Anisotropic Non-gaussianity with noncommutative spacetime

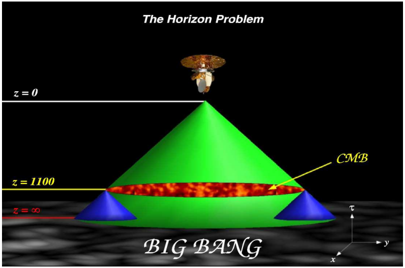
Akhilesh Nautiyal

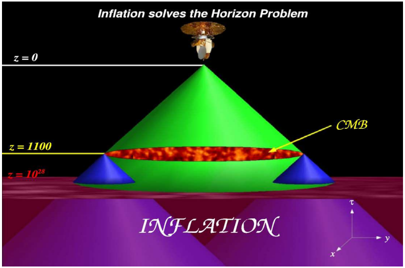
Institute of Mathematical Sciences Chennai

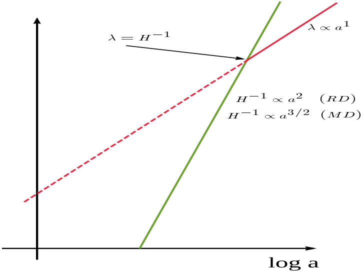
arXiv:1303.4159

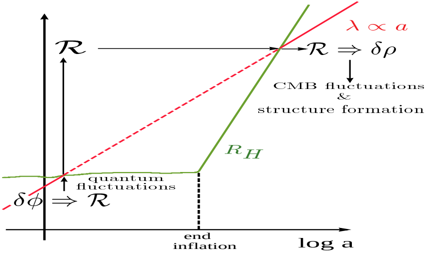
Outline of the talk

1. A brief review of inflation
2. Noncommutativity
3. Twisted quantum fields
4. Perturbations with ADM formalism
5. Two-point function (Power spectrum)
6. Three-point function
7. Implications for observations
8. Conclusions


The Horizon Problem
z = 0
CMB
z = 1100
z = ∞
BIG BANG
τ
y
x


Inflation solves the Horizon Problem
z = 0
CMB
z = 1100
z = 10^28
INFLATION
τ
y
x


$\lambda = H^{-1}$
$\lambda \propto a^1$
$H^{-1} \propto a^2$ (RD)
$H^{-1} \propto a^{3/2}$ (MD)
log a


$\lambda \propto a$
$\mathcal{R}$
$\mathcal{R} \Rightarrow \delta\rho$
CMB fluctuations
&
structure formation
R_H
quantum
fluctuations
$\delta\phi \Rightarrow \mathcal{R}$
end
inflation
log a

Inflation

- Inflation solves horizon problem and flatness problem.
- Rapid accelerated expansion during the early universe $a \propto e^{Ht}$, horizon H^{-1} remains nearly constant $\rightarrow \dot{H} = -4\pi G\left(\rho + p\right) \Rightarrow p \sim -\rho$.
- For scalar field $(\varphi(x,t) = \phi(t) \, + \delta\phi(x,t) \,)$

$$\rho = \frac{\dot{\phi}^2}{2} + V(\phi), \;\; p = \frac{\dot{\phi}^2}{2} - V(\phi) \tag{1}$$

- The dynamics of the inflaton field $\phi(t)$ is given by

$$\ddot{\phi} + 3H\dot{\phi} + V' = 0 \tag{2}$$

- For inflation the kinetic energy of ϕ $<<$ potential energy.$\rightarrow$ $\epsilon = \frac{M_P^2}{16\pi}\left(\frac{V'}{V}\right)^2 << 1$ and $\ddot{\phi} << 3H\dot{\phi} \rightarrow \eta_V = \frac{M_P^2}{8\pi}\left(\frac{V''}{V}\right) << 1$.
- The duration of inflation is given as $N = \int_{t_i}^{t_f} H \, dt$.
- At the end of inflation universe reheats to the GUT scale.

- ϵ and η can also be defined as

$$\epsilon = -\frac{\dot{H}}{H^2}, \ \eta = \frac{\dot{\epsilon}}{\epsilon H} \tag{3}$$

where $\eta = -2\eta_V + 4\epsilon$.

- Quantum fluctuations during inflation → scalar and tensor perturbations in the metric → CMB anisotropy and structures in the universe.
- The scalar perturbations are described by two-point correlation function for curvature perturbation in Fourier space called as "Power spectrum" whose amplitude and scale (momentum) dependence are determined by two-point correlation function of temperature anisotropy of CMB.
- Standard inflationary models predict adiabatic, nearly scale-invariant and gaussian perturbations which are consistent with the observations.
- Testing Non-gaussianity is a major goal of on-going Planck and other future experiments and is determined by non-zero higher order correlation function
- Planck puts tight bounds on non-gaussianity but detects statistical anisotropy.
- Inflation occurs above GUT scale and stretches out length scales of the order of Planck length to the current hubble scales so it provides a window to see the new physics effects on Planck scale.

