

# Probing quantum gravity and non-locality through $R^2$ -like inflation

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Based on [arXiv:2003.00629 \[hep-th\]](#) (JHEP06(2020)152), [arXiv:1711.08864 \[hep-th\]](#) (JHEP 03 (2018) 071), [arXiv:2005.09550 \[hep-th\]](#) (Int.J.Mod.Phys.D 29 (2020) 14, 2043018) and ongoing works [arXiv:2209.XXXX](#), [arXiv:2209.XXXX](#) in collaboration with **Alexey S. Koshelev, Alexei A. Starobinsky.**

August 18, 2022

# Plan of the talk

- Brief introduction to inflationary cosmology, current status and UV-physics
- $R + R^2$ -inflation and foundations
- Beyond  $R + R^2$ -gravity and emergence of non-locality!
- Predictions of generalized non-local  $R^2$ -like inflation
- New revelations on probing the physics of early Universe.
- Distinguishing non-local  $R^2$ -like inflation with respect to EFT of inflation

# General relativity and beyond

- GR has been the most successful theory (perfect with solar system and also perfect with many astrophysical observations) and even today it surprises us with its utmost predictions (Recent LIGO achievements+ BH shadow).
- But GR leads to singularities (big bang and black holes) which direct us to modify it at high energies.
- Singularities in GR strongly indicate existence of high curvature regimes (since  $R, R_{\mu\nu}, W_{\mu\nu\rho\sigma} \sim \frac{1}{t^n}$  or  $\frac{1}{r^n}$ )

# Planck CMB map: Scale Invariance

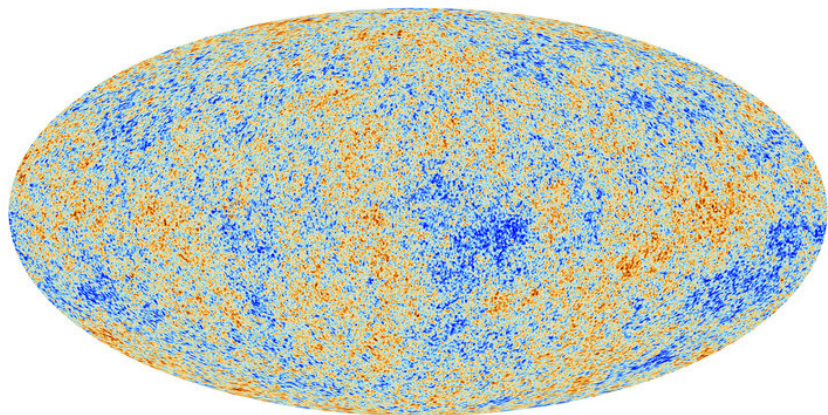


Figure: Planck 2015 CMB map with  $\frac{\Delta T}{T} \sim 10^{-5}$

# Inflationary paradigm

(A. A. Starobinsky, A. H. Guth, A. D. Linde and F. Mukhanov )

Homogeneous, isotropic and spatially flat geometry

⇒ Friedmann-Lemaître-Robertson-Walker (FLRW) metric

$$ds^2 = -dt^2 + \frac{a(t)^2}{1-kr^2} dr^2 + r^2 d\Omega^2$$

The Universe scale factor  $a(t)$  increases exponentially ( $N = 50 - 60$  number of e-foldings)

⇒ Hubble parameter  $H = \frac{1}{a} \frac{da}{dt}$  almost constant

⇒ Comoving Hubble radius  $(aH)^{-1}$  decreases i.e.,  $\frac{d}{dt} \left( \frac{1}{aH} \right) < 0$ .

⇒ Slow-roll conditions  $\epsilon = -\frac{\dot{H}}{H^2} \ll 1$ ,  $\eta = \frac{\dot{\epsilon}}{H\epsilon} \ll 1$ .  $\Updownarrow$

Modifying General Relativity or addition of hypothetical matter fields

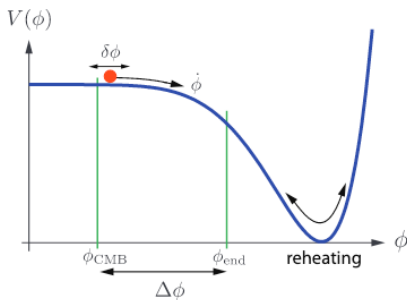
$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{1}{m_{\text{P}}^2} T_{\mu\nu},$$

$\Updownarrow$   $\Updownarrow$   
Scaloron Inflaton(s)



Primordial fluctuations:

# Standard demonstration of inflation



**Figure:** There exists some scalar field and it rolls down the hill causing expansion of Universe (Picture Credit: Baumann lectures on inflation)

Quantum gravity  $\Leftrightarrow$  *Inflation*  $\Leftrightarrow$  Standard Model of particle physics.

