Curvature couplings of the Inflaton

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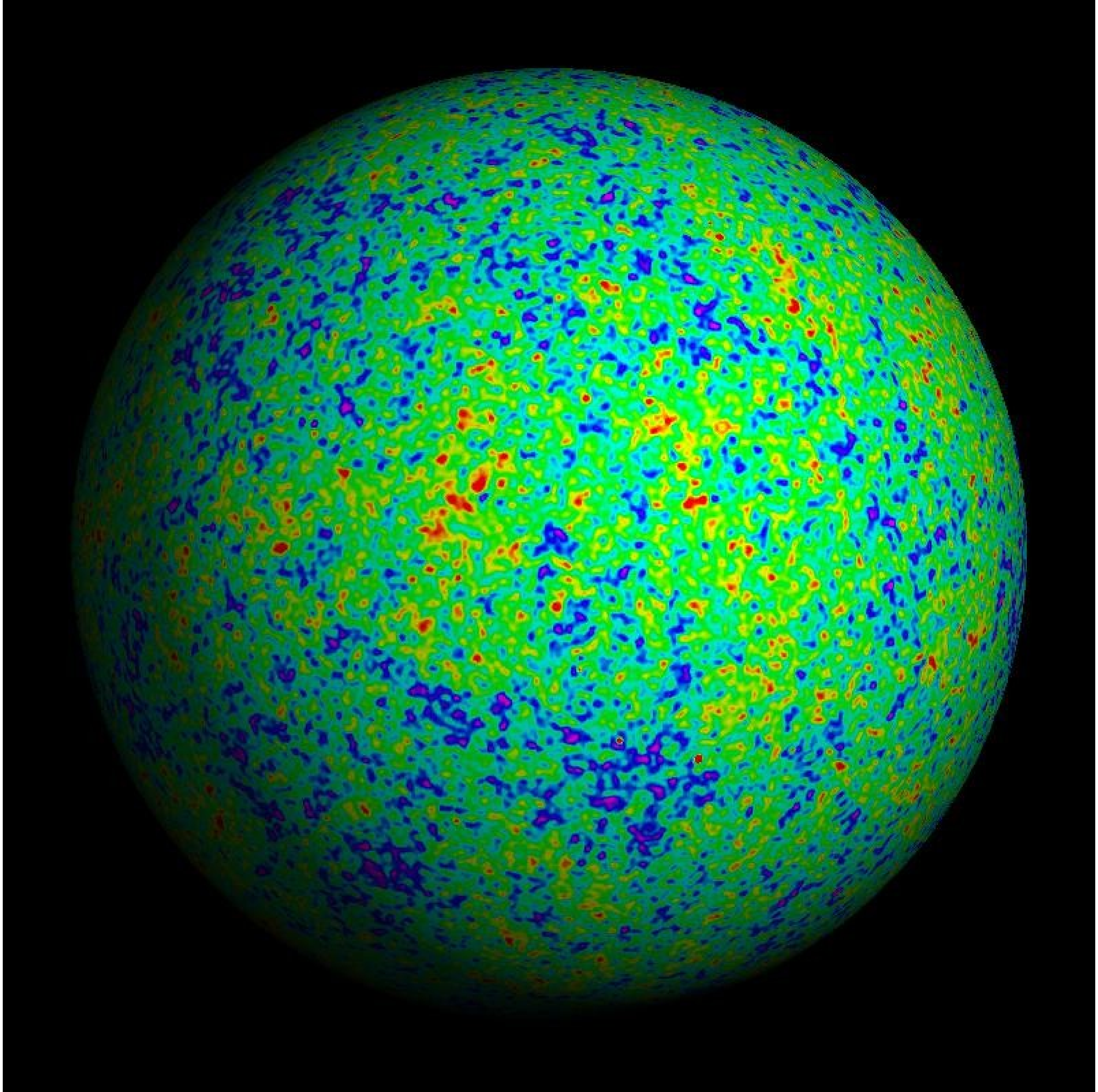
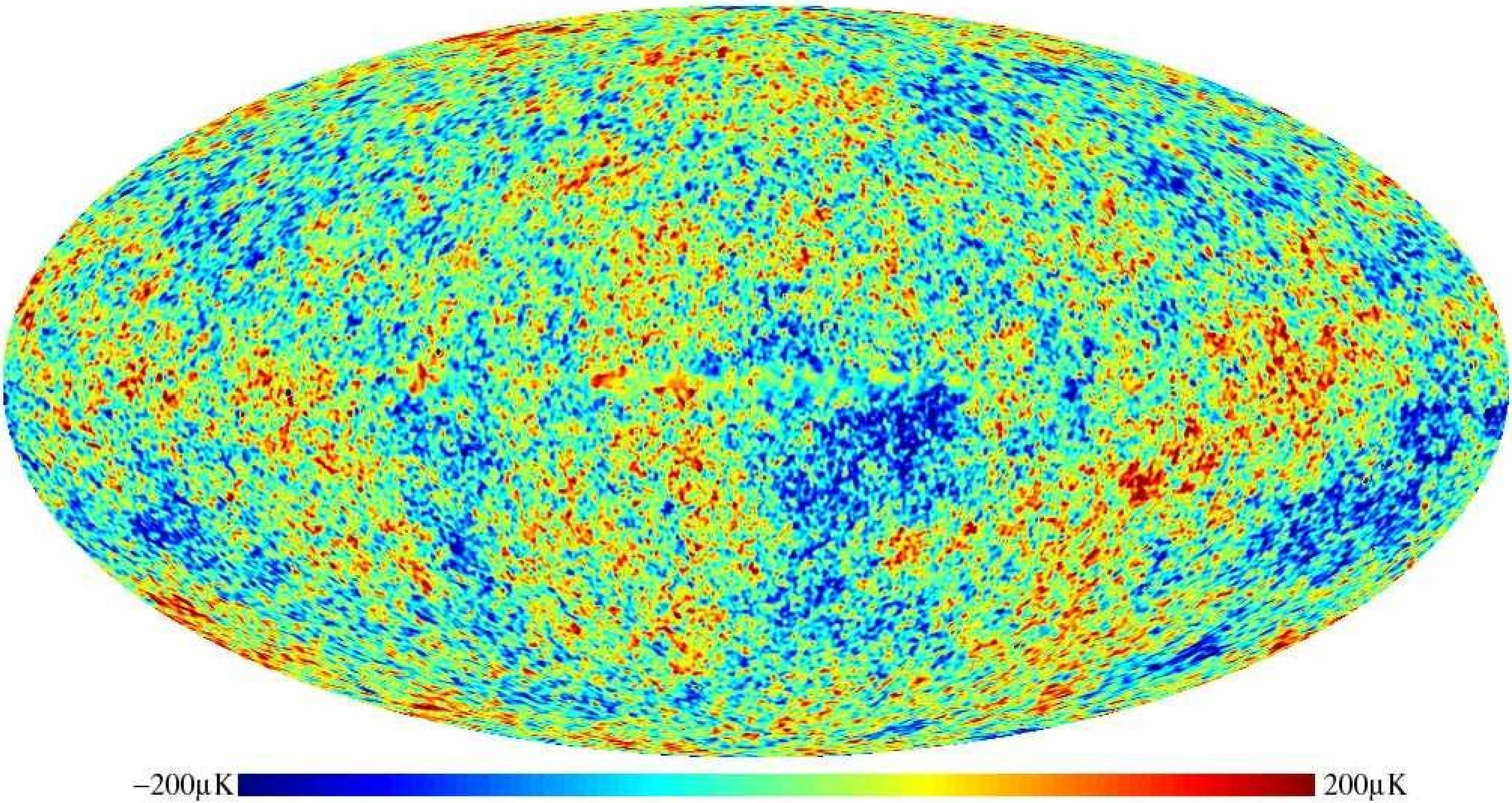
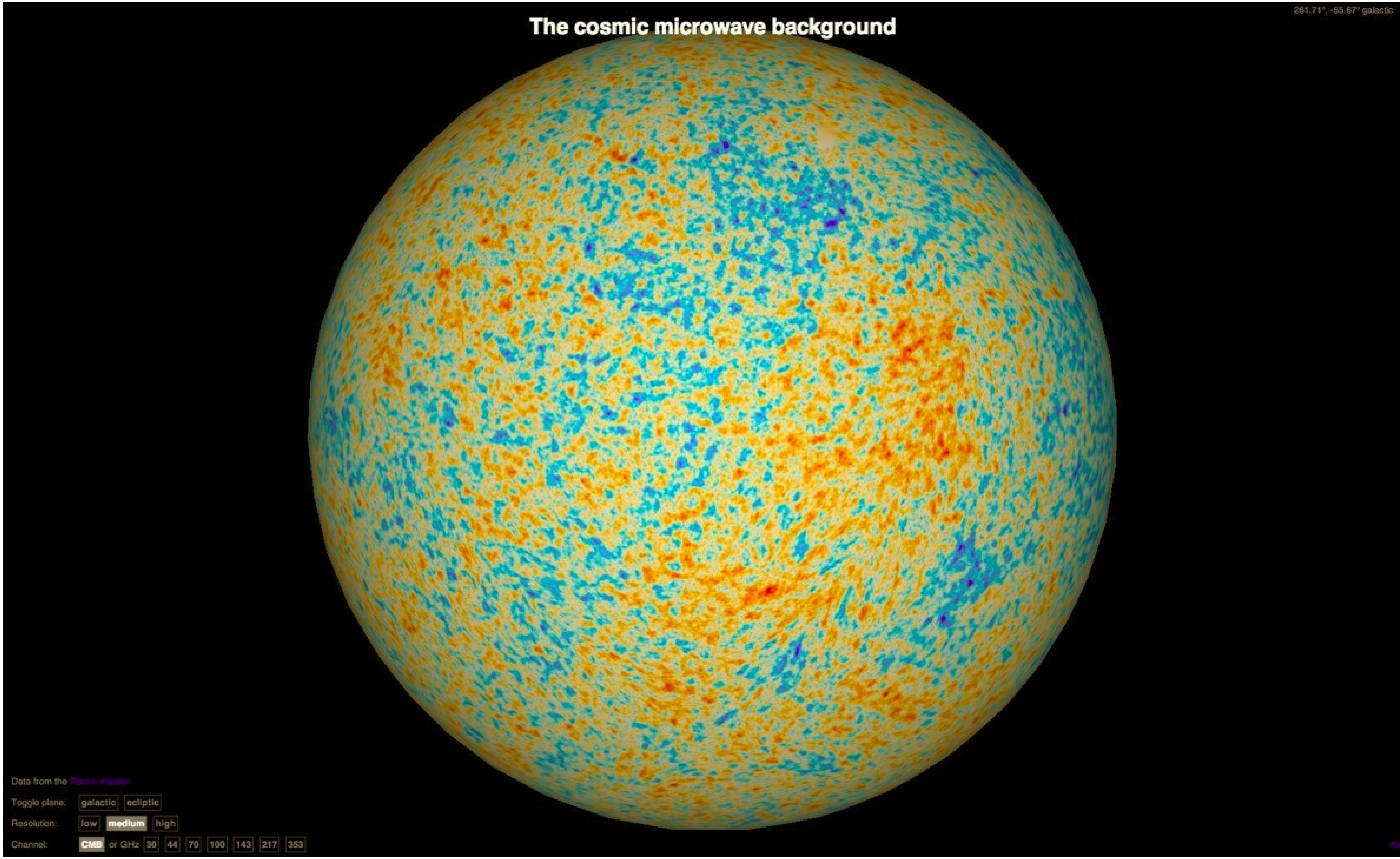