Noncommutativity

- Noncommutativity is motivated by Heisenberg uncertainty principle and Einstein's gravity.
- It arises in certain theories of gravity and string theory.
- We consider the Groenewold-Moyal (GM) plane defined as

$$[\tilde{x}_\mu, \tilde{x}_\nu] = i\theta_{\mu\nu} \tag{4}$$

- $\theta_{\mu\nu}$ is constant, real antisymmetric matrix and $\tilde{x}_\mu(x) = x_\mu$ in some chosen coordinate system.
- We take this to be comoving coordinates so

$$\theta^{ph}_{0i} = a(t)\theta_{0i}, \ \theta^{ph}_{ij} = a^2(t)\theta_{ij} \tag{5}$$

- $\theta_{\mu\nu}$ doesn't transform as a tensor so breaks Lorentz invariance
- We deform Poincare symmetry to make commutation relations invariant under this symmetry.

- Poincare algebra acts on funcions in Minkowski space as

$$P_\alpha f(x) = -i\partial_\alpha f(x),\ M_{\alpha\beta} f(x) = -i\left(x_\alpha \partial_\beta - x_\alpha \partial_\beta\right) f(x). \tag{6}$$

- The algebra of function is commutative with commutative multiplication

$$m_0\left(f \otimes g\right)(x) = f(x)g(x). \tag{7}$$

- The coproduct acting on the tensor product $f \otimes g$ is

$$\Delta_0(X) = I \otimes X + X \otimes I \tag{8}$$

- Commutative multiplication is changed in GM plane as

$$m_\theta\left(f \otimes g\right)(x) = \left(f \star g\right)(x) = m_0\left[\mathcal{F}_\theta f \otimes g\right](x) \tag{9}$$

where twist element $\mathcal{F}_\theta = \exp\left[-\frac{i}{2}\theta^{\alpha\beta} P_\alpha \otimes P_\beta\right]$. So

$$\left(f \star g\right)(x) = \exp\left[\frac{i}{2}\theta_{\mu\nu}\frac{\partial}{\partial x_\mu}\frac{\partial}{\partial y_\nu}\right] f(x)g(y)\big|_{x=y} \tag{10}$$

- Twisted coproduct $\rightarrow$ compatible with $\star$ product defined as $\Delta_\theta = \mathcal{F}_\theta^{-1}\Delta_o\mathcal{F}_\theta$.

Twisted quantum fields

- For multi-particle state twisted coproduct $\rightarrow$ not compatible with the statistics operator defined as $\tau_0 (\phi \otimes \chi) = (\chi \otimes \phi)$ to construct symmetric and antisymmetric states i.e

$$\phi \otimes \chi_{S,A} = \frac{1 \pm \tau_0}{2} \phi \otimes \chi \tag{11}$$

- Define deformed statistics operator $\tau_\theta = \mathcal{F}_\theta^{-1} \tau_0 \mathcal{F}_\theta$
- In terms of quantum fields we defined deformed quantum field as

$$\phi_\theta = \int \frac{d^3 p}{(2\pi)^3} \left(a_{\vec{p}} e^{-ip\cdot x} + a_{\vec{p}}^\dagger e^{ip\cdot x} \right) \tag{12}$$

where $a_{\vec{p}} = c_{\vec{p}} e^{-\frac{i}{2} p_\mu \theta^{\mu\nu} P_\nu}$, $a_{\vec{p}}^\dagger = c_{\vec{p}}^\dagger e^{\frac{i}{2} p_\mu \theta^{\mu\nu} P_\nu}$

- In terms of usual quantum field

$$\phi_\theta = \phi_0 e^{\frac{1}{2} \overleftarrow{\partial} \wedge P} \tag{13}$$

where $\overleftarrow{\partial} \wedge P = \overleftarrow{\partial}_\mu \theta^{\mu\nu} P_\nu$ and $(\phi_\theta \star \phi_\theta)(x) = \phi_\theta(x) e^{\frac{i}{2} \overleftarrow{\partial_x} \wedge \overrightarrow{\partial_y}} \phi_\theta(y)\Big|_{x=y}$.

