Equations of state in the curved spacetime of compact degenerate stars

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White Dwarfs and Neutron Stars

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- This process releases a huge amount of energy which results in an increase in temperature even more and it also exerts thermal pressure which keeps the sphere of the gas stable from further collapse. A stable equilibrium is reached eventually which forms the second phase of the star's lifetime which may continue for a millions of years till there is sufficient hydrogen to sustain the nuclear reactions in its core.
- Once the hydrogen is insufficient to keep the process continuing, the gravitational pull wins over the exerted thermal pressure, resulting in the collapse of the star. This is the final phase of the star.

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- Because of the compactness of these stars, the number density of matter constituent particles are extremely high which results in a degenerate matter. For example, the average mass density of white dwarf Sirius B is $2.8 \times 10^9 Kg/m^3$ which has observed mass around $1M_\odot$, radius $0.008R_\odot$ and temperature 25922K.

Motivation

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- General relativity plays an important role for these stars due to their compact size and significant mass which cannot be overlooked.
- The lapse function of a spherical star at the boundary is given by $1 - 2GM/R$. For white dwarfs like Sirius B, $2GM/R \sim 10^{-3}$ whereas for a $2M_\odot$ neutron star of radius 12Km, $2GM/R \sim .5$.

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- **On the other hand, in the literature, the equation of states used in studying these stars are invariably computed in flat spacetime.** This leads one to grossly underestimate the mass limits of these compact stars.

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- **Therefore, an EOS computed in the global flat spacetime cannot capture the effects of strong gravity on the matter field dynamics within these stars despite using Einstein's equations.**
- It necessitates a first principle derivation of the **EOS using the curved spacetime (curved EOS) of these stars**. Using the curved EOS, we compute the mass-radius relation of these stars.

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- Despite being a semi-classical theory, it becomes successful in describing phenomena like Hawking radiation, Unruh effect, Primordial density perturbation in different models of cosmic inflation, and so on.
- The aim of this thesis is to show the effect of curved spacetimes of compact degenerate stars on the matter field dynamics using QFTCS and its consequence on the mass-radius relations, and angular momentum of these stars.

Spacetime within spherical stars

- In *natural units*, the invariant line element within a static spherical star can be written as

$$ds^2 = -e^{2\Phi(r)}dt^2 + e^{2\nu(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \ . \qquad (1)$$

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- In particular, these equations lead to $e^{-2\nu(r)} = (1 - 2G\mathcal{M}/r)$ and

$$\frac{d\Phi}{dr} = \frac{G(\mathcal{M} + 4\pi r^3 P)}{r(r - 2G\mathcal{M})} \ , \quad \frac{dP}{dr} = -(\rho + P)\frac{d\Phi}{dr} \ , \tag{2}$$

  where $d\mathcal{M} = 4\pi r^2 \rho dr$.

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- In a new set of coordinates $X = e^{\nu(r_0)} r \sin\bar{\theta} \cos\phi$, $Y = e^{\nu(r_0)} r \sin\bar{\theta} \sin\phi$, and $Z = e^{\nu(r_0)} r \cos\bar{\theta}$ along with $\bar{\theta} = e^{-\nu(r_0)}\theta$, the metric within the box reduces to

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- **The above metric is flat over a scale which is sufficient to describe the nuclear interactions. At the same time, it shows that the metric is not globally flat as it carries information about the large scale radial variation of the metric function Φ.**

Ideal neutron stars and white dwarfs with non-interacting fermions

- An ideal neutron star and a white dwarf consist of non-interacting degenerate gas of fermions. For the mathematical generalization, we consider the action $S = \sum_I S_{\psi_I}$ where the index I runs over different types of 4-component Dirac spinor field ψ_I.

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- We assume *perfect fluid* form for the stress-energy tensor, which implies $\langle \hat{T}_{\mu\nu} \rangle = (\rho + P) u_\mu u_\mu + P g_{\mu\nu}$.

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- These components of stress-energy tensor can be obtained from the corresponding partition function in a given small region, which is given by

$$\mathcal{Z} = \mathrm{Tr}[e^{-\beta(\hat{H} - \sum_I \mu_I \hat{N}_I)}] \ , \tag{4}$$

where μ_I, $\hat{N}_I$ are the chemical potential and number operator of I^{th} spinor field, and $\beta = \frac{1}{k_B T}$.

