MEMORIES OF A SQUARE PULSE

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MEMORY IN COMMON SENSE

Memory is the faculty of the mind by which data or information is encoded, stored, and retrieved when needed. It is the retention of information over time for the purpose of influencing future action...

– Wikipedia on human memory.

Memory is an organism's ability to store, retain, and recall information.

– Wikipedia on memory for 'other' uses.

MEMORY IN MATTER

'Memory formation in matter is a theme of broad intellectual relevance; it sits at the interdisciplinary crossroads of physics, biology, chemistry, and computer science....'

Keim et al, RMP (2019).

Huge variety:

Memory of direction, memory of duration, memory of input, hysteresis, cyclic driving, shape memory, echoes, memristors, associative memory, initial conditions....

Most of them involve a remembrance of an influence or experience of the past.

MEMORY EFFECTS IN PHYSICS

Memory effects have to do with permanent changes induced by radiation on physical configurations, like a collection of test particles with certain positions and velocities. They are classical observable effects in the low-energy region of gravity and gauge theories.

- Web output for 'memory effects in physics'

GRAVITATIONAL WAVE

MEMORY

HISTORY

• **Zel'dovich and Polnarev:**

The idea of observing a burst-like gravitational wave through the displacement of freely falling bodies after the wave has passed was put forward in 1974 (already 50 years!). Linearised gravity.

. . . another, nonresonance, type of detector is possible, consisting of two noninteracting bodies (such as satellites). . . . the value of h_{ik} after the encounter of two objects differs from the value before the encounter. As a result the distance between a pair of free bodies should change, and in principle this effect might possibly serve as a nonresonance detector.....

Radiation of gravitational waves from a cluster of superdense stars, Soviet Astronomy(1974)

• **Braginsky and Grishchuk:**

Introduced 'memory effect' in GW physics in 1985. 'Change in deviation $\propto$ Change in h_{ij}.'Linearised gravity.

.... *In the memory effect, the distance between a pair of bodies is different from the initial distance in the presence of a gravitational radiation pulse.* ....

• **Braginsky and Thorne, Gibbons and Hawking: Linearised gravity.**

Braginsky and Thorne, in 1987, made a distinction between two types of bursts, namely, one without memory and one with memory. The same distinction had been much made earlier by Gibbons and Hawking (1971) but without explicit mention of the memory concept.

• Christodoulou, Blanchet and Damour:

In the 1990s, a nonlinear memory was discovered, independently, by Christodoulou (1991) and by Blanchet and Damour (1992). It arises from the contribution of the emitted gravitational waves to the changing quadrupole and higher mass moments. They obtained a permanent displacement.

• Strominger, Pasterski, Zhiboedov (2014):

Asymptotic symmetries, soft theorems, memory; infrared triangle.

Asymptotic symmetry BMS group and memory (Vacuum transitions); memory and soft theorems (Fourier transform); asymptotic symmetry and soft theorems (Ward identities).

• Gibbons, Horvathy, Duval, Zhang (2017):

Uses exact pp-wave geometry for GWs. Displacement and velocity memory in pp-wave spacetimes with different pulse shapes. We will focus on this approach in the next part of this talk.

• Flanagan, Grant, Nichols, Harte (2019-):

Persistent gravitational wave observables. Attempt to define different observables which show permanent changes. Done both in the perturbative and non-perturbative regimes.

• Review by M. Favata (2010): 1003.3486, see also Mitman et al, 2405:08868.

- Observational status:

- Gravitational wave memory due to bursts. Arms of GW interferometers may experience a permanent change in ‘effective proper length’ due to a GW burst.

Can we isolate any cumulative d.c. effect caused by the many events that have hit the apparatus?

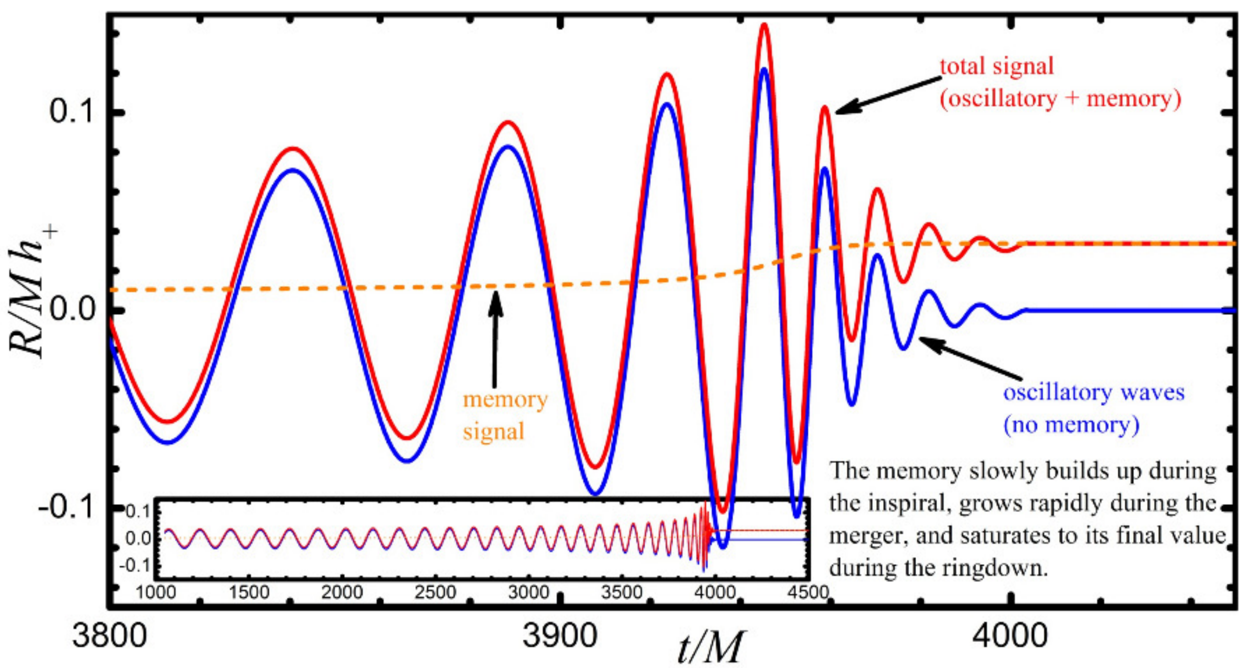
No detections yet as stated in Cheung, Lasky, Thrane, CQG (2024)

It seems one requires $\mathcal{O}(2000)$ BBH merger data for a significant observable effect!

WHAT TO DETECT? PROSPECTS

- The expected signal:

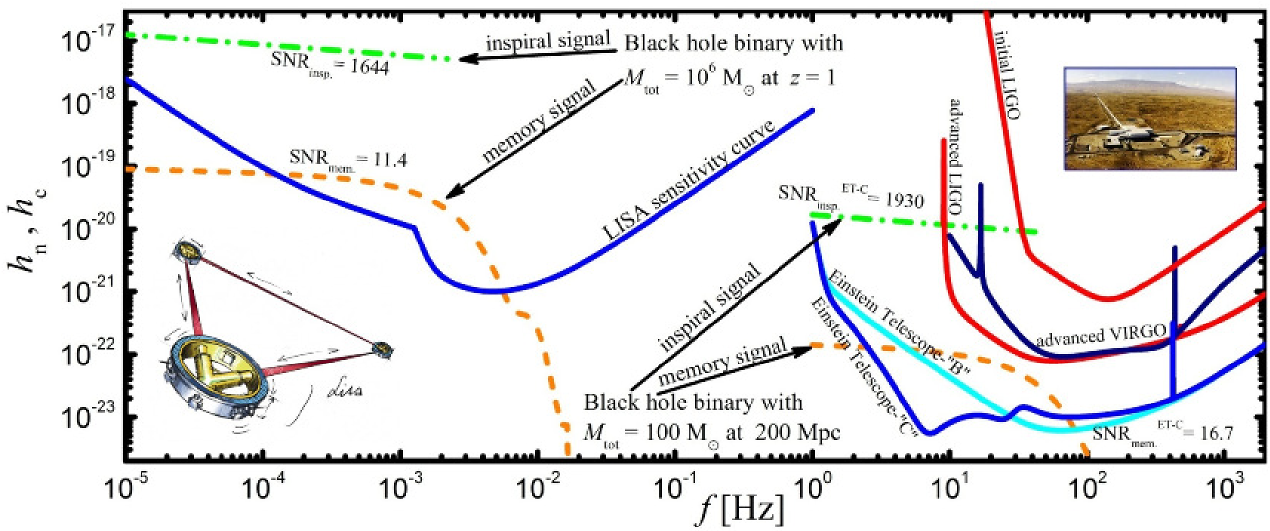
From a talk by M. Favata.



- Overall tiny DC effect.

- **Possible detections:**

From a talk by M. Favata.



- **Two cases: (1) BH Binary with $M_{tot} = 10^6 M_{sun}$ at $z = 1$; (2) BH Binary with $M_{tot} = 100 M_{Sun}$ at 200 Mpc.**

- **For (2), Strain Amplitude is 10^{-23} (detector arm 10^3 to 10^4 m) and change in length due to memory signal is 10^{-19} m.**

• Detectability (Favata (2009)):

• Memory from a GW that passed through a region in the distant past (long before the start of the observation) is itself difficult to detect.