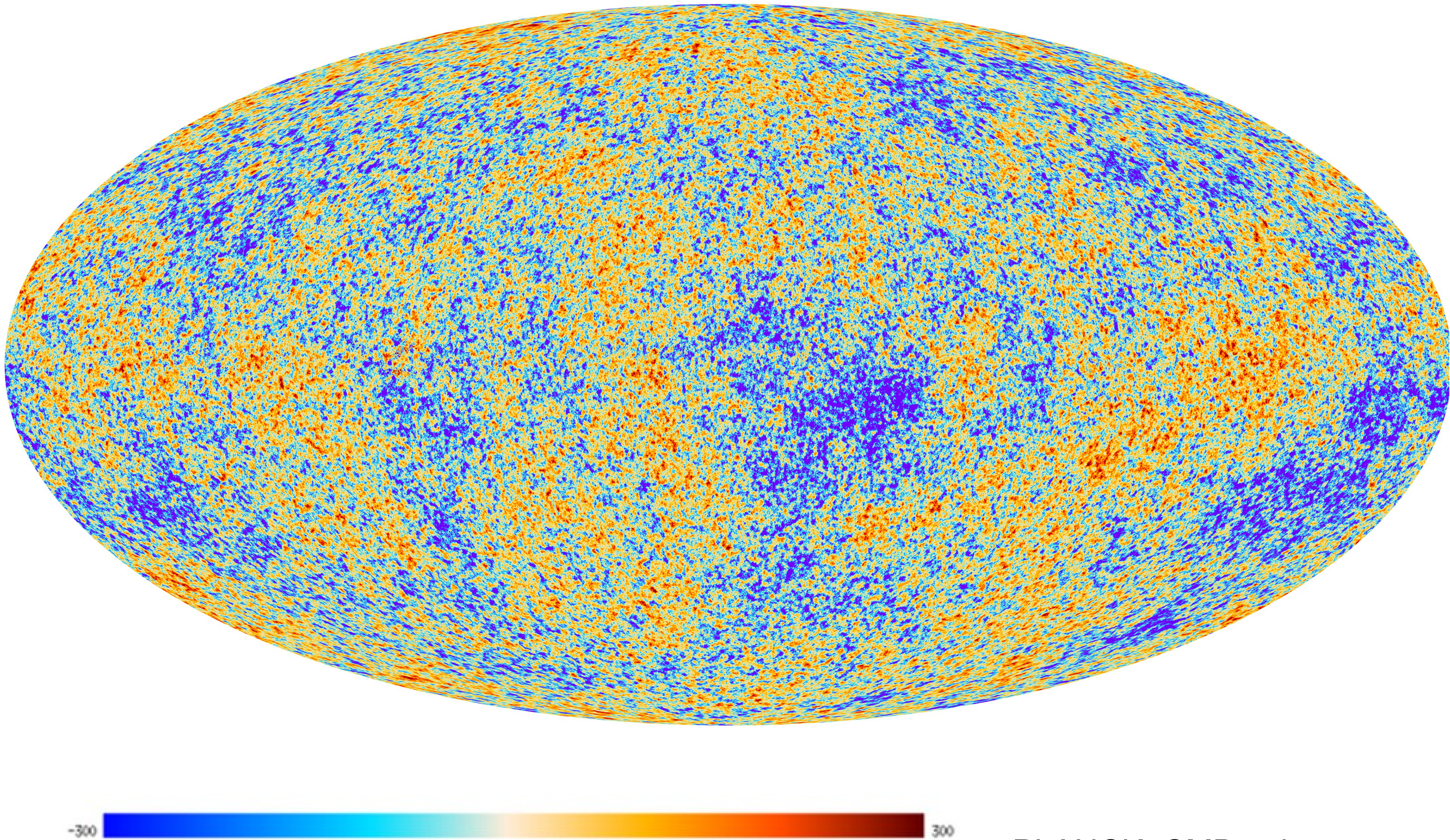
CMB Anisotropy
WMAP



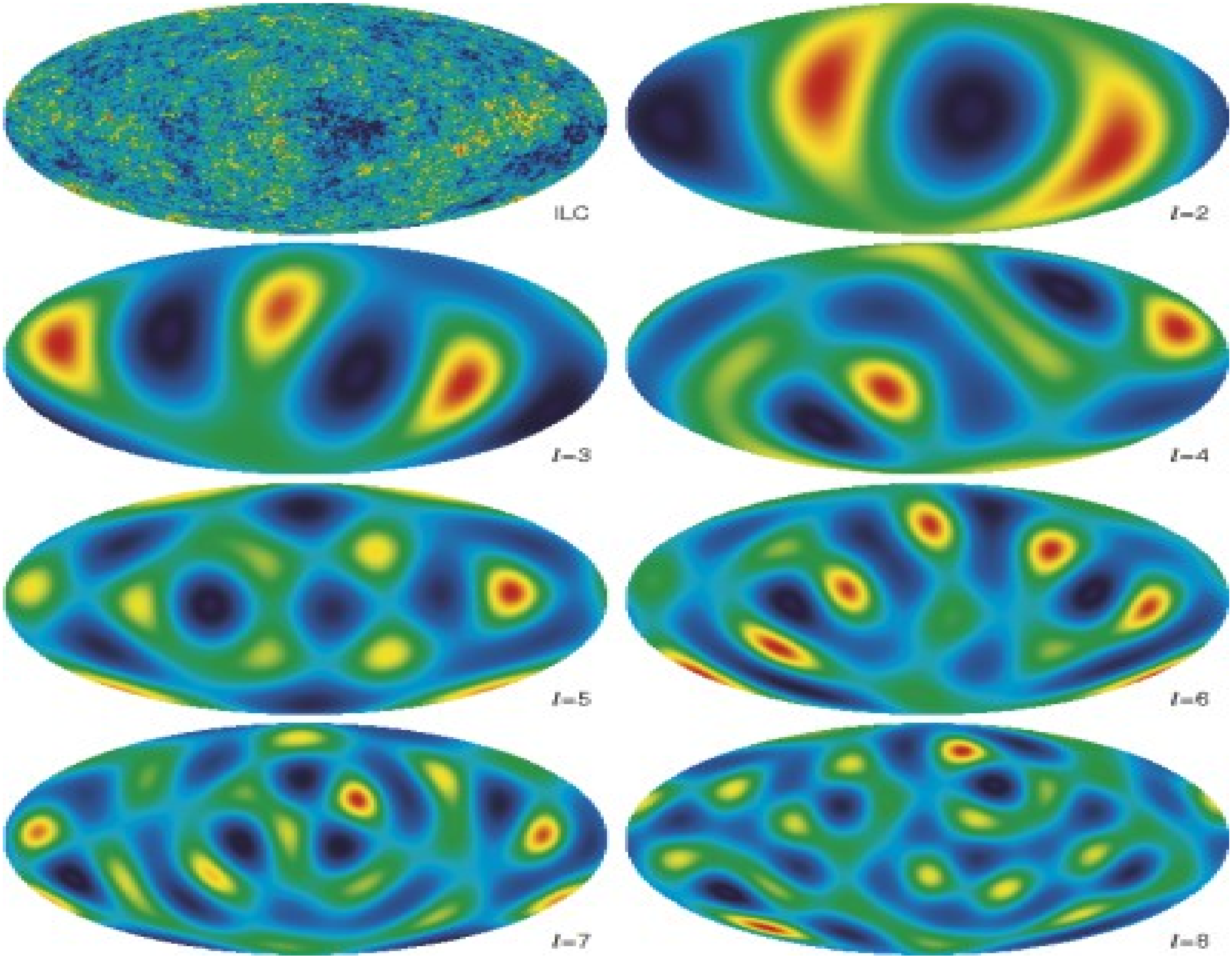
WMAP 3 year



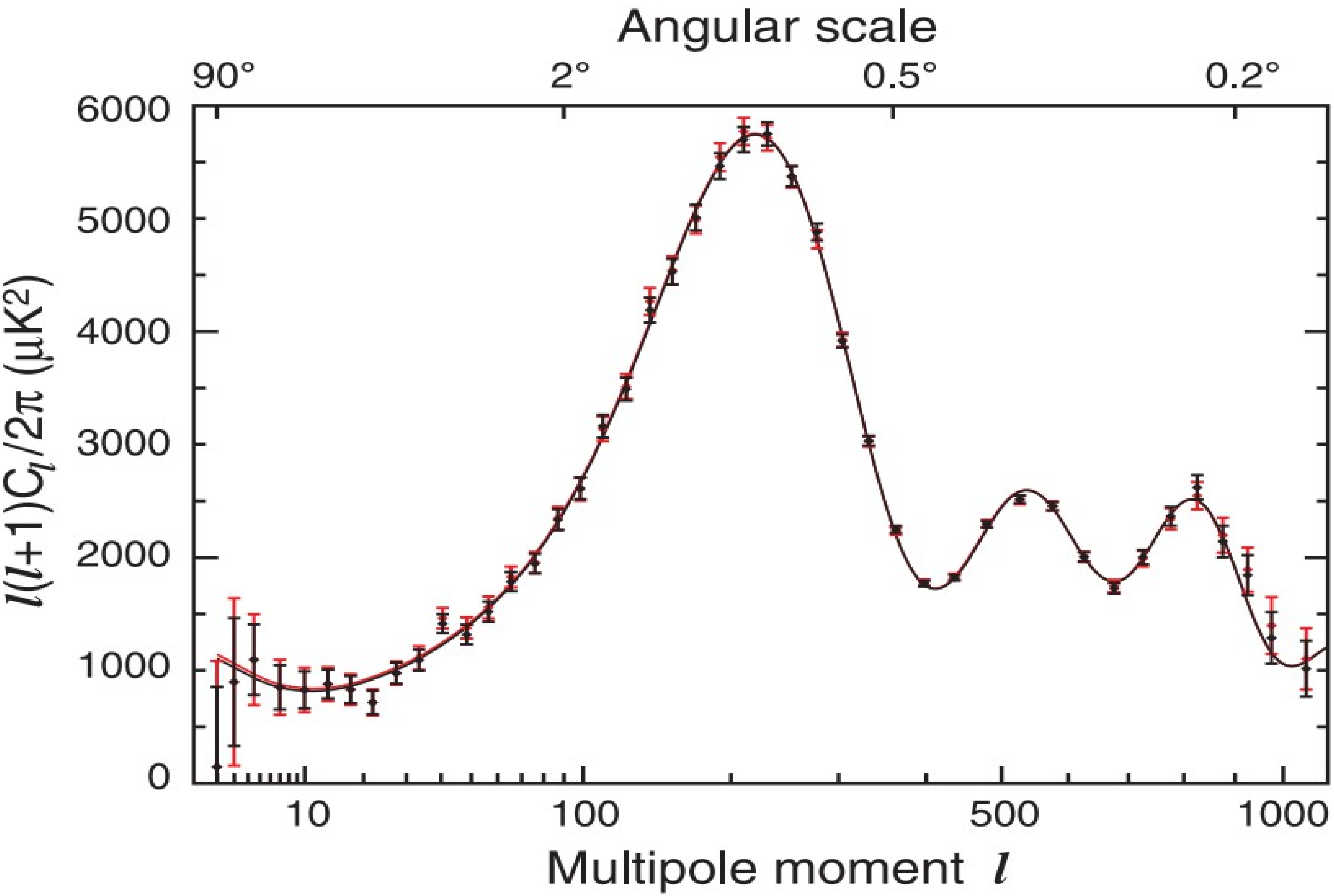
CMB Anisotropy, PLANCK



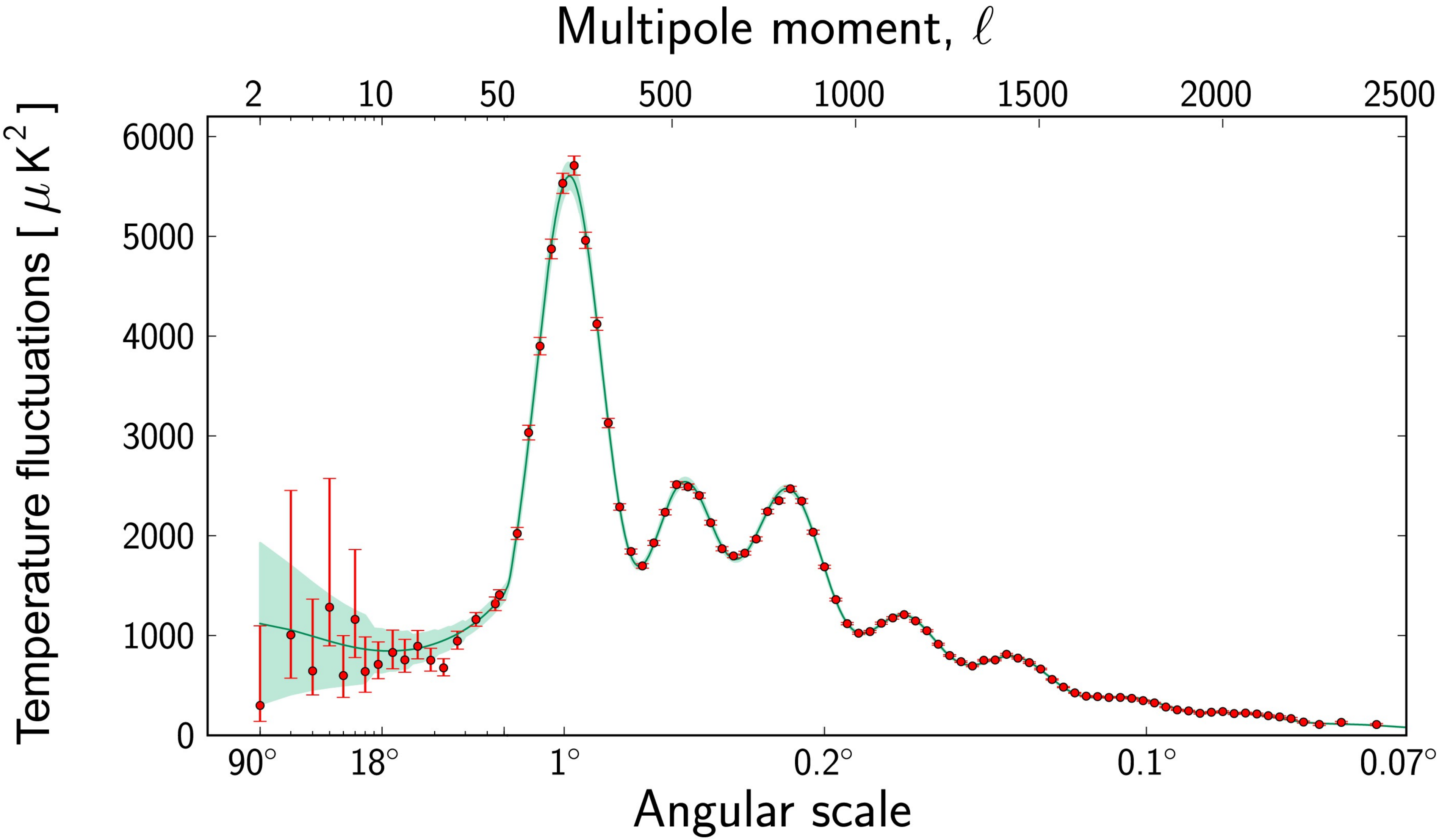
PLANCK, CMB anisotropy



ILC
l=2
l=3
l=4
l=5
l=6
l=7
l=8



WMAP 9 year data



Multipole moment, ℓ
2
10
50
500
1000
1500
2000
2500
Temperature fluctuations [μK^2]
6000
5000
4000
3000
2000
1000
0
90°
18°
1°
