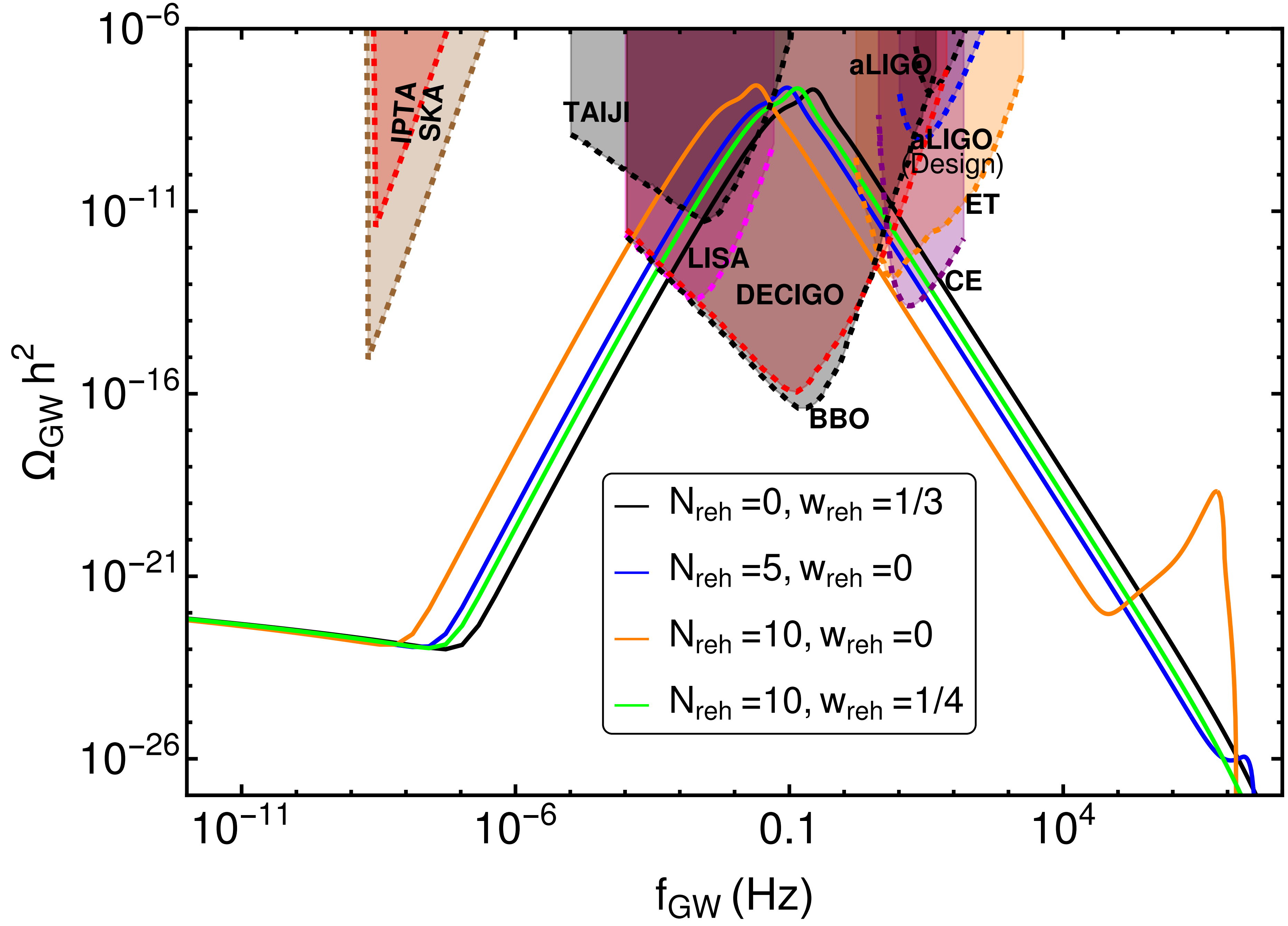
Induced secondary GWs



$$\Omega_{\rm GW} \sim f_{\rm GW}^3$$

- A universal IR scaling of GW spectrum, Cai, Pi & Sasaki, 2019
- IR scaling with log corrections, Yuan, Chen & Huang, 2019

Induced secondary GWs — effects of reheating



N. Bhaumik & **RKJ**, Phys. Rev. D 104, 023531 (2021)

PBH observational imprints: early domination and evaporation

Ultralight PBHs — Hawking evaporation

- **Non-spinning PBH**

$$T^{\rm S}_{\rm PBH} = \frac{M^2_{\rm Pl}}{M_{\rm PBH}} \simeq 1.053\,{\rm GeV}\left(\frac{10^{13}\,{\rm g}}{M_{\rm PBH}}\right)$$

$$\Delta t^{\rm S}_{\rm PBH} \approx \frac{160 M^3_{\rm PBH}}{\pi\ \mathcal{G}\ \overline{g_{*,H}}\ M^4_{Pl}}$$

- **Spinning PBH**

$$T_{\rm PBH} = \frac{M^2_{\rm Pl}}{M_{\rm PBH}}\left(\frac{2\sqrt{1-a_*^2}}{1+\sqrt{1-a_*^2}}\right)$$

$$a_* = J M^2_{\rm Pl}/M^2_{\rm PBH}$$

$$\Delta t_{\rm PBH} = \Delta t^{\rm S}_{\rm PBH}\ \mathcal{F}(a_*, M_{\rm PBH})$$