• What is actually observable is the buildup of memory over some observation time.

• LISA appears to have better chances because we have 'free fall'. In LIGO/VIRGO etc. mirrors are fixed via servo-mechanisms.

• Memory manifest in 'different' incarnations of Minkowski spacetime. The link with BMS symmetries and asymptotic symmetry group. How?

• Choose TT coordinates. z axis along wave direction. Late time metric:

$$ds^2 = -dt^2 + (1 + h_+)dx^2 + (1 - h_+)dy^2 + 2h_\times dx dy + dz^2$$

Take h_+, $h_\times$ constant near detector for a d.c. memory signal.

Transform to a new set of coordinates given by (T, X, Y, Z):

$$T = t \;\; ; \;\; Z = z$$

$$X = x + \frac{h_\times - \sqrt{h_+}\sqrt{1 - h_+^2 - h_\times^2}}{1 + h_+}\, y$$

$$Y = \sqrt{h_+}\, x + \frac{h_\times\sqrt{h_+} + \sqrt{1 - h_+^2 - h_\times^2}}{1 + h_+}\, y$$

where $ds^2 = -dT^2 + dX^2 + dY^2 + dZ^2$.

Memory manifest in the coordinate transformation → asymptotic symmetries.

OUR APPROACH (THEORETICAL)

• Motivated by gravitational wave physics research. But we do not use linearised gravity.

• Do trajectories remember that they have encountered a burst or pulse in the past? How? Is it quantified, measurable?

• To develop a model:

→ we need a spacetime with gravitational waves near the detector.

→ a way to represent pulses or bursts

→ study individual or relative motion of test objects.

→ Infer about permanent changes from such analyses.

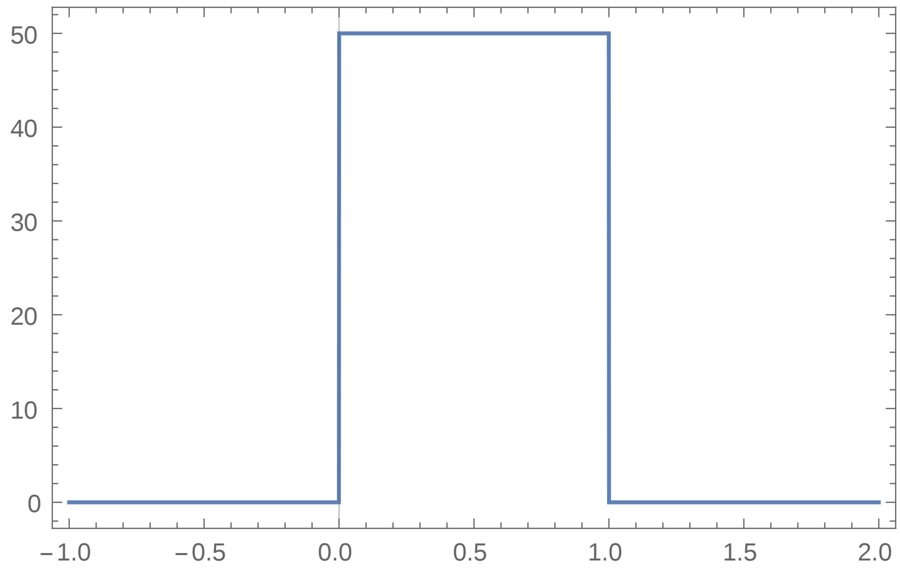
WHAT WE DO

- Spacetime: plane parallel wave (pp-waves).
- Pulse/burst: Square or sech-squared.
- Test objects: particles or test strings.
- What to study: propagation from past to future.
- Compare: situations much before and much after the pulse. Quantify any difference.

The main question:

Does the pulse leave an imprint in the future behaviour of test objects? How?

THE SQUARE PULSE



• **The Fourier transform of a square pulse is a 'Sinc $\frac{\omega}{2}$' function for a pulse between $(-\frac{1}{2}, \frac{1}{2})$, of unit height. It is peaked around $\omega \sim 0$, i.e. low energies, 'soft'. Generic pulses behave similarly.**

We look for different signatures and scenarios which characterise memories of a square pulse.

THE PP-WAVE SPACETIME

• **Vacuum solutions of the Einstein equations of GR (Brinkmann coordinates)**

$$ds^2 = -H(u,x,y)du^2 - 2dudv + dx^2 + dy^2$$

where, $u = t - z$, $v = t + z$.

• $H_{,xx} + H_{,yy} = 0$ **from EE. This leads to**

$$H(u,x,y) = \frac{1}{2}A_+(u)[x^2 - y^2] + A_\times(u)xy$$

$A_+(u)$ **and** $A_\times(u)$ **are the two polarisations. These are free functions.**

• **The Riemann tensor:**

$$R_{xuxu} = \frac{1}{2}A_+(u) = -R_{yuyu}; R_{xuyu} = \frac{1}{2}A_\times(u)$$

•**The Riemann curvature changes with** u **and has a fixed value on a** u **=const. hypersurface. Fixed** u **hypersurfaces are the planar wavefronts.**

• One can also write the metric using the BJR coordinates through which one can see the link with the perturbative h_{ij}.

$$ds^2 = -2dudV + a_{ij}(u)dX^i dX^j$$

• u is the null coordinate. Considering $u = t - z$, curvature disturbances propagate along z with the speed of light.

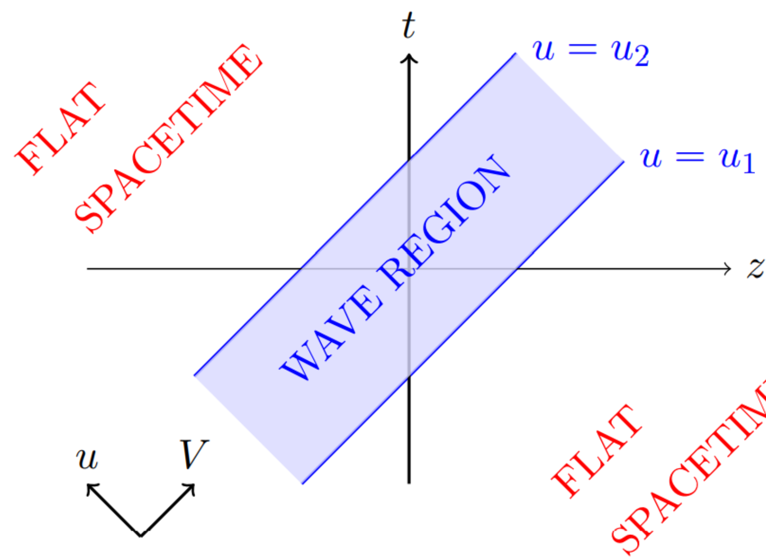
• The spacetime gets distorted in the space orthogonal (i.e. along X^1, X^2) to the direction of propagation.

• Constant u hypersurfaces correspond to planar wavefronts. Hence, a plane gravitational wave spacetime.

• The function $a_{ij}(u)$ encodes the gravitational wave field. If $a_{ij} = I_{2\times2}$, the full metric is manifestly Minkowski .

• In the transverse traceless (TT) gauge, linearized plane waves can be written in this coordinate system using $a_{ij} = \delta_{ij} + h_{ij}^{TT}$.

• Coordinate transformations between BJR and Brinkmann are known (see Gibbons et al (PRD, 2017)).



• Also known as a sandwich wave.

We will use $A_+(u)$ only and assume it as a square pulse.

MEMORY IN GEODESICS

- Note that u **behaves as an affine parameter** for geodesics. v variation gives $\ddot{u} = 0$.

- Geodesic eqns (x,y) in Brinkmann coordinates having both non-zero polarizations.

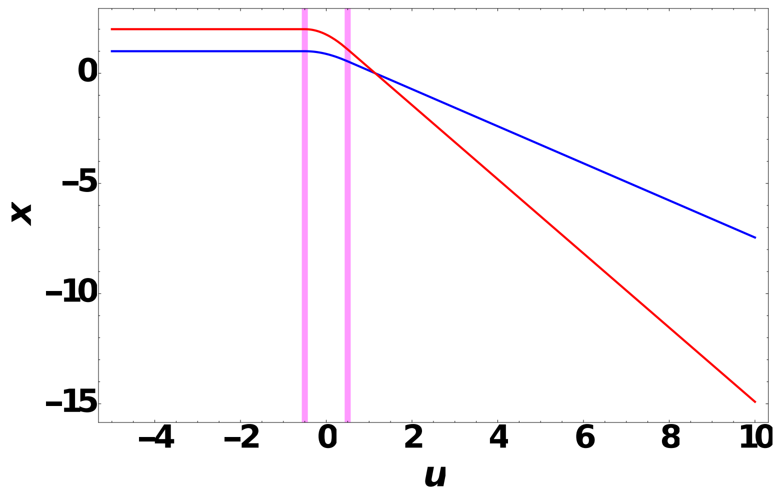
$$\ddot{x} = -\frac{1}{2}A_{+}(u)x - \frac{1}{2}A_{\times}(u)y$$
$$\ddot{y} = \frac{1}{2}A_{+}(u)y - \frac{1}{2}A_{\times}(u)x$$

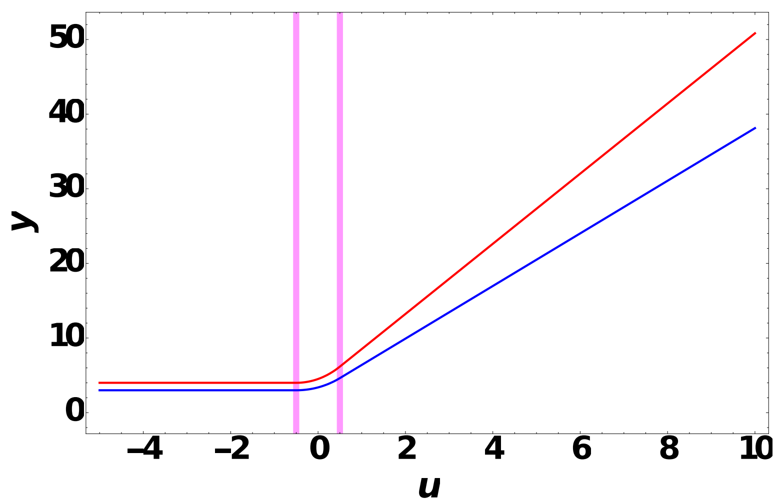
- v eqn. can be solved once x,y are known.