0.2°
0.1°
0.07°
Angular scale

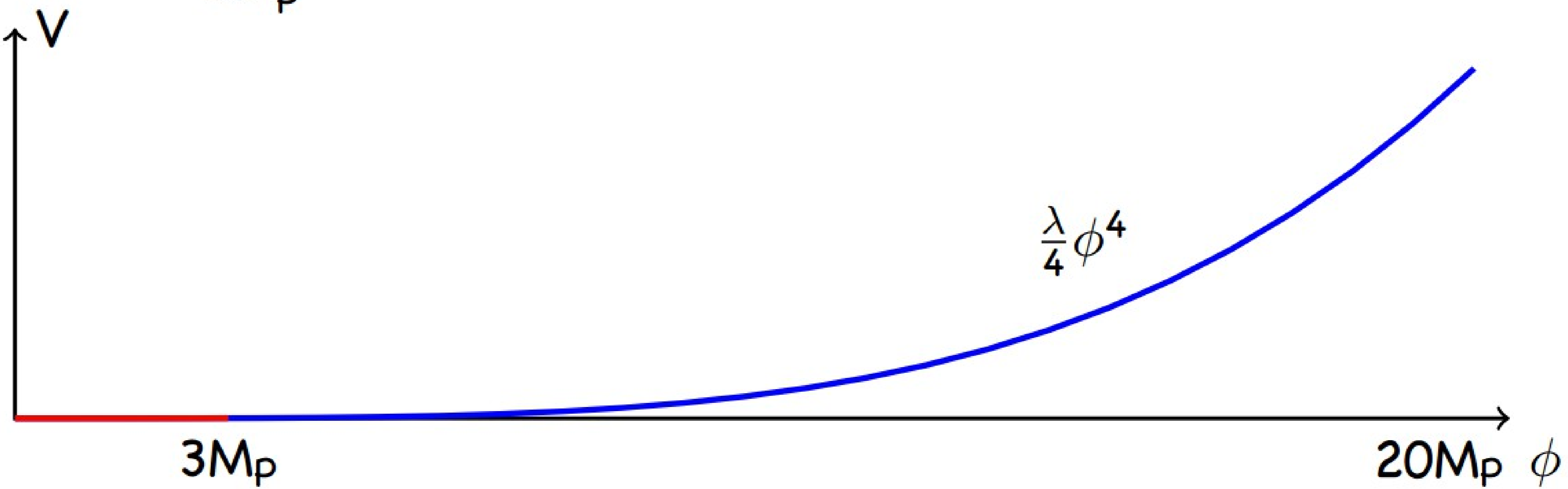
PLANCK 2013

Theory of Cosmic Inflation solves the problem of explaining super-horizon correlations.

It also provides a mechanism for genration of scale invariant density perturbations.

Slow roll inflation

$$\mathcal{H}^2 \simeq \frac{1}{3M_P^2}\left(V(\phi) + \dot{\phi}^2/2\right) , \qquad \ddot{\phi} + 3\mathcal{H}\dot{\phi} + V'(\phi) = 0$$



