Initial Conditions for Inflation in an FRW Universe

@ Thursday Seminar, IIT Madras

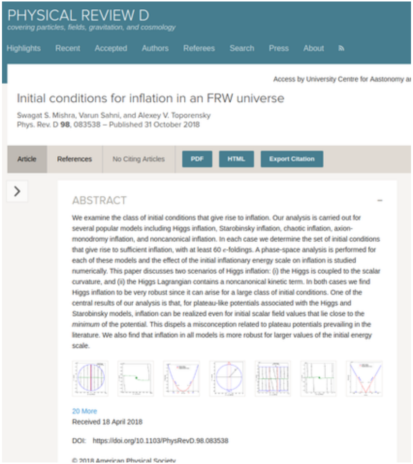
22 November, 2018

Swagat Saurav Mishra (SRF CSIR)
Inter-University Centre for Astronomy and Astrophysics(IUCAA), Pune

In Collaboration with: Prof. Varun Sahni (IUCAA), Prof. Aleksey Toporensky (Moscow State University)

Other Collaborators: Yuri Shtanov (BITP, Ukraine), Armaan Shafieloo (KASI, Korea), Sanil Unnikrishnan (St. Stephens, Delhi), Satadru Bag (IUCAA), Ujjaini Alam (ISI, Kolkata)

Published in Phys. Rev. D 98, 083538 [arXiv:1801.04948]


PHYSICAL REVIEW D
covering particles, fields, gravitation, and cosmology

Highlights Recent Accepted Authors Referees Search Press About

Access by University Centre for Aastonomy a

Initial conditions for inflation in an FRW universe

Swagat S. Mishra, Varun Sahni, and Alexey V. Toporensky
Phys. Rev. D **98**, 083538 – Published 31 October 2018

Article References No Citing Articles PDF HTML Export Citation

ABSTRACT

We examine the class of initial conditions that give rise to inflation. Our analysis is carried out for several popular models including Higgs inflation, Starobinsky inflation, chaotic inflation, axion-monodromy inflation, and noncanonical inflation. In each case we determine the set of initial conditions that give rise to sufficient inflation, with at least 60 e-foldings. A phase-space analysis is performed for each of these models and the effect of the initial inflationary energy scale on inflation is studied numerically. This paper discusses two scenarios of Higgs inflation: (i) the Higgs is coupled to the scalar curvature, and (ii) the Higgs Lagrangian contains a noncanonical kinetic term. In both cases we find Higgs inflation to be very robust since it can arise for a large class of initial conditions. One of the central results of our analysis is that, for plateau-like potentials associated with the Higgs and Starobinsky models, inflation can be realized even for initial scalar field values that lie close to the *minimum* of the potential. This dispels a misconception related to plateau potentials prevailing in the literature. We also find that inflation in all models is more robust for larger values of the initial energy scale.

20 More

Received 18 April 2018

DOI: https://doi.org/10.1103/PhysRevD.98.083538

Why bother about for Initial Conditions!

Why bother about for Initial Conditions!

1. Traditional Physics aims at discovering laws of evolution (dynamics) of a system in nature given Initial conditions.

Why bother about for Initial Conditions!

1. Traditional Physics aims at discovering laws of evolution (dynamics) of a system in nature given Initial conditions.
2. Initial conditions are to be given.

Why bother about for Initial Conditions!

1. Traditional Physics aims at discovering laws of evolution (dynamics) of a system in nature given Initial conditions.
2. Initial conditions are to be given.
3. Cosmology, the study of ORIGIN, evolution and the fate of the universe, unfortunately can not afford to ignore initial conditions. It has to explain the initial conditions.

Why bother about for Initial Conditions!

1. Traditional Physics aims at discovering laws of evolution (dynamics) of a system in nature given Initial conditions.
2. Initial conditions are to be given.
3. Cosmology, the study of ORIGIN, evolution and the fate of the universe, unfortunately can not afford to ignore initial conditions. It has to explain the initial conditions.
4. Standard Hot Big Bang Cosmology seemed (or seems) to be suggesting that universe started out with a set of very finely tuned unnatural initial conditions.

Why bother about for Initial Conditions!

1. Traditional Physics aims at discovering laws of evolution (dynamics) of a system in nature given Initial conditions.
2. Initial conditions are to be given.
3. Cosmology, the study of ORIGIN, evolution and the fate of the universe, unfortunately can not afford to ignore initial conditions. It has to explain the initial conditions.
4. Standard Hot Big Bang Cosmology seemed (or seems) to be suggesting that universe started out with a set of very finely tuned unnatural initial conditions.
5. Inflationary Cosmology was proposed as an extension to the standard Hot Expanding Universe Theory in order to set up the stage for the hot Big Bang phase providing the required initial conditions.

Why bother about for Initial Conditions!

1. Traditional Physics aims at discovering laws of evolution (dynamics) of a system in nature given Initial conditions.
2. Initial conditions are to be given.
3. Cosmology, the study of ORIGIN, evolution and the fate of the universe, unfortunately can not afford to ignore initial conditions. It has to explain the initial conditions.
4. Standard Hot Big Bang Cosmology seemed (or seems) to be suggesting that universe started out with a set of very finely tuned unnatural initial conditions.
5. Inflationary Cosmology was proposed as an extension to the standard Hot Expanding Universe Theory in order to set up the stage for the hot Big Bang phase providing the required initial conditions.

Why bother about for Initial Conditions!

1. Traditional Physics aims at discovering laws of evolution (dynamics) of a system in nature given Initial conditions.
2. Initial conditions are to be given.
3. Cosmology, the study of ORIGIN, evolution and the fate of the universe, unfortunately can not afford to ignore initial conditions. It has to explain the initial conditions.
4. Standard Hot Big Bang Cosmology seemed (or seems) to be suggesting that universe started out with a set of very finely tuned unnatural initial conditions.
5. Inflationary Cosmology was proposed as an extension to the standard Hot Expanding Universe Theory in order to set up the stage for the hot Big Bang phase providing the required initial conditions.

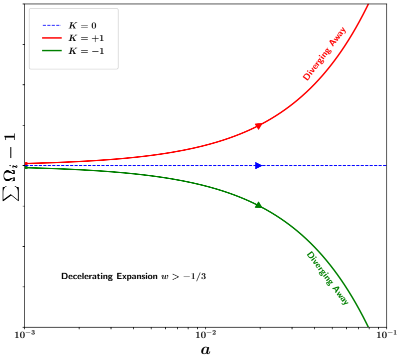
So Cosmic Inflation should better happen starting from generic Initial Conditions!

Need for Cosmic Inflation

1) **Flatness Problem** :

$$\sum_i \Omega_i(a) - 1 = \frac{K}{a^2 H^2}$$

$\sum_i \Omega_i(a)$ diverges away from 1 if
$\ddot{a} < 0$



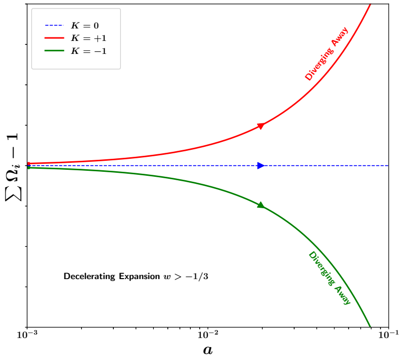
Need for Cosmic Inflation

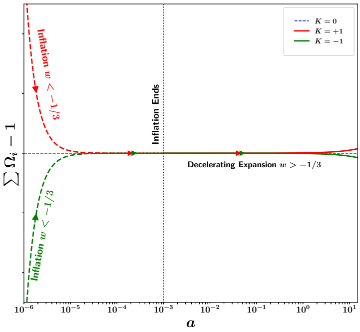
1) **Flatness Problem** :

$$\sum_i \Omega_i(a) - 1 = \frac{K}{a^2 H^2}$$

$\sum_i \Omega_i(a)$ diverges away from 1 if $\ddot{a} < 0$

Long enough period of accelerated expansion drives $\sum_i \Omega_i(a) \longrightarrow 1$





Need for Cosmic Inflation

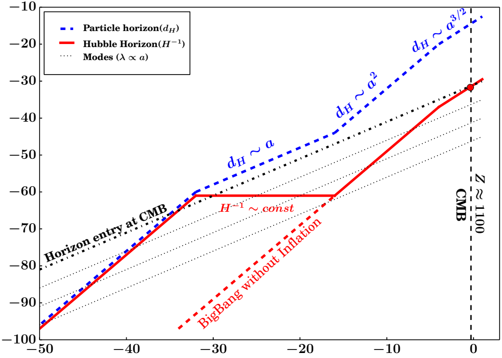
2) **Horizon Problem and Super-Hubble Correlations**:

Need for Cosmic Inflation

2) **Horizon Problem and Super-Hubble Correlations**:
Why CMB has the same T on angular scales greater than 1^0 ?

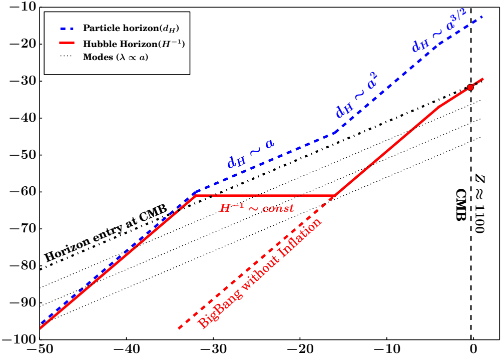
Need for Cosmic Inflation

2) **Horizon Problem and Super-Hubble Correlations**:
Why CMB has the same T on angular scales greater than 1^0 ? Why CMB fluctuations are correlated beyond Hubble scale?



Need for Cosmic Inflation

2) **Horizon Problem and Super-Hubble Correlations**:
Why CMB has the same T on angular scales greater than 1^0 ? Why CMB fluctuations are correlated beyond Hubble scale?

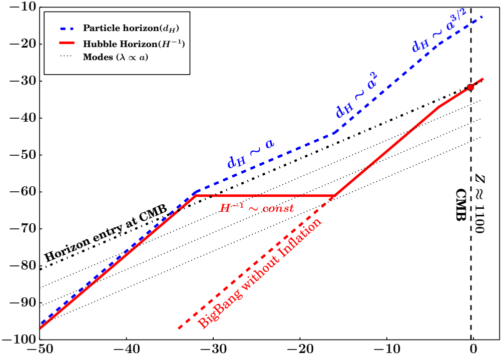


3)Generating almost (**but not exactly**) scale invariant predominantly Gaussian adiabatic primordial fluctuations

$$\Delta^2_{\mathcal{R}} = A_s \left(\frac{K}{K_*}\right)^{n_S-1}$$

$A_s = 2.2 \times 10^{-9}$ and $r \leq 0.06$.