- We can solve **analytically for x, y** with $A_{\times}(u) = 0$ and **$A_{+}(u)$ as a square pulse of height A_0 and width $2a$**.

- Match **x, y and $\dot{x}$, $\dot{y}$** at the boundaries for full profile.

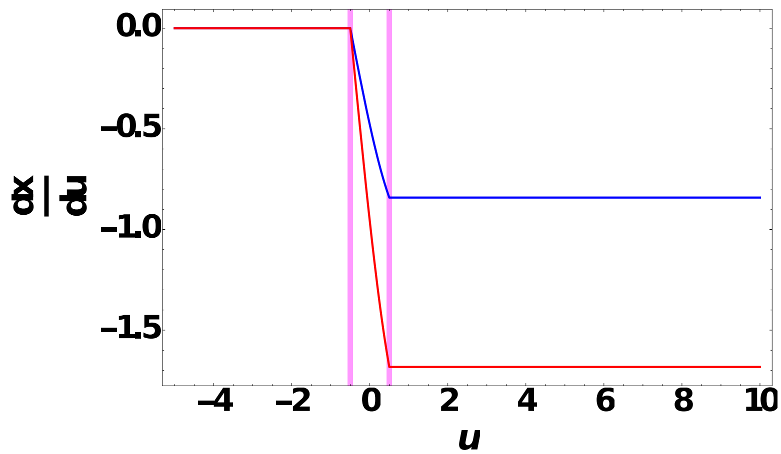
• x(u), y(u) evolution. Constant separation before. Vertical lines indicate pulse location.

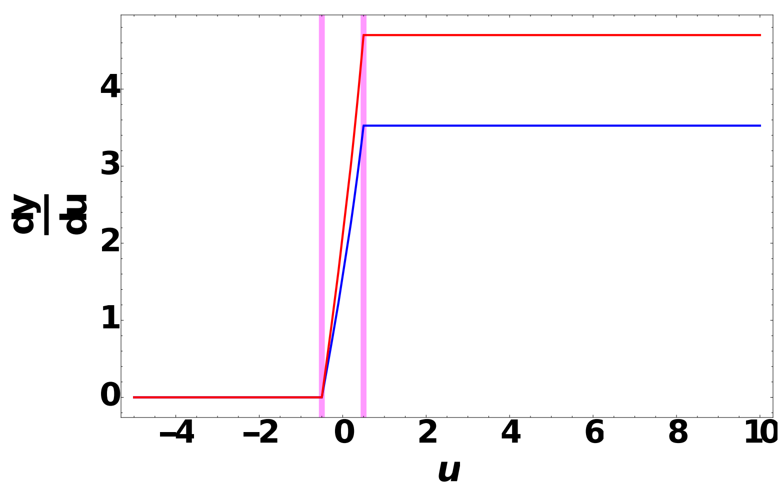




Change in separation after the pulse. Zero separation along x. Displacement memory.

- $\dot{x}(u)$, $\dot{y}(u)$ evolution.





Change in velocity along x and y. Velocity memory effect.

• **Behaviour of a ring of particles following geodesics, for a ‘plus’ pulse.**

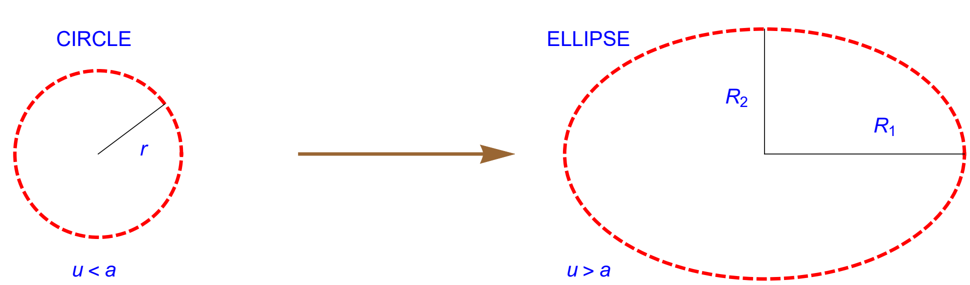
→ **In Region-I ($u \le -a$), the solutions are:** $x(u) = \alpha = r\cos\phi$ **and** $y(u) = \beta = r\sin\phi$. **Thus, the locus corresponding to the initial configuration is a circle:** $x^2 + y^2 = r^2$.

→ **In Region-III ($u \ge a$), we have,**

$$x(u) = r[\cos 2\xi - (\nu - 1)\xi \sin 2\xi]\cos\phi = R_1 \cos\phi$$
$$y(u) = r[\cosh 2\xi + (\nu - 1)\xi \sinh 2\xi]\sin\phi = R_2 \sin\phi$$

→ **Here** $\xi = aA_0, u = \nu a (\nu > 1)$. **One can check that** $R_1 \neq R_2$. **Hence, after the pulse departs, the loci becomes an ellipse.**

$$\frac{x^2}{R_1^2} + \frac{y^2}{R_2^2} = 1$$



• The nature of the ellipse is determined by $\xi = aA_0$. Thus, the character of the pulse determines the change in shape of the configuration.

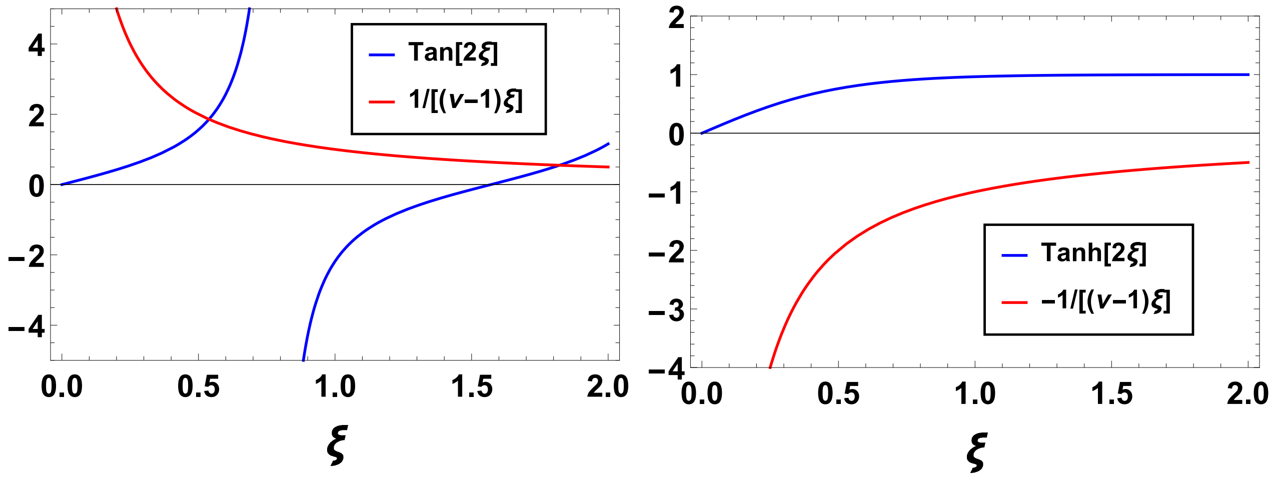
• To observe focusing we have to check whether R_1 and/or R_2 vanish. Setting $R_1 = 0$ we get,

$$\tan(2\xi) = \frac{1}{(\nu - 1)\xi}$$

Similarly, setting $R_2 = 0$ gives,

$$\tanh(2\xi) = -\frac{1}{(\nu - 1)\xi}$$

• The above eqns. are transcendental and hence, analytic solutions are not possible. We try to find solutions using plots.