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$$S_{\psi_I} = -\int d^4x\sqrt{-g}\;\bar{\psi}_I[i\gamma^a e^{\mu}{}_a\mathcal{D}_\mu + m_I]\psi_I \;, \tag{5}$$

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- $e^\mu{}_a$ are the tetrad components satisfying $e^\mu{}_a e^\nu{}_b \eta^{ab} = g^{\mu\nu}$ where $\eta_{ab} = \text{diag}(-1,1,1,1)$.

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- e^μ_a are the tetrad components satisfying $e^\mu{}_a e^\nu{}_b\eta^{ab} = g^{\mu\nu}$ where $\eta_{ab} = \text{diag}(-1, 1, 1, 1)$. On the other hand, $\gamma^\mu \equiv \gamma^a e^\mu{}_a$ satisfy $\{\gamma^\mu, \gamma^\nu\} = -2g^{\mu\nu}\mathbb{I}$.

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where k runs over $1, 2, 3$. The corresponding conserved charge then becomes $Q_I = \int d^3x \sqrt{-g} j_I^0 = \int d^3x \bar{\psi}_I \gamma^0 \psi_I$.

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- **The reduced action (6) should be viewed as an effective field action in a locally Minkowski spacetime. Moreover, it carries information about the box-specific, fixed metric function Φ.**

Partition function

- The partition function corresponding to the this model that represents the non-interacting spinor degrees of freedom within the box is given by

$$\ln \mathcal{Z} = \sum_{I} \ln \mathcal{Z}_{\psi_I} \ . \tag{7}$$

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- Using the degeneracy condition $\beta\mu_l \gg 1$, we obtain the following expression

$$\ln \mathcal{Z}_{\psi_l} = \frac{\beta V e^{-3\Phi}}{24\pi^2}\left[2\mu_l\mu_{lm}^3 - 3\tilde{m}_l^2\bar{\mu}_{lm}^2 + \frac{48\mu_l\mu_{lm}}{\beta^2}\right] \ , \tag{8}$$

where V denotes the spatial volume of the box, $\tilde{m}_l = m_l e^{\Phi}$, $\mu_{lm} \equiv \sqrt{\mu_l^2 - \tilde{m}_l^2}$ and $\bar{\mu}_{lm}^2 \equiv \mu_l\mu_{lm} - \tilde{m}_l^2 \ln(\frac{\mu_l + \mu_{lm}}{\tilde{m}_l})$.

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$$P = e^{\Phi} \sum_l \frac{m_l^4}{24\pi^2} \left[\sqrt{(b_l n)^{2/3} + 1} \left\{ 2(b_l n) - 3(b_l n)^{1/3} \right\} + 3 \ln \left\{ (b_l n)^{1/3} + \sqrt{(b_l n)^{2/3} + 1} \right\} \right] , \quad (9)$$

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where the constant $b_I = 3\pi^2 \bar{b}_I / m_I^3$ and the temperature correction is neglected.
- Ignoring the temperature corrections as earlier, the expression of energy density ρ within the box is given by

$$\rho = -P + e^{\Phi} \sum_I \frac{m_I^4}{3\pi^2} \sqrt{(b_I n)^{2/3} + 1} \, (b_I n) \, . \tag{10}$$

EOS for white dwarfs and ideal neutron stars

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- In white dwarfs, the nuclei have negligible motion. Hence, the energy density due to the nuclei is $n m_u e^{\Phi}$ which needs to be included in total energy density ρ. Here m_u is the atomic mass unit.
- The EOS for an *ideal* neutron star follows from the Eqs. (9, 10), if one chooses the fermions to be the neutrons, and sets $\bar{b}_I = 1$.