2) **Horizon Problem and Super-Hubble Correlations**:
Why CMB has the same T on angular scales greater than 1^0 ? Why CMB fluctuations are correlated beyond Hubble scale?



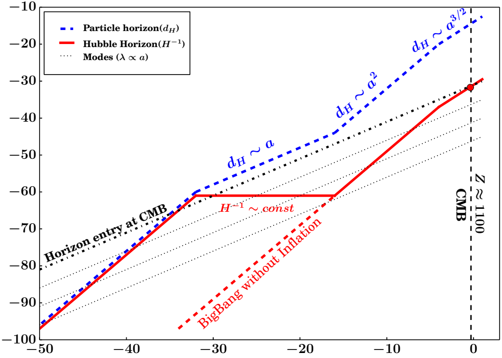
3)Generating almost (**but not exactly**) scale invariant predominantly Gaussian adiabatic primordial fluctuations

$$\Delta^2_{\mathcal{R}} = A_s \left(\frac{K}{K_*}\right)^{n_S - 1}$$

$A_s = 2.2 \times 10^{-9}$ and $r \leq 0.06$.
4)Getting rid of unobserved relics of phase transitions from earlier epochs.

Need for Cosmic Inflation

2) **Horizon Problem and Super-Hubble Correlations**:
Why CMB has the same T on angular scales greater than 1^0 ? Why CMB fluctuations are correlated beyond Hubble scale?



3)Generating almost (**but not exactly**) scale invariant predominantly Gaussian adiabatic primordial fluctuations

$$\Delta^2_{\mathcal{R}} = A_s \left(\frac{K}{K_*}\right)^{n_S - 1}$$

$A_s = 2.2 \times 10^{-9}$ and $r \leq 0.06$.
4)Getting rid of unobserved relics of phase transitions from earlier epochs.
A period of $50 - 60$ e-foldings of accelerated expansion (inflation) is required.
How to achieve a brief period of Inflation?

Inflationary Dynamics of a Scalar Field

Action of a scalar field minimally coupled to gravity

$$S[\phi] = \int \mathrm{d}^4 x \sqrt{-g}\, \mathcal{L}(F, \phi)$$

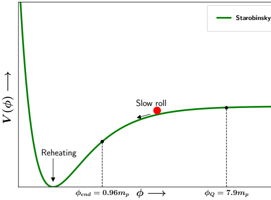
with $F = \frac{1}{2}\partial_\mu \phi\, \partial^\mu \phi$ which for a canonical kinetic term leads to

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi), \quad p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

And Einstein's equations imply

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \left(\frac{8\pi G}{3}\right)\rho_\phi = \frac{1}{3m_p^2}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right),$$

$$\frac{\ddot{a}}{a} = -\left(\frac{4\pi G}{3}\right)\left(\rho_\phi + 3\,p_\phi\right) = -\frac{1}{3m_p^2}\left(\dot{\phi}^2 - V(\phi)\right)$$



The dynamics of the scalar field is governed by

$$\ddot{\phi} + 3\,H\dot{\phi} + V'(\phi) = 0$$

while the equation of state is

$$w_\phi = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)}$$

Inflationary Dynamics

For an extended period of inflation,

$$\epsilon_{_H} = -\frac{\dot{H}}{H^2} < 1 \ , \quad \eta_{_H} = -\frac{\ddot{\phi}}{H\dot{\phi}} < 1$$

Primordial Power-spectra

$$\Delta^2_{\mathcal{R}} = A_s \left(\frac{K}{K_*}\right)^{n_{_S}-1}$$

$$\Delta^2_t = A_t \left(\frac{K}{K_*}\right)^{n_{_t}-1}$$

with

$$A_s = \frac{1}{8\pi^2}\left(\frac{H_*}{m_p}\right)^2 \frac{1}{\epsilon_{_H}}, \ A_t = \frac{2}{\pi^2}\left(\frac{H_*}{m_p}\right)^2$$

Inflationary Dynamics

For an extended period of inflation,

$$\epsilon_H = -\frac{\dot{H}}{H^2} < 1 \; , \quad \eta_H = -\frac{\ddot{\phi}}{H\dot{\phi}} < 1$$

Primordial Power-spectra

$$\Delta^2_{\mathcal{R}} = A_s \left(\frac{K}{K_*}\right)^{n_S - 1}$$

$$\Delta^2_t = A_t \left(\frac{K}{K_*}\right)^{n_t - 1}$$

with

$$A_s = \frac{1}{8\pi^2}\left(\frac{H_*}{m_p}\right)^2 \frac{1}{\epsilon_H}, \; A_t = \frac{2}{\pi^2}\left(\frac{H_*}{m_p}\right)^2$$

Scalar Spectral Index

$$n_S = 1 + 2\eta_H - 4\epsilon_H$$

Tensor to Scalar Ratio

$$r = 16\epsilon_H$$

Inflationary Dynamics

For an extended period of inflation,

$$\epsilon_H = -\frac{\dot{H}}{H^2} < 1 \, , \quad \eta_H = -\frac{\ddot{\phi}}{H\dot{\phi}} < 1$$

Primordial Power-spectra

$$\Delta^2_{\mathcal{R}} = A_s \left(\frac{K}{K_*}\right)^{n_S - 1}$$

$$\Delta^2_t = A_t \left(\frac{K}{K_*}\right)^{n_t - 1}$$

with

$$A_s = \frac{1}{8\pi^2}\left(\frac{H_*}{m_p}\right)^2 \frac{1}{\epsilon_H}, \; A_t = \frac{2}{\pi^2}\left(\frac{H_*}{m_p}\right)^2$$

Scalar Spectral Index

$$n_S = 1 + 2\eta_H - 4\epsilon_H$$

Tensor to Scalar Ratio

$$r = 16\epsilon_H$$

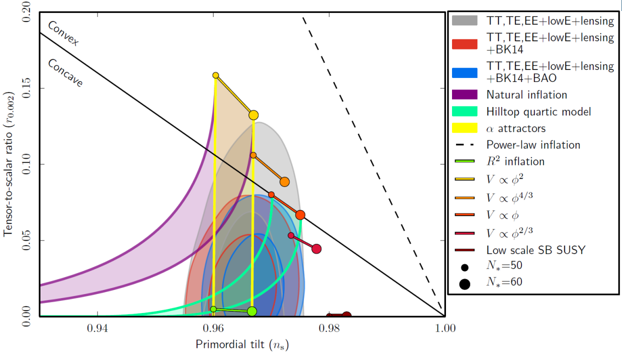
Potential slow-roll parameters

$$\epsilon_V(\phi) = \frac{m_p^2}{2}\left(\frac{V'}{V}\right)^2$$

$$\eta_V(\phi) = m_p^2\left(\frac{V''}{V}\right)$$

Slow-roll conditions corresponds to $\epsilon_H, \eta_H \ll 1$, where
$\epsilon_H \simeq \epsilon_V$, $\eta_H \simeq \eta_V - \epsilon_V$

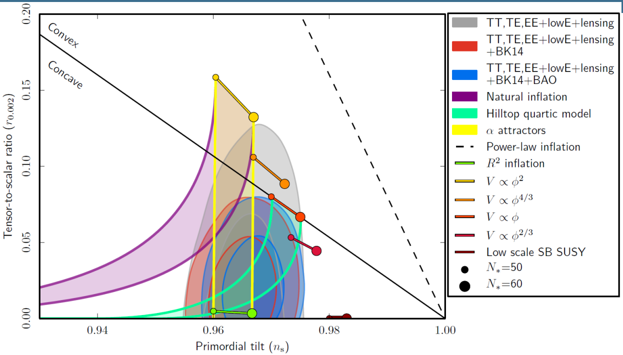
What do we know from CMB Observations



$$n_S \in [0.953, 0.977],$$

$$r \lesssim 0.06$$

What do we know from CMB Observations



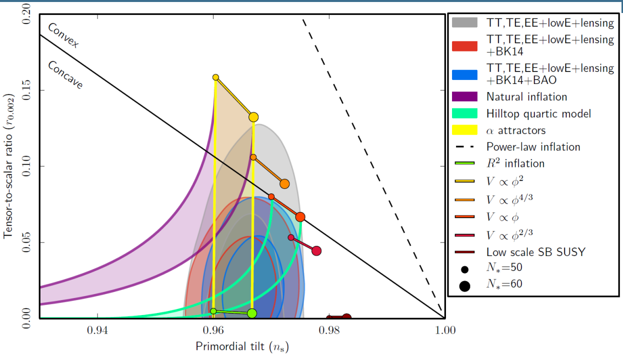
$$n_S \in [0.953, 0.977],$$

$$r \lesssim 0.06$$

which translates to

$$\epsilon_H \lesssim 0.00375,$$

$$|\eta_H| < 0.02$$



$$n_S \in [0.953, 0.977],$$

$$r \leq 0.06$$

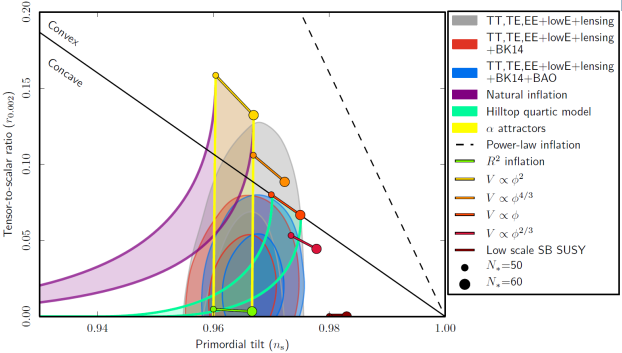
which translates to

$$\epsilon_H \leq 0.00375,$$

$$|\eta_H| < 0.02$$

CMB Observations favour concave potentials over convex potentials.