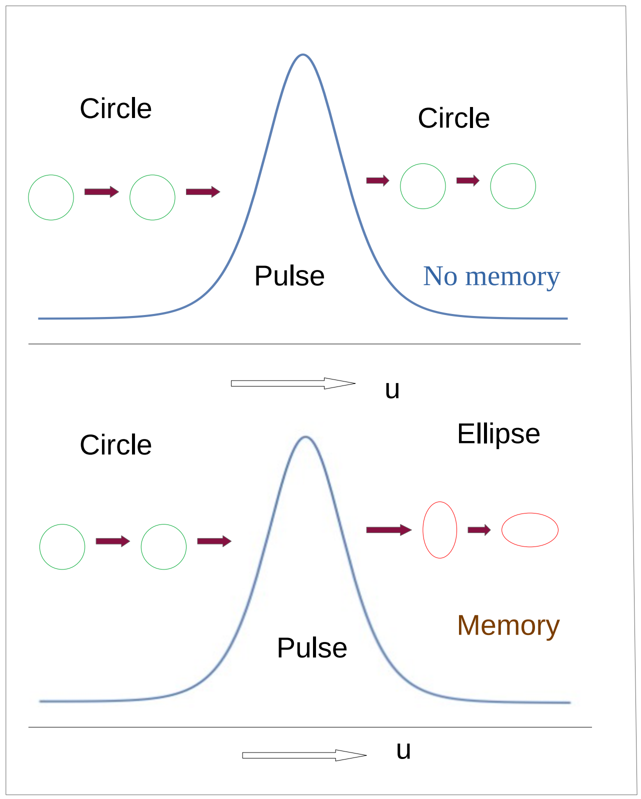


• The figures show that there exist solutions for ξ when $R_1 = 0$. But, no solution exists for $R_2 = 0$. Thus, we find that the ellipse degenerates to a straight line along the y-direction.

•This is consistent with our previous x, y plots, where we had shown the formation of a benign caustic as the geodesic separation along x-direction vanishes (i.e. $x_2 - x_1 = 0$ in $u > a$) at a finite u-value.

• One may calculate the expansion, shear and rotation (the kinematic variables) corresponding to this two-dimensional deformation. See IC, SK (EPJP, 2022).

Qualitative picture: ring of particles, memory as a permanent change of shape.



We have 'realised' this picture using geodesics and geodesic deviation in pp-wave spacetimes. IC & SK, EPJP (2022). We now look at test strings.

MEMORY IN CLOSED STRINGS

- Ring of particles $\rightarrow$ continuum generalisation $\rightarrow$ closed string.

- Look at string propagation in a pp-wave geometry where we have a square pulse.

- Question: Imagine a closed circular string in the past. What happens to it after it interacts with a pulse?

- Check out the string profile in the future when there is no pulse.

- Is there a permanent change? In what sense?

• 4D background metric g_{ij}. Embedding $x^i(\tau, \sigma)$, where τ, σ are world sheet coordinates. Nambu-Goto string.

• String equations of motion and constraints:

$$\ddot{x}^i - x^{i''} + \Gamma^i_{jk} \left(\dot{x}^j \dot{x}^k - x^{j'} x^{k'} \right) = 0$$

$$g_{ij} \dot{x}^i x^{j'} = 0 \quad , \quad g_{ij} \dot{x}^i \dot{x}^j + g_{ij} x^{i'} x^{j'} = 0$$

dot, prime $\rightarrow$ differentiation w.r.t. τ,σ. Induced metric $\gamma_{ab} = g_{ij} \partial_a x^i \partial_b x^j$ diagonal.

• Solve eqns. and constraints in a background pp-wave with a chosen pulse in g_{ij}.

• Some earlier works on strings in pp-wave spacetimes: Amati and Klimcik (1988), Horowitz and Steif (1990), de Vega and Sanchez (1992), de Vega, Ramon-Medrano and Sanchez (1993).

• Recent: Li ˇska and von Unge (2022).

- **Goal:** For a square pulse, find $x^i(\tau, \sigma)$ in the regions **before** the pulse arrives, **during** its existence and **after** it departs.
- **pp-wave spacetime:**

$$ds^2 = F(u, x, y)du^2 - 2dudv + dx^2 + dy^2$$

$$F(u, x, y) = W(u)(x^2 - y^2)$$

- **x, y equations:**

$$\left(\partial_\tau^2 - \partial_\sigma^2\right) x = \frac{p^2}{2} \partial_x F$$
$$\left(\partial_\tau^2 - \partial_\sigma^2\right) y = \frac{p^2}{2} \partial_y F$$

- **Procedure:** $u = p\tau$ solves u equation. Given F one can find x and y, which can be used to find v. Choose the W in F and solve for x, y and obtain v.

- **Separation of variables:**

$$x(\tau, \sigma) = (\cos k_1 \sigma)\, x(\tau)$$
$$y(\tau, \sigma) = (\sin k_1 \sigma)\, y(\tau)$$

- **Equations for $x(\tau)$ and $y(\tau)$:**

$$\ddot{x} + \left(k_1^2 - p^2 W(\tau)\right) x = 0$$
$$\ddot{y} + \left(k_1^2 + p^2 W(\tau)\right) y = 0$$

where the dot denotes differentiation w.r.t. τ (or, equivalently, u – except a constant factor 'p').

- **Initial profile: In the remote past when $W(\tau)$ is zero we assume**

$$x(\tau, \sigma) = R \cos k_1 \tau \, cos k_1 \sigma \quad (1)$$
$$y(\tau, \sigma) = R \cos k_1 \tau \, sin k_1 \sigma \quad (2)$$

which yields a circle with radius $R \cos k_1 \tau$, in the past.

- **Future profiles:**

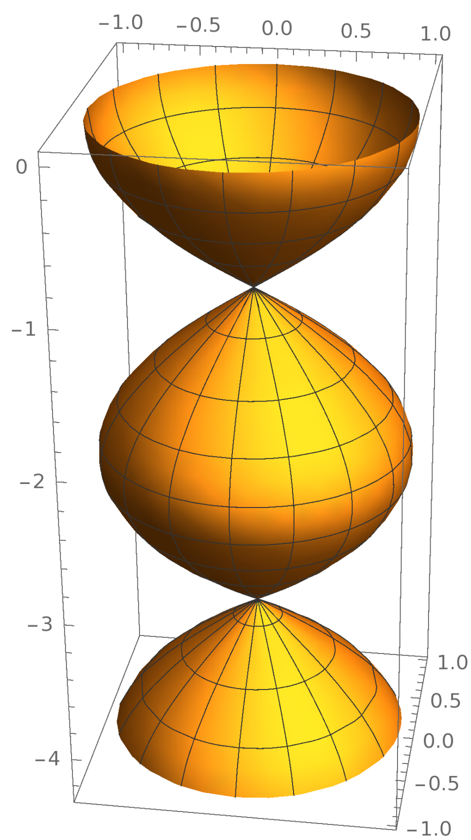
We now focus on the full profiles, especially in future.

Pulsating string before the pulse arrives.

Region $\tau \leq 0$:

$$u = \frac{k_1 R}{\sqrt{2}}\tau \quad ; \quad v = \frac{k_1 R}{\sqrt{2}}\tau$$

$$x = R \cos k_1\tau \cos k_1\sigma \quad ; \quad y = R \cos k_1\tau \sin k_1\sigma$$



Singular worldsheet. Circular string.

String during the pulse duration.

Region $0 \leq \tau \leq T$**:**

$$u = \frac{k_1 R}{\sqrt{2}} \tau \, ;$$

$$x = R \cos k_2 \tau \cos k_1 \sigma \, ; \; y = R \cos k_3 \tau \sin k_1 \sigma$$

where,

$$k_2 = \sqrt{k_1^2 - W_0 p^2} \quad ; \quad k_3 = \sqrt{k_1^2 + W_0 p^2}$$

$$v(\tau, \sigma) = -\frac{R}{2\sqrt{2}} \frac{\cos 2k_1 \sigma}{k_1^2} G_0(\tau) + H_0(\tau)$$

where,

$$G_0(\tau) = k_1 k_2 \cos k_2 \tau \sin k_2 \tau - k_1 k_3 \cos k_3 \tau \sin k_3 \tau$$

and

$$H_0(\tau) = \frac{k_1 R}{\sqrt{2}} \tau + \frac{k_1^2 - k_2^2}{2\sqrt{2} k_1} R \left(\frac{\sin(2k_2 \tau)}{2k_2} - \frac{\sin(2k_3 \tau)}{2k_3} \right)$$

Note the profiles for $\tau \leq 0$ **and** $0 \leq \tau \leq T$ **match at** $\tau = 0$**.**

String after the pulse departs.