Mass-radius relation of white dwarfs

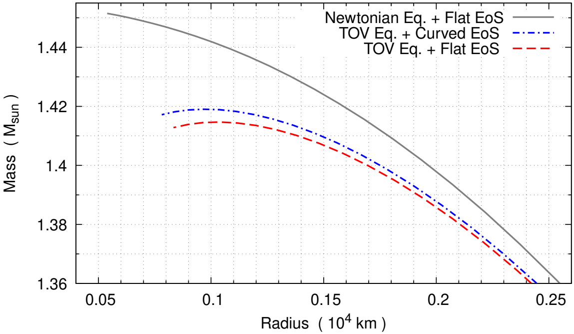

Figure: The mass-radius relations for the white dwarfs with $A/Z = 2$. The curved EOS leads the maximum mass limit to increase from around $1.415 M_{\odot}$ to $1.419 M_{\odot}$

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Mass-radius relation of ideal neutron star

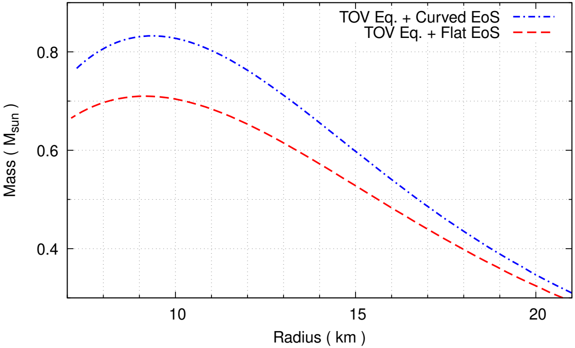

Figure: The mass-radius relations for ideal neutron stars. The maximum mass limit increases by $\sim 16.9\%$, from around $0.71 M_\odot$ to $0.83 M_\odot$ and the corresponding radius increases by $\sim 2.2\%$, from approximately 9.2 km to 9.4 km, due to the usage of curved EOS.

The $\sigma - \omega$ model of nuclear matter

- The action describing $\sigma - \omega$ model of nuclear matter inside neutron star is given by

$$S = \int d^4x \sqrt{-g}\ \mathcal{L} = \int d^4x \sqrt{-g}\,[\mathcal{L}_D + \mathcal{L}_M]\ , \tag{11}$$

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$$\mathcal{L}_\omega = -g^{\mu\rho}g^{\nu\lambda}(\nabla_{[\mu}\omega_{\nu]})(\nabla_{[\rho}\omega_{\lambda]}) - \frac{1}{2}m_\omega^2 g^{\mu\nu}\omega_\mu\omega_\nu \;, \tag{14}$$

$$\mathcal{L}_{\omega i} = \frac{\zeta\; g_\omega^4}{4!}(g^{\mu\nu}\omega_\mu\omega_\nu)^2 + \sum_{l=n,p} g_\omega\omega_\mu e^{\mu}{}_a\bar{\psi}_l\gamma^a\psi_l \;. \tag{15}$$

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- The Euler-Lagrange equation for the σ meson then becomes

$$\bar{\sigma} \equiv m_{\sigma} \langle \hat{\sigma} \rangle = \bar{g}_{\sigma} \sum_{I=n,p} n_I^S \ , \ \text{where} \ \ \bar{g}_{\sigma} = \left(\frac{g_{\sigma}}{m_{\sigma}} \right) \ , \tag{17}$$

and $n_I^S = \langle \bar{\psi}_I \psi_I \rangle$ is the pseudo-scalar number density of the baryon.

RMF approximation

- Similarly, by using the RMF approximation, the Euler-Lagrange equation for the temporal component of the ω meson, leads to

$$\bar{\omega} + \frac{\zeta\, \bar{g}_\omega^4}{6}\, \bar{\omega}^3 = \bar{g}_\omega \sum_{I=n,p} n_I \ , \quad \text{where} \quad \bar{g}_\omega = \left(\frac{g_\omega}{m_\omega}\right) \ , \tag{18}$$

where $\bar{\omega} = m_\omega \langle \hat{\omega}^0 \rangle e^{\Phi}$ and $n_I = \langle \bar{\psi}_I \gamma^0 \psi_I \rangle = \langle \psi_I^\dagger \psi_I \rangle$.

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- Using the locally flat coordinates (t, X, Y, Z) inside the box, together with the RMF approximation, the action for meson fields reduce to

$$S_M = \int d^4x \Big[\mathcal{L}_{\sigma\omega} + \sum_{I=n,p} \bar{\psi}_I e^\Phi (\bar{g}_\sigma \bar{\sigma} - \gamma^0 \bar{g}_\omega \bar{\omega}) \psi_I \Big] \ , \tag{19}$$

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where

$$\mathcal{L}_{\sigma\omega} = e^\Phi \left[-\frac{\bar{\sigma}^2}{2} + \frac{\bar{\omega}^2}{2} + \frac{\zeta\,\bar{g}_\omega^4}{24}\,\bar{\omega}^4\right] \ . \tag{20}$$