What do we know from CMB Observations



$$n_S \in [0.953, 0.977],$$

$$r \lesssim 0.06$$

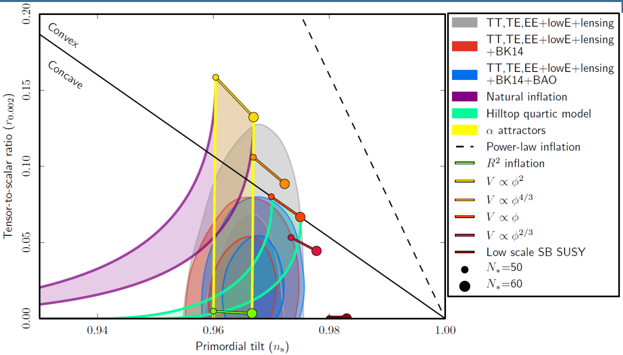
which translates to

$$\epsilon_H \lesssim 0.00375,$$

$$|\eta_H| < 0.02$$

CMB Observations favour concave potentials over convex potentials. Tight constraints on the upper bound of r imply asymptotically flat plateau like potentials are favoured.

What do we know from CMB Observations



$$n_S \in [0.953, 0.977],$$

$$r \lesssim 0.06$$

which translates to

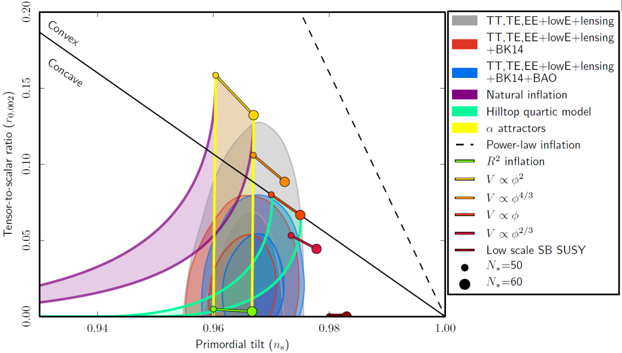
$$\epsilon_H \lesssim 0.00375,$$

$$|\eta_H| < 0.02$$

CMB Observations favour concave potentials over convex potentials. Tight constraints on the upper bound of r imply asymptotically flat plateau like potentials are favoured.

Issue of initial conditions for Inflation becomes more important!!

What do we know from CMB Observations



$$n_S \in [0.953, 0.977],$$

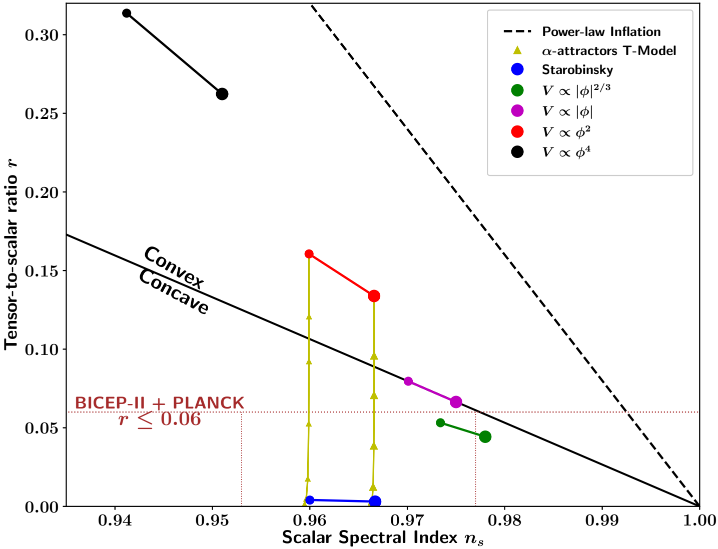
$$r \leq 0.06$$

which translates to

$$\epsilon_H \leq 0.00375,$$

$$|\eta_H| < 0.02$$

CMB Observations favour concave potentials over convex potentials. Tight constraints on the upper bound of r imply asymptotically flat plateau like potentials are favoured.
Issue of initial conditions for Inflation becomes more important!! In this talk, we are not going to discuss about inhomogeneous initial conditions.


Power-law Inflation
α-attractors T-Model
Starobinsky
$V \propto |\phi|^{2/3}$
$V \propto |\phi|$
$V \propto \phi^2$
$V \propto \phi^4$
Convex
Concave
BICEP-II + PLANCK
$r \leq 0.06$
0.30
0.25
0.20
0.15
0.10
0.05
0.00
Tensor-to-scalar ratio r
0.94
0.95
0.96
0.97
0.98
0.99
1.00
Scalar Spectral Index n_s

Initial Conditions for Inflation in an FRW Universe

Inflation $\Rightarrow$ Accelerated Expansion $\Rightarrow \epsilon_H < 1$. When the energy budget of the universe is dominated by the inflaton field, this implies,

$$w_\phi < -\frac{1}{3} \quad \text{or} \quad \dot{\phi}^2 < V(\phi)$$

Even though this condition seems restrictive, it is actually quite generic and can be quantified, at least for monotonically increasing potentials.

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

Role of Friction term $H\dot{\phi}$ and existence of inflationary separatrix (indicating inflationary trajectories are local attractors at least for large field inflation) [Brandenberger 2016]

Hamilton-Jacobi Formalism: $H_{,\phi}^2 - \frac{3}{2}H^2 = -\frac{1}{2}V(\phi)$, $dN = \frac{|d\phi|}{\sqrt{2\epsilon_V}}$

$$\delta H(\phi) = \delta H(\phi_i)e^{-3(N-N_i)}$$

Methodology

- We explicitly calculate the set of initial conditions that yield **adequate inflation** i.e $N_e \geq 60$, where $a(t_e) = a(t_i)e^{N_e}$ so $N_e = \int_{t_i}^{t_e} H dt$
- For a given fixed initial energy scale $\rho(\phi_i)$ of inflation (or equivalently H_i), the first Friedmann equation becomes the equation of a circle

$$R^2 = X^2 + Y^2$$

where

$$R = \sqrt{6}\frac{H}{m_p}, \quad X = \hat{\phi}\frac{\sqrt{2V(\phi)}}{m_p^2}, \quad Y = \frac{1}{m_p^2}\frac{d\phi}{dt}$$

with $\hat{\phi} = \frac{\phi}{|\phi|}$ (this ensures that X and ϕ have same sign).

Methodology

- We explicitly calculate the set of initial conditions that yield **adequate inflation** i.e $N_e \geq 60$, where $a(t_e) = a(t_i)e^{N_e}$ so $N_e = \int_{t_i}^{t_e} H dt$
- For a given fixed initial energy scale $\rho(\phi_i)$ of inflation (or equivalently H_i), the first Friedmann equation becomes the equation of a circle

$$R^2 = X^2 + Y^2$$

where

$$R = \sqrt{6}\frac{H}{m_p}, \quad X = \hat{\phi}\frac{\sqrt{2V(\phi)}}{m_p^2}, \quad Y = \frac{1}{m_p^2}\frac{d\phi}{dt}$$

with $\hat{\phi} = \frac{\phi}{|\phi|}$ (this ensures that X and ϕ have same sign). So fixing initial energy scale of inflation is equivalent to fixing the radius of the circle R while varying X and Y **uniformly**.

Methodology

- We explicitly calculate the set of initial conditions that yield **adequate inflation** i.e $N_e \geq 60$, where $a(t_e) = a(t_i)e^{N_e}$ so $N_e = \int_{t_i}^{t_e} H dt$
- For a given fixed initial energy scale $\rho(\phi_i)$ of inflation (or equivalently H_i), the first Friedmann equation becomes the equation of a circle

$$R^2 = X^2 + Y^2$$

where

$$R = \sqrt{6}\frac{H}{m_p}, \quad X = \hat{\phi}\frac{\sqrt{2V(\phi)}}{m_p^2}, \quad Y = \frac{1}{m_p^2}\frac{d\phi}{dt}$$

with $\hat{\phi} = \frac{\phi}{|\phi|}$ (this ensures that X and ϕ have same sign). So fixing initial energy scale of inflation is equivalent to fixing the radius of the circle R while varying X and Y **uniformly**. Our analysis [Mishra, Sahni and Toporensky PRD 2018] is a natural and generalized extension of the original work by [Belinsky et. al 1985].

Initial Conditions for Power-law Potentials: Quadratic Chaotic

We begin explaining our methodology for power-law potentials through the example of simple Quadratic Chaotic Inflation

Initial Conditions for Power-law Potentials: Quadratic Chaotic

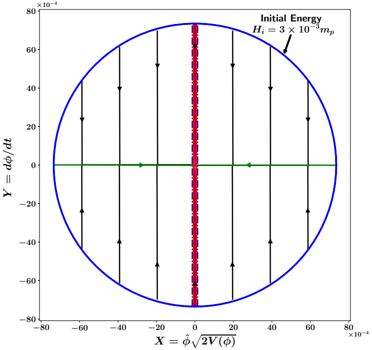
We begin explaining our methodology for power-law potentials through the example of simple Quadratic Chaotic Inflation

$$V(\phi) = \frac{1}{2}m^2\phi^2, \quad m = 5.96 \times 10^{-6}\ m_p$$

Initial Conditions for Power-law Potentials: Quadratic Chaotic

We begin explaining our methodology for power-law potentials through the example of simple Quadratic Chaotic Inflation

$$V(\phi) = \frac{1}{2}m^2\phi^2, \quad m = 5.96 \times 10^{-6}\ m_p$$

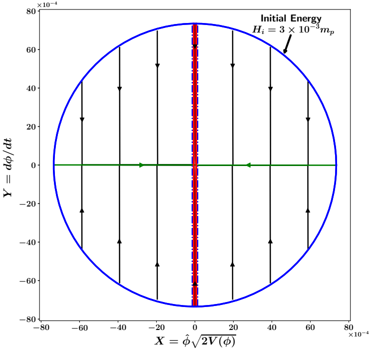


Phase-space Analysis

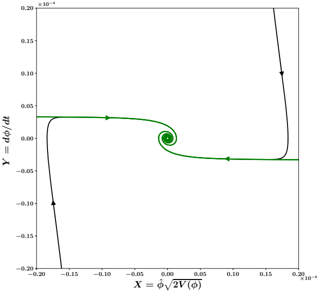
Initial Conditions for Power-law Potentials: Quadratic Chaotic

We begin explaining our methodology for power-law potentials through the example of simple Quadratic Chaotic Inflation

$$V(\phi) = \frac{1}{2}m^2\phi^2, \quad m = 5.96 \times 10^{-6}\ m_p$$



Phase-space Analysis

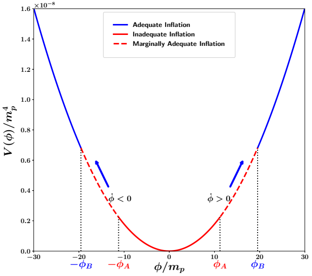


Zoomed View: Attractor Slow-roll Trajectories

Initial Conditions for Power-law Potentials: Quadratic Chaotic

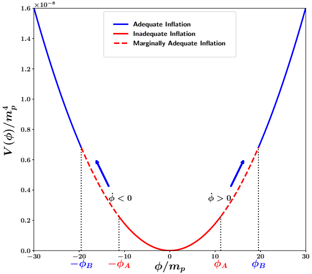
Initial Conditions for Power-law Potentials: Quadratic Chaotic