$$\frac{dM(t)}{dt} = -\varepsilon(M(t), a(t))\frac{M^4_{\rm Pl}}{M(t)^2}\,,$$

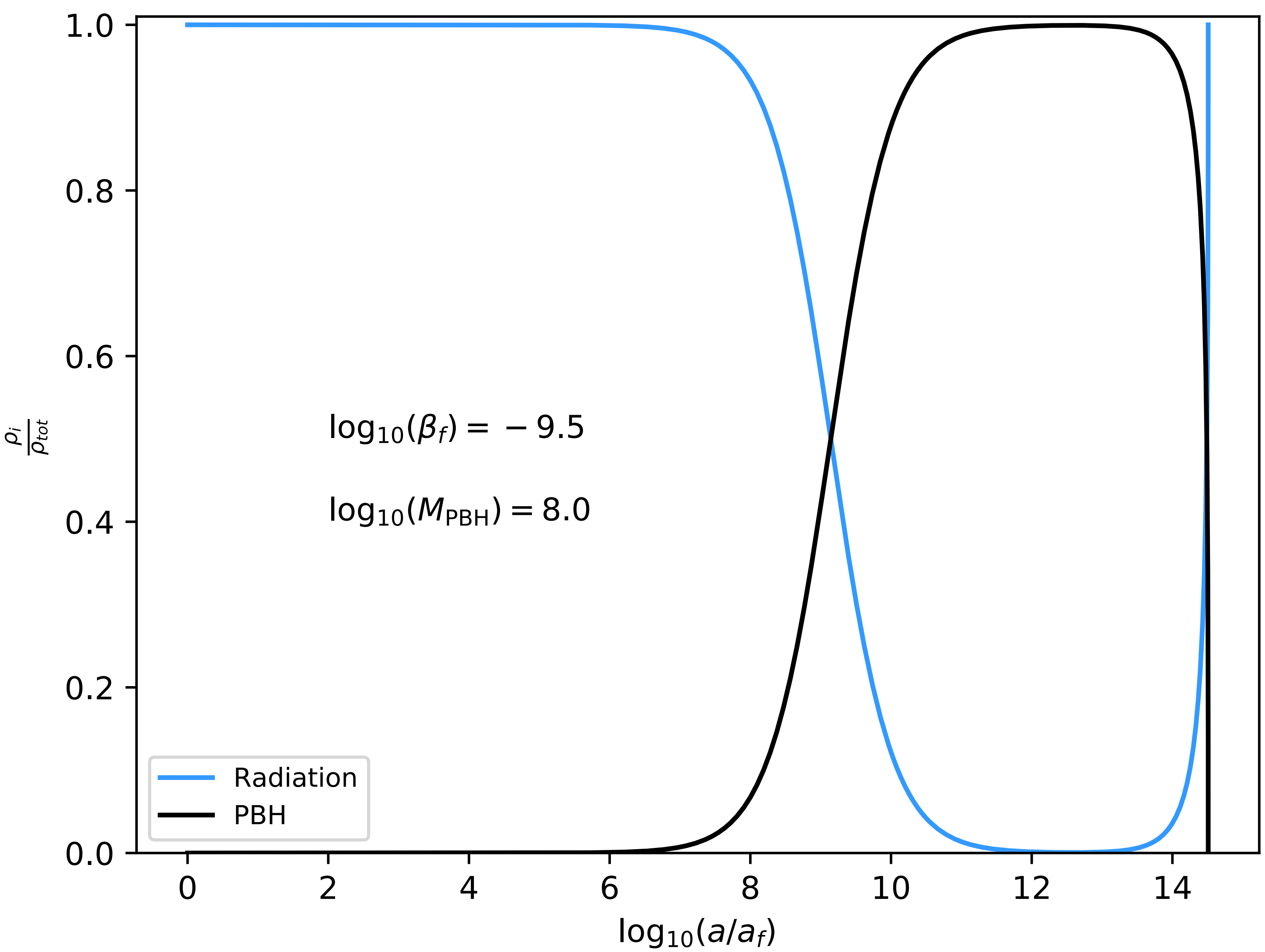
$$\frac{da(t)}{dt} = -a(t)\Big[\gamma(M(t), a(t)) - 2\varepsilon(M(t), a(t))\Big]\frac{M^4_{\rm Pl}}{M(t)^3}$$

BlackHawk

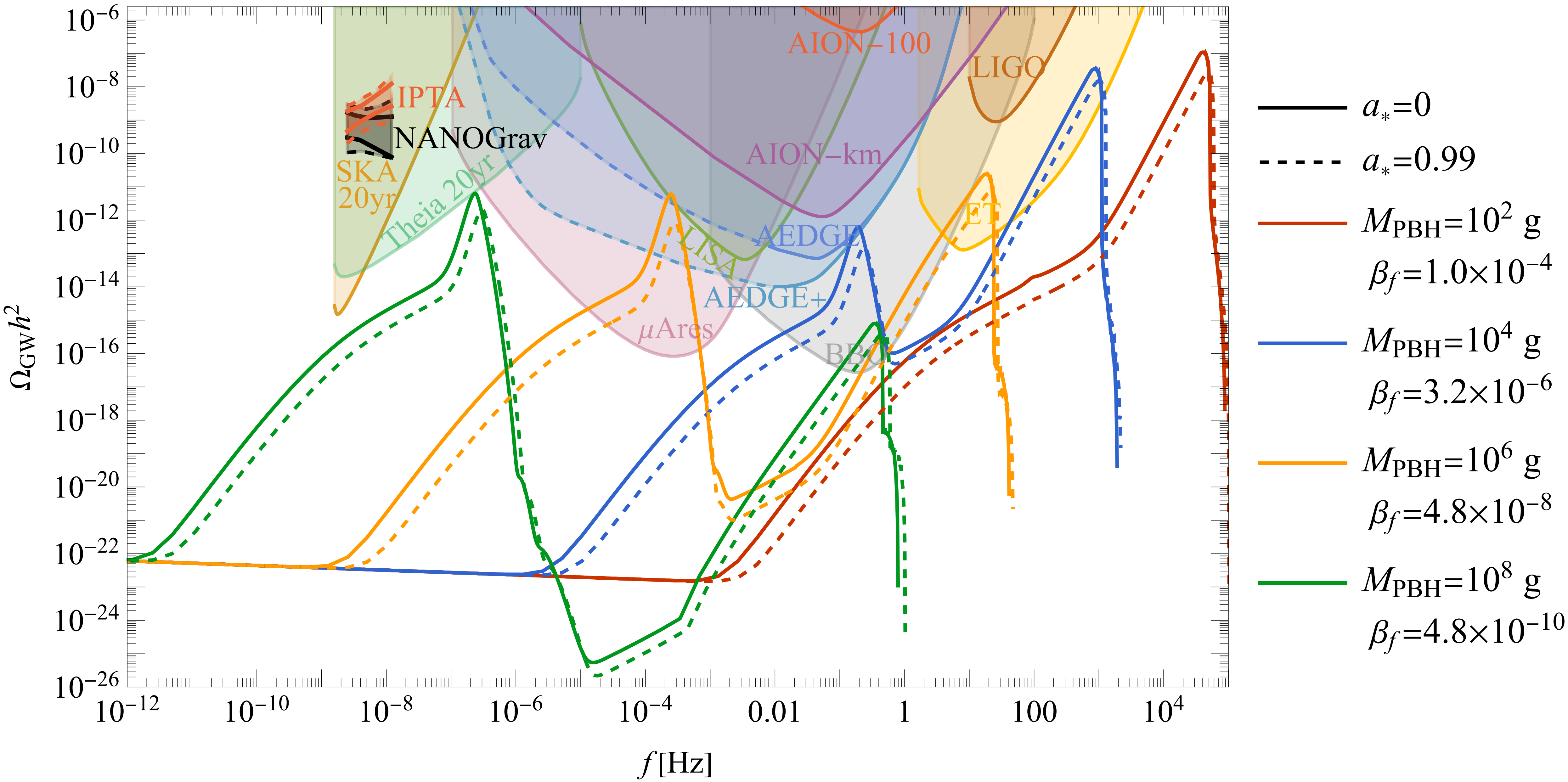
N. Bhaumik, A. Ghoshal, **RKJ** & M. Lewicki, JHEP 05, 169 (2023)