Perturbations with ADM formalism

- The action for scalar field

$$S = \int d^4x\sqrt{-g}\left(\frac{{M_p}^2}{2}R + \mathcal{L}\right). \quad (14)$$

- The background is described by FRW metric

$$ds^2 = -dt^2 + a^2(t)\left(dx^2 + dy^2 + dz^2\right). \quad (15)$$

- To do perturbation theory, the metric in ADM formalism is

$$ds^2 = -N^2dt^2 + h_{ij}\left(dx^i + N^i dt\right)\left(dx^j + N^j dt\right) \quad (16)$$

  where N is laps function, N_i, N_j are shift vectors and h_{ij} is metric of three-dimensional hypersurface of constant time.

- We use comving gauge defined as

$$h_{ij} = a^2e^{2\zeta}\delta_{ij}, \ \delta\phi = 0. \quad (17)$$

- The action becomes

$$S = \frac{1}{2}\int dt d^3x\sqrt{h}\left(NR^{(3)} - 2NV(\phi) + N^{-1}\dot{\phi}^2 + N^{-1}\left(E_{ij}E^{ij} - E^2\right)\right) \tag{18}$$

Here $R^{(3)}$ is Ricci scalar calculated using the three-dimensional metric h_{ij} and E_{ij} is related to the extrinsic curvature of the constant time hypersurface

$$E_{ij} = \frac{1}{2}\left(\dot{h}_{ij} - \nabla_j N_i - \nabla_i N_j\right). \tag{19}$$

- Varying the action we get

$$\begin{aligned} R^{(3)} - 2V - N^{-2}(E_{ij}E^{ij} - E^2) - N^{-2}\dot{\phi}^2 = 0, \\ \nabla_j\left[N^{-1}\left(E^j_i - \delta^j_i E\right)\right] = 0. \end{aligned} \tag{20}$$

- Decompose N_i into irrotational and incompressible parts as $N_i = \tilde{N}_i + \partial_i\psi$ where $\partial_i\tilde{N}^i = 0$ and expand N, ψ and $\tilde{N}^i$ into powers of ζ as

$$\begin{aligned} N &= 1 + \alpha_1 + \alpha_2 + ... \\ \tilde{N}_i &= \tilde{N}_i^{(1)} + \tilde{N}_i^{(2)} + ... \\ \psi &= \psi_1 + \psi_2 + ... \end{aligned} \tag{21}$$

- Using these expansions constraint equations can be solved order by order. And at first order

$$\alpha_1 = \frac{\dot{\zeta}}{H},\ \tilde{N_i}^{(1)} = 0,\ \psi_1 = -\frac{\zeta}{H} + \chi,\ \partial^2\chi = a^2\epsilon\dot{\zeta} \tag{22}$$

Here $\partial^2 = \delta^{ij}\partial_i\partial_j$ and the use of suitable choice of boundary conditions has been made to put $N_i^{(1)} = 0$.

- To compute power spectrum and bispectrum, one needs to expand action up to 3rd order in ζ. For this we only need N and N_i up to first order. We get

$$S_2 = \int dt d^3x \left[a^3\epsilon\dot{\zeta}^2 - a\epsilon(\partial\zeta)^2\right] \tag{23}$$

$$\begin{aligned} S_3 = \int dt d^3x \Big[&-a\epsilon\zeta(\partial\zeta)^2 - a^3\epsilon\dot{\zeta}^3 + 3a^3\epsilon\zeta\dot{\zeta}^2 \\ &+\frac{1}{2a}\left(3\zeta - \frac{\dot{\zeta}}{H}\right)\left(\partial_i\partial_j\psi\partial^i\partial^j\psi - \partial^2\psi\partial^2\psi\right) - 2a^{-1}\partial_i\psi\partial_i\zeta\partial^2\psi\Big] \end{aligned} \tag{24}$$

Two-point function (Power spectrum)

- To compute power spectrum we use S_2 which in conformal time $d\tau = \frac{dt}{a}$ is

$$S_2 = \int d\tau d^3x a^2 \epsilon \left[\zeta'^2 - (\partial\zeta)^2\right] \tag{25}$$

- ζ can be expanded as

$$\zeta(\vec{x},\tau) = \int \frac{d^3k}{(2\pi)^3}\zeta(\vec{k},\tau)e^{i\vec{k}\cdot\vec{x}} = \int \frac{d^3k}{(2\pi)^3}\left(u(\vec{k},\tau)a_{\vec{k}} + u^\star(-\vec{k},\tau)a^\dagger_{-\vec{k}}\right)e^{i\vec{k}\cdot\vec{x}}, \tag{26}$$

with equation of motion

$$\zeta'' + 2\frac{z'}{z}\zeta' - \partial^2\zeta = 0. \tag{27}$$

Here $z^2 = 2a^2\epsilon$.