Partition function

- By using the reduced actions (6, 19) one can split the partition function as

$$\ln \mathcal{Z} = \beta V \mathcal{L}_{\sigma\omega} + \sum_{I} \ln \mathcal{Z}_{\psi_I} \ . \tag{21}$$

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- The partition function involving I^{th} spinor can be expressed as $\mathcal{Z}_{\psi_I} = \int \mathcal{D}\bar{\psi}_I \mathcal{D}\psi_I \, e^{-S^{\beta}_{\psi_I}}$ where
$$S^{\beta}_{\psi_I} = \int_0^{\beta} d\tau \int d^3x \bar{\psi}_I \left[-\gamma^0(\partial_\tau + \mu_I^*) + e^{\Phi}(i\gamma^k\partial_k + m_I^*) \right] \psi_I \ . \tag{22}$$

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- In the eq. (22), the *effective* chemical potential is

$$\mu_I^* = \mu_I - \bar{g}_\omega \bar{\omega} \, e^{\Phi} (\delta_I^p + \delta_I^n) \ , \tag{23}$$

whereas the *effective* mass is given by

$$m_I^* = m_I - \bar{g}_\sigma \bar{\sigma} \, (\delta_I^p + \delta_I^n) \ . \tag{24}$$

Partition function

- Doing integration over the Grassmann variables, one can evaluate the partition function for the l^{th} spinor as

$$\ln \mathcal{Z}_{\psi_l} = \frac{\beta V e^{-3\Phi}}{24\pi^2}\left[2\mu_l^* \mu_{lm}^3 - 3\bar{m}_l^2\,\bar{\mu}_{lm}^2\right], \;\; \bar{m}_l = e^{\Phi} m_l^* \;, \tag{25}$$

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where $\mu_{Im} \equiv \sqrt{\mu_I^{*2} - \bar{m}_I^2}$ and $\bar{\mu}_{Im}^2 \equiv \mu_I^* \mu_{Im} - \bar{m}_I^2 \,\text{asinh}(\mu_{Im}/\bar{m}_I)$.

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- The number density of l^{th} spinor and its pseudo-scalar number density are expressed as

$$n_l = \frac{e^{-3\Phi}}{3\pi^2}\, \mu_{lm}^3 \quad , \quad n_l^S = \frac{e^{-3\Phi}}{2\pi^2}\, \bar{m}_l \bar{\mu}_{lm}^2 \; . \qquad (26)$$

Pressure and energy density

- For this grand canonical ensemble, the total pressure $P = (\beta V)^{-1} \ln \mathcal{Z}$ becomes

$$P = P_{\sigma\omega} + \sum_{I} P_{I} \ , \tag{27}$$

where the term involving only meson contributions is

$$P_{\sigma\omega} = e^{\Phi} \left[-\frac{\bar{\sigma}^2}{2} + \frac{\bar{\omega}^2}{2} + \frac{\zeta \, \bar{g}_{\omega}^4}{24} \, \bar{\omega}^4 \right] \ . \tag{28}$$

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- On the other hand, the pressure contribution involving l^{th} spinor is given by

$$P_l = \frac{e^{\Phi} m_l^{*4}}{24\pi^2} \left[\sqrt{(b_l n_l)^{\frac{2}{3}} + 1} \left\{ 2(b_l n_l) - 3(b_l n_l)^{\frac{1}{3}} \right\} + \ 3 \,\text{asinh} \left\{ (b_l n_l)^{\frac{1}{3}} \right\} \right] \ , \tag{29}$$

where the constant $b_l = 3\pi^2 / m_l^{*3}$.

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- Nevertheless, these number densities, corresponding to the three fermions are subjected to the constraints $n_p = n_e$ and $\mu_n = \mu_p + \mu_e$.
- For convenience of numerical evaluation, we define the scaled coupling constants of mesons as $\tilde{g}_{\sigma} = \bar{g}_{\sigma} m_n$, $\tilde{g}_{\omega} = \bar{g}_{\omega} m_n$.