# Inflationary observables, consistency relations and new physics

- The key observables of inflationary paradigm are related to two and 3-point correlations of primordial fluctuations.
- The two-point correlations give the scalar power spectrum  $\mathcal{P}_{\mathcal{R}*} \sim 10^{-9}$  and its tilt  $n_s \approx 1 - \frac{2}{N}$ , the tensor power spectrum is usually expressed through tensor-to-scalar ratio  $r = \frac{\mathcal{P}_T}{\mathcal{P}_R} < 0.036$  and the tilt of tensor power spectrum ( $n_t$ ) is not YET measured (**BICEP/Keck Array 2021**).
- 3-point correlations give non-Gaussianities (also called bispectrum) measured by the parameter  $f_{NL}$  and the current constraints are  $f_{NL}^{\text{loc}} = 0.8 \pm 5.0$ ,  $f_{NL}^{\text{equi}} = -4 \pm 43$ ,  $f_{NL}^{\text{ortho}} = -26 \pm 21$ , at 68% CL.
- In the case of single field inflation there are so-called consistency relations given by (Tensor and Maldacena consistency relations)

$$r = -8n_t, \quad f_{NL}^{\text{sq}} = \frac{5}{12}(1 - n_s).$$

# How to generate non-Gaussianities (NGs)?

Violate one of these (E. Komatsu et al, 2009)

- **Single Field:** There is only one quantum field responsible for driving inflation and for generating the primordial seeds for structures.
- **Canonical Kinetic Energy:** The kinetic energy of the quantum field is such that the speed of propagation of fluctuations is equal to the speed of light.
- **Slow Roll:**  $\epsilon_V = \frac{1}{2} \left( \frac{V'}{V} \right)^2 \ll 1$ ,  $\eta_V = \frac{V''}{V} \ll 1$
- **Initial Vacuum State:** The quantum field was in the preferred adiabatic vacuum state (also sometimes called the Bunch-Davies vacuum?) just before the quantum fluctuations were generated during inflation.

**None of the above??**



# Shapes of NGs

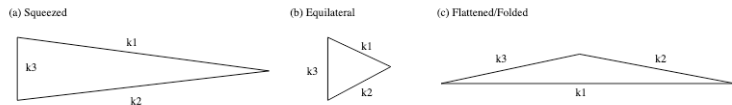
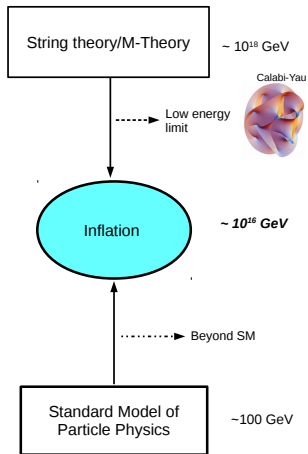
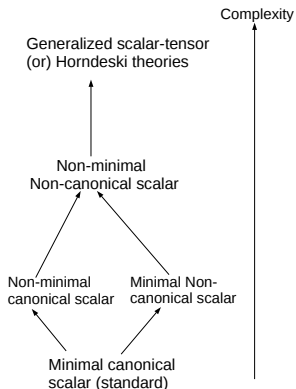


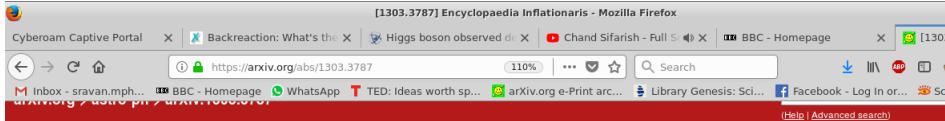
FIG. 1: Bispectrum shapes,  $B(k_1, k_2, k_3)$ , which can be characterized by triangles formed by three wave vectors. The shape (a) has the maximum signal at the squeezed configuration,  $k_3 \ll k_2 \approx k_1$ , and can be produced by models of inflation involving multiple fields. The shape (b) has the maximum signal at the equilateral configuration,  $k_1 = k_2 = k_3$ , and can be produced by non-canonical kinetic terms of quantum fields. The shape (c) has the maximum signal at the flattened configuration,  $k_1 \approx 2k_2 \approx 2k_3$ , and can be produced by non-vacuum initial conditions.

Figure: Physics of NG? from E. Komatsu et al, 2009

# Inflationary model building: Top down vs Bottom up



# Encyclopedia $\sim$ 300 models



Astrophysics > Cosmology and Nongalactic Astrophysics

## Encyclopaedia Inflationaris

Jerome Martin, Christophe Ringeval, Vincent Vennin

(Submitted on 15 Mar 2013 (v1), last revised 3 Sep 2013 (this version, v3))

The current flow of high accuracy astrophysical data, among which are the Cosmic Microwave Background (CMB) measurements by the Planck satellite, offers an unprecedented opportunity to constrain the inflationary theory. This is however a challenging project given the size of the inflationary landscape which contains hundreds of different scenarios. Given that there is currently no observational evidence for primordial non-Gaussianities, isocurvature perturbations or any other non-minimal extension of the inflationary paradigm, a reasonable approach is to consider the simplest models first, namely the slow-roll single field models with minimal kinetic terms. This still leaves us with a very populated landscape, the exploration of which requires new and efficient strategies. It has been customary to tackle this problem by means of approximate model independent methods while a more ambitious alternative is to study the inflationary scenarios one by one. We have developed the new publicly available runtime library ASPIC to implement this last approach. The ASPIC code provides all routines needed to quickly derive reheating consistent observable predictions within this class of scenarios. ASPIC has been designed as an evolutive code which presently supports 74 different models, a number that may be compared with three or four representing the present state of the art. In this paper, for each of the ASPIC models, we present and collect new results in a systematic manner, thereby constituting the first Encyclopaedia Inflationaris. Finally, we discuss how this procedure and ASPIC could be used to determine the best model of inflation by means of Bayesian inference.