Ultralight PBHs — early matter dominated epoch

$$a_{eMD}(\tau) = \frac{a_f(\tau+\tau_m)^2}{4\tau_f\tau_m}, \quad a_{RD}(\tau) = \frac{a_f(\tau_r+\tau_m)(2\tau-\tau_r+\tau_m)}{4\tau_f\tau_m} \simeq a(\tau_r)\left(2\frac{\tau}{\tau_r}-1\right)$$

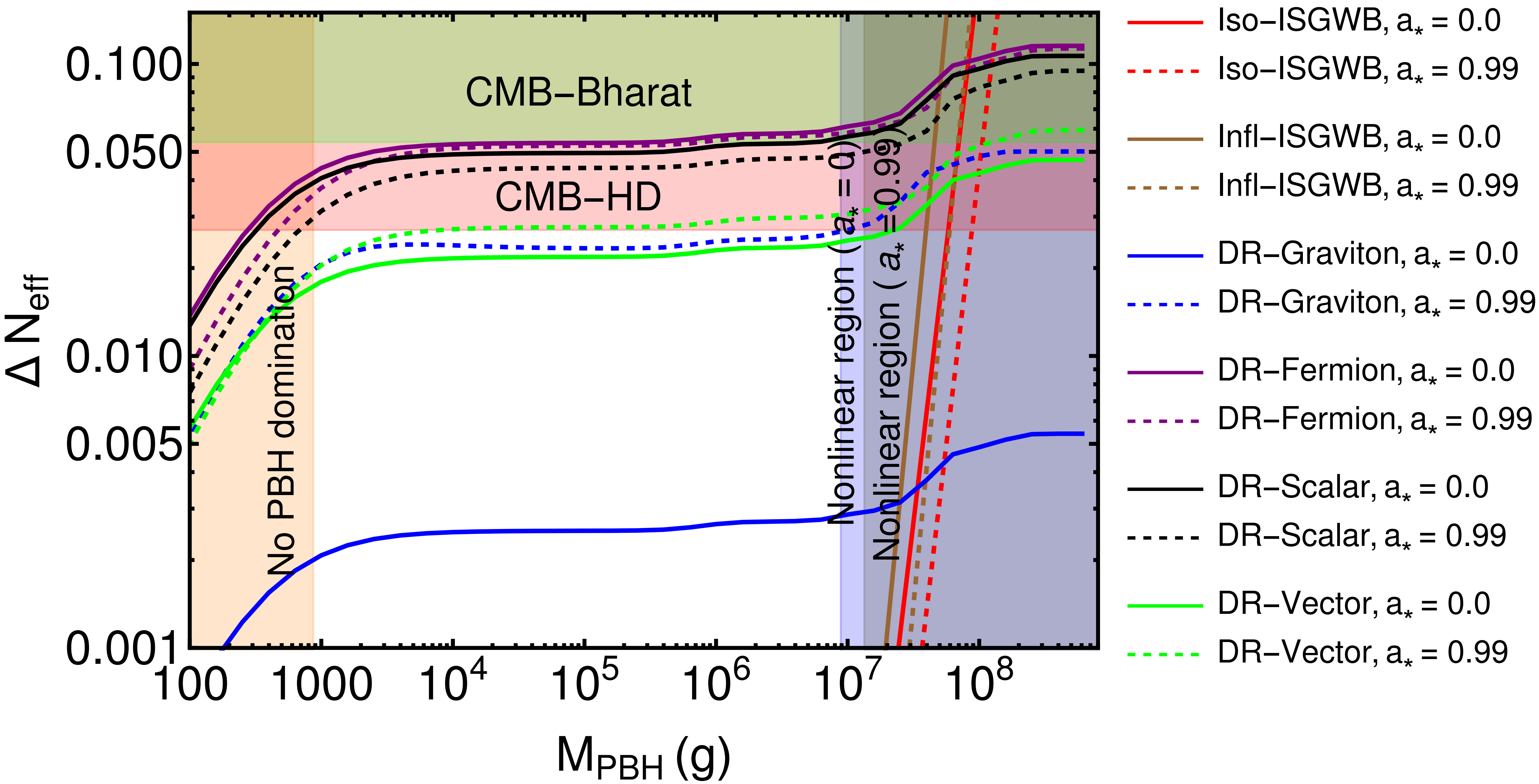