Relating CMB observables to Inflation potential

$$A_s \approx \frac{V}{24\pi^2 M_{pl}^4 \epsilon_V}$$

$$A_t \approx \frac{2V}{3\pi^2 M_{pl}^4}$$

$$n_s - 1 \approx 2\eta_V - 6\epsilon_V$$

$$\epsilon_V \equiv \frac{M_{pl}^2}{2}\left(\frac{V_{,\phi}}{V}\right)^2 , \qquad \eta_V \equiv \frac{M_{pl}^2 V_{,\phi\phi}}{V}$$

PLANCK 2013

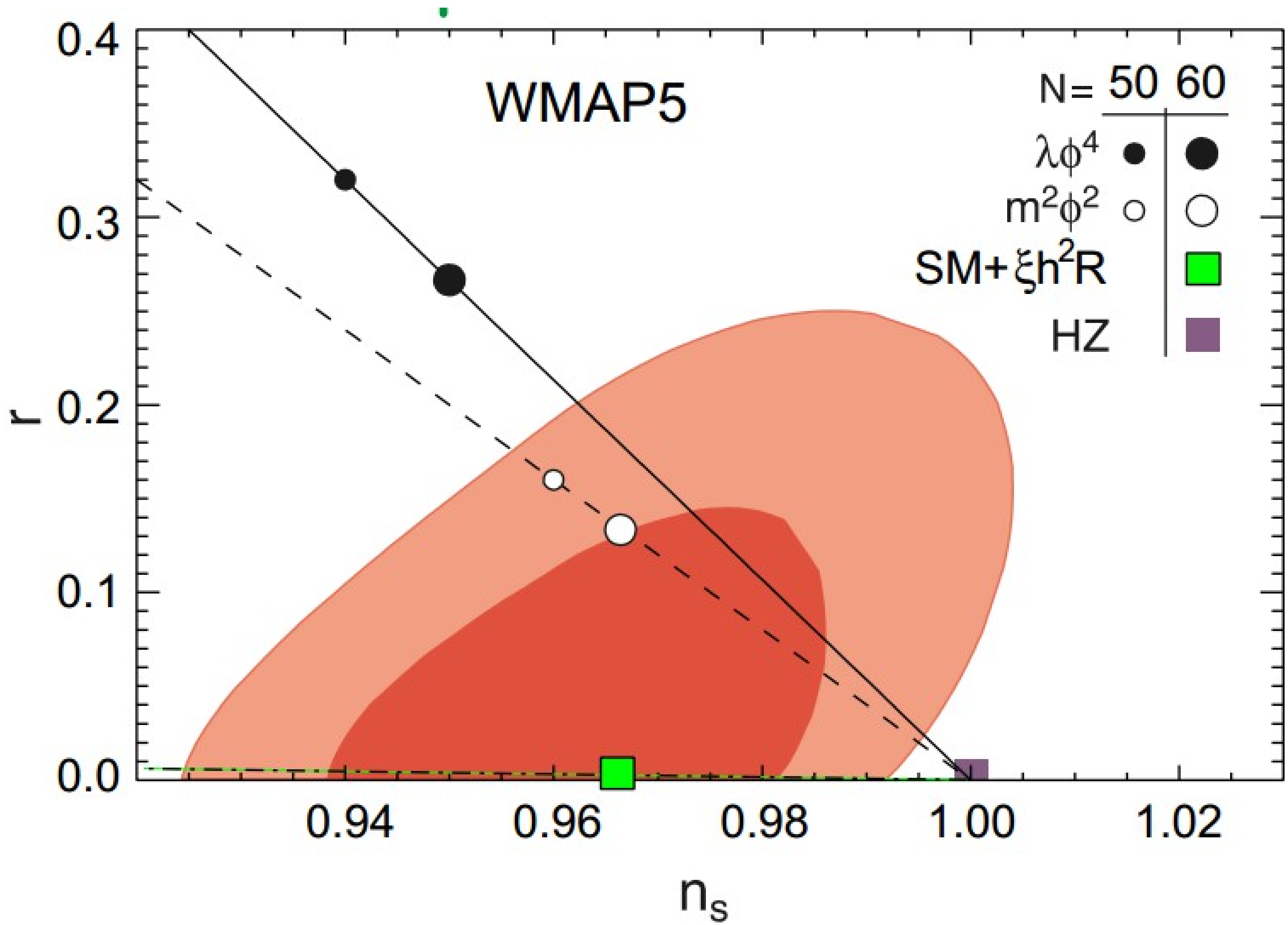
$$\Delta_{\mathcal{R}}^2 = 2.1955^{+0.533}_{-0.585} \times 10^{-9}$$