Comments: 368 pages, 192 figures, uses jcapub. Theoretical justifications, new models and references added

Subjects: **Cosmology and Nongalactic Astrophysics (astro-ph.CO)**; General Relativity and Quantum Cosmology (gr-qc); High Energy Physics -

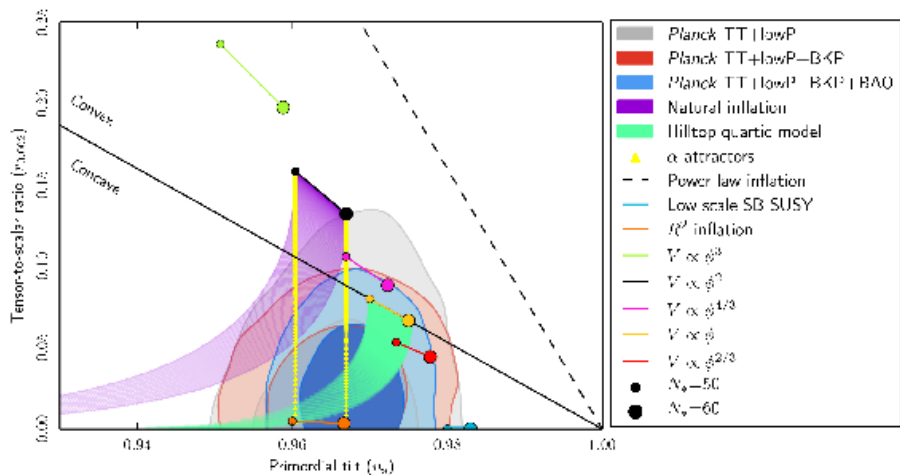


Figure: Spectral index Vs. tensor to scalar ratio

# Starobinsky inflation: A phase of unstable de Sitter

The  $R + R^2$  gravity is

$$S_{R+R^2}^{\text{local}} = \int d^4x \sqrt{-g} \left[ \frac{M_p^2}{2} R + \frac{f_0}{2} R^2 \right],$$

$f_0 = \frac{M_p^2}{6M^2}$ . Two regimes

- Higher curvature regime:  $M^2 \ll R \ll M_p^2$  or  $M_p^2 \ll f_0 R$
- Low-curvature regime:  $M_p^2 \gg f_0 R$
- Inflation happens in high curvature regime and particle production happens in a low-curvature regime [Starobinsky \(Quantum Gravity 1981 proceedings\)](#)
- de-Sitter phase is unstable both to the future and also to the past, [Anomaly driven inflation, Hawking, Hertog and Reall \(2001\)](#).

## $R^2$ inflation or Starobinsky inflation

- $\frac{M_p^2}{2}R + \frac{M_p^2}{12M^2}R^2$  is the first model of inflation (A.A. Starobinsky, 1980) with the simplest one parameter extension of GR.
- Inflation in this model is achieved by growth of scale factor in the following manner (given by the solution of trace-equation  $\square R = M^2 R$ )

$$\begin{aligned}a(t) &\approx a_0(t_s - t)^{-1/6} e^{-r_1(t_s - t)^2/12}, \\H = \frac{\dot{a}}{a} &= \frac{r_1(t_s - t)}{6} + \frac{1}{6(t_s - t)} + \dots, \\ \bar{R}_{\text{dS}} = 6(\dot{H} + 2H^2) &= \frac{r_1^2(t_s - t)^2}{3} - \frac{r_1}{3} + \frac{4}{3(t_s - t)^2} + \dots,\end{aligned}$$

where  $t_s$  mark the end of inflation corresponds to the slow-roll parameter  $\epsilon = \frac{-\dot{H}}{H^2} \sim 1$

- During inflation  $H$  is nearly constant and  $\epsilon \ll 1$  which is nothing but "quasi de Sitter (dS)" expansion.

- In the light of recent CMB data, Starobinsky inflation stands out to be the best fit with

$$n_s = 1 - \frac{2}{N}, \quad r = \frac{12}{N^2}.$$

- The scalaron mass is constrained as  $M \sim 5.5 \times 10^{-5} M_P$  and the Hubble parameter is  $H_{\text{inf}} \sim \mathcal{O}(10)M$ .
- This model features a graceful exit and power-law expansion stage with  $a(t) \propto t^{2/3} \left(1 + \frac{2}{3t} \sin(Mt)\right)$  modulated by small oscillations (Starobinsky (1980,1981,1984), *Fundamental Interactions*, MGPI Press, Moscow, 1984, p. 55-79).
- $R^2$  model in Einstein frame gives a scalar field with an exponentially flat potential  $V \sim \left(1 - e^{-\sqrt{\frac{2}{3}} \frac{\varphi}{M_P}}\right)^2$
- Several Starobinsky like inflationary scenarios with  $V \sim \left(1 - e^{-\sqrt{\frac{2}{3B}} \frac{\varphi}{M_P}}\right)^2$  (which gives  $r = \frac{12B}{N^2}$ ) were recently found in the context of String theory/SUGRA.

# Foundations of $R^2$ model

- $R^2$  inflation was first motivated from conformal anomaly of gravity which in short given by

$$G_{\mu\nu} = 8\pi G \langle T_{\mu\nu} \rangle,$$

where  $\langle T_{\mu\nu} \rangle$  is the expectation value of the quantum energy momentum tensor.