Doubly peaked GWs — with and without PBH spin



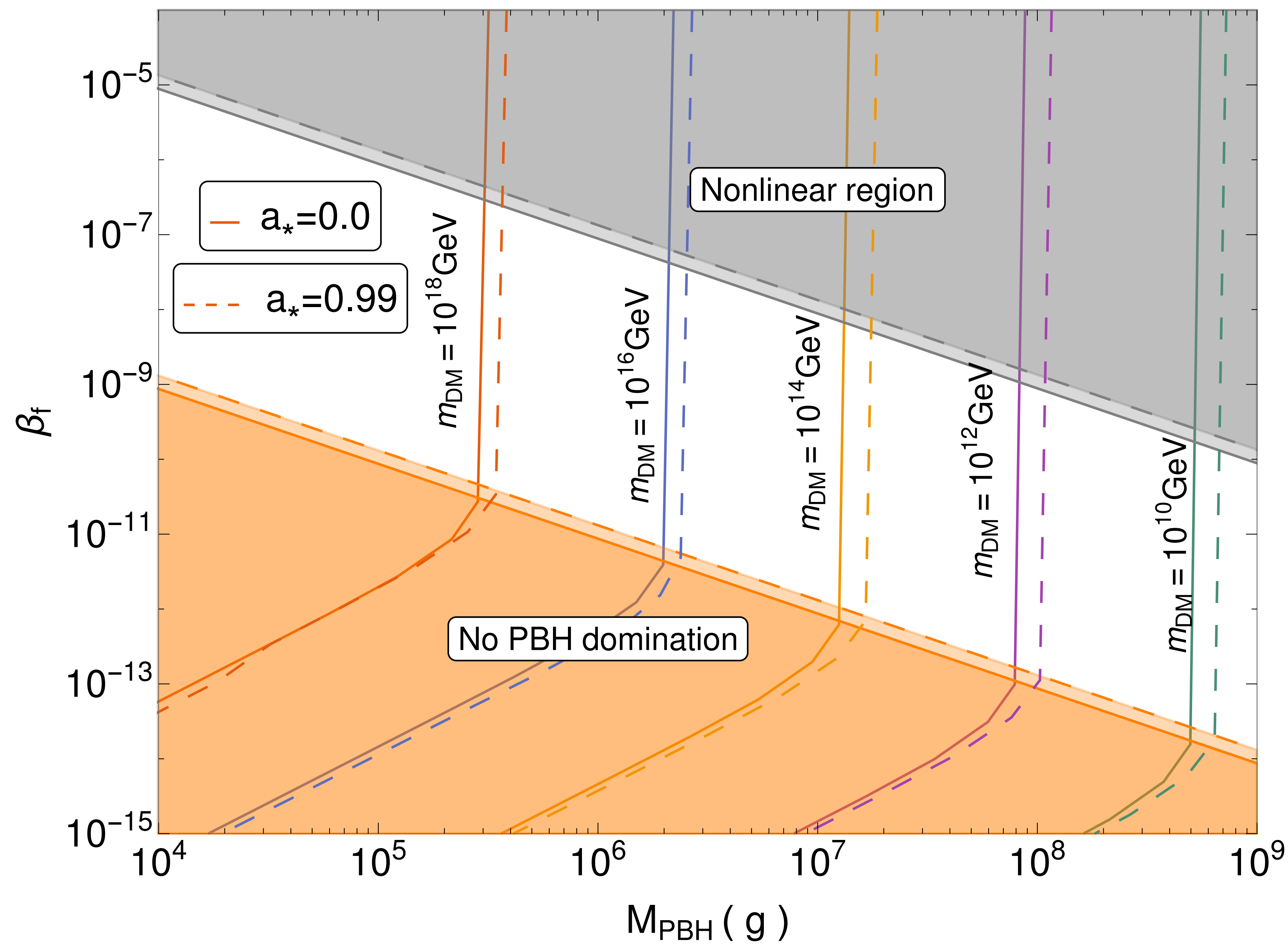
N. Bhaumik, A. Ghoshal, **RKJ** & M. Lewicki, JHEP 05, 169 (2023)

Extra relativistic dof from PBH evaporation



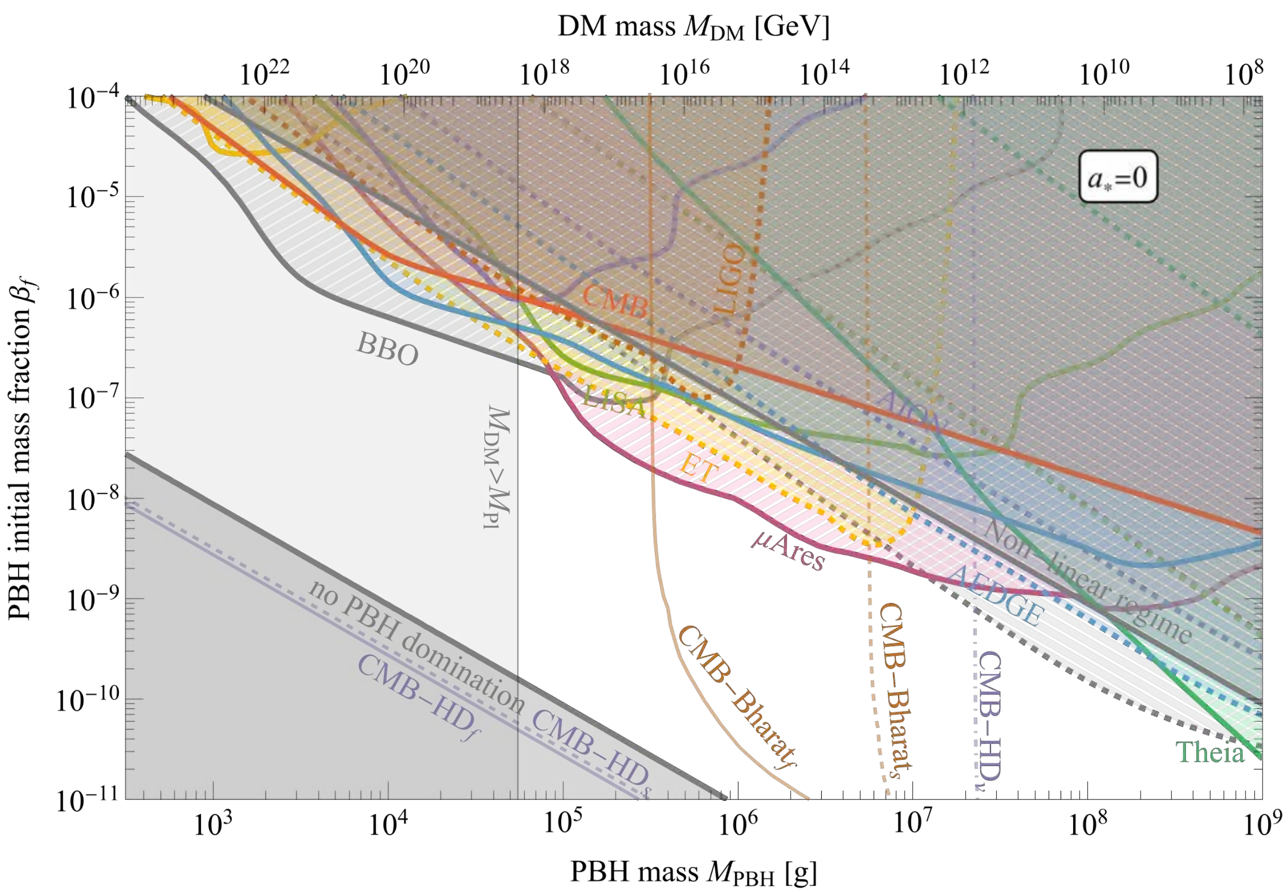
N. Bhaumik, A. Ghoshal, **RKJ** & M. Lewicki, JHEP 05, 169 (2023)