Region $\tau \geq T$:

$$x(\tau,\sigma) = R\left(cos(k_2T)cos[k_1(\tau-T)] - \frac{k_2}{k_1}sin(k_2T)sin[k_1(\tau-T)]\right)cos(k_1\sigma)$$

$$y(\tau,\sigma) = R\left(cos(k_3T)cos[k_1(\tau-T)] - \frac{k_3}{k_1}sin(k_3T)sin[k_1(\tau-T)]\right)sin(k_1\sigma)$$

$$u = \frac{k_1 R}{\sqrt{2}}\tau \quad ; \quad v(\tau,\sigma) = -\frac{R}{2\sqrt{2}}\frac{\cos 2k_1\sigma}{k_1^2}G(\tau) + H(\tau)$$

where,

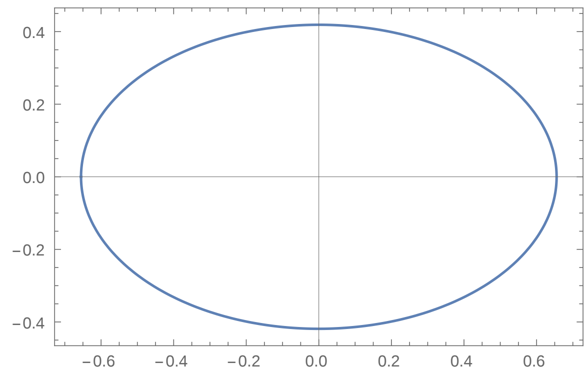
$$k_2 = \sqrt{k_1^2 - W_0 p^2} \quad ; \quad k_3 = \sqrt{k_1^2 + W_0 p^2}$$

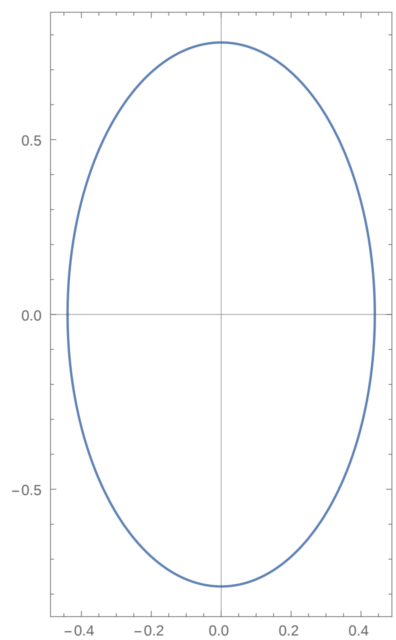
$$\begin{aligned} G(\tau) = \frac{1}{2}&\left(k_1^2\cos^2 k_2T - k_2^2\sin^2 k_2T\right)\sin\,2k_1(\tau-T) \\ &+k_1k_2\cos k_2T\sin k_2T\cos 2k_1(\tau-T) \\ &-\frac{1}{2}\left(k_1^2\cos^2 k_3T - k_3^2\sin^2 k_3T\right)\sin\,2k_1(\tau-T) \\ &-k_1k_3\cos k_3T\sin k_3T\cos 2k_1(\tau-T) \end{aligned}$$

$$\begin{aligned} H(\tau) = \frac{R}{2\sqrt{2}k_1}&\left(k_1^2\cos^2 k_2T + k_2^2\sin^2 k_2T + k_1^2\cos^2 k_3T + k_3^2\sin^2 k_3T\right)(\tau-T) \\ &+\frac{k_1R}{\sqrt{2}}T + \frac{k_1^2-k_2^2}{2\sqrt{2}k_1}R\left(\frac{\sin(2k_2T)}{2k_2} - \frac{\sin(2k_3T)}{2k_3}\right) \end{aligned}$$

The profiles for $\tau \geq T$ and $0 \leq \tau \leq T$ match at $\tau = T$. v has a derivative discontinuity.

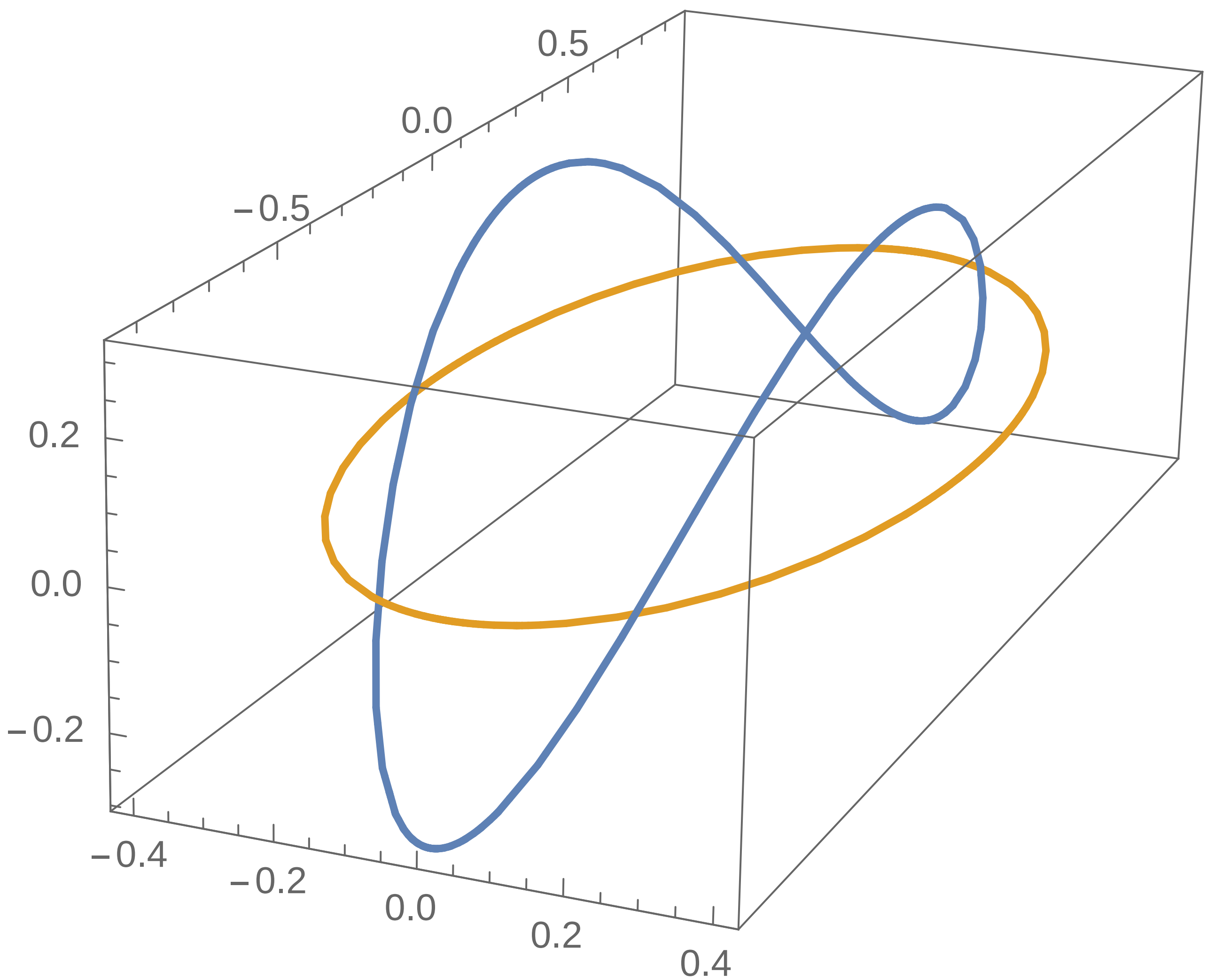
xy profile after the wave departs. Initially circular.





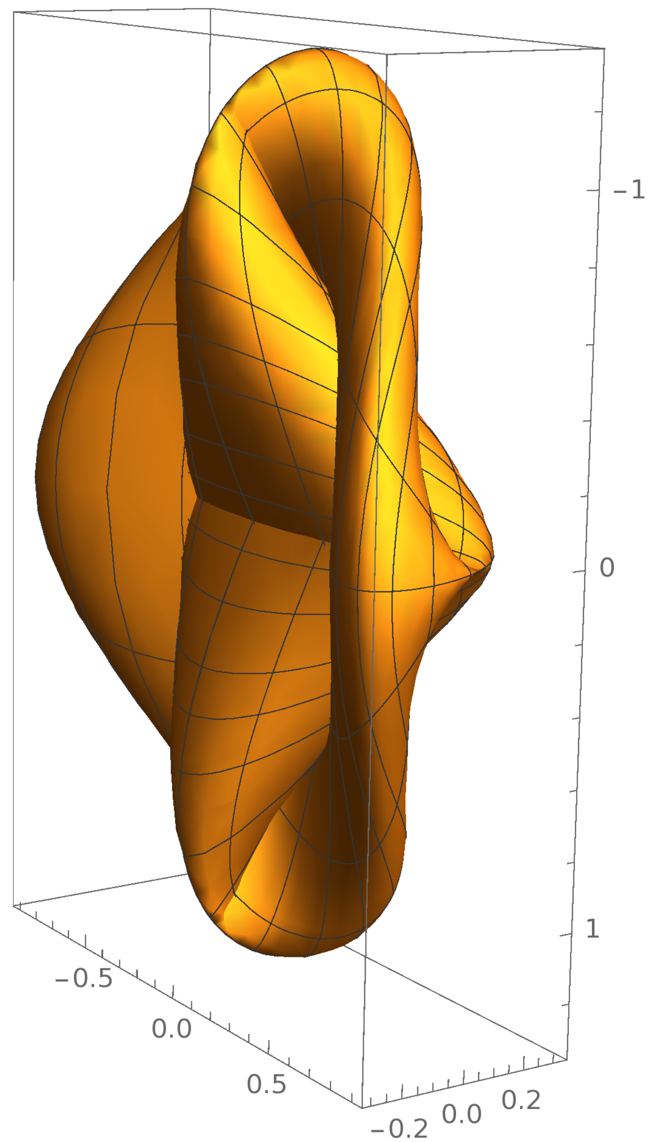
Change of shape. Circle$\rightarrow$ ellipse