- In  $D = 4$  it is known that

$$\langle T_{\mu}^{\mu} \rangle = b \left( W + \frac{2}{3} \square R \right) + b' \mathcal{G} + \delta \square R \implies \square R = M^2 R,$$

where  $W$ ,  $\mathcal{G} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma}$  are Weyl square and the Gauss-Bonnet terms. The coefficients depend on the no. of massless (conformal) scalar and vector fields.

Neglecting contribution from GB term we get  $\square R \approx M^2 R$ .



## Beyond $R + R^2$

By adding higher curvatures? L. Sebastiani and R. Myrzakulov (2015)

$$S_{R^3} = \int d^4x \sqrt{-g} \left[ \frac{M_p^2}{2} R + \frac{f_0}{2} \left\{ R^2 + \frac{f_1}{6M^2} R^3 \right\} \right]$$

$$S_{R^4} = \int d^4x \sqrt{-g} \left[ \frac{M_p^2}{2} R + \frac{f_0}{2} \left\{ R^2 + \frac{f_2}{4M^4} R^4 \right\} \right]$$

where  $f_1 \sim 10^{-5}$  and  $f_2 \sim 10^{-7}$  Motivated from Asymptotic safety

$$S_{AS} = \int d^4x \sqrt{-g} \left[ \frac{M_p^2}{2} R + \frac{a}{2} \frac{R^2}{1 + b \ln \left( \frac{R}{\mu} \right)} \right].$$

and  $a < 0.1$

$$S_{R^2}^6 = \frac{M_p^2}{2} \int \left[ R + \alpha R^2 + \gamma R \square R \right] \quad (1)$$

# Planck data and lessons from $f(R)$

Planck data says ,  $\mathcal{P}_{\mathcal{R}} \sim A_s \left( \frac{k}{k_*} \right)^{n_s-1}$

$$n_s = 1 - \frac{2}{N}$$

$$\left. \frac{dn_s}{d \ln k} \right|_{k=k_*} = -0.0045 \pm 0.0067 \text{ at } 68\% \text{ CL} .$$

We do not know

$$\left\{ r < 0.036, n_t = \frac{d \ln P_T}{d \ln k}, f_{\text{NL}} \right\}$$

**Questions: Scale-invariance in tensor-sector? and Non-Gaussianities?**

# Generalized quantum gravity action with $R^2$ -like inflation

In general, any higher curvature extension of GR is constructed through adding all possible curvature invariants involving the following three curvature tensors

$$\mathbb{R} = \left\{ R \quad R_{\mu\nu} \quad W_{\mu\nu\rho\sigma} \right\},$$

We can choose either Riemann tensor or Weyl tensor

$$R_{\mu\nu\rho\sigma} \square_s^n R^{\mu\nu\rho\sigma} = W_{\mu\nu\rho\sigma} \square_s^n W^{\mu\nu\rho\sigma} + 2R_{\mu\nu} \square_s^n R^{\mu\nu} - \frac{1}{3} R \square_s^n R.$$

**Question: What is the most general action that admits  $\square R = M^2 R$  as a solution (in FLRW)**

Since during inflation  $R_{\mu\nu} \approx g_{\mu\nu} \frac{R}{4}$  we omit Ricci tensor from the construction

## Higher derivative curvature invariants:

- Stelle's 4th order gravity (Renormalizable but has a tensor-ghost, therefore non-Unitary theory)

$$S_4 = \int d^4x \sqrt{-g} \left[ \frac{M_p^2}{2} R + \frac{f_0}{2} R^2 + \frac{f_{W0}}{2} W_{\mu\nu\rho\sigma} W^{\mu\nu\rho\sigma} \right],$$

- With new prescriptions ghost problem may be solved ([Works of D. Anselmi, Carl Bender and Phillip Mannheim, A. Strumia and A. Salvio etc..](#) )
- We can extend Stelle gravity by adding  $\mathbb{R} \square_s \mathbb{R}, \mathbb{R} \square_s^2 \mathbb{R}, \dots$  we end up by a non-local gravity

$$S_q^{\text{Non-local}} = \int d^4x \sqrt{-g} \left[ \frac{M_p^2}{2} R + \frac{1}{2} R \mathcal{F}_R(\square_s) R + \frac{1}{2} W_{\mu\nu\rho\sigma} \mathcal{F}_C(\square_s) W^{\mu\nu\rho\sigma} \right]$$

where  $\square_s = \frac{\square}{\mathcal{M}_s^2}$  See [A. S. Koshelev, KSK, A. Mazumdar and A. Starobinsky \(2020\)](#) and [L. Buoninfante \(2021\)](#), [Buoninfante, KSK \(2022\)](#) for some fundamental motivations for non-local gravity.