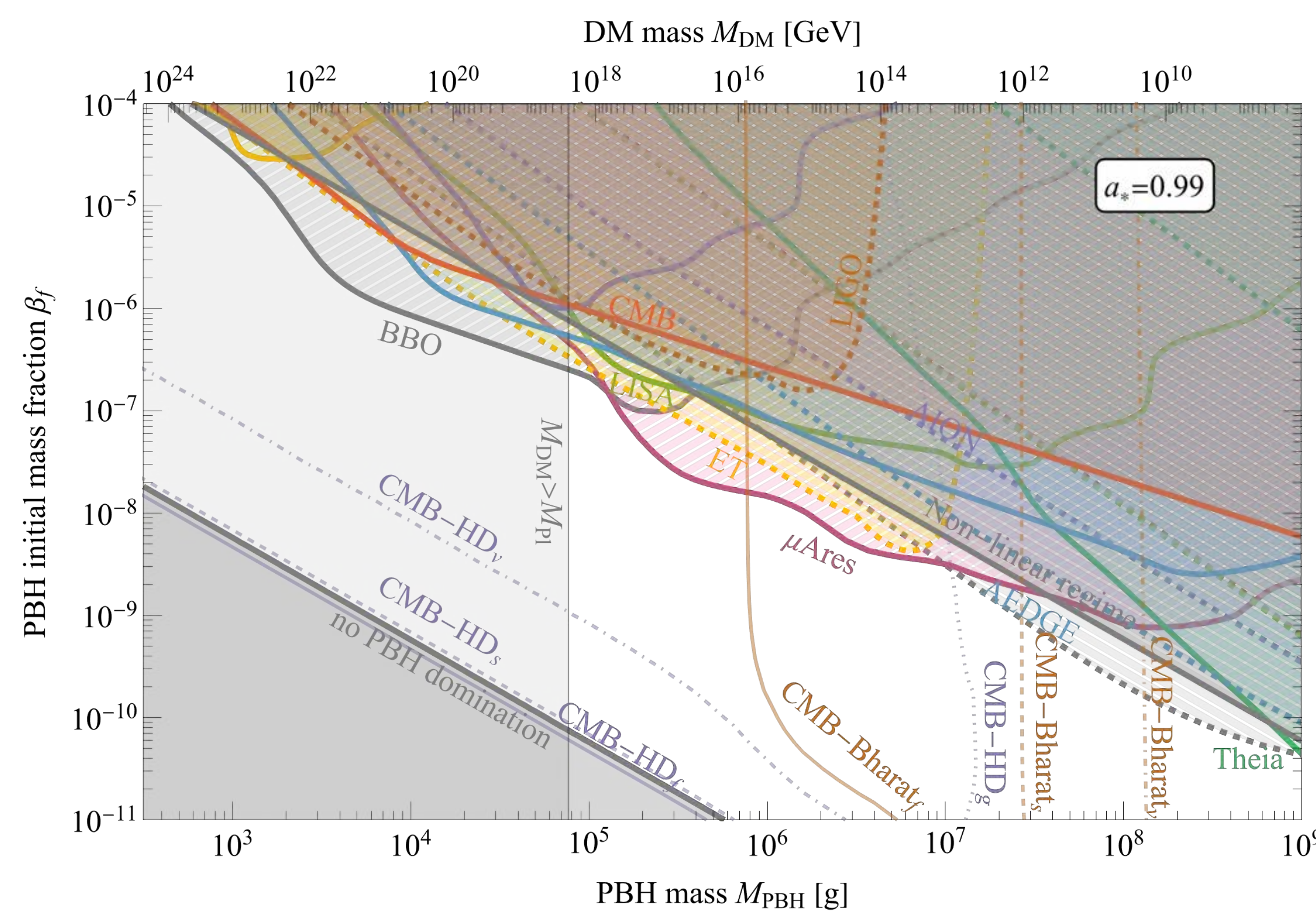
Heavy dark matter from PBH evaporation



N. Bhaumik, A. Ghoshal, **RKJ** & M. Lewicki, JHEP 05, 169 (2023)

Future GWs detector sensitivities