Range of Initial Conditions in the Field Space

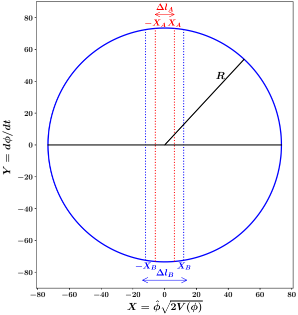


Initial Conditions for Power-law Potentials: Quadratic Chaotic

Range of Initial Conditions in the Field Space

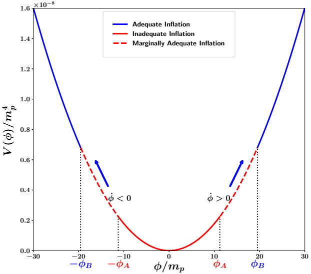


Defining the 'Measure'

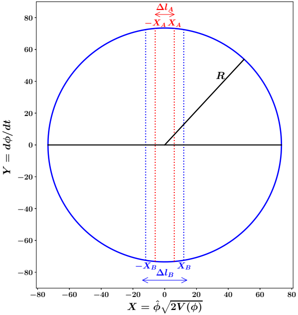


Initial Conditions for Power-law Potentials: Quadratic Chaotic

Range of Initial Conditions in the Field Space



Defining the 'Measure'



H_i (in m_p)	ϕ_A (in m_p)	ϕ_B (in m_p)	$f_A = 2\frac{\Delta l_A}{l}$	$f_B = 2\frac{\Delta l_B}{l}$
3×10^{-3}	11.22	19.55	5.80×10^{-3}	1.01×10^{-2}
3×10^{-2}	9.33	21.38	4.83×10^{-4}	1.11×10^{-3}
3×10^{-1}	7.47	23.27	3.86×10^{-5}	1.20×10^{-4}

Initial Conditions for Power-law Potentials: Axion-Monodromy

Axion-Monodromy potential is given by
[Silverstein, Westphal, McAllister 2008-09]

$$V(\phi) = V_0 \left| \frac{\phi}{m_p} \right|^p + \Lambda^4 \left(\cos \frac{\phi}{f} - 1 \right)$$

Initial Conditions for Power-law Potentials: Axion-Monodromy

Axion-Monodromy potential is given by [Silverstein, Westphal, McAllister 2008-09]

$$V(\phi) = V_0 \left| \frac{\phi}{m_p} \right|^p + \Lambda^4 \left(\cos \frac{\phi}{f} - 1 \right)$$

Monotonicity $\Rightarrow$

$$b \left| \frac{\phi}{m_p} \right|^{1-p} \sin \frac{\phi}{f} < 1 \ ,$$

where $b = \frac{1}{p} \frac{\Lambda^4}{V_0} \frac{m_p}{f} < 1$.

Initial Conditions for Power-law Potentials: Axion-Monodromy

Axion-Monodromy potential is given by [Silverstein, Westphal, McAllister 2008-09]

$$V(\phi) = V_0 \left|\frac{\phi}{m_p}\right|^p + \Lambda^4 \left(\cos\frac{\phi}{f} - 1\right)$$

Monotonicity $\Rightarrow$

$$b\left|\frac{\phi}{m_p}\right|^{1-p} \sin\frac{\phi}{f} < 1 \ ,$$

where $b = \frac{1}{p}\frac{\Lambda^4}{V_0}\frac{m_p}{f} < 1$. Consistency with CMB observations imply $b \ll 1$ and $f \ll m_p$, so the oscillatory Axionic term is insignificant.

Initial Conditions for Power-law Potentials: Axion-Monodromy

Axion-Monodromy potential is given by [Silverstein, Westphal, McAllister 2008-09]

$$V(\phi) = V_0 \left|\frac{\phi}{m_p}\right|^p + \Lambda^4 \left(\cos\frac{\phi}{f} - 1\right)$$

Monotonicity $\Rightarrow$

$$b\left|\frac{\phi}{m_p}\right|^{1-p} \sin\frac{\phi}{f} < 1 \ ,$$

where $b = \frac{1}{p}\frac{\Lambda^4}{V_0}\frac{m_p}{f} < 1$. Consistency with CMB observations imply $b \ll 1$ and $f \ll m_p$, so the oscillatory Axionic term is insignificant. We have considered two cases namely $p = 2/3,\ 1$

Initial Conditions for Power-law Potentials: Axion-Monodromy

Axion-Monodromy potential is given by [Silverstein, Westphal, McAllister 2008-09]

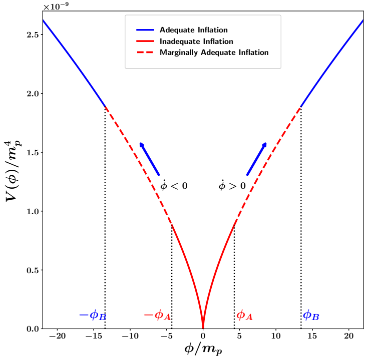
$$V(\phi) = V_0 \left|\frac{\phi}{m_p}\right|^p + \Lambda^4 \left(\cos\frac{\phi}{f} - 1\right)$$

Monotonicity $\Rightarrow$

$$b\left|\frac{\phi}{m_p}\right|^{1-p} \sin\frac{\phi}{f} < 1 \ ,$$

where $b = \frac{1}{p}\frac{\Lambda^4}{V_0}\frac{m_p}{f} < 1$. Consistency with CMB observations imply $b \ll 1$ and $f \ll m_p$, so the oscillatory Axionic term is insignificant. We have considered two cases namely $p = 2/3,\ 1$

Fractional Monodromy Inflation $p = 2/3$

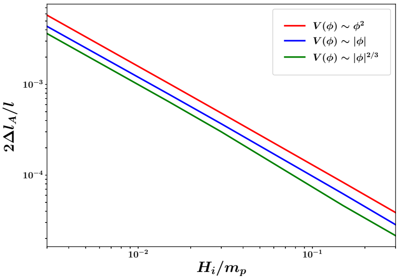


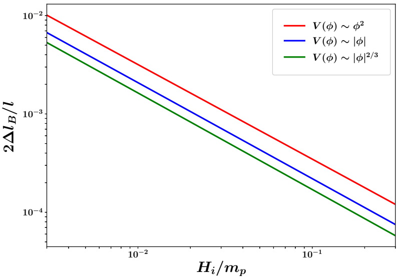
Range of Initial field values

Comparison of Power-law Potentials

$f_A = 2\frac{\Delta l_A}{l}$

$f_B = 2\frac{\Delta l_B}{l}$

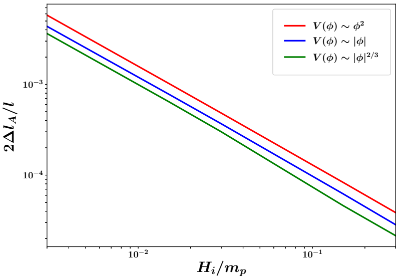


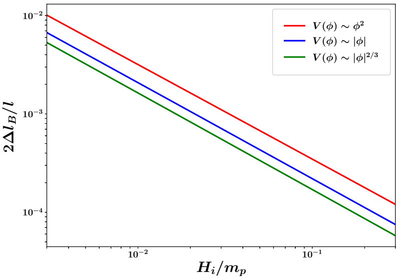


Comparison of Power-law Potentials

$f_A = 2\frac{\Delta l_A}{l}$

$f_B = 2\frac{\Delta l_B}{l}$

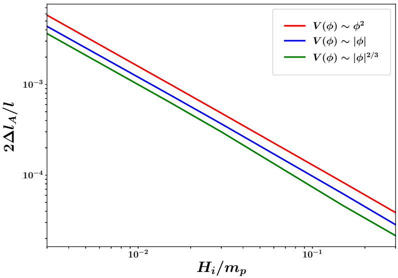


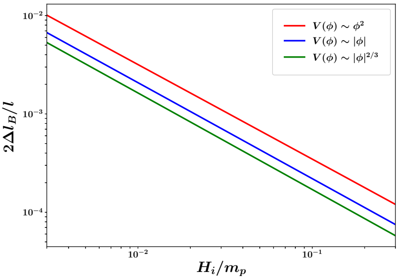


1. For **Concave** potentials, Inflation is more likely as compared to **Convex** potentials.

Comparison of Power-law Potentials

$f_A = 2\frac{\Delta l_A}{l}$ $f_B = 2\frac{\Delta l_B}{l}$

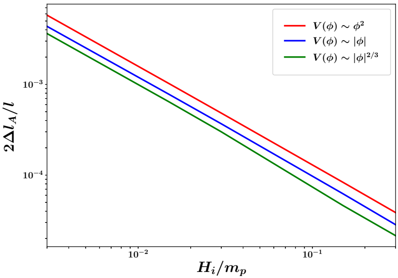


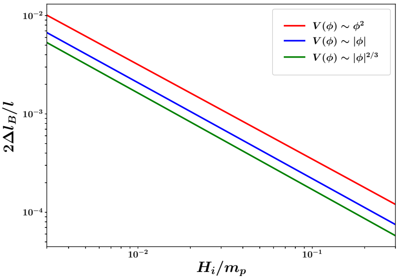


1. For **Concave** potentials, Inflation is more likely as compared to **Convex** potentials.
2. Degree of generality of inflation increases with higher initial energy scale H_i.
3. Results are not dependent on the Measure

Comparison of Power-law Potentials

$f_A = 2\frac{\Delta l_A}{l}$ $f_B = 2\frac{\Delta l_B}{l}$





1. For **Concave** potentials, Inflation is more likely as compared to **Convex** potentials.
2. Degree of generality of inflation increases with higher initial energy scale H_i.
3. Results are not dependent on the Measure (**small caveat**).

But [Planck2018] favours **Asymptotically Flat (Plateau-like)**

Asymptotically Flat (Plateau-like) Potentials

- They typically possess one or two plateau wings.