## $R^2$ -like inflation in Non-local gravity

- EOM of AID gravity even though looks complicated they can be solved by a simple equation which is exactly the trace equation of local  $R^2$  gravity

$$\square R = M^2 R \implies \mathcal{F}_R \left( \frac{\square}{\mathcal{M}_s^2} \right) R = \mathcal{F}_R \left( \frac{M^2}{\mathcal{M}_s^2} \right) R.$$

Since the CMB observations indicate scale invariance we expect

$$M \ll M_s \lesssim \mathcal{M}_s.$$

- Using the above trace equation the trace equation for non-local gravity become

$$\left[ M_P^2 - 6\lambda M^2 \mathcal{F}_R \left( \frac{M^2}{\mathcal{M}_s^2} \right) \right] R - \lambda \mathcal{F}_R^{(1)} \left( \frac{M^2}{\mathcal{M}_s^2} \right) (\partial^\mu R \partial_\mu R + 2M^2 R^2) = 0$$

- It was showed that the only solution of the above equation for  $R \neq 0$  (See [Appendix C of 1711.08864](#)) is

$$\mathcal{F}_R^{(1)} \left( \frac{M^2}{\mathcal{M}_s^2} \right) = 0, \quad \frac{M_P^2}{2\lambda} = 3M^2 \mathcal{F}_1, \quad \text{where } \mathcal{F}_1 \equiv \mathcal{F}_R \left( \frac{M^2}{\mathcal{M}_s^2} \right)$$

# Towards generalized non-local $R^2$ -like inflation

$$S_{W^2}^{\text{Non-local}} = \int d^4x \sqrt{-g} \left[ \left( \frac{M_p^2}{2M_s^2} + f_0 \frac{R}{M_s^2} \right) W_{\mu\nu\rho\sigma} \mathcal{F}_W(\square_s, R) W^{\mu\nu\rho\sigma} \right]$$

where  $\mathcal{F}_W(\square_s, R)$  is a general analytic function of d'Alembertian and the Ricci scalar which can be expanded as

$$\mathcal{F}_C(\square_s, R/M_s^2) = \sum_{m,n=0}^{\infty} f_{C\{m,n\}} \square_s^n \left( \frac{R}{M_s^2} \right)^m + \sum_{m,n=0}^{\infty} f_{C\{n,m\}} \left( \frac{R}{M_s^2} \right)^n \square_s^m,$$

where  $f_{C\{m,n\}} \neq f_{C\{n,m\}}$  are arbitrary dimensionless coefficients. One can add cubic order curvature invariants of the form

$$\begin{aligned} & \square_s R \square_s R \square_s R \\ & \square_s^2 R \square_s^2 R \square_s^2 R \\ & \vdots \\ & \square_s^n R \square_s^n R \square_s^n R \end{aligned}$$

# Generalized non-local $R^2$ -like inflation

This is the most general action that admits  $R^2$ -like inflation (using several identities from [Barvinsky et al 1994](#))

$$\begin{aligned} S_H^{\text{Non-local}} = & \int d^4x \sqrt{-g} \left( \frac{M_p^2}{2} R \right. \\ & + \frac{1}{2} \left[ R \mathcal{F}_R(\square_s) R + \frac{f_0 \lambda_c}{M_s^2} \mathcal{L}_1(\square_s) R \mathcal{L}_2(\square_s) R \mathcal{L}_3(\square_s) R \right. \\ & + \left. \left( \frac{M_p^2}{2M_s^2} + f_0 \frac{R}{M_s^2} \right) W_{\mu\nu\rho\sigma} \mathcal{F}_W \left( \square_s, \frac{R}{M_s^2} \right) W^{\mu\nu\rho\sigma} \right. \\ & + \frac{f_0 \lambda_W}{M_s^2} \mathcal{C}_1(\square_s) W_{\mu\nu\rho\sigma} \mathcal{C}_2(\square_s) W^{\mu\nu\gamma\lambda} \mathcal{C}_3(\square_s) W_{\gamma\lambda}{}^{\rho\sigma} \\ & \left. \left. + \frac{f_0 \lambda_W}{M_s^2} \mathcal{D}_1(\square_s) W_{\mu\nu\rho\sigma} \mathcal{D}_2(\square_s) W^{\mu\nu\rho\sigma} \mathcal{D}_3(\square_s) R \right] + \dots \right), \end{aligned}$$

## Form factors and parameter space

$$\left\{ \mathcal{F}_R(\square_s), \mathcal{F}_W\left(\square_s, \frac{2R}{3\mathcal{M}_s^2}\right), \mathcal{L}_i(\square_s), \mathcal{C}_i(\square_s) \right\}$$

with

$$\mathcal{F}_R\left(\frac{M^2}{\mathcal{M}_s^2}\right) = f_0 = \frac{M_p^2}{6M^2}, \quad \mathcal{F}_R^\dagger\left(\frac{M^2}{\mathcal{M}_s^2}\right) = 0, \quad \mathcal{L}_i\left(\frac{M^2}{\mathcal{M}_s^2}\right) = 0.$$

where  $\dagger$  denotes derivative with respect to the argument.

$$\mathcal{F}_R(\square_s) = f_0 M^2 \frac{1 - \left(1 - \frac{\square_s}{M^2}\right) e^{\gamma_S(\square_s)}}{\square_s}$$

$$\mathcal{F}_W\left(\square_s, \frac{R}{\mathcal{M}_s^2}\right) = \frac{e^{\gamma_T\left(\square_s - \frac{2}{3}\frac{R}{\mathcal{M}_s^2}\right)} - 1}{\square_s - \frac{2R}{3\mathcal{M}_s^2}}.$$

$$\gamma_S\left(\frac{M^2}{\mathcal{M}_s^2}\right) = 0 \implies \gamma_S(\square_s) = \left(\square_s - \frac{M^2}{\mathcal{M}_s^2}\right) p_i(\square_s).$$