String shape before and after: xyz' profile



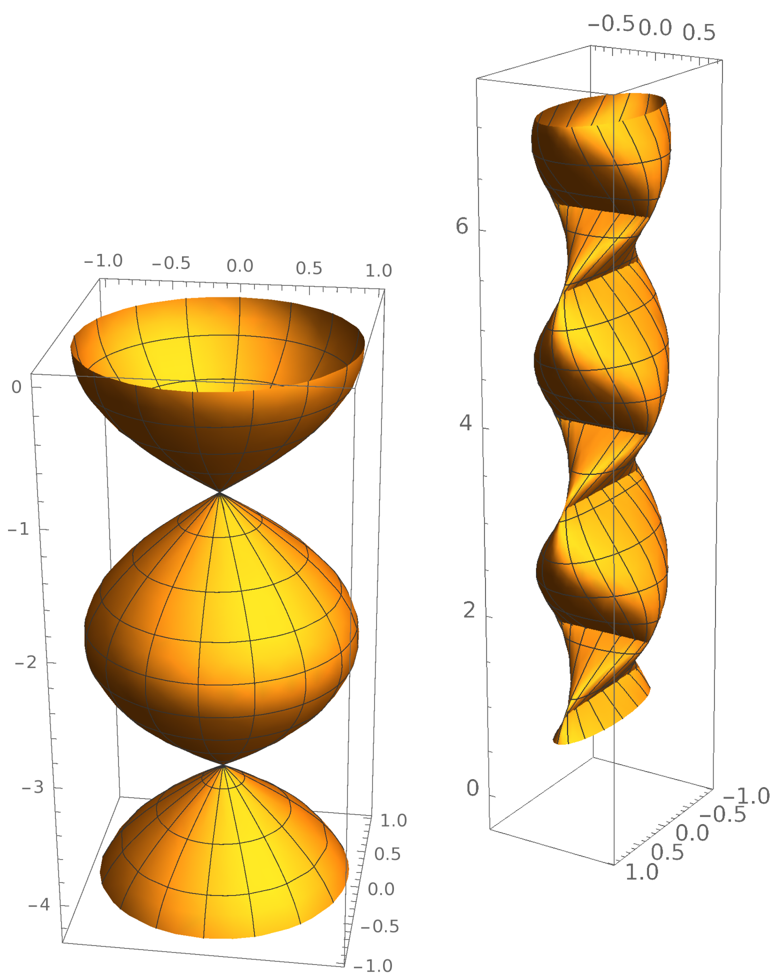
Yellow → before, Blue → after.

xyz' profile over extended time.



Note compact shape. Wrapped strings.

The xyu profiles. Before (left), after(right).



No singularities on worldsheet. Note the degenerate ellipses along x, y in the right.

Worldsheet geometry

• **Induced worldsheet metric (before the pulse):**

$$ds^2_{\tau\leq 0} = R^2 k_1^2 \cos^2 k_1\tau \left(-d\tau^2 + d\sigma^2\right)$$

The metric is degenerate (zero determinant) at all $\tau = \frac{(2n+1)\pi}{2k_1}$ ($n = 0, 1, 2....$).

For $\tau \geq T$ metric is given as:

$$ds^2 = \Omega^2(\tau,\sigma)\left(-d\tau^2 + d\sigma^2\right)$$

where

$$\Omega^2(\tau,\sigma) = R^2 \sin^2 k_1\sigma \left[k_1 \cos k_2T \cos k_1(\tau - T) - k_2 \sin k_2T \sin k_1(\tau - T)\right]^2 + R^2 \cos^2 k_1\sigma \left[k_1 \cos k_3T \cos k_1(\tau - T) - k_3 \sin k_3T \sin k_1(\tau - T)\right]^2$$

Is this metric never degenerate? Possible if $\Omega^2(\tau,\sigma)$ is not zero for any value of τ, σ.

Ω^2 is a sum of two squares, it can only be zero if the individual terms are both zero.

This seems possible, if

$$k_2 \tan\, k_2 T = k_3 \tan k_3 T = \frac{k_1}{\nu}$$

where $\tan k_1(\tau_c - T) = \nu$**. Also, we have** $k_2 < k_3$ **and** $k_2^2 + k_3^2 = 2k_1^2$**.**

For example, assuming, in appropriate units, $k_1 = \frac{5}{\sqrt{2}}$**,** $k_2 = 3$ **and** $k_3 = 4$**, we can easily see that** $T = m\pi$ **solves the first equation. Using** $m = 1$**, we get one value** $\tau_c = \pi(1 + \frac{\sqrt{2}}{10})$ **(others are there too) from the second equation. In fact,** $T = m\pi$ **will always satisfy the first equation as long as** k_2**,** k_3 **are integers satisfying the previously stated constraints.**

Hence, at such τ **values, for the specific** T**, one does get** $\Omega^2 = 0$**, but, for all other** T **one can have** $\Omega^2 \neq 0$ **everywhere.**

Therefore, if the pulse is such that its width T **is different from the set of** T **values which yield** $\Omega^2 = 0$ **at some** $\tau = \tau_c$**, one does indeed end up with a non-degenerate metric.**

- Worldsheet Ricci scalar

The Ricci scalar $\mathcal{R}$ of the two dimensional worldsheet also undergoes a change in character after the passage of the pulse.

Recall that the expression for the Ricci scalar in terms of the conformal factor Ω^2 is

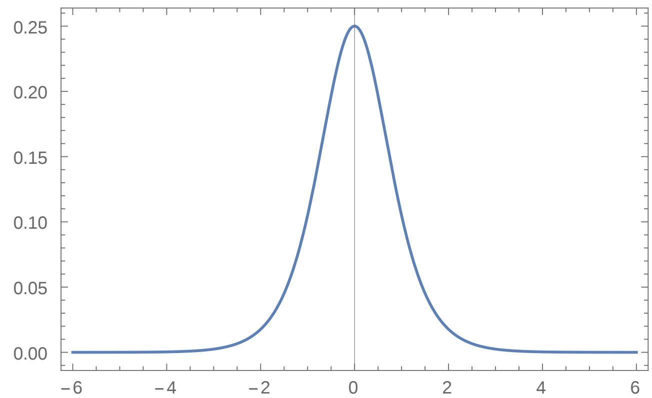
$$\mathcal{R} = -\frac{2}{\Omega^2}\left(-\frac{\partial^2}{\partial\tau^2} + \frac{\partial^2}{\partial\sigma^2}\right)\ln\Omega$$

The denominator factor in the above expression can lead to a worldsheet singularity where $\Omega^2 = 0$.

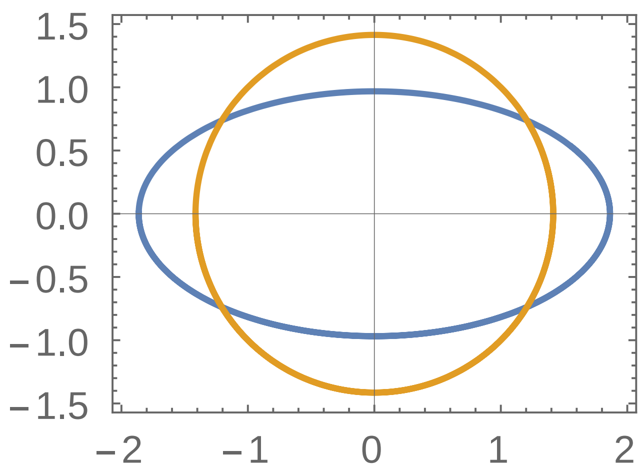
Thus, singularities always exist prior to the arrival of the pulse, but may get removed (depending on the value of T for the pulse) after its passage.

Sech-squared pulse

Pulse:



xy profile before and much after:



Shape change from circle (yellow) to ellipse (blue). More details in AD & SK (PRD (2024).

RESULT

• A closed string in a pp-wave spacetime which has a degenerate, singular world-sheet and circular xy plane profiles,

can, after meeting a GW pulse, evolve

• into a string with a non-degenerate, non-singular worldsheet metric and non-circular xy plane profiles.

The GW pulse inflicts a permanent change in the character of the string and the world-sheet.

The string carries the memory of the pulse permanently as it evolves into the future!

More details in AD & SK, Phys. Rev. D (October 2024).

ELECTROMAGNETIC MEMORY

ELECTROMAGNETIC MEMORY

- In analogy with gravitational wave memory, electromagnetic memory is conventionally defined also through a 'permanent change'— the so-called 'velocity kick'.

- First briefly discussed by Grishchuk and Polnarev (1989) and recently re-analysed by Bieri and Garfinkle (2013).

- Defined (for a specific case where the force is $q\mathrm{E}$ and unit mass) using the following simple equation:

$$\mathbf{v}_{\infty} - \mathbf{v}_{-\infty} = q \int_{-\infty}^{\infty} \mathbf{E}\, dt$$

E is the electric field, usually taken as a radiation field due to a source far away.

• If one considers an electric field which is non-zero and constant only over a small interval (0,T) in time, then

$$\mathbf{v}_{\infty} - \mathbf{v}_{-\infty} = q\mathbf{E_0}T$$

The velocity at negative infinity has a different value compared to the value at positive infinity.

• Implies a jump or a kick in the velocity, imparted to the charged test particle by the electric pulse.

• Conventionally one considers the electric field due to radiation—say, electric dipole radiation (far away from the source) – and its effect on a test charge.