Asymptotically Flat (Plateau-like) Potentials

- They typically possess one or two plateau wings.
- Because of the extreme flatness of the plateau, they produce low tensor to scalar ratio r.

Asymptotically Flat (Plateau-like) Potentials

- They typically possess one or two plateau wings.
- Because of the extreme flatness of the plateau, they produce low tensor to scalar ratio r.
- Energy scale of observable inflation is also comparatively lower (Since scalar power is fixed, lower ϵ_H implies lower H_I).

Asymptotically Flat (Plateau-like) Potentials

- They typically possess one or two plateau wings.
- Because of the extreme flatness of the plateau, they produce low tensor to scalar ratio r.
- Energy scale of observable inflation is also comparatively lower (Since scalar power is fixed, lower ϵ_H implies lower H_I).
- Typical height of the plateau $V_0 \leq 10^{-10}\ m_p^4$.

Asymptotically Flat (Plateau-like) Potentials

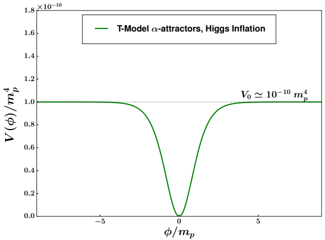
- They typically possess one or two plateau wings.
- Because of the extreme flatness of the plateau, they produce low tensor to scalar ratio r.
- Energy scale of observable inflation is also comparatively lower (Since scalar power is fixed, lower ϵ_H implies lower H_I).
- Typical height of the plateau $V_0 \leq 10^{-10}\ m_p^4$. And this is the main source of the recent intense debates related to the initial conditions for inflation.

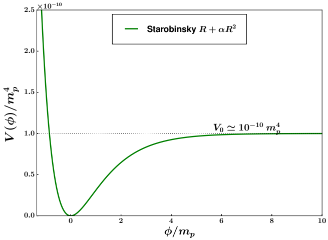
Asymptotically Flat (Plateau-like) Potentials

- They typically possess one or two plateau wings.
- Because of the extreme flatness of the plateau, they produce low tensor to scalar ratio r.
- Energy scale of observable inflation is also comparatively lower (Since scalar power is fixed, lower ϵ_H implies lower H_I).
- Typical height of the plateau $V_0 \leq 10^{-10}\ m_p^4$. And this is the main source of the recent intense debates related to the initial conditions for inflation.
- Most of them are Physically well motivating along with a proper theory of reheating. And they satisfy observations extremely well !!

Asymptotically Flat (Plateau-like) Potentials

- They typically possess one or two plateau wings.
- Because of the extreme flatness of the plateau, they produce low tensor to scalar ratio r.
- Energy scale of observable inflation is also comparatively lower (Since scalar power is fixed, lower ϵ_H implies lower H_I).
- Typical height of the plateau $V_0 \leq 10^{-10}\ m_p^4$. And this is the main source of the recent intense debates related to the initial conditions for inflation.
- Most of them are Physically well motivating along with a proper theory of reheating. And they satisfy observations extremely well !!





Standard Model Higgs Inflation

- Attractive to use minimal degrees of freedom of nature.

Standard Model Higgs Inflation

- Attractive to use minimal degrees of freedom of nature.
- Standard Model Higgs potential (Mexican Hat)

$$U(\phi) = \frac{\lambda}{4}\left(\phi^2 - \sigma^2\right)^2$$

Standard Model Higgs Inflation

- Attractive to use minimal degrees of freedom of nature.
- Standard Model Higgs potential (Mexican Hat)

$$U(\phi) = \frac{\lambda}{4}\left(\phi^2 - \sigma^2\right)^2$$

where $\lambda = 0.1$ and VEV of Higgs, $\sigma = 246$ GeV, is negligible during inflation. So $U(\phi) \simeq \frac{\lambda}{4}\phi^4$.

Standard Model Higgs Inflation

- Attractive to use minimal degrees of freedom of nature.
- Standard Model Higgs potential (Mexican Hat)

$$U(\phi) = \frac{\lambda}{4}\left(\phi^2 - \sigma^2\right)^2$$

  where $\lambda = 0.1$ and VEV of Higgs, $\sigma = 246$ GeV, is negligible during inflation. So $U(\phi) \simeq \frac{\lambda}{4}\phi^4$.
- Higgs coupling λ is too high to produce small quantum fluctuations during inflation. And any way ϕ^4 is ruled our by observation.

Standard Model Higgs Inflation

- Attractive to use minimal degrees of freedom of nature.
- Standard Model Higgs potential (Mexican Hat)

$$U(\phi) = \frac{\lambda}{4}\left(\phi^2 - \sigma^2\right)^2$$

  where $\lambda = 0.1$ and VEV of Higgs, $\sigma = 246$ GeV, is negligible during inflation. So $U(\phi) \simeq \frac{\lambda}{4}\phi^4$.
- Higgs coupling λ is too high to produce small quantum fluctuations during inflation. And any way ϕ^4 is ruled our by observation.
- Modifications are required.

Standard Model Higgs Inflation

- Attractive to use minimal degrees of freedom of nature.
- Standard Model Higgs potential (Mexican Hat)

$$U(\phi) = \frac{\lambda}{4}\left(\phi^2 - \sigma^2\right)^2$$

where $\lambda = 0.1$ and VEV of Higgs, $\sigma = 246$ GeV, is negligible during inflation. So $U(\phi) \simeq \frac{\lambda}{4}\phi^4$.

- Higgs coupling λ is too high to produce small quantum fluctuations during inflation. And any way ϕ^4 is ruled our by observation.
- Modifications are required. In general there have been two types of modification of the inflationary scenario to get Inflation from SM Higgs.

Standard Model Higgs Inflation

- Attractive to use minimal degrees of freedom of nature.
- Standard Model Higgs potential (Mexican Hat)

$$U(\phi) = \frac{\lambda}{4}\left(\phi^2 - \sigma^2\right)^2$$

  where $\lambda = 0.1$ and VEV of Higgs, $\sigma = 246$ GeV, is negligible during inflation. So $U(\phi) \simeq \frac{\lambda}{4}\phi^4$.
- Higgs coupling λ is too high to produce small quantum fluctuations during inflation. And any way ϕ^4 is ruled our by observation.
- Modifications are required. In general there have been two types of modification of the inflationary scenario to get Inflation from SM Higgs.
  1. Higgs field is non-minimally coupled to Gravity [Bezrukov, Shaposhnokov 2008].

Standard Model Higgs Inflation

- Attractive to use minimal degrees of freedom of nature.
- Standard Model Higgs potential (Mexican Hat)

$$U(\phi) = \frac{\lambda}{4}\left(\phi^2 - \sigma^2\right)^2$$

where $\lambda = 0.1$ and VEV of Higgs, $\sigma = 246$ GeV, is negligible during inflation. So $U(\phi) \simeq \frac{\lambda}{4}\phi^4$.

- Higgs coupling λ is too high to produce small quantum fluctuations during inflation. And any way ϕ^4 is ruled our by observation.
- Modifications are required. In general there have been two types of modification of the inflationary scenario to get Inflation from SM Higgs.
  1. Higgs field is non-minimally coupled to Gravity [Bezrukov, Shaposhnokov 2008].
  2. Higgs field has a non-canonical Kinetic term [Unnikrishnan, Sahni JCAP 2012], [Mishra, Sahni and Toporensky PRD 2018].

Standard Model Higgs Inflation

- Attractive to use minimal degrees of freedom of nature.
- Standard Model Higgs potential (Mexican Hat)

$$U(\phi) = \frac{\lambda}{4}\left(\phi^2 - \sigma^2\right)^2$$

where $\lambda = 0.1$ and VEV of Higgs, $\sigma = 246$ GeV, is negligible during inflation. So $U(\phi) \simeq \frac{\lambda}{4}\phi^4$.

- Higgs coupling λ is too high to produce small quantum fluctuations during inflation. And any way ϕ^4 is ruled our by observation.
- Modifications are required. In general there have been two types of modification of the inflationary scenario to get Inflation from SM Higgs.
  1. Higgs field is non-minimally coupled to Gravity [Bezrukov, Shaposhnokov 2008].
  2. Higgs field has a non-canonical Kinetic term [Unnikrishnan, Sahni JCAP 2012], [Mishra, Sahni and Toporensky PRD 2018].

In both cases Standard Model Higgs can act like the inflaton field generating necessary initial conditions for structure formation.

Non-minimal Higgs Inflation

Action in the Jordan frame

$$S_J = \int d^4x\sqrt{-g}\left[f(\phi)R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - U(\phi)\right]$$

Non-minimal Higgs Inflation

Action in the Jordan frame

$$S_J = \int d^4x\sqrt{-g}\left[f(\phi)R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - U(\phi)\right]$$

where $U(\phi) = \frac{\lambda}{4}\left(\phi^2 - \sigma^2\right)^2$, $f(\phi) = \frac{1}{2}(m^2 + \xi\phi^2)$ and $m^2 = m_p^2 - \xi\sigma^2$ leads to

$$f(\phi) \simeq \frac{1}{2}(m_p^2 + \xi\phi^2) = \frac{m_p^2}{2}\left(1 + \frac{\xi\phi^2}{m_p^2}\right) .$$

Non-minimal Higgs Inflation

Action in the Jordan frame

$$S_J = \int d^4x\sqrt{-g}\left[f(\phi)R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - U(\phi)\right]$$

where $U(\phi) = \frac{\lambda}{4}\left(\phi^2 - \sigma^2\right)^2$, $f(\phi) = \frac{1}{2}(m^2 + \xi\phi^2)$ and $m^2 = m_p^2 - \xi\sigma^2$ leads to

$$f(\phi) \simeq \frac{1}{2}(m_p^2 + \xi\phi^2) = \frac{m_p^2}{2}\left(1 + \frac{\xi\phi^2}{m_p^2}\right) .$$

After doing a conformal transformation $g_{\mu\nu} \longrightarrow \hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$

Non-minimal Higgs Inflation

Action in the Jordan frame

$$S_J = \int d^4x\sqrt{-g}\left[f(\phi)R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - U(\phi)\right]$$

where $U(\phi) = \frac{\lambda}{4}\left(\phi^2 - \sigma^2\right)^2$, $\quad f(\phi) = \frac{1}{2}(m^2 + \xi\phi^2)$ and $\quad m^2 = m_p^2 - \xi\sigma^2$
leads to

$$f(\phi) \simeq \frac{1}{2}(m_p^2 + \xi\phi^2) = \frac{m_p^2}{2}\left(1 + \frac{\xi\phi^2}{m_p^2}\right) .$$