## Form factors and parameter space

$$\mathcal{L}_i(\square_s) = \sum_{n=0}^{\infty} L_{in} \square_s^n = \left( e^{\ell_i(\square_s)} - 1 \right).$$

where  $\ell_i(\square_s)$  are entire functions that satisfy following property

$$\ell_i(\square_s) = \left( \square_s - \frac{M^2}{\mathcal{M}_s^2} \right) P_i(\square_s),$$

$$C_i(\square_s) = e^{c_i(\square_s)} - 1.$$

The finite parameters here determined by the entire functions which can be finite degree polynomials.

$$\left\{ \gamma_S(\square_s), \gamma_T(\square_s), \ell_i(\square_s), c_i(\square_s) \right\}.$$

# Power spectrum predictions of generalized non-local $R^2$ -like inflation

- Sound speed of scalar and tensor degrees of freedom is Unity
- Scalar power spectrum and tilts are

$$\mathcal{P}_{\mathcal{R}} \approx \frac{1}{3f_0 \bar{R}_{\text{dS}}} \frac{H^2}{16\pi^2 \epsilon^2} \Bigg|_{k_* = a_* H_*}, \quad n_s - 1 \equiv \frac{d \ln \mathcal{P}_{\mathcal{R}}}{d \ln k} \Bigg|_{k_* = a_* H_*} \approx -\frac{2}{N},$$

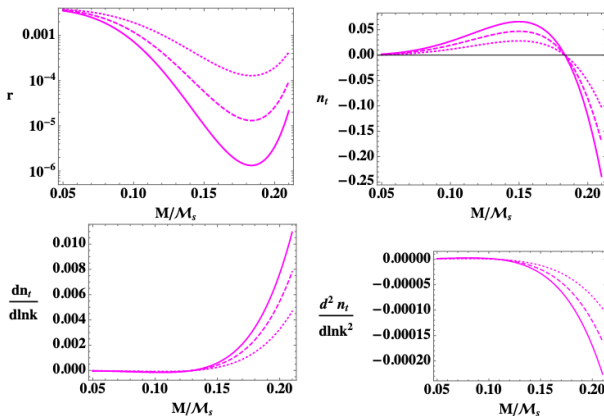
- Tensor to scalar ratio is

$$r = \frac{12}{N^2} e^{-2\gamma_T \left( -\frac{\bar{R}_{\text{dS}}}{2\mathcal{M}_s^2} \right)} \Bigg|_{k_* = a_* H_*}.$$

The tensor spectral index is

$$n_t \equiv \frac{d \ln \mathcal{P}_T}{d \ln k} \Bigg|_{k_* = a_* H_*} \approx -\frac{3}{2N^2} - \left( \frac{2}{N} + \frac{3}{2N^2} \right) \frac{\bar{R}_{\text{dS}}}{2\mathcal{M}_s^2} \gamma_T^\dagger \left( -\frac{\bar{R}_{\text{dS}}}{2\mathcal{M}_s^2} \right)$$

## Case of $r > 0.004$ and $N = 55$



**Figure:** Predictions of  $r$ ,  $n_t$  and running and running of the running of tensor spectral index. Tensor consistency relation  $r = -8n_t$  is violated.

## Case of $r < 0.004$

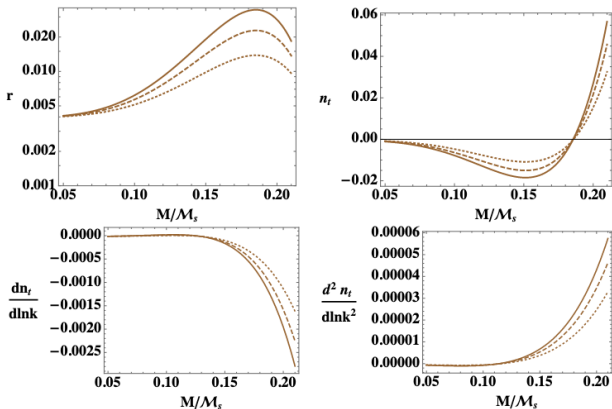
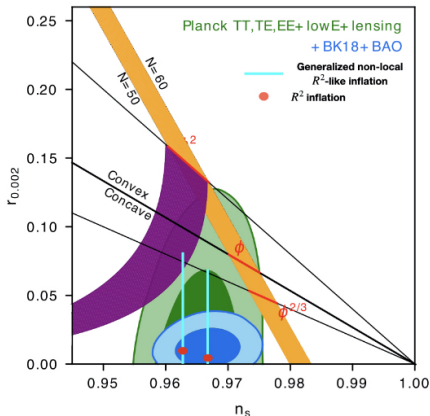


Figure: Predictions of  $r$ ,  $n_t$  and running and running of the running of tensor spectral index. Tensor consistency relation  $r = -8n_t$  is violated.