DIPOLE FIELD EXAMPLE

$$m\ddot{\vec{x}} = q\vec{E} \quad ; \quad \Delta\vec{v} = \frac{q}{m}\int_{-\infty}^{\infty} \vec{E}\, dt$$

Nonrelativistic, far from source, dipole moment $\vec{p}(t) = p(t)\hat{\mathrm{k}}$, magnetic field small, ignored.

$$\vec{E}_{rad} = \frac{\mu_0}{4\pi r}\ddot{p}(t_r)\sin\theta\,\hat{\theta}$$

$$\Delta v_\theta = \frac{q}{m}\frac{\mu_0 \sin\theta}{4\pi r}\left(\frac{dp}{dt}(t\to\infty) - \frac{dp}{dt}(t\to-\infty)\right)$$

Consider systems which at large positive and negative times consist of widely separated charges each moving at constant velocity. This gives,

$$\frac{dp}{dt} = \sum_k q_k v_k$$

Thus, we find a velocity kick

$$\Delta v_\theta = \frac{q}{m}\frac{\mu_0 \sin\theta}{4\pi r}\left(\sum_k q_k v_k(t\to\infty) - \sum_k q_k v_k(t\to-\infty)\right)$$

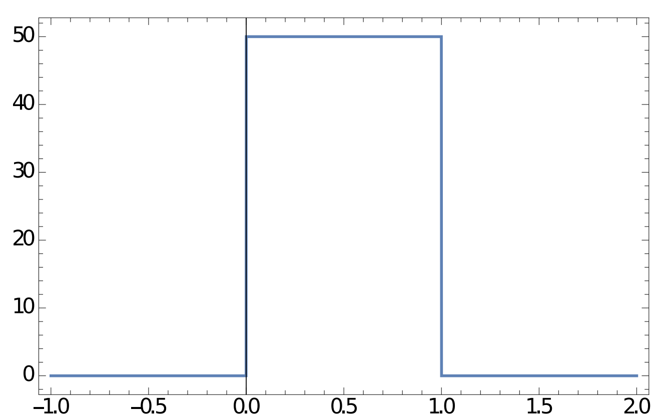
PULSE INDUCED MEMORY-LIKE EFFECT?

• $\mathrm{B} = B\,\hat{\mathrm{k}}$ **(B constant). Motion of a charge is a circle (cyclotron motion). Add an electric field in the xy plane:**

$$\mathbf{E} = E(t)\left(\cos\alpha\hat{\mathbf{i}} + \sin\alpha\hat{\mathbf{j}}\right)$$

where E is chosen as the following function (a square pulse):

$$\begin{aligned} E(t) &= 0, \qquad -\infty \le t \le 0 \\ &= E, \qquad 0 \le t \le T \\ &= 0 \qquad\qquad t \ge T \end{aligned} \tag{3}$$



• **How does the motion change after the pulse is switched off?**

• **Equations of motion of a test charge, in the interval $(0, T)$ (where the electric pulse acts), are given as $m\frac{d\mathbf{v}}{dt} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B}$. In component form:**

$$m\frac{dv_x}{dt} = qE\cos\alpha + qv_yB$$
$$m\frac{dv_y}{dt} = qE\sin\alpha - qv_xB$$
$$m\frac{dv_z}{dt} = 0$$

q **and** m **are the charge and mass of the particle and** $\mathbf{v} = (v_x, v_y, v_z)$ **is its velocity vector.**

• **In the other two regions, i.e. for $t \leq 0$ and $t \geq T$, the same equations hold with $E = 0$.**

• **The v_z equation is trivially solved and we assume $v_z = 0$ by setting the integration constant to zero, thereby restricting the motion of the charge to the xy plane.**

- **Solutions for x, y and v_x, v_y in all three equations can be found.**

- **Initial circular path (before pulse):**

$$x^{(I)} = A_0 \sin \omega_B t - D_0 \cos \omega_B t + C_0$$
$$y^{(I)} = A_0 \cos \omega_B t + D_0 \sin \omega_B t + C_0'$$

$$\left(x^{(I)} - C_0\right)^2 + \left(y^{(I)} - C'^0\right)^2 = A_0^2 + D_0^2$$

Centre at (C_0, C_0'), radius $\sqrt{A_0^2 + D_0^2}$.

- **Final circle (after pulse):**

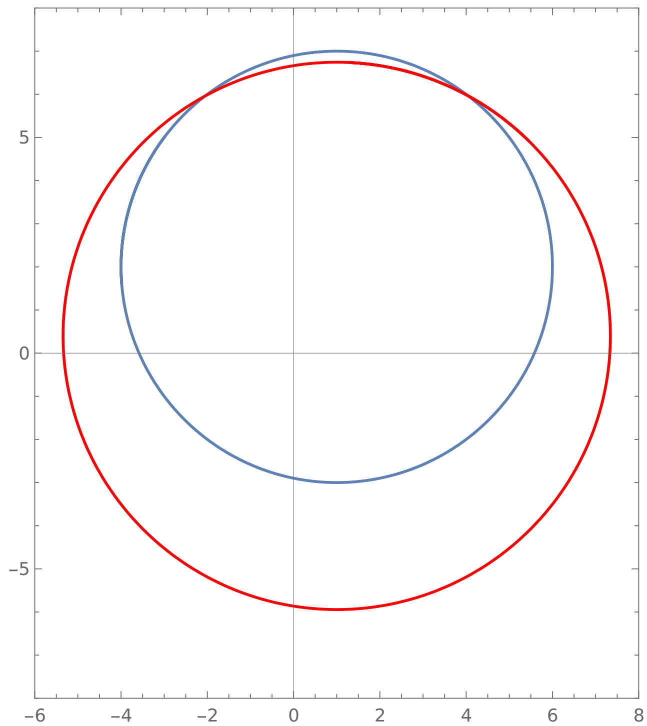
$$\left(x^{(III)} - \left(C_0 + \frac{E}{B} (\sin \alpha) T\right)\right)^2 + \left(y^{(III)} - \left(C_0' - \frac{E}{B} (\cos \alpha) T\right)\right)^2 = R_f^2$$

Centre at $\left(C_0 + \frac{E}{B} (\sin \alpha) T, C_0' - \frac{E}{B} (\cos \alpha) T\right)$.

Radius:

$$R_f^2 = \left[A_0 + \frac{2E}{\omega_B B} \sin \frac{\omega_B T}{2} \cos\left(\frac{\omega_B T}{2} + \alpha\right)\right]^2 + \left[D_0 + \frac{2E}{\omega_B B} \sin \frac{\omega_B T}{2} \sin\left(\frac{\omega_B T}{2} + \alpha\right)\right]^2$$

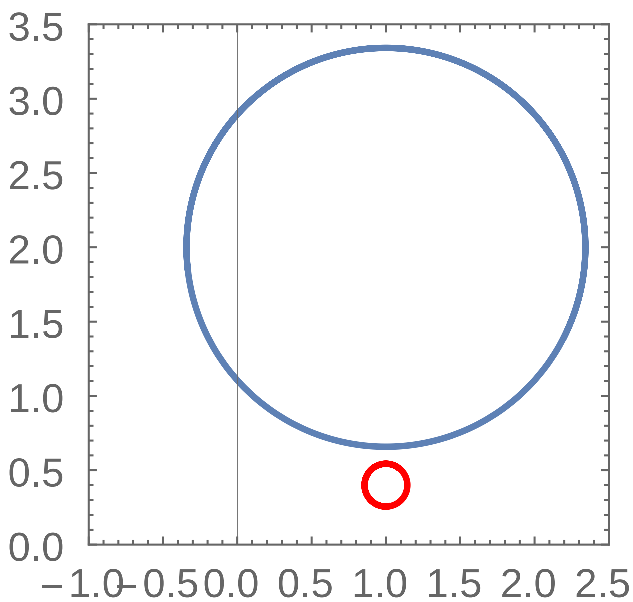
Before the pulse (blue) and after (red):



The blue (red) curve traces the motion of the charged particle before (after) the electric pulse is injected. Parameter values (in respective units): $A_0 = 3$, $D_0 = 4$, $C_0 = 1$, $C_0' = 2$, $\omega_B = 1$, $\alpha = 2\pi$, $T = 2$, $\frac{E}{B} = 0.8$. **Centre of blue curve at** $(1, 2)$, **radius 5 units. Red curve centre at** $(1, 0.4)$ **and radius** 6.28 **units.**

Larger radius, shift of centre.

Before the pulse (blue) and after (red):



Parameter values (in respective units) are: $A_0 = -0.6$, $D_0 = -1.2$, $C_0 = 1$, $C_0' = 2$, $\omega_B = 1$, $\alpha = 2\pi$, $T = 2$, $\frac{E}{B} = 0.8$. **Centre of blue curve at** $(1, 2)$**, radius 1.342 units. Red curve centre at** $(1, 0.4)$ **and radius** 0.144 **units. Here, the radius is smaller.**

Smaller radius, shift of centre.