After doing a conformal transformation $g_{\mu\nu} \longrightarrow \hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$ with
$\Omega^2 = \frac{2}{m_p^2}f(\phi) = 1 + \frac{\xi\phi^2}{m_p^2}$

Non-minimal Higgs Inflation

Action in the Jordan frame

$$S_J = \int d^4x\sqrt{-g}\left[f(\phi)R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - U(\phi)\right]$$

where $U(\phi) = \frac{\lambda}{4}\left(\phi^2 - \sigma^2\right)^2$, $f(\phi) = \frac{1}{2}(m^2 + \xi\phi^2)$ and $m^2 = m_p^2 - \xi\sigma^2$ leads to

$$f(\phi) \simeq \frac{1}{2}(m_p^2 + \xi\phi^2) = \frac{m_p^2}{2}\left(1 + \frac{\xi\phi^2}{m_p^2}\right) .$$

After doing a conformal transformation $g_{\mu\nu} \longrightarrow \hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$ with $\Omega^2 = \frac{2}{m_p^2}f(\phi) = 1 + \frac{\xi\phi^2}{m_p^2}$ and a field redefinition $\phi \to \chi$

Non-minimal Higgs Inflation

Action in the Jordan frame

$$S_J = \int d^4x\sqrt{-g}\left[f(\phi)R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - U(\phi)\right]$$

where $U(\phi) = \frac{\lambda}{4}\left(\phi^2 - \sigma^2\right)^2, \quad f(\phi) = \frac{1}{2}(m^2 + \xi\phi^2)$ and $\quad m^2 = m_p^2 - \xi\sigma^2$ leads to

$$f(\phi) \simeq \frac{1}{2}(m_p^2 + \xi\phi^2) = \frac{m_p^2}{2}\left(1 + \frac{\xi\phi^2}{m_p^2}\right) .$$

After doing a conformal transformation $g_{\mu\nu} \longrightarrow \hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$ with $\Omega^2 = \frac{2}{m_p^2}f(\phi) = 1 + \frac{\xi\phi^2}{m_p^2}$ and a field redefinition $\phi \to \chi$ we end up at the Einstein frame where action is given by

$$S_E = \int d^4x\sqrt{-\hat{g}}\left[\frac{m_p^2}{2}\hat{R} - \frac{1}{2}\hat{g}^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - V(\chi)\right] ,$$

Non-minimal Higgs Inflation

Action in the Jordan frame

$$S_J = \int d^4x\sqrt{-g}\left[f(\phi)R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - U(\phi)\right]$$

where $U(\phi) = \frac{\lambda}{4}\left(\phi^2 - \sigma^2\right)^2$, $f(\phi) = \frac{1}{2}(m^2 + \xi\phi^2)$ and $m^2 = m_p^2 - \xi\sigma^2$ leads to

$$f(\phi) \simeq \frac{1}{2}(m_p^2 + \xi\phi^2) = \frac{m_p^2}{2}\left(1 + \frac{\xi\phi^2}{m_p^2}\right) .$$

After doing a conformal transformation $g_{\mu\nu} \longrightarrow \hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$ with $\Omega^2 = \frac{2}{m_p^2}f(\phi) = 1 + \frac{\xi\phi^2}{m_p^2}$ and a field redefinition $\phi \to \chi$ we end up at the Einstein frame where action is given by

$$S_E = \int d^4x\sqrt{-\hat{g}}\left[\frac{m_p^2}{2}\hat{R} - \frac{1}{2}\hat{g}^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - V(\chi)\right] ,$$

where

$$V(\chi) = \frac{U[\phi(\chi)]}{\Omega^4} \quad \text{and} \quad \frac{\partial\chi}{\partial\phi} = \pm\frac{1}{\Omega^2}\sqrt{\Omega^2 + \frac{6\xi^2\phi^2}{m_p^2}}$$

Higgs Potential in the Einstein frame

Approximate form of the potential is given by [Mishra, Sahni and Toporensky PRD 2018]

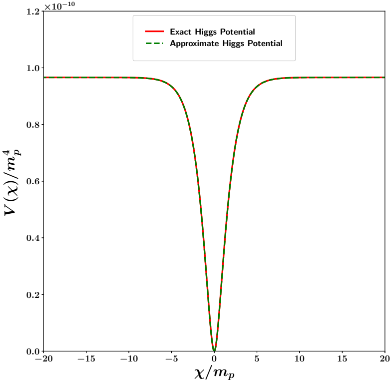
$$V(\chi) \simeq V_0 \left(1 - \exp\left[-\sqrt{\frac{2}{3}}\frac{|\chi|}{m_p}\right]\right)^2$$

where $V_0 = \frac{\lambda m_p^4}{4\xi^2} = 9.6 \times 10^{-11}\ m_p^4$,
$\Rightarrow \xi = 1.6 \times 10^4$

Higgs Potential in the Einstein frame

Approximate form of the potential is given by [Mishra, Sahni and Toporensky PRD 2018]

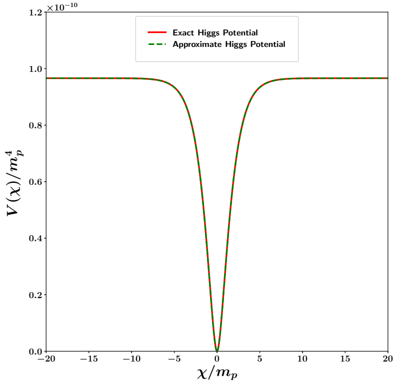
$$V(\chi) \simeq V_0 \left(1 - \exp\left[-\sqrt{\frac{2}{3}}\frac{|\chi|}{m_p}\right]\right)^2$$

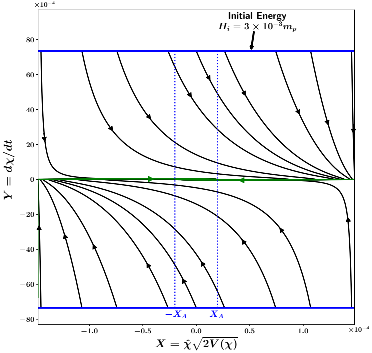


Higgs Potential in the Einstein frame

Approximate form of the potential is given by [Mishra, Sahni and Toporensky PRD 2018]

$$V(\chi) \simeq V_0 \left(1 - \exp\left[-\sqrt{\frac{2}{3}}\frac{|\chi|}{m_p}\right]\right)^2$$

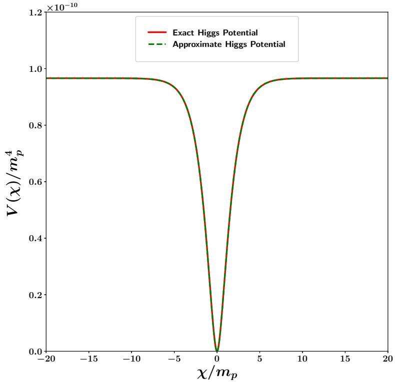


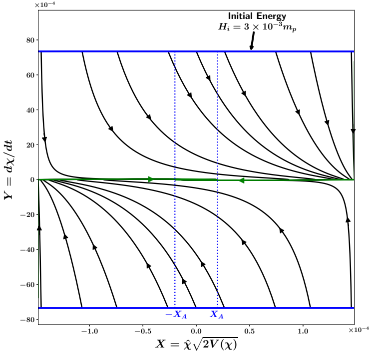


Higgs Potential in the Einstein frame

Approximate form of the potential is given by [Mishra, Sahni and Toporensky PRD 2018]

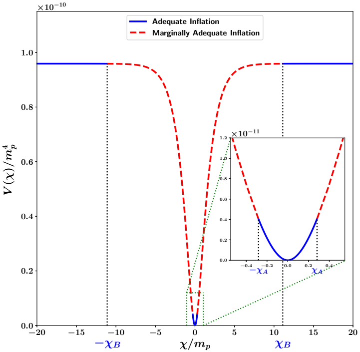
$$V(\chi) \simeq V_0 \left(1 - \exp\left[-\sqrt{\frac{2}{3}}\frac{|\chi|}{m_p}\right]\right)^2$$



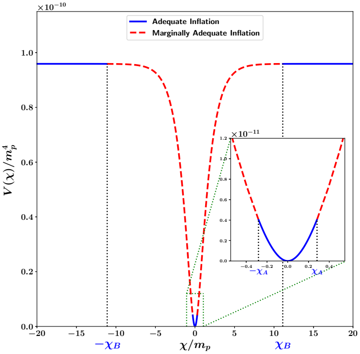


Slanted trajectories in the phase-space indicate that starting from $\phi = 0$, it is possible to converge on to the inflationary separatrix to obtain adequate inflation.

Range of initial field values



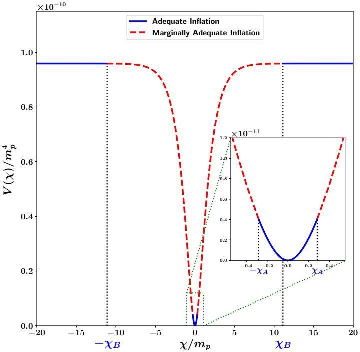
Range of initial field values



Also applicable to the T-Model of α-attractors [Kallosh, Linde and Roest JCAP(2013)]

$$V(\phi) = V_0 \tanh^2\left(\lambda \frac{\phi}{m_p}\right)$$

Range of initial field values

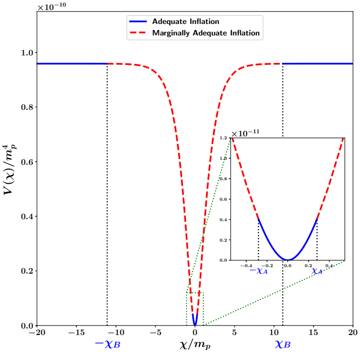


Also applicable to the T-Model of α-attractors [Kallosh, Linde and Roest JCAP(2013)]

$$V(\phi) = V_0 \tanh^2\left(\lambda \frac{\phi}{m_p}\right)$$

Starobinsky Inflation: [Starobinsky 1980, Phys. Lett 91B]