Kinematic behaviour of curved EOS

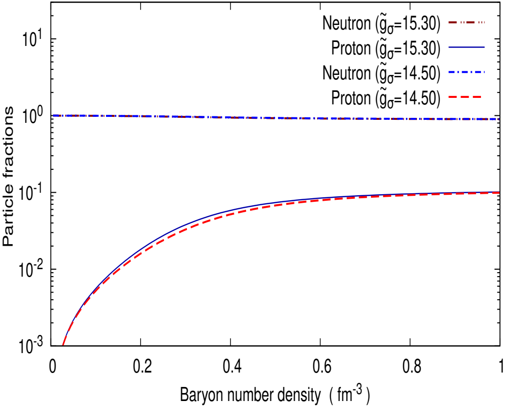

Figure: Particle fractions as a function of baryon number density N for different values of $\tilde{g}_\sigma$.

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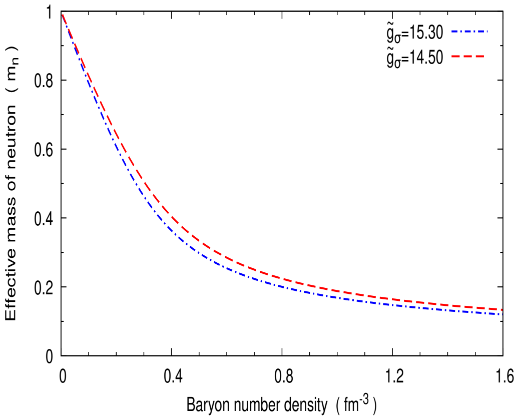

Figure: Effective mass of the neutron as a function of baryon number density N for different values of coupling constant $\tilde{g}_\sigma$.

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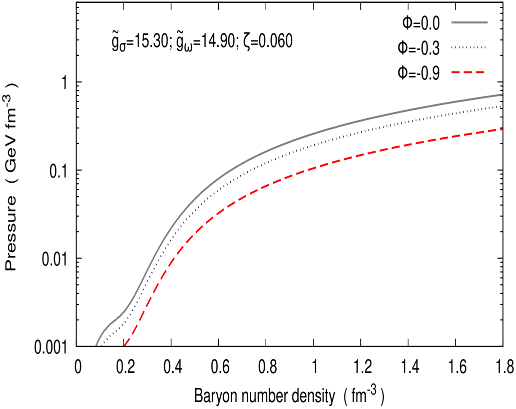

Figure: Plot of the pressure P as a function of baryon number density N for different kinematical values of Φ.

Kinematic behaviour of curved EOS

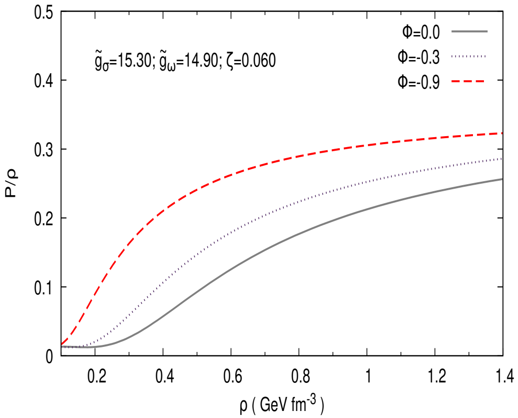

Figure: Ratio between the pressure and the energy density, (P/ρ), is plotted as a function of ρ for different kinematical values of the metric function Φ.

Mass radius relations

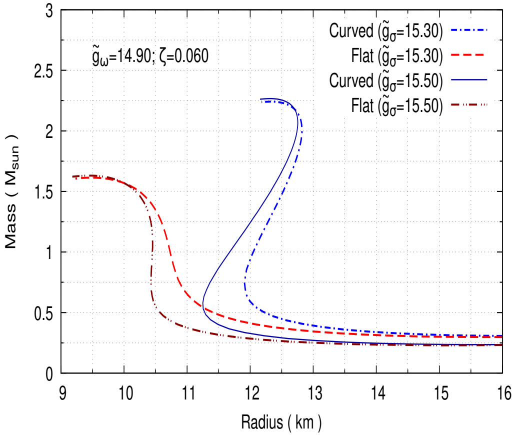

Figure: Plot of the mass-radius relations for different values of the parameter $\tilde{g}_\sigma$.