$$n_s = .9603 \pm .0073$$

$$\lambda \sim 10^{-12}$$

$$r = \frac{\mathcal{P}_{\mathrm{t}}(k_*)}{\mathcal{P}_{\mathcal{R}}(k_*)} < 0.12 \text{ at } 95\% \text{ CL}$$

Energy scale of inflation: $V_* < (1.94 \times 10^{16}\mathrm{GeV})^4$



WMAP5
N= 50 60
λφ⁴
m²φ²
SM+ξh²R
HZ
r
0.4
0.3
0.2
0.1
0.0
0.94
0.96
0.98
1.00
1.02
nₛ

Inflaton with non-minimal curvature coupling

$$S = \int d^4x \sqrt{-g}\left[\frac{M_{pl}^2 + \xi\phi^2}{2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{\lambda}{4}\left(\phi^2 - \phi_0^2\right)^2\right]$$

$$\hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}\ , \qquad \Omega^2 = 1 + \frac{\xi\phi^2}{M_P^2}$$

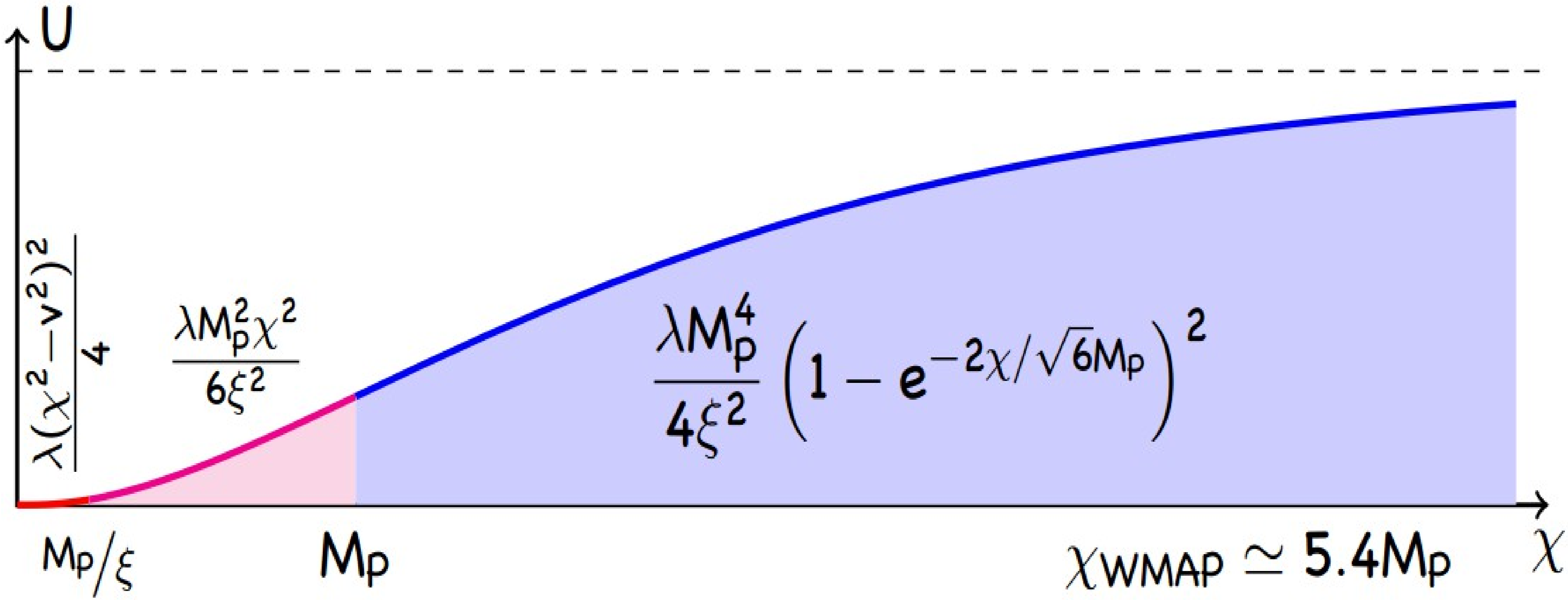
$$S = \int d^4x\sqrt{-\tilde{g}}\left(-\frac{M_P^2}{2}\tilde{R} + \frac{M_P^2(M_P^2 + (6\xi+1)\xi\phi^2)}{(M_P^2 + \xi\phi^2)^2}\frac{(\partial_\mu\phi)^2}{2} - \frac{\lambda M_P^4\phi^4}{4(M_P^2 + \xi\phi^2)^2}\right)$$