Pair of trajectories, separation

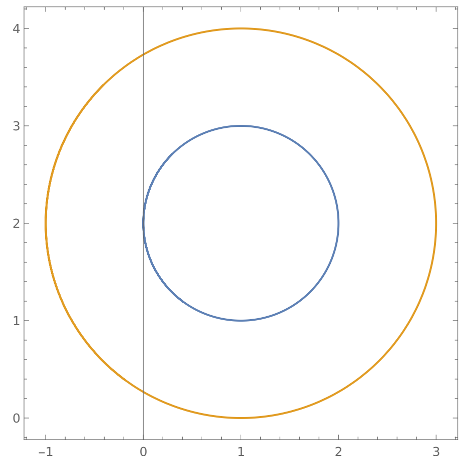
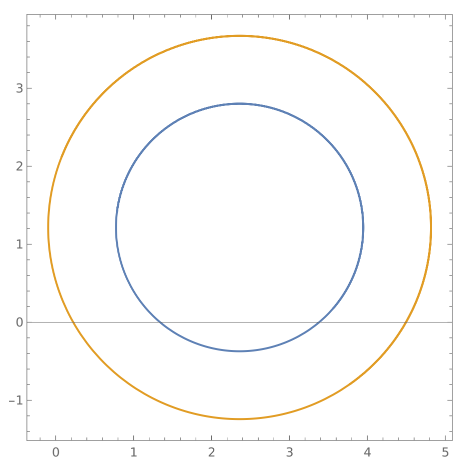
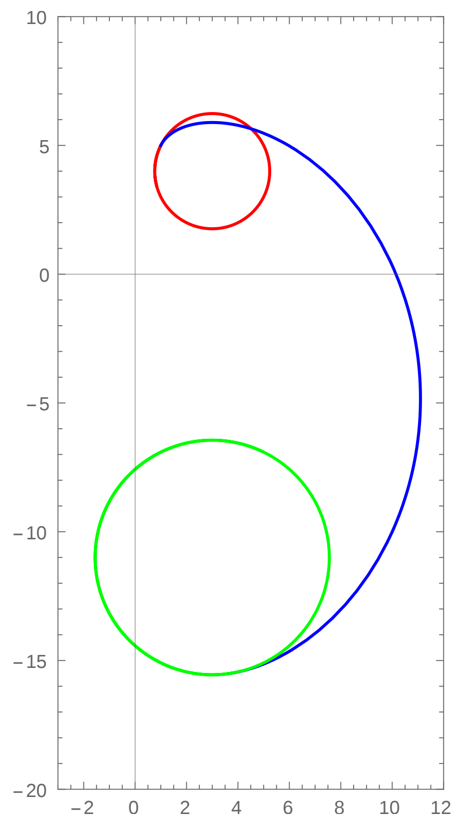


Figure above: initial circles (before pulse) of radii R_{i1} (blue) and R_{i2} (yellow). Figure below: final circles (after pulse) of radii R_{f1} (blue) and R_{f2} (yellow). Chosen values are $A_{01} = D_{01} = 1/\sqrt{2}$, $A_{02} = D_{02} = \sqrt{2}$, $C_{01} = C_{02} = 1$, $C'_{01} = C'_{02} = 2$, $\omega_B = 1$, $T = \pi$, $\frac{E}{B} = 0.5$, $\alpha = \frac{\pi}{4}$. $\Delta R_i \neq \Delta R_f$ and $\Delta v_i \neq \Delta v_f$. $\Delta R_i = 1$, $\Delta R_f = 0.8219$. Concentric centre of the two particles initially at $(1, 2)$. After pulse, shifts to $(2.11, 0.89)$.



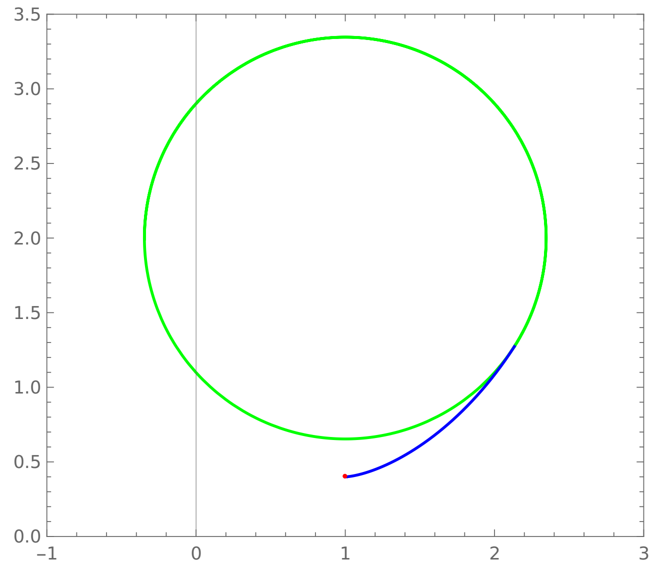
Change of separation, relative velocity

Evolution: Initial circle (green),during pulse (blue), final circle (red):



The red circle (pre-pulse) is from $t = -7$ to $t = 0$ (clockwise). The blue curve is from $t = 0$ to $t = 5$ and the green circle (post-pulse) is from $t = 5$ to $t = 15$ (clockwise). The points $t = 0$ and $t = 5$ are the locations where the red, blue and the blue, green curves meet tangentially. Chosen parameters are: $\omega_B = 1$, $T = 5$, $\alpha = 2\pi$, $A_0 = 1$, $D_0 = 2$, $C_0 = 3$ and $C_0' = 4$, $\frac{E}{\omega_B B} = 3$.

Particle stuck and static later.



The circular path (green) before the arrival of the pulse. The blue (red) curve traces the motion of the charged particle during the period when the electric pulse is present. The red dot at the end of the blue curve shows the static particle after the pulse has departed. Parameter values chosen (in respective units) are: $A_0 = -0.727$, $D_0 = -1.133$, $C_0 = 1$, $C_0' = 2$, $\omega_B = 1$, $\alpha = 2\pi$, $T = 2$, $\frac{E}{B} = 0.8$. The centre of the green curve is at $(1, 2)$ and its radius is 1.342 units. The location of the static particle at late times is at $(1, 0.4)$.

- **With properly tuned initial conditions one may end up with x, y as constant.**

- **Some numbers:**

We assume:

$m = 3.3 \times 10^{-27}$ **kg (deuteron),** $q = 1.6 \times 10^{-19}$ **C,** $B = 1.5$ **Tesla,** $\alpha = 2\pi$**.**

Thus $\omega_B = 72.7 \times 10^6$ **rad/s which is in the radio frequency range (around 12 MHz).**

We assume $T = 10$ **nanoseconds and** $E = 50$ **MegaVolts/m. The constants are chosen as** $A_0 = 0.30$ **m,** $D_0 = 0.40$ **m** $C_0 = 0.60$ **m and** $C_0' = -0.10$ **m.**

The chosen values yield the following:

Initial trajectory: Centre at $(0.60, -0.10)$**, Radius** $R_i = 0.50$**m and** $v_i = 0.12$**c.**

Final trajectory: Centre at $(0.60, -0.43)$**, Radius** $R_f = 0.64$**m and** $v_f = 0.16$**c.**

All the velocities are non-relativistic in value.

• What current J produces the electric pulse?

The electric pulse is written as

$$\mathbf{E} = E\left[\Theta(t) - \Theta(t-T)\right]\left(\cos\,\alpha\,\hat{\mathbf{i}} + \sin\,\alpha\,\hat{\mathbf{j}}\right).$$

This gives J (in vacuum) equal to

$$\mathbf{J} = \epsilon_0 E\left[\delta(t) - \delta(t-T)\right]\left(\cos\,\alpha\,\hat{\mathbf{i}} + \sin\,\alpha\,\hat{\mathbf{j}}\right).$$

The current spike at $t = 0$, creates the uniform electric field sustained till $t = T$. At $t = T$, an opposite spike reduces the net electric field to zero.

• Permanent change which persists. Caused by a time-dependent pulse.

• Not quite a wave-induced memory as seen with the dipole field or in GW cases discussed.

• Could possibly be seen in cyclotron experiments.

Ref: SK (in preparation)

CONCLUDING SUMMARY

• In a pp-wave geometry with a square pulse profile, geodesic motion shows displacement and velocity memory.

• Treating a ring of particles using geodesics one obtains the change of shape from a circle to an ellipse, caused by a square pulse.

• The evolution of a test string in a pp-wave, using string equations of motion and constraints with a square pulse, display a change of shape of a closed string from a circle to an ellipse.

• Worldsheet features can show a change due to GW pulse in a pp-wave, in terms of a degenerate, singular worldsheet geometry in the past to a nonsingular, non-degenerate worldsheet in the future.

• In the motion of test charge in a constant magnetic and a short duration electric pulse one notices a change in the radius of cyclotron motion, a shift of the centre and a velocity kick.

Pairs of trajectories also show a change in relative separation.

On the whole, these results are similar to a velocity kick as well as a displacement memory-like effect triggered by a square electric pulse.

SOME OPEN ISSUES

• One can construct newer examples using test particles and strings with various pulse shapes in a pp-wave geometry.

• It is important to also figure out a 'no memory' example. What pulses will have no memory?

• It is also possible to use more general spacetimes with non-planar wavefronts (eg. Kundt waves) and re-do these studies.

• Making these studies observationally relevant remains a bigger challenge.

• In the electromagnetic memory context one may look for studying the memory-like effect in cyclotron motion with smooth electric field pulses, such as a Gaussian or a sech-squared.