Mass radius relations

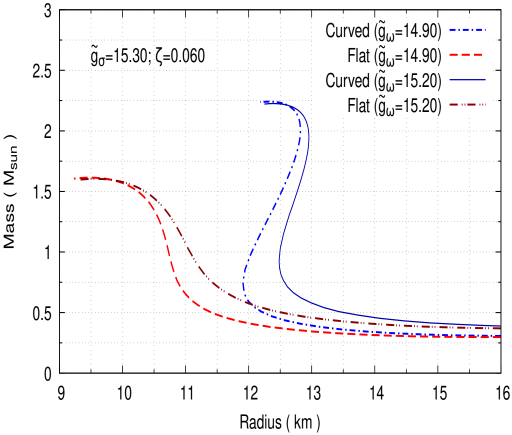

Figure: Plot of the mass-radius relations for different values of the parameter $\tilde{g}_\omega$.

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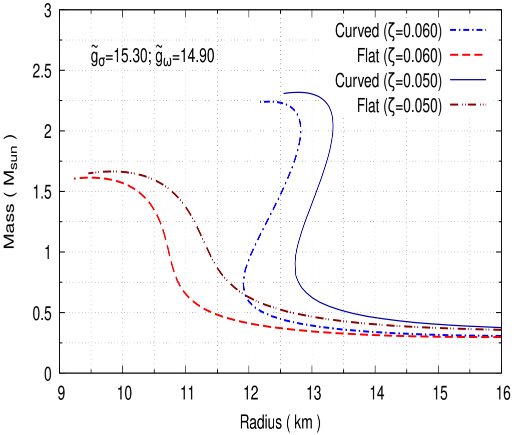

Figure: Plot of the mass-radius relations for different values of the parameter ζ.

Slowly rotating star

- The spacetime metric of a slowly rotating, axially symmetric star can be represented, in the *natural units* $c = \hbar = 1$, by an invariant line element

$$ds^2 = -e^{2\Phi}dt^2 + e^{2\nu}dr^2 + r^2[d\theta^2 + \sin^2\theta(d\varphi - \omega dt)^2] \, , \qquad (33)$$

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- The *exterior* vacuum Einstein equation corresponding to the metric (33) can be solved exactly as

$$e^{2\Phi} = e^{-2\nu} = 1 - \frac{2GM}{r} \ ; \ \omega = \frac{2GJ}{r^3} \ , \qquad (34)$$

where constant J and M represent the angular momentum and the 'spherical' mass of the star.

Einstein's equations

- In this case, apart from the TOV equations we obtain one additional equation for the frame-dragging angular velocity ω which follows from the $t - \varphi$ component of the Einstein equation and is given by

$$\frac{1}{r^4}\frac{d}{dr}\left(r^4 j \frac{d\omega}{dr}\right) + \frac{4}{r}\frac{dj}{dr}(\omega - \Omega) = 0 \ , \tag{35}$$

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  where $j = e^{-(\nu+\Phi)}$.
- The solutions of the differential equations are subject to the boundary conditions $e^{2\Phi(R)} = (1-2GM/R)$ and $\omega'(R) = -3\omega(R)/R$ where $\omega' = (d\omega/dr)$ and R denotes the radius of the 'spherical' part.

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- In the computation of thermodynamic observables, we neglected contributions $\mathcal{O}(\omega^2)$ as we are considering slow rotation case.

Reduced action

- The metric within the box becomes

$$g_{\mu\nu} = \begin{bmatrix} -e^{2\Phi} & \omega Y & -\omega X & 0 \\ \omega Y & 1 & 0 & 0 \\ -\omega X & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} , \tag{36}$$

where $\Phi = \Phi(r_0)$, $\omega = \omega(r_0)$.

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where $\Phi = \Phi(r_0)$, $\omega = \omega(r_0)$.

- Consequently, within the box the Dirac action reduces to

$$S_\psi = -\int d^4x \; \bar{\psi}[i\gamma^0 \partial_t + e^{\Phi}(i\gamma^k \partial_k + m) - \omega\gamma^0 \hat{J}_Z]\psi \;, \tag{37}$$

where $\hat{J}_Z = (\hat{L}_Z + \frac{1}{2}\Sigma_3)$ with $\hat{L}_Z = -i(X\partial_Y - Y\partial_X)$ and $\Sigma_3 = \sigma^3 \otimes \mathbb{I}_2$.