# Tensor to scalar ratio

The tensor to scalar ratio in  $R^2$ -like inflation in AID gravity gives

$$r = \frac{12}{N^2} e^{-2\gamma_T \left( \frac{-\bar{R}}{2\mathcal{M}_s^2} \right)} \Bigg|_{k=aH}$$



# Modified consistency relation and possibility of blue tilt

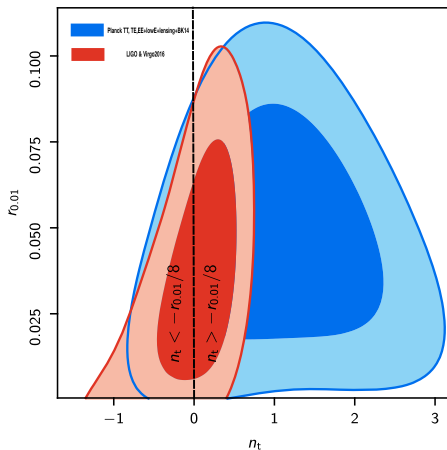
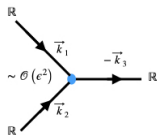


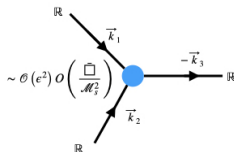
Figure: The  $(n_t, r)$  plane of latest Planck 2018.

# Beyond two-point correlations: Scalar Non-Gaussianities

- Beyond two point correlations it is interesting to see if there will be non-Gaussianities in non-local  $R^2$ -like inflation.
- How non-locality effects the interactions of various curvature modes ? and Can they be detectable ?



Cubic interactions in local  $R^2$  inflation



Cubic interactions in Non-local  $R^2$ -like inflation

**Figure:** In the above plot  $\mathbb{R} = \{\mathcal{R}, \partial\mathcal{R}, \mathcal{D}^\mu\partial\mathcal{R}\}$  imply various tree level interactions of different modes of the curvature perturbation in the local  $R^2$  and the non-local  $R^2$ -like inflation.  $O\left(\frac{\square}{\mathcal{M}_s^2}\right)$  is some analytic non-local operator.

## Reduced bispectrum $f_{NL}$

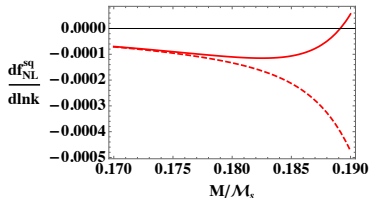
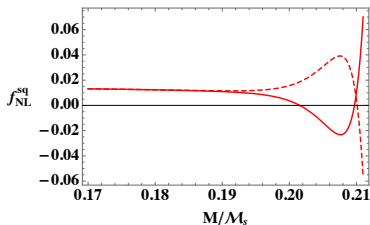
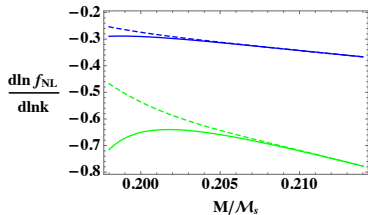
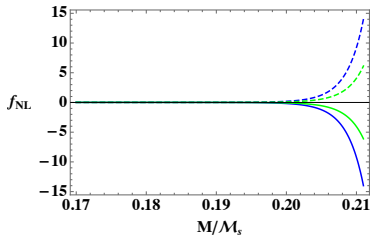
- To calculate bi-spectrum, first we expand our action to cubic order to the leading order in slow-roll parameter. We consider the mode functions  $\mathcal{R} \approx -\Psi/\epsilon$  given by  $\square\mathcal{R} = M^2\mathcal{R}$
- New Interactions arise via the commutation relation in dS approximation  $\square\nabla_\mu\mathcal{R} = \nabla_\mu\square\mathcal{R} + \frac{\bar{R}}{4}\nabla_\mu\mathcal{R}$ .
- Our obtained cubic order action in  $\mathcal{R}$  of AID gravity in the leading order slow-roll approximation is

$$\delta^{(3)}S_{(S)} = 4\epsilon M_p^2 \int d\tau d^3x \left\{ T_1^* \mathcal{R} \nabla \mathcal{R} \cdot \nabla \mathcal{R} + T_2^* \mathcal{R} \mathcal{R}'^2 + T_3^* \mathcal{H}^2 \mathcal{R}^3 \right. \\ \left. + T_4^* \mathcal{H} \mathcal{R} \mathcal{R} \mathcal{R}' + T_5^* \mathcal{H}^{-1} \nabla \mathcal{R} \cdot \nabla \mathcal{R} \mathcal{R}' + T_6^* \mathcal{H}^{-1} \mathcal{R}'^3 \right. \\ \left. + T_7^* \mathcal{H}^{-2} \mathcal{R}' \nabla \mathcal{R} \cdot \nabla \mathcal{R}' \right\},$$

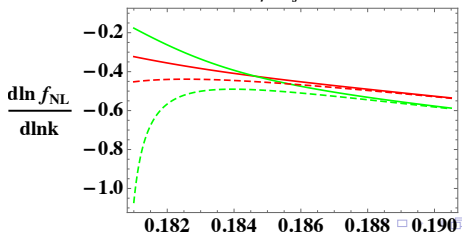
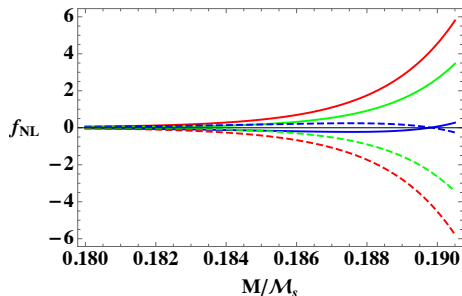
where  $T_i$ 's are dimensionless constants.