Range of initial field values



Starobinsky Inflation: [Starobinsky 1980, Phys. Lett 91B]Jordan frame action

$$S = \int d^4x \sqrt{-g}\frac{m_p^2}{2}\left[R + \frac{1}{6m^2}R^2\right]$$

takes the form

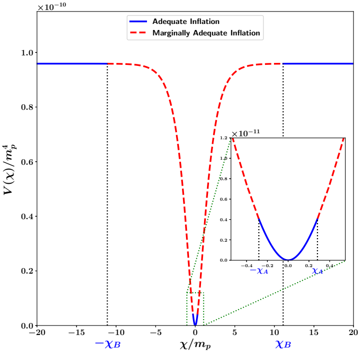
$$S_E = \int d^4x \sqrt{-g}\left[\frac{m_p^2}{2}\hat{R} - \frac{1}{2}\hat{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right]$$

with $V(\phi) = \frac{3}{4}m^2 m_p^2\left(1 - e^{-\sqrt{\frac{2}{3}}\frac{\phi}{m_p}}\right)^2$

Also applicable to the T-Model of α-attractors [Kallosh, Linde and Roest JCAP(2013)]

$$V(\phi) = V_0 \tanh^2\left(\lambda\frac{\phi}{m_p}\right)$$

Range of initial field values



Starobinsky Inflation: [Starobinsky 1980, Phys. Lett 91B]Jordan frame action

$$S = \int d^4x \sqrt{-g} \frac{m_p^2}{2} \left[R + \frac{1}{6m^2} R^2 \right]$$

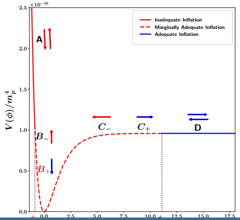
takes the form

$$S_E = \int d^4x \sqrt{-g} \left[\frac{m_p^2}{2} \hat{R} - \frac{1}{2} \hat{g}^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right]$$

with $V(\phi) = \frac{3}{4} m^2 m_p^2 \left(1 - e^{-\sqrt{\frac{2}{3}} \frac{\phi}{m_p}} \right)^2$

Also applicable to the T-Model of α-attractors [Kallosh, Linde and Roest JCAP(2013)]

$$V(\phi) = V_0 \tanh^2 \left(\lambda \frac{\phi}{m_p} \right)$$



Asymptotically flat Potentials

Asymptotically flat Potentials

- Favoured by Observations.

Asymptotically flat Potentials

- Favoured by Observations.
- Inflation can happen starting from close to the minimum of the potential.

Asymptotically flat Potentials

- Favoured by Observations.
- Inflation can happen starting from close to the minimum of the potential.
- Large range of initial field values yield adequate inflation.

Asymptotically flat Potentials

- Favoured by Observations.
- Inflation can happen starting from close to the minimum of the potential.
- Large range of initial field values yield adequate inflation.
- Difficulty in defining 'measure'.

Asymptotically flat Potentials

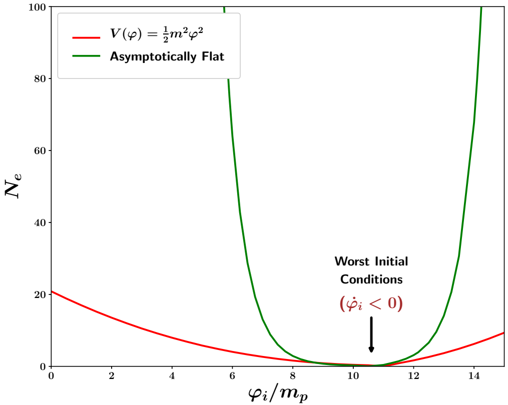
- Favoured by Observations.
- Inflation can happen starting from close to the minimum of the potential.
- Large range of initial field values yield adequate inflation.
- Difficulty in defining 'measure'.
- Dynamics is very different from Power-law potentials.

Asymptotically flat Potentials

- Favoured by Observations.
- Inflation can happen starting from close to the minimum of the potential.
- Large range of initial field values yield adequate inflation.
- Difficulty in defining 'measure'.
- Dynamics is very different from Power-law potentials. WHY!!

Asymptotically flat Potentials

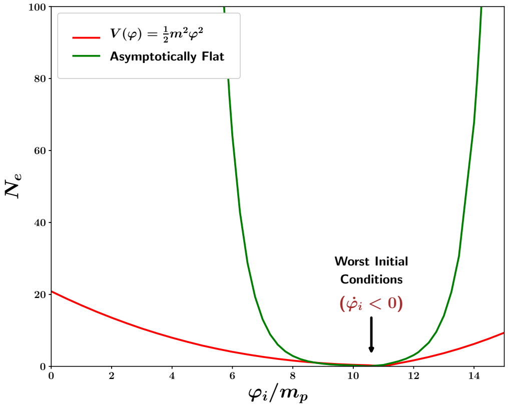
- Favoured by Observation
- Inflation can happen starting from close to the minimum of the potentia
- Large range of initial fiel values yield adequate inflation.
- Difficulty in defining 'measure'.
- Dynamics is very differen from Power-law potential WHY!!



Asymptotically flat Potentials

- Favoured by Observation
- Inflation can happen starting from close to the minimum of the potentia
- Large range of initial fiel values yield adequate inflation.
- Difficulty in defining 'measure'.
- Dynamics is very differen from Power-law potential

WHY!!



However Positive Spatial Curvature is a serious problem for Asymptotically Flat potentials [Steinhardt, Ijjas and Loeb 2014]

Effect of Large Positive Spatial Curvature $K = +1$

Effect of Large Positive Spatial Curvature $K = +1$

Friedmann Equation

$$H^2 = \frac{1}{3m_p^2}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right) - \frac{1}{a^2}$$

Effect of Large Positive Spatial Curvature $K = +1$

Friedmann Equation

$$H^2 = \frac{1}{3m_p^2}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right) - \frac{1}{a^2}$$

gives an upper bound on the potential

$$\frac{V(\varphi)}{m_p^4} \leq \frac{3}{a^2 m_p^2} \ .$$

Effect of Large Positive Spatial Curvature $K = +1$

Friedmann Equation

$$H^2 = \frac{1}{3m_p^2}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right) - \frac{1}{a^2}$$

gives an upper bound on the potential

$$\frac{V(\varphi)}{m_p^4} \leq \frac{3}{a^2 m_p^2}\ .$$

Turn over point is described by $H = 0$ and $a = a_b$. Collapse happens when $a < a_b$ ($\frac{\ddot{a}}{a} < 0$) while bounce happens when $a > a_b$ ($\frac{\ddot{a}}{a} > 0$) where

$$a_b m_p = \sqrt{\frac{2}{V(\phi)/m_p^4}}$$

Effect of Large Positive Spatial Curvature $K = +1$

Friedmann Equation

$$H^2 = \frac{1}{3m_p^2}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right) - \frac{1}{a^2}$$

gives an upper bound on the potential

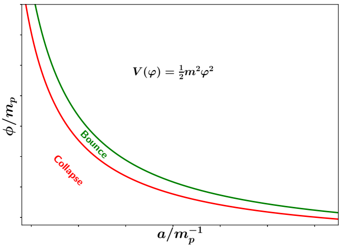
$$\frac{V(\varphi)}{m_p^4} \leq \frac{3}{a^2 m_p^2} \, .$$

Turn over point is described by $H = 0$ and $a = a_b$. Collapse happens when $a < a_b$ ($\frac{\ddot{a}}{a} < 0$) while bounce happens when $a > a_b$ ($\frac{\ddot{a}}{a} > 0$) where

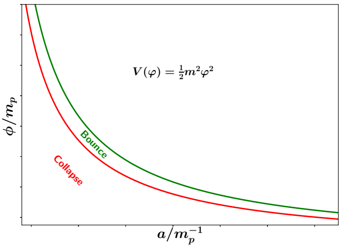
$$a_b m_p = \sqrt{\frac{2}{V(\phi)/m_p^4}}$$

So starting from $H_i = 0$ at the Planck boundary $\rho_\phi = m_p^4$, $a_i m_p = \sqrt{3}$, we have
Collapse: $\frac{V(\varphi)}{m_p^4} < \frac{2}{a^2 m_p^2}$ **Bounce**: $\frac{2}{a^2 m_p^2} < \frac{V(\varphi)}{m_p^4} \leq \frac{3}{a^2 m_p^2}$

Trouble with asymptotically flat potentials for $K = +1$

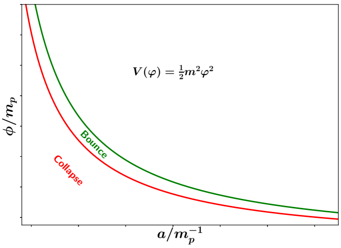


Trouble with asymptotically flat potentials for $K = +1$



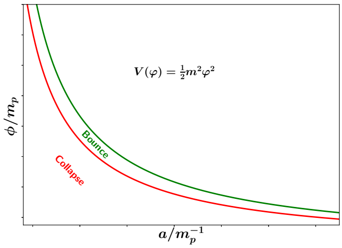
At $H_i = 0$, for any given initial value of ρ_ϕ (hence a), there is a range of values of ϕ for which universe bounces back and inflation is possible.

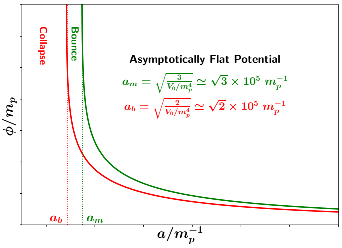
Trouble with asymptotically flat potentials for $K = +1$



At $H_i = 0$, for any given initial value of ρ_ϕ (hence a), there is a range of values of ϕ for which universe bounces back and inflation is possible. For higher $H_i > 0$, probability of bounce increases (quantifiable).

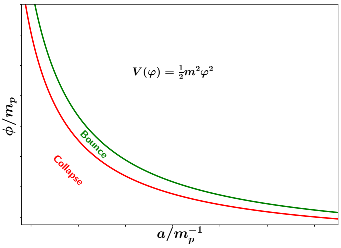
Trouble with asymptotically flat potentials for $K = +1$

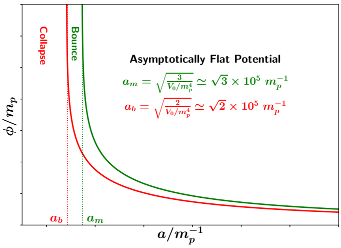




At $H_i = 0$, for any given initial value of ρ_ϕ (hence a), there is a range of values of ϕ for which universe bounces back and inflation is possible. For higher $H_i > 0$, probability of bounce increases (quantifiable).