Partition function

- In order to evaluate the partition function, it is convenient to split it as $\ln \mathcal{Z}_\psi = \ln \mathcal{Z}_0 + \ln \mathcal{Z}_L$ where $\mathcal{Z}_0 = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi \; e^{-S_0^\beta}$ with

$$S_0^\beta = \int_0^\beta d\tau \int d^3x \bar{\psi}\Big[-\gamma^0\big(\partial_\tau + \mu + \frac{\omega}{2}\Sigma_3\big) + e^\Phi\big(i\gamma^k\partial_k + m\big)\Big]\psi. \quad (38)$$

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- On the other hand, $\ln \mathcal{Z}_L$ contains contributions from the orbital angular momentum operator $\hat{L}_Z$ and can be expressed as a perturbative series

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Partition function

- In order to evaluate the partition function, it is convenient to split it as $\ln \mathcal{Z}_\psi = \ln \mathcal{Z}_0 + \ln \mathcal{Z}_L$ where $\mathcal{Z}_0 = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi \ e^{-S_0^\beta}$ with

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- It can be shown that the leading order terms in $\ln \mathcal{Z}_L$ are $\mathcal{O}(\omega^2)$ which we neglect henceforth for a slowly-rotating star.

Pressure and energy density

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- Here $n_\pm$ satisfy the following relations

$$
n = n_+ + n_-, \ \ me^\Phi\sqrt{(bn_+)^{2/3}+1} - me^\Phi\sqrt{(bn_-)^{2/3}+1} = \omega. \quad (42)
$$

where the constant $b = (6\pi^2/m^3)$.

Kinematical behaviour of EOS

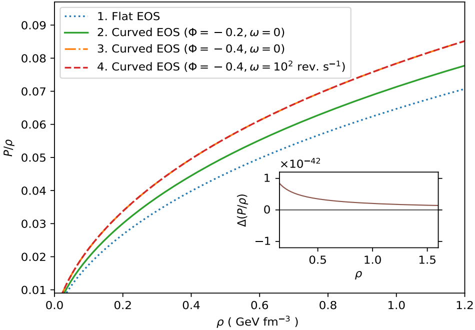

Figure: Plot of the ratio P/ρ as a function of the energy density ρ for different kinematical values of the metric functions Φ and ω. The curves 2 and 3 with different values of ω are indistinguishable as earlier. The sharper fall at lower densities of the curve 5 arises due to the effect of threshold number density n_0.

Mass radius relation

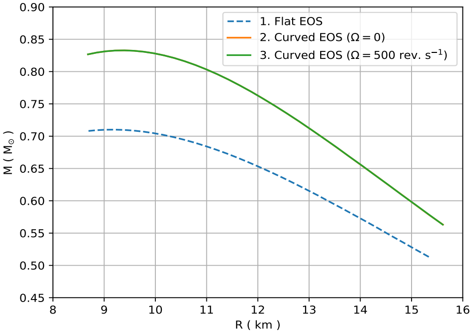

Figure: Comparison of the mass-radius relations for a slowly-rotating ideal neutron star whose degenerate core is made of an ensemble of non-interacting neutrons. The effect of gravitational time dilation itself leads to an increase of maximum mass limit from 0.71 $M_\odot$ to 0.83 $M_\odot$. The enhancement of mass limit due to the dragging of inertial frames is however extremely small for any physically plausible values of stellar fluid angular velocity Ω.

Angular momentum plot

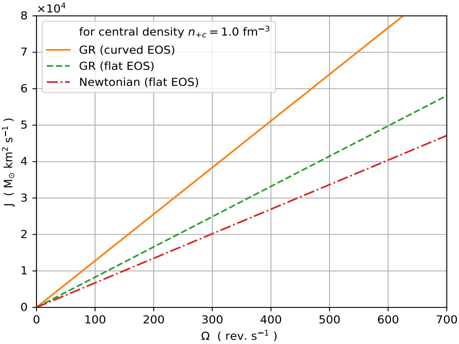

Figure: Comparison of the angular momenta of a slowly rotating ideal neutron star as a function of the angular velocity Ω of the stellar fluid. The Newtonian expression for the angular momenta is computed as $J = \Omega \int drd\theta d\varphi \; \rho r^4 \sin^3\theta$ where energy density $\rho = \rho(r)$ is taken to be the same as found by solving TOV equations with the flat EOS.

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- Here we show that the genesis of seed magnetism is a direct consequence of **spin-degeneracy breaking of fermions caused by the curved spacetime of rotating neutron stars, principally due to the dragging of inertial frames**.