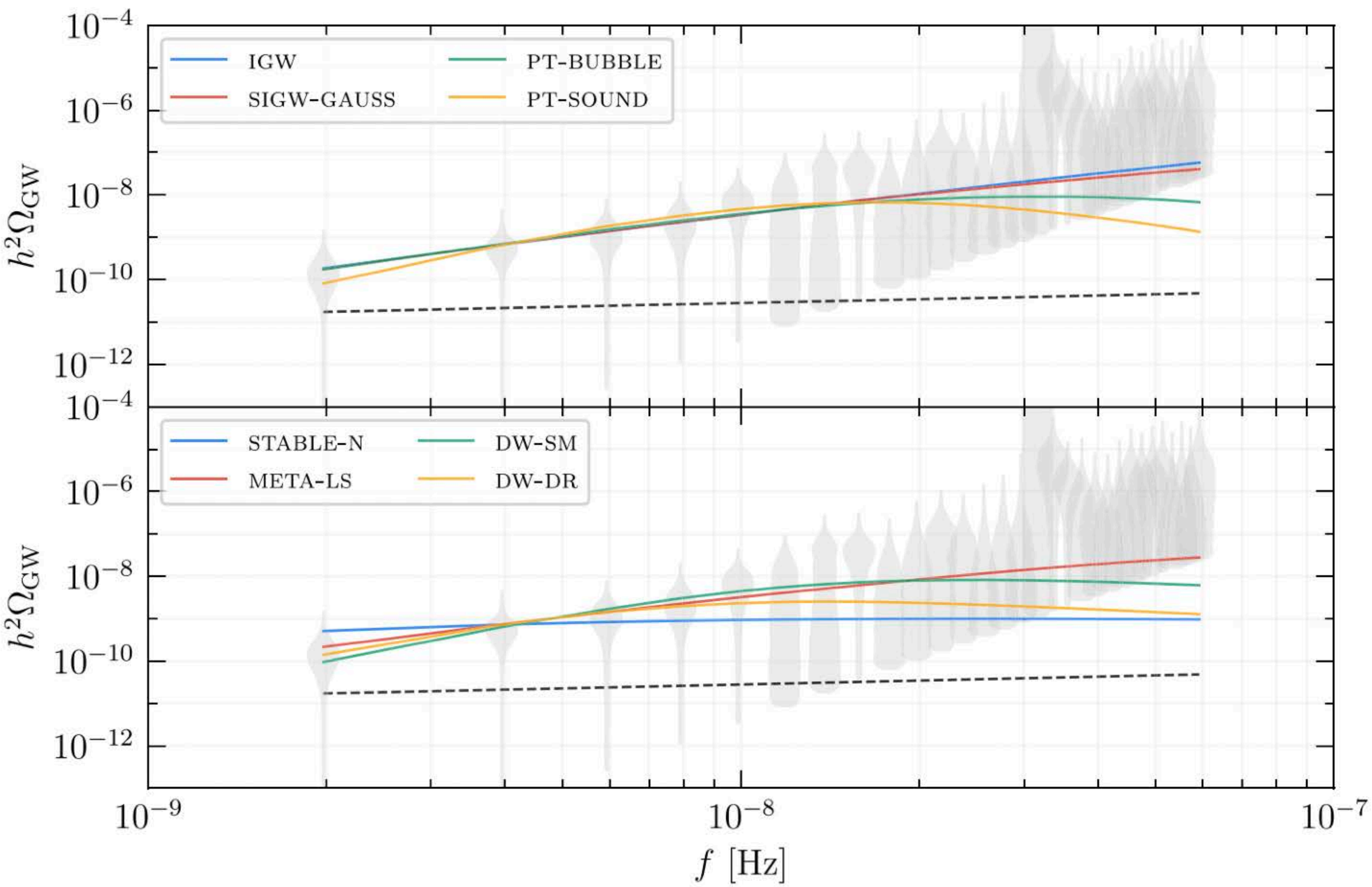


N. Bhaumik, A. Ghoshal, **RKJ** & M. Lewicki, JHEP 05, 169 (2023)

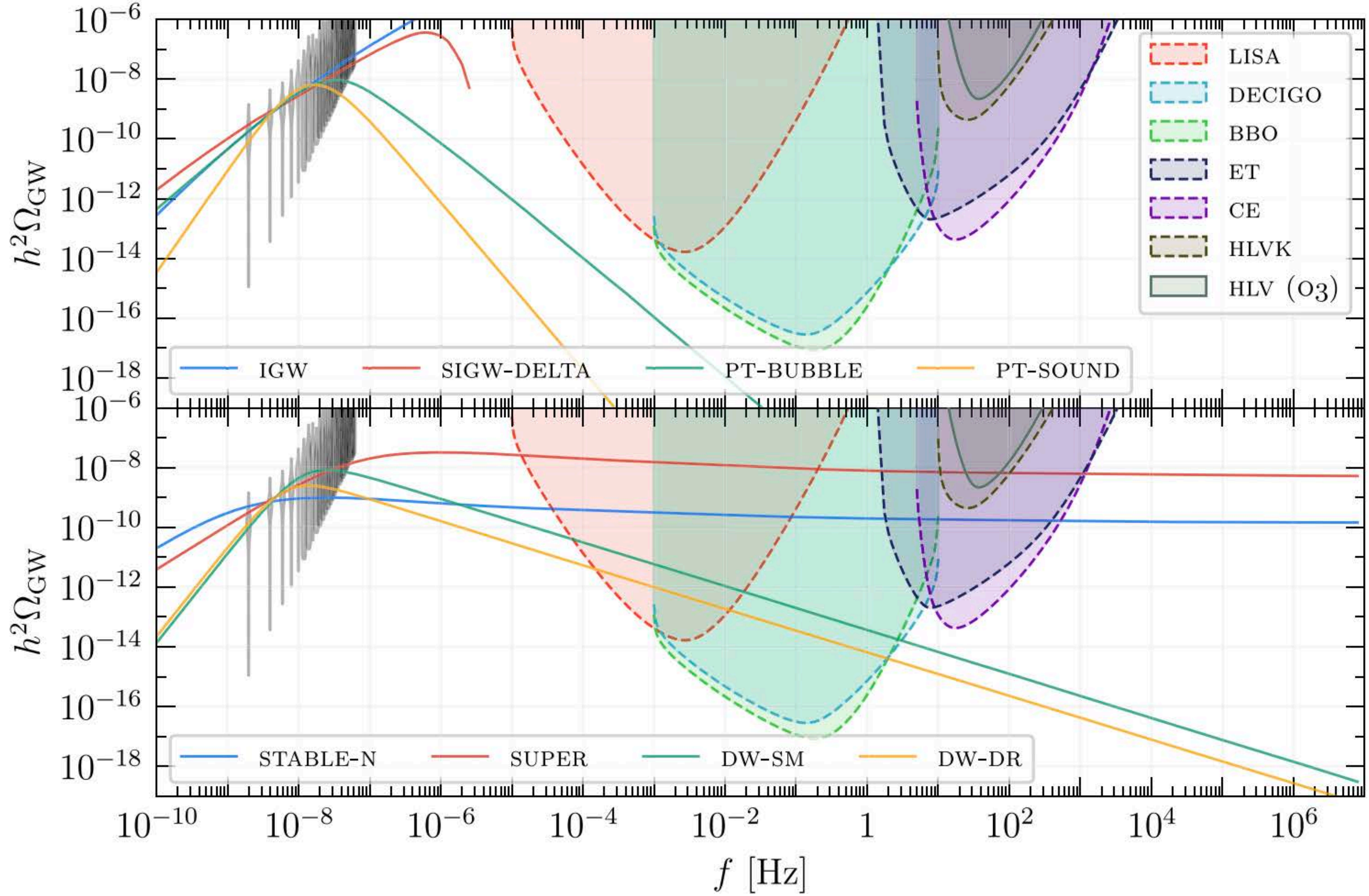
Stochastic GWs — PTA detection