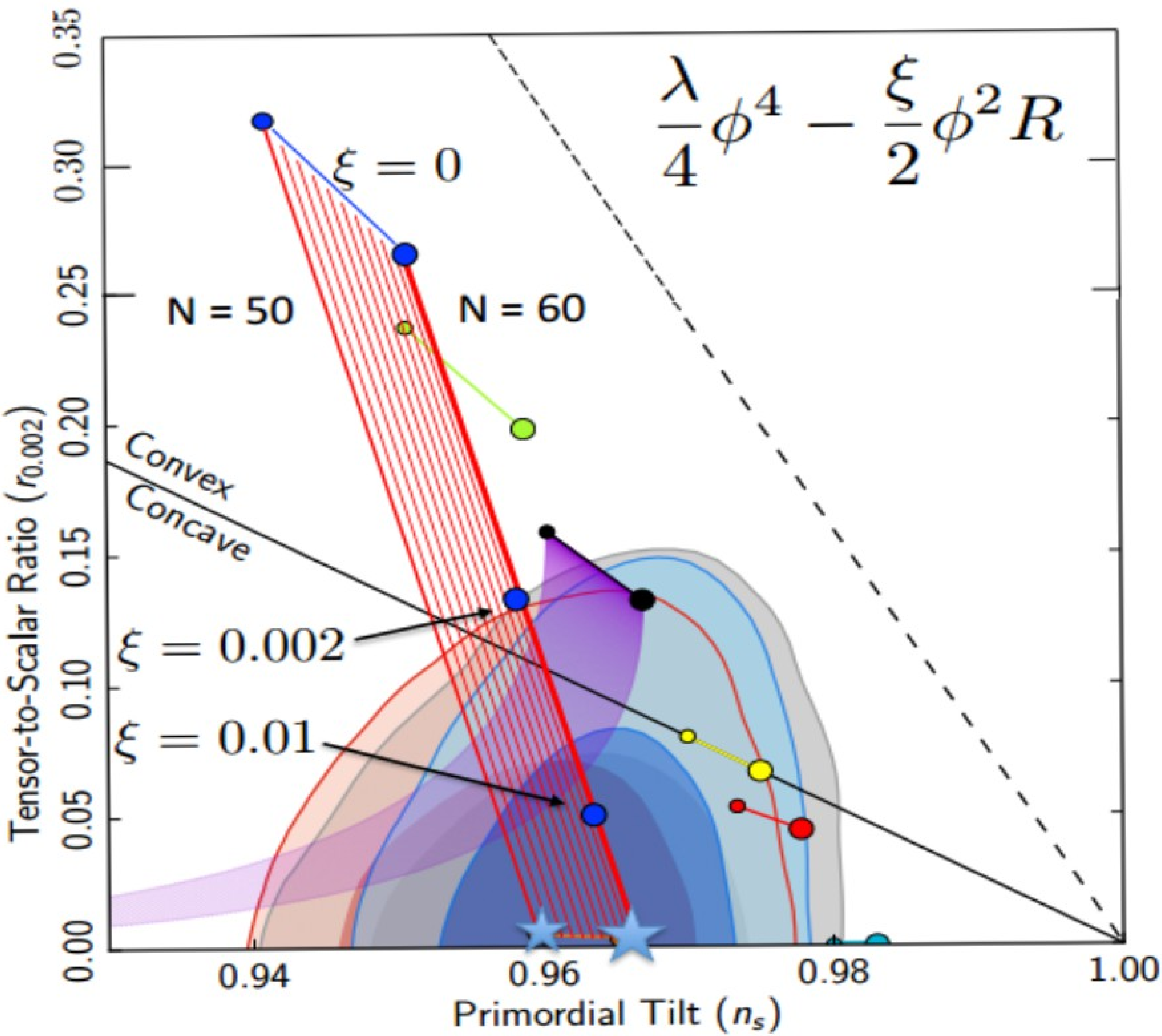
$$\frac{d\chi}{d\phi} = \frac{M_P\sqrt{M_P^2 + (6\xi+1)\xi\phi^2}}{M_P^2 + \xi\phi^2}$$

$$S_E = \int d^4x\sqrt{-\hat{g}}\left\{ -\frac{M_P^2}{2}\hat{R} + \frac{\partial_\mu\chi\partial^\mu\chi}{2} - U(\chi)\right\}$$

$$U(\chi) = \frac{\lambda M_P^4}{4\xi^2}\left(1 - \exp\left(-\frac{2\chi}{\sqrt{6}M_P}\right)\right)^2$$

Effective potential in curvature coupling models





$\frac{\lambda}{4}\phi^4 - \frac{\xi}{2}\phi^2 R$
$\xi = 0$
N = 50
N = 60
Convex
Concave
$\xi = 0.002$
$\xi = 0.01$
Tensor-to-Scalar Ratio ($r_{0.002}$)
Primordial Tilt (n_s)
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.94
0.96
0.98
1.00