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- The partition function of a rotating ideal neutron star is expressed as

$$\ln \mathcal{Z}_\psi = \sum_{\mathrm{k}} \left[\ln\left(1 + e^{-\beta(\varepsilon-\mu_+)}\right) + \ln\left(1 + e^{-\beta(\varepsilon-\mu_-)}\right) \right] . \qquad (43)$$

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$$n = n_+ + n_- \ , \quad \text{with} \quad n_\pm = \frac{1}{V} \sum_{\mathrm{k}} \frac{1}{e^{\beta(\varepsilon - \mu_\pm)} + 1} \ . \tag{44}$$

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- The net magnetic moment is defined by $\mathfrak{M} = \mu_{\mathbf{D}}(n_+ - n_-)$ where $\mu_{\mathbf{D}}$ is the *magnitude* of the magnetic moment of the neutrons. The corresponding magnetic moment then can be expressed as

$$\mathfrak{M} = \mu_{\mathbf{D}} \frac{\beta\omega}{V} \sum_{\mathrm{k}} \frac{e^{\beta(\varepsilon-\mu)}}{(e^{\beta(\varepsilon-\mu)} + 1)^2} + \mathcal{O}(\omega^2) . \qquad (45)$$

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- Consider a test magnetic field B along the Z-direction. The coupling between neutron field ψ and the electromagnetic field A_μ is described by Pauli-Dirac interaction term $S_I = \int d^4x\sqrt{-g}\ \bar{\psi}\left[\frac{1}{2}\mu_{\mathbf{D}}\sigma^{\mu\nu}F_{\mu\nu}\right]\psi$ where $\sigma^{\mu\nu} = \frac{i}{2}e^{\mu}{}_{a}e^{\nu}{}_{b}[\gamma^a, \gamma^b]$ and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.

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- The magnetic susceptibility is found out to be

$$\chi = 2{\mu_{\mathbf{D}}}^2\frac{\beta e^{\Phi}}{V}\sum_{\mathrm{k}}\frac{e^{\beta(\varepsilon-\mu)}}{(e^{\beta(\varepsilon-\mu)}+1)^2} + \mathcal{O}(\omega)\ . \tag{47}$$

Primeval seed magnetic field of ideal neutron stars

- Therefore, the resultant magnetic field in the small box is given by

$$B = \frac{\mathfrak{M}}{\chi} = \frac{1}{2\mu_{\mathbf{D}}} \frac{\omega}{e^{\Phi}} + \mathcal{O}(\omega^2) \ . \tag{48}$$

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- **The presence of the above non-zero seed magnetic field is crucial in a newly born proto-neutron star (PNS) where seed field can be amplified almost exponentially by turbulent dynamo mechanism.** For example, a seed field of 0.1 Gauss, as seen here, can be amplified to around 10^{11} Gauss, a typically observed field strength in neutron stars. **Further, due to the expected differential rotation inside a PNS, the seed field itself could be stronger.**

Primeval seed magnetic field of ideal neutron stars

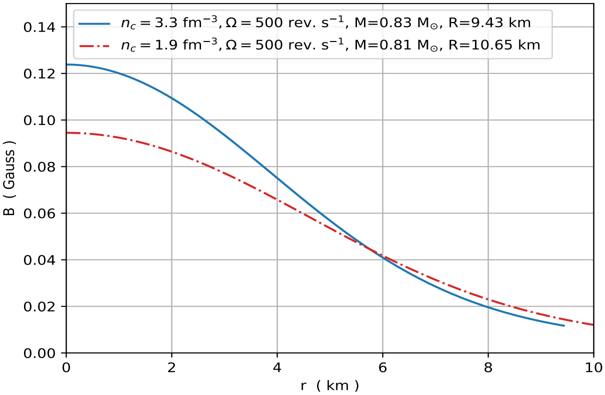

Figure: Primeval seed magnetic field strength inside an ideal neutron star as a function of r. For a central neutron number density of 3.3 fm^{-3}, the star has a seed magnetic field of around 0.12 Gauss at its center. Here we have taken $\mu_{\mathbf{D}} = 9.66 \times 10^{-31}$ J/Gauss for neutrons.

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- Given the flat EOS are widely used in the neutron star literature, the result presented here would imply significant alterations of various existing predictions.

Thank you for your attention