THE ASTROPHYSICAL JOURNAL LETTERS, 951:L11 (56pp), 2023 July 1 https://doi.org/10.3847/2041-8213/acdc91

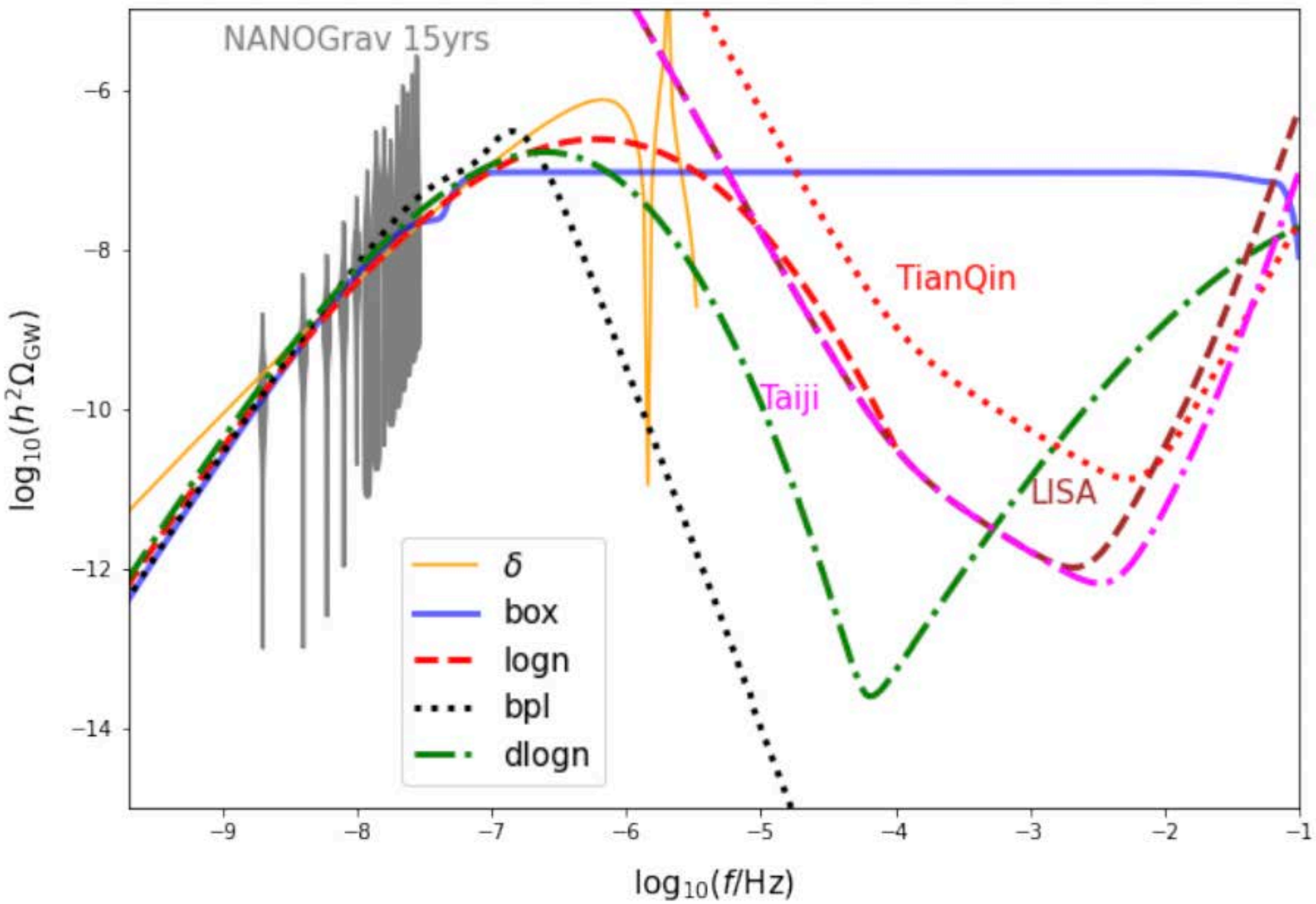
The NANOGrav 15 yr Data Set: Search for Signals from New Physics