Trouble with asymptotically flat potentials for $K = +1$

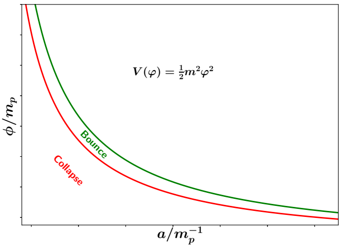


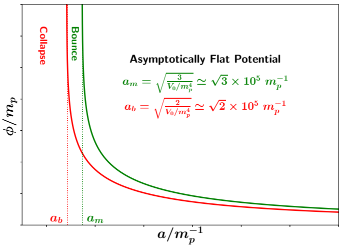


At $H_i = 0$, for any given initial value of ρ_ϕ (hence a), there is a range of values of ϕ for which universe bounces back and inflation is possible. For higher $H_i > 0$, probability of bounce increases (quantifiable).

For $H_i = 0$, **Collapse is inevitable**. While for $H_i > 0$, bounce becomes possible only if $a_i > 1600\ a_i$ which puts H_i close to its flat universe value upto 6 decimal places.

Trouble with asymptotically flat potentials for $K = +1$





At $H_i = 0$, for any given initial value of ρ_ϕ (hence a), there is a range of values of ϕ for which universe bounces back and inflation is possible. For higher $H_i > 0$, probability of bounce increases (quantifiable).

For $H_i = 0$, **Collapse is inevitable**. While for $H_i > 0$, bounce becomes possible only if $a_i > 1600\ a_i$ which puts H_i close to its flat universe value upto 6 decimal places. Very Bad for Inflation

This is a serious issue and needs to be resolved! [Mishra, Sahni and Toporensky (in preparation)]

Possible Resolution

Consider the Margarita Potential
[Bag, Mishra and Sahni JCAP 2017]

$$V(\phi) = V_0 \tanh^{\left(\frac{\lambda_1 \phi}{m_p}\right)} \cosh\left(\frac{\lambda_2 \phi}{m_p}\right), \;\; \lambda_1 > \lambda_2$$

which has three asymptotes

$$\begin{aligned}
\text{Exponential wing} \quad V(\varphi) &\simeq \frac{V_0}{2} \exp\left(\lambda_2 |\varphi|/m_p\right), \quad \frac{|\varphi|}{m_p} \gg \frac{1}{\lambda_2}, \\
\text{Flat wing:} \quad V(\varphi) &\simeq V_0 + \frac{1}{2} m_2^2 \varphi^2, \quad \frac{1}{\lambda_1} \ll \frac{|\varphi|}{m_p} \ll \frac{1}{\lambda_2}, \\
\text{Oscillatory region:} \quad V(\varphi) &\simeq \frac{1}{2} m_1^2 \varphi^2, \quad \frac{|\varphi|}{m_p} \ll \frac{1}{\lambda_1},
\end{aligned}$$

where $m_1^2 = \frac{2V_0\lambda_1^2}{m_p^2}$ and $m_2^2 = \frac{V_0\lambda_2^2}{m_p^2}$
with $m_1 \gg m_2$.

Possible Resolution

Consider the Margarita Potential
[Bag, Mishra and Sahni JCAP 2017]

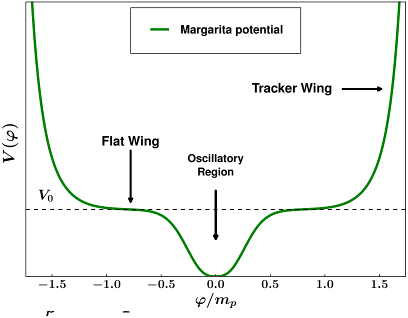
$$V(\phi) = V_0 \tanh^{\left(\frac{\lambda_1 \phi}{m_p}\right)} \cosh\left(\frac{\lambda_2 \phi}{m_p}\right), \;\; \lambda$$

which has three asymptotes

$$\text{Exponential wing} \quad V(\varphi) \simeq \frac{V_0}{2} \, \text{ex}$$

$$\text{Flat wing:} \quad V(\varphi) \simeq V_0 +$$

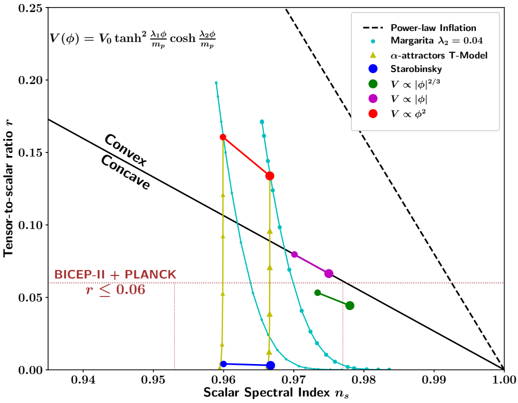
$$\text{Oscillatory region:} \quad V(\varphi) \simeq \frac{1}{2} m_1^2 \varphi$$



where $m_1^2 = \frac{2V_0\lambda_1^2}{m_p^2}$ and $m_2^2 = \frac{V_0\lambda_2^2}{m_p^2}$
with $m_1 \gg m_2$. We need $\lambda_1 > \lambda_2$
and $\lambda_2 < \sqrt{2}$

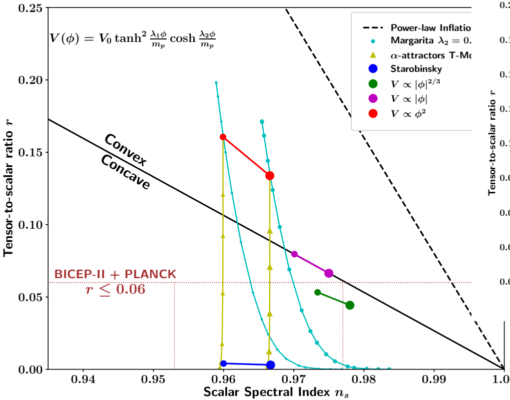
Margarita Potential r vs n_s

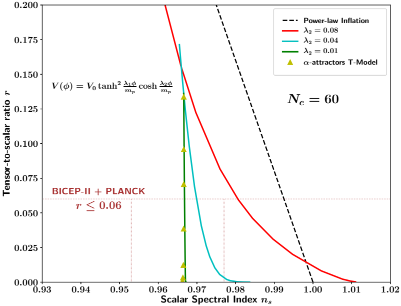
[Mishra, Sahni and Toporensky (in preparation)]



Margarita Potential r vs n_s

[Mishra, Sahni and Toporensky (in preparation)]





Concluding Remarks: Take Office Messages

- Initial Conditions for inflation is an important issue to be considered.

Concluding Remarks: Take Office Messages

- Initial Conditions for inflation is an important issue to be considered.
- Inflationary phase (slow-roll trajectory) is a local attractor for large field models.

Concluding Remarks: Take Office Messages

- Initial Conditions for inflation is an important issue to be considered.
- Inflationary phase (slow-roll trajectory) is a local attractor for large field models.
- Planck 2018 data prefers/favours concave potentials which are plateau-like.

Concluding Remarks: Take Office Messages

- Initial Conditions for inflation is an important issue to be considered.
- Inflationary phase (slow-roll trajectory) is a local attractor for large field models.
- Planck 2018 data prefers/favours concave potentials which are plateau-like.
- For Plateau-like potentials, inflation can happen starting from larger set of initial conditions as compared to power-law potentials. Inflation can happen even starting from close to the minimum of the potential.

Concluding Remarks: Take Office Messages

- Initial Conditions for inflation is an important issue to be considered.
- Inflationary phase (slow-roll trajectory) is a local attractor for large field models.
- Planck 2018 data prefers/favours concave potentials which are plateau-like.
- For Plateau-like potentials, inflation can happen starting from larger set of initial conditions as compared to power-law potentials. Inflation can happen even starting from close to the minimum of the potential.
- Positive spatial curvature is a serious problem for plateau-like potentials and needs close attention.

Concluding Remarks: Take Office Messages

- Initial Conditions for inflation is an important issue to be considered.
- Inflationary phase (slow-roll trajectory) is a local attractor for large field models.
- Planck 2018 data prefers/favours concave potentials which are plateau-like.
- For Plateau-like potentials, inflation can happen starting from larger set of initial conditions as compared to power-law potentials. Inflation can happen even starting from close to the minimum of the potential.
- Positive spatial curvature is a serious problem for plateau-like potentials and needs close attention.
- Possible modifications of the potential is required for large field values (or multi-field inflation is desirable.).

Concluding Remarks: Take Office Messages

- Initial Conditions for inflation is an important issue to be considered.
- Inflationary phase (slow-roll trajectory) is a local attractor for large field models.
- Planck 2018 data prefers/favours concave potentials which are plateau-like.
- For Plateau-like potentials, inflation can happen starting from larger set of initial conditions as compared to power-law potentials. Inflation can happen even starting from close to the minimum of the potential.
- Positive spatial curvature is a serious problem for plateau-like potentials and needs close attention.
- Possible modifications of the potential is required for large field values (or multi-field inflation is desirable.).
- Plateau-potentials with an initial exponential-type modification has advantages both theoretically and observationally.

Concluding Remarks: Take Office Messages

- Initial Conditions for inflation is an important issue to be considered.
- Inflationary phase (slow-roll trajectory) is a local attractor for large field models.
- Planck 2018 data prefers/favours concave potentials which are plateau-like.
- For Plateau-like potentials, inflation can happen starting from larger set of initial conditions as compared to power-law potentials. Inflation can happen even starting from close to the minimum of the potential.
- Positive spatial curvature is a serious problem for plateau-like potentials and needs close attention.
- Possible modifications of the potential is required for large field values (or multi-field inflation is desirable.).
- Plateau-potentials with an initial exponential-type modification has advantages both theoretically and observationally.
- Initial conditions should also be thoroughly analysed for alternatives to Inflation like String Gas Cosmology, Matter Bounce Models, Ekpyrotic Models, Emergent Scenarios etc.

References

1. Initial Conditions for Inflation in an FRW Universe, Mishra, Sahni and Toporensky, PRD 98, 083538(2018) [arXiv:1801:04948]
2. Initial Conditions for inflation: A short Review, Robert Brandenberger, Int. J. Mod. Phys. D 26, 1740002(2017)
3. Beginning of Inflation in an inhomogeneous universe, East, Kleban, Linde, Senatore, JCAP 09, (2016) 010
4. Guth and Nomura, Phys. Lett. B 733, 112 (2014)
5. Carrasco, Kallosh and Linde, PRD 92, 063519(2015)
6. Initial Conditions for inflation in an FRW Universe - II Mishra, Sahni and Toporensky (in preparation).