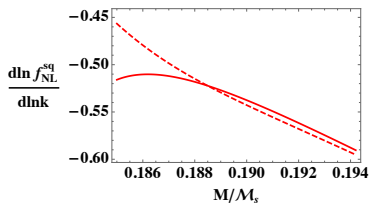
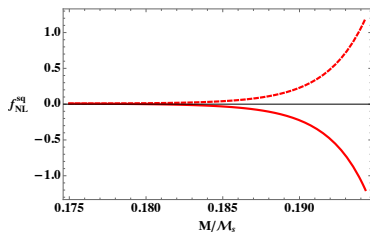
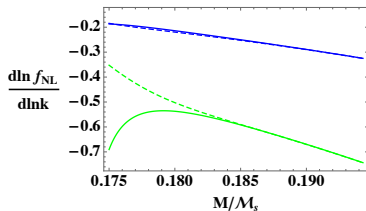
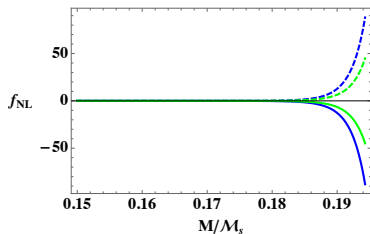
# Non-local $R^2$ -like inflation with peak in the equilateral limit



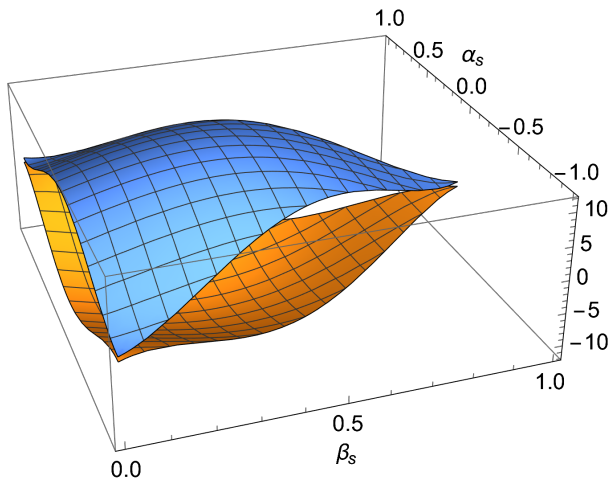
# Non-local $R^2$ -like inflation with peak in the squeezed limit and orthogonal limit



# Non-local $R^2$ -like inflation with equilateral and orthogonal limits and $f_{\text{NL}} \sim 1$



# Shape of Non-Gaussianities



## In a nutshell

- We obtain several shapes of non-Gaussianities without violating the standard nature of inflaton such as sound speed, slow-roll and adiabatic initial conditions.
- We obtain new shapes of non-Gaussianities and large running contrary to the EFT of inflation.
- Non-Gaussianities we obtain are scale dependent,  
$$f_{\text{NL}} \left( k_1, k_2, k_3, K, e^{\gamma_S \left( \frac{\bar{R}_{\text{dS}}}{4\mathcal{M}_s^2} \right)}, \gamma_S^\dagger \left( \frac{\bar{R}_{\text{dS}}}{4\mathcal{M}_s^2} \right), \mathcal{L}_i \left( \frac{\bar{R}_{\text{dS}}}{4\mathcal{M}_s^2} \right) \right)$$
- Most importantly for the first time we obtain sizeable non-Gaussianities in a geometric theory of inflation.

# Generalized non-local $R^2$ -like inflation versus EFT of inflation

- EFT of inflation [Cheung et al 2008](#) promotes the new physics of inflation in terms of EFT (slowly varying) parameters

$$\left\{ H_{\text{inf}}, \epsilon_E, \eta_E, c_s, c_t \right\}$$

According to EFTI violation consistency relations must be encoded in non-trivial sound speeds which are either constant or slowly-varying.

- In our case we do not have any non-trivial sound speeds but violation of consistency relations and large PNGs are result of non-locality!
- According to [Weinberg's version of EFTI \(2008\)](#) one must also consider inflaton couplings to Weyl square

$$\int d^4x \sqrt{-g} \left[ f_1 \left( \frac{\phi}{\Lambda} \right) W^{\mu\nu\rho\sigma} W_{\mu\nu\rho\sigma} \right] \sim \int d^4x \sqrt{-g} \left[ R W^{\mu\nu\rho\sigma} W_{\mu\nu\rho\sigma} \right]$$

that gives  $c_t \neq 1$  [Bauman et al \(2015\)](#) but this is result of truncation.

# Conclusions

- We obtain most general theory of gravity consistent with observations and new predictions.
- We obtain  $r < 0.036$  with tensor power spectrum modification. We get a violation of tensor consistency relation  $r = -8n_t$  and also we can have blue tilt. This proves inflation is consistent with blue-tilt.
- We obtain all shapes of scalar non-Gaussianities predicted also from EFT of inflation. Furthermore, we get more predictions compared to EFT.
- What about tensor non-Gaussianities? (yes, work in progress.)
- Generalized non-local  $R^2$ -like inflation is an interesting target for CMBS4, LISA and LSS observations such as 21cm.

Thank you for your attention

Stay tuned to arXiv for further results!