Stochastic GWs — PTA detection

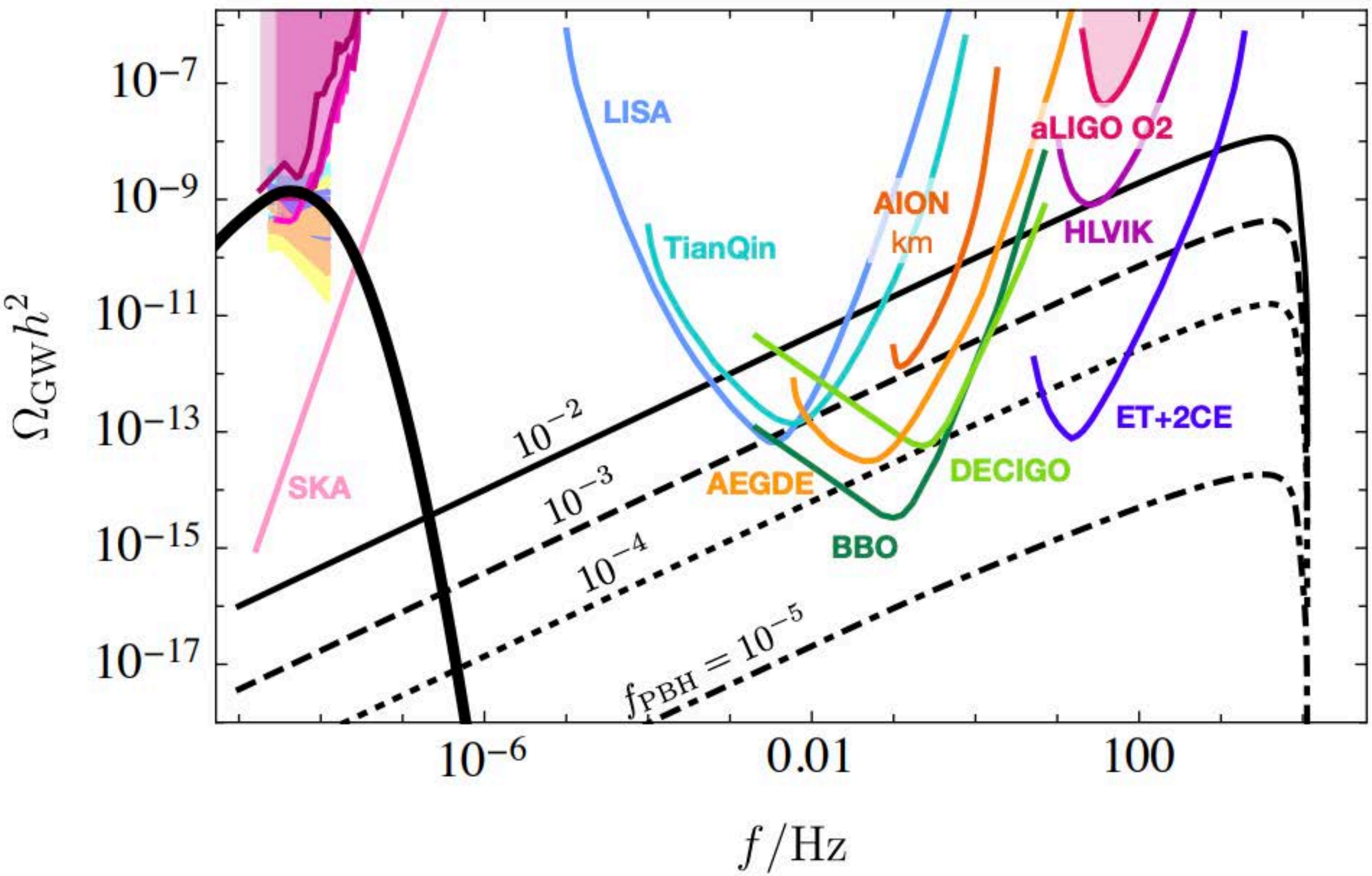


Stochastic GWs — PTA detection



You, Yi & Wu, *2307.04419*

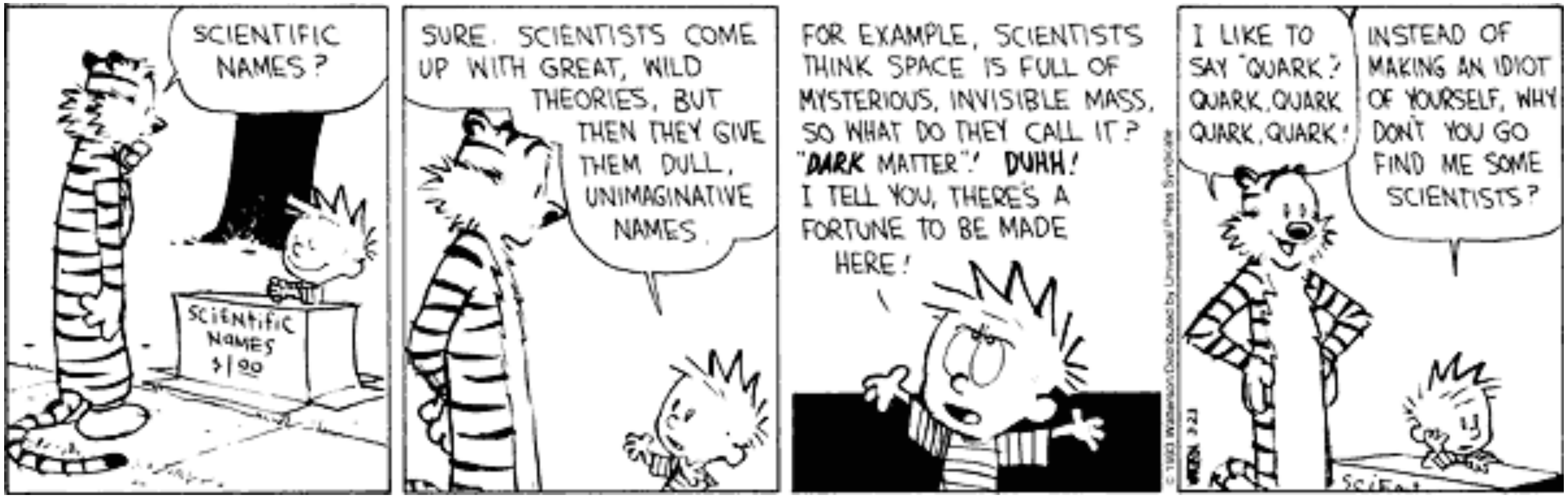
Stochastic GWs — PTA detection



Kohri & Tarada, *2009.11853*

Conclusions

- PBHs are novel candidates for the cold DM in the universe — an important probe of small scale dynamics during inflation
- Inflation can produce significant abundance of PBHs — inflection point models are very useful — model dependent results
- Interesting observational implications — induced GWs on scales probed by LISA, DECIGO or BBO
- A testable prediction with LISA — non-observation of such GWs at LISA may rule out PBHs as DM
- Hawking evaporation can lead to dark radiation and dark matter — constraints from future observations.
- NANOGrav PTA results — stochastic induced GW — one of the most probable explanation.



Thank you.

Acknowledgments:

INDIAN INSTITUTE OF SCIENCE
भारतीय विज्ञान संस्थान

SERB INDIA

Infosys Foundation

CEFIPRA