Standard model Higgs as Inflaton

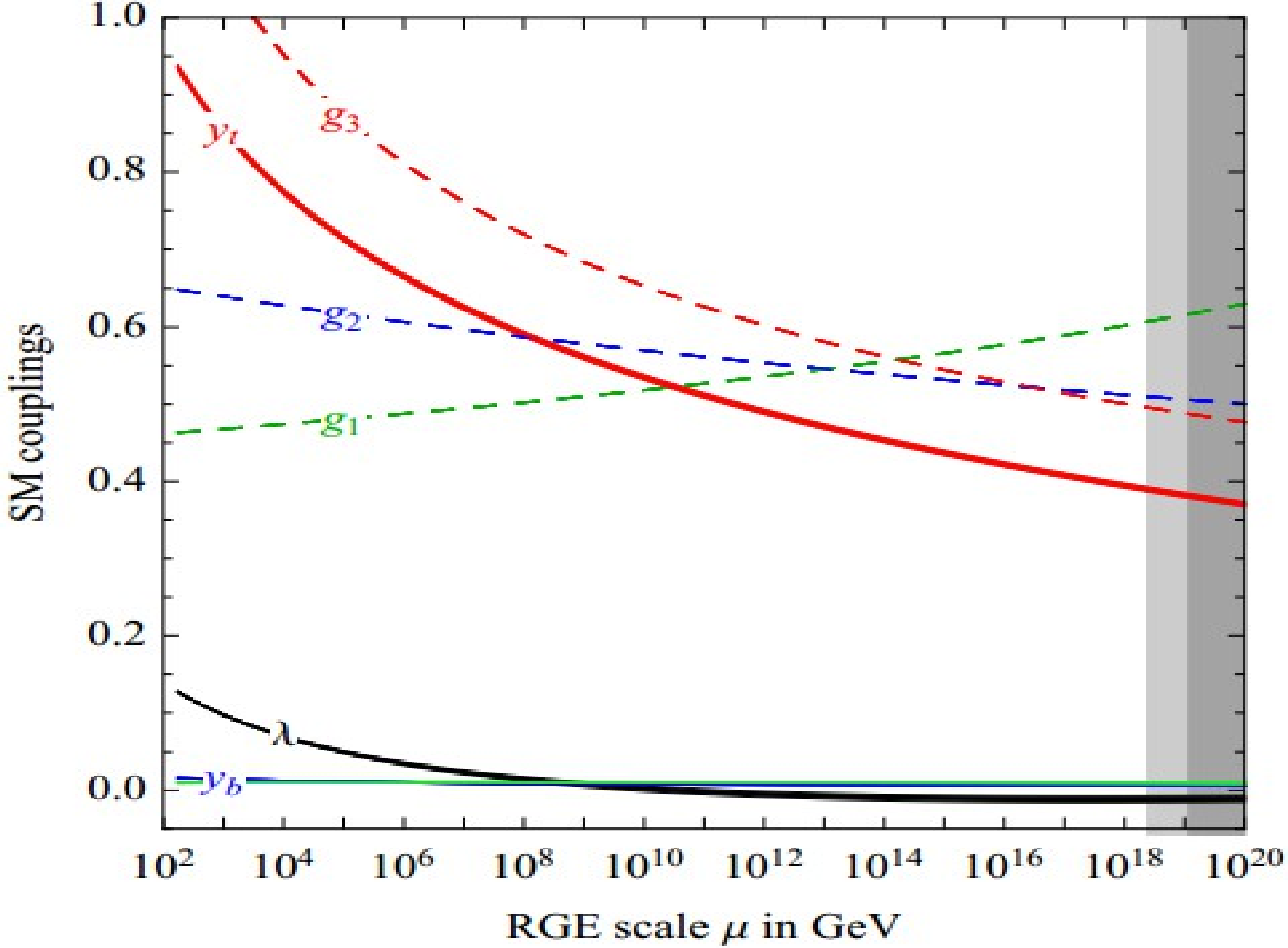
Bezrukov and Shaposhnikov 2008

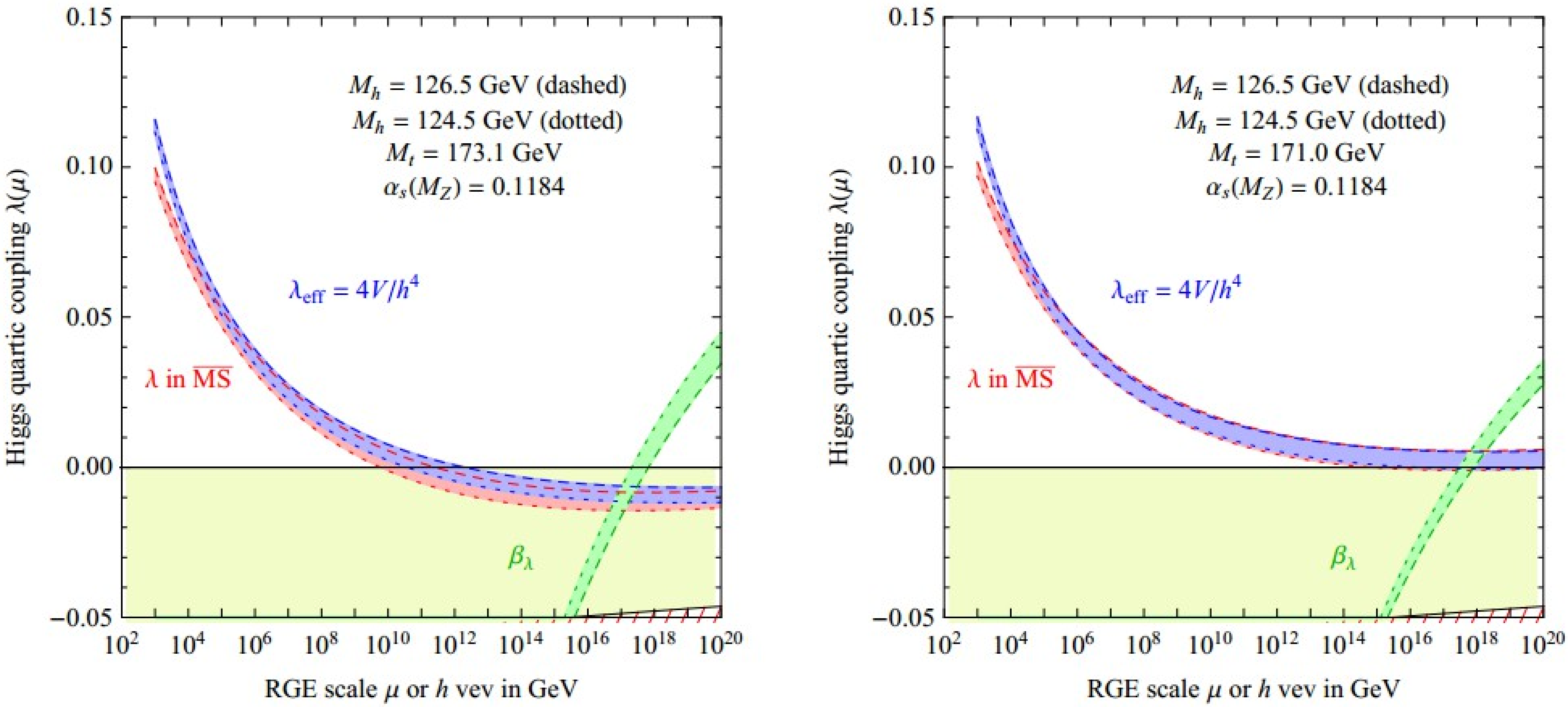
$$S_J = \int d^4x\sqrt{-g}\left\{ -\frac{M^2+\xi h^2}{2}R + \frac{\partial_\mu h \partial^\mu h}{2} - \frac{\lambda}{4}\left(h^2 - v^2\right)^2 \right\}$$

$$\xi \simeq 49000\sqrt{\lambda} = 49000\frac{m_H}{\sqrt{2}v}$$

$$r = 16\epsilon \simeq 0.0033.$$

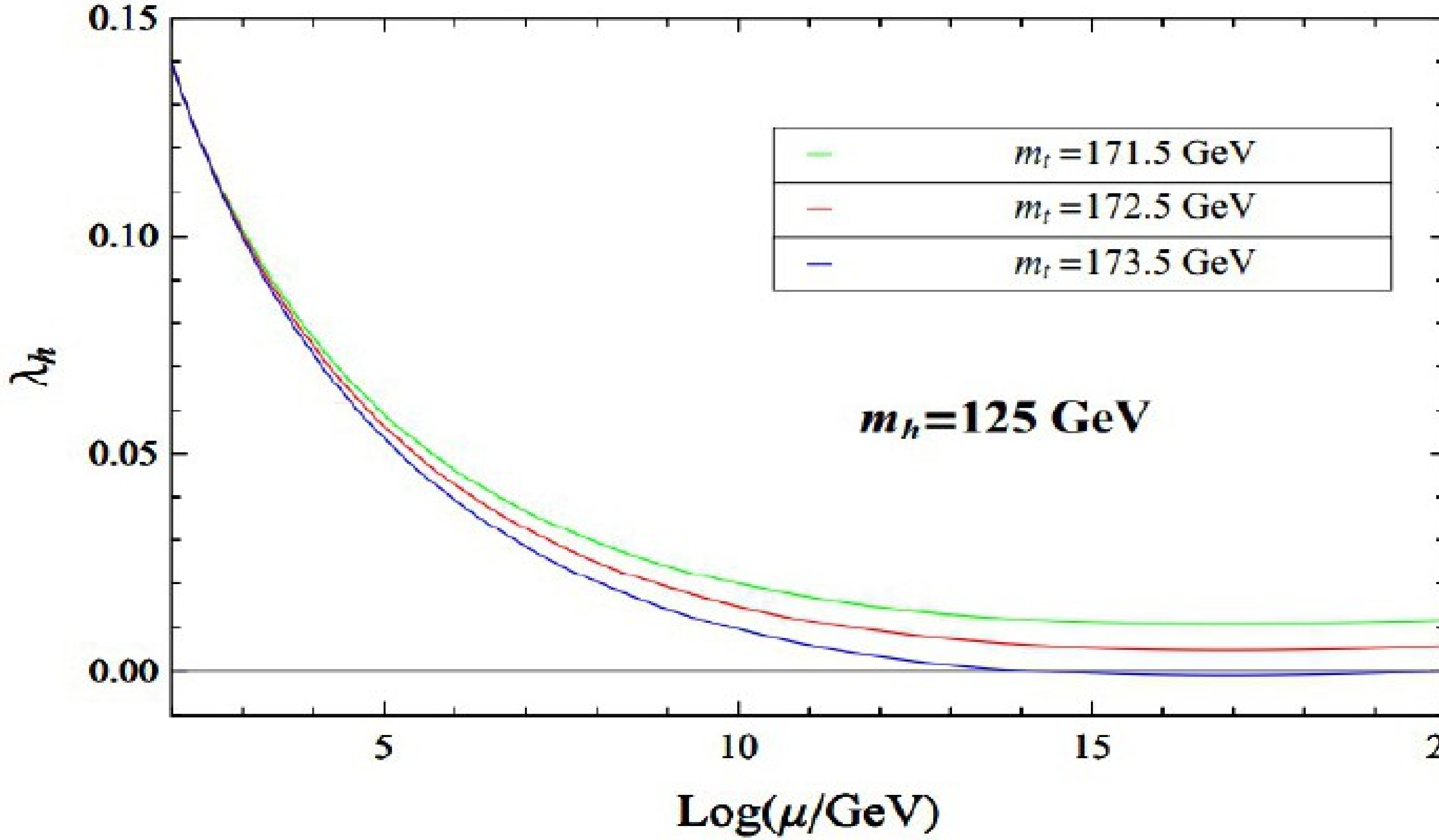
Running of the couplings in the Standard Model

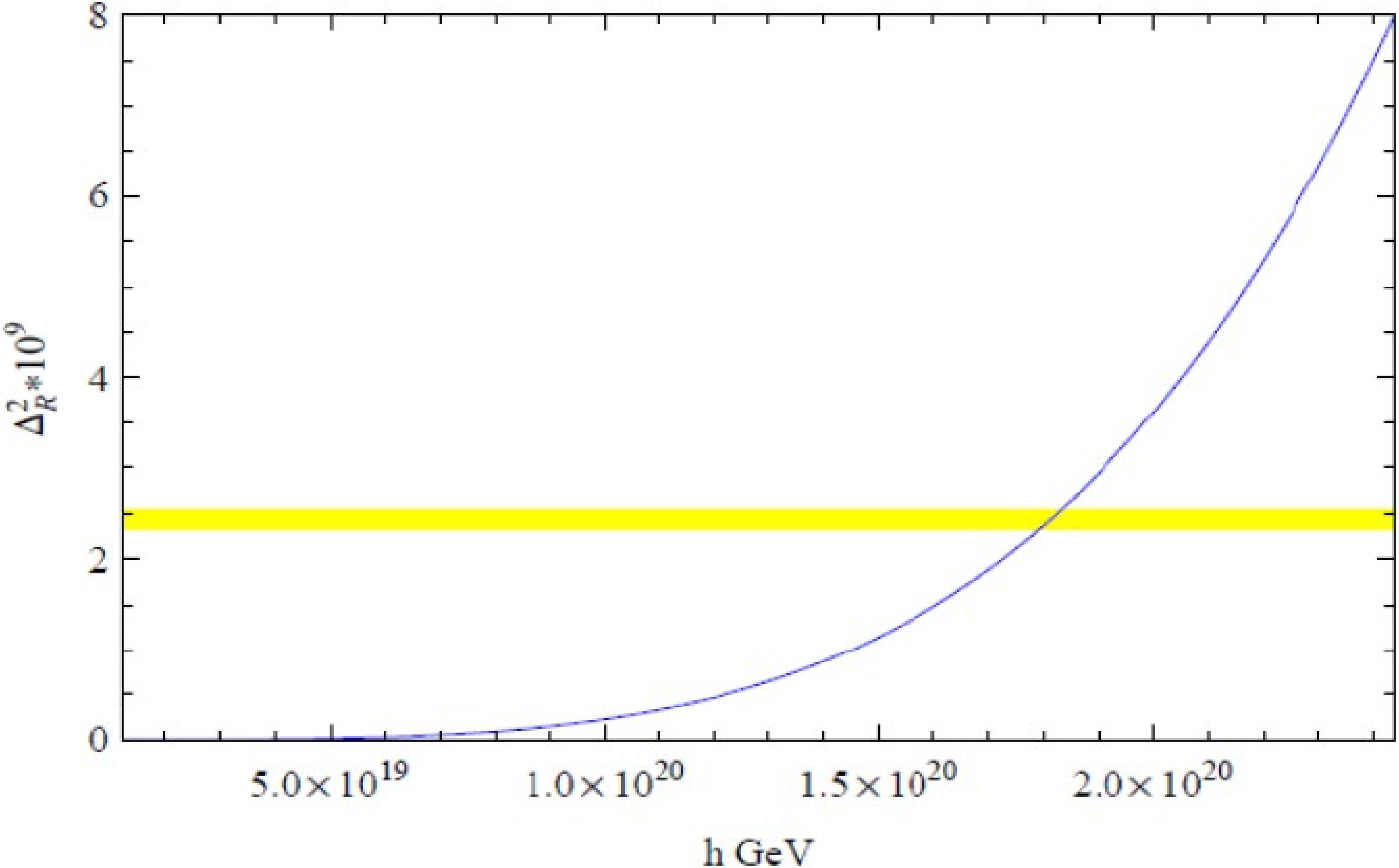




Degrassi et al 2013

Moumita Das, Joydeep Chakrabortty, SM
arXiv 1207.2027





$\Delta_R^2 * 10^9$
8
6
4
2
0
5.0×10^{19}
1.0×10^{20}
1.5×10^{20}
2.0×10^{20}
h GeV

Moumita Das

f(R) theories are equivalent to scalar tensor theories

$$S = \frac{1}{2\kappa}\int d^4x\sqrt{-g}\,f(R) + S^{(m)}$$

$$\psi(\phi) \equiv f'(\phi)$$

$$S = \frac{1}{2\kappa}\int d^4x\sqrt{-g}\,[\psi(\phi)R - V(\phi)] + S^{(m)}$$

$$V(\phi) = \phi f'(\phi) - f(\phi)$$

Starobinsky model

$$S_{\mathrm{S}} = \frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}\,\left(M_{\mathrm{P}}^2 R + \frac{M_{\mathrm{P}}^2}{6M^2}R^2\right)$$

$$\tilde{g}_{\mu\nu} = (1+\varphi/3M^2)g_{\mu\nu}$$

$$\varphi' = \sqrt{\frac{3}{2}}\ln\left(1+\frac{\varphi}{3M^2}\right);$$

$$S = \frac{1}{2}\int d^4x\sqrt{-\tilde{g}}\left[\tilde{R} + (\partial_\mu\varphi')^2 - \frac{3}{2}M^2(1-e^{-\sqrt{2/3}\varphi'})^2\right]$$

Effective potential in the Staorbinsky model

$$V = \frac{3}{4}M^2\left(1 - e^{-\sqrt{2/3}\varphi'}\right)^2 M_P^2$$

is of the same form as the Higgs inflation model

$$U(\chi) = \frac{\lambda M_P^4}{4\xi^2}\left(1 - \exp\left(-\frac{2\chi}{\sqrt{6}M_P}\right)\right)^2$$

Distinguishing Higgs Inflation from Staroninsky Inflation from observation

$$T_H^{\rm reh} \simeq 6 \times 10^{13}\,{\rm GeV}\ , \qquad T_{R^2}^{\rm reh} = 3.1 \times 10^{9}\,{\rm GeV}$$

$$N_* \approx 57 - \frac{1}{3}\log\frac{10^{13}\,{\rm GeV}}{T_r}$$

$$N_H = 57.66, \qquad N_{R^2} = 54.37.$$

$$\text{Higgs-inflation: } n_s = 0.967, \quad r = 0.0032,$$

$$R^2\text{-inflation: } n_s = 0.965, \quad r = 0.0036.$$

$$n_s = .9603 \pm .0073$$

F. L. Bezrukova, D. S. Gorbunov, 2012

Can we differentiate between HI and SI with non-Gaussinity ?

$$\langle \mathcal{R}(\mathbf{k}_1)\, \mathcal{R}(\mathbf{k}_2) \rangle = (2\pi)^3 \, \frac{2\pi^2}{k^3} \mathcal{P}_{\mathcal{R}}(k)\, \delta^3(\mathbf{k}_1 + \mathbf{k}_2)$$

$$\langle \mathcal{R}(\mathbf{k}_1)\, \mathcal{R}(\mathbf{k}_2)\, \mathcal{R}(\mathbf{k}_3) \rangle = (2\pi)^3 B_{\mathcal{R}}(k_1, k_2, k_3) \delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)$$

Higgs Inflation in $f(\Phi, R)$ Theory

$$S = \int d^4x\sqrt{-g}\Big[-\frac{{M_p}^2}{2}R\left(1+\frac{\xi\Phi^a R^{b-1}}{M_p^{a+2b-2}}\right) + \frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi + \frac{\lambda\Phi^4}{4}\Big]$$

Girish Chakravarty, SM, Naveen Singh ,

arXiv:1303.3870 [astro-ph.CO]

Amplitude of curvature purterbation

$$\Delta_{\mathcal{R}} = \frac{1}{\sqrt{Q_s}} \left(\frac{H}{2\pi} \right)$$

$$Q_s = \frac{\dot{\phi}^2 + 3\dot{F}^2/(2\kappa^2 F)}{\left(H + \dot{F}/(2F)\right)^2}$$

$$F = \partial f/\partial R = 1 + \frac{\xi b \Phi^a R^{b-1}}{M_p^{a+2b-2}}$$

Tensor amplitude

$$P_T = \frac{2}{\pi^2}\left(\frac{H}{M_P}\right)^2 \frac{1}{F}$$

$$r \simeq \frac{8\kappa^2 Q_s}{F} \simeq 0.002$$

Parameters fit with the PLANCK+WMAP data

λ	0.1	10^{-2}	10^{-3}	10^{-4}	10^{-5}
$\phi_f\|_{(\phi_E=1M_p)}$	$0.0146M_p$	$0.0253M_p$	$0.044M_p$	$0.077M_p$	$0.134M_p$
$\phi_J\|_{(\phi_E=13M_p)}$	$3.566M_p$	$6.187M_p$	$10.77M_p$	$18.8M_p$	$32.77M_p$
a	3.56398962	3.27512990	3.02576940	2.80956100	2.62085100
b	0.21800513	0.36243484	0.48711456	0.59521700	0.68956620
a+2b	3.999999	3.999999	3.999998	3.999995	3.99998

TABLE II: The values of parameters (a,b) are evaluated in the Jordan frame at $\xi = 1$

$$S = \int d^4x\sqrt{-g}\left[-\frac{f(\Phi,R)}{2\kappa^2} + \frac{1}{2}g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi + V(\Phi)\right]$$

$$\frac{1}{\kappa^2}f(\Phi,R) = \frac{1}{\kappa^2}R + \frac{\xi\Phi^a R^b}{M_p^{a+2b-4}} \;\; ; \;\; V(\Phi) = \frac{\lambda\Phi^4}{4}$$

Scale invariant spectrum requires scale invariant interaction terms !

THANK YOU