- Define $v_{\vec{k}} = z\zeta(\vec{k},\tau)$ to get

$$v''_{\vec{k}} + \left(k^2 - \frac{z''}{z}\right)v_{\vec{k}} = 0. \tag{28}$$

- The solution $v_{\vec{k}}$ can be obtained assuming Bunch Davies initial conditions as

$$v_{\vec{k}} = \frac{1}{\sqrt{2k}} \left(1 - \frac{i}{k\tau}\right) e^{-ik\tau}. \tag{29}$$

- Hence the basis function $u(\vec{k}, \tau)$ is

$$u(\vec{k}, \tau) = \frac{v_{\vec{k}}}{z} = \frac{iH}{\sqrt{4\epsilon k^3}} (1 + ik\tau) e^{-ik\tau}. \tag{30}$$

- Due to noncommutativity $\zeta_\theta(\vec{x}, t) = \zeta(\vec{x}, t) e^{\frac{1}{2} \overleftarrow{\partial}_\mu \wedge P_\nu}$. Two-point correlation function of deformed quantum field ζ will be

$$\begin{aligned} \langle \zeta_\theta(\vec{x}, t) \zeta_\theta(\vec{y}, t') \rangle &= \langle \zeta(\vec{x}, t) e^{\frac{1}{2} \overleftarrow{\partial}_{x\mu} \wedge P_\nu} \zeta(\vec{y}, t') e^{\frac{1}{2} \overleftarrow{\partial}_{y\mu} \wedge P_\nu} \rangle \\ &= \langle \zeta(\vec{x}, t) \zeta(\vec{y}, t') \rangle e^{-\frac{i}{2} \overleftarrow{\partial}_{x\mu} \wedge \overrightarrow{\partial}_{y\nu}}, \end{aligned} \tag{31}$$

where $[P_\mu, \zeta] = -i\partial_\mu \zeta$ is used.

- Taking Fourier transform

$$
\begin{aligned}
\langle \zeta_\theta(\vec{x},t)\zeta_\theta(\vec{y},t')\rangle &= \int \frac{d^3k d^3k'}{(2\pi)^6} \langle 0|\zeta(\vec{k},t)\zeta(\vec{k'},t')|0\rangle \\
&\times\ e^{-\frac{i}{2}\overleftarrow{\partial}_{x\mu}\wedge\overrightarrow{\partial}_{y\nu}} e^{i\left(\vec{k}\cdot\vec{x}+\vec{k'}\cdot\vec{y}\right)} \\
&= \int \frac{d^3k d^3k'}{(2\pi)^6} \langle 0|\zeta(\vec{k},t)\zeta(\vec{k'},t')|0\rangle \\
&\times\ e^{-\frac{i}{2}\left(\partial_t\theta^{0i}\partial_{\vec{y}}+\partial_{\vec{x}}\theta^{io}\partial_{t'}+\partial_{\vec{x}}\wedge\partial_{\vec{y}}\right)} e^{i\left(\vec{k}\cdot\vec{x}+\vec{k'}\cdot\vec{y}\right)} \\
&= \int \frac{d^3k d^3k'}{(2\pi)^6} \langle 0|\zeta(\vec{k},t)\zeta(\vec{k'},t')|0\rangle \\
&\times\ e^{\left(\frac{i}{2}\vec{k}\wedge\vec{k'}+\frac{\overrightarrow{\theta^0}\cdot\vec{k'}}{2}\partial_t-\frac{\overrightarrow{\theta^0}\cdot\vec{k}}{2}\partial_{t'}\right)} e^{i\left(\vec{k}\cdot\vec{x}+\vec{k'}\cdot\vec{y}\right)} \\
&= \int \frac{d^3k d^3k'}{(2\pi)^6} \langle 0|\zeta\left(\vec{k},t+\frac{\overrightarrow{\theta^0}\cdot\vec{k'}}{2}\right)\zeta\left(\vec{k'},t'-\frac{\overrightarrow{\theta^0}\cdot\vec{k}}{2}\right)|0\rangle \\
&\times\ e^{\frac{i}{2}\vec{k}\wedge\vec{k'}} e^{i\left(\vec{k}\cdot\vec{x}+\vec{k'}\cdot\vec{y}\right)}
\end{aligned}
\tag{32}
$$

Here $\overrightarrow{\theta^0} = \theta^{0i}$.

- In momentum space

$$\langle 0|\zeta_\theta(\vec{k},t)\zeta_\theta(\vec{k'},t')|0\rangle = e^{\frac{i}{2}\vec{k}\wedge\vec{k'}}\langle 0|\zeta\left(\vec{k},t+\frac{\vec{\theta^0}\cdot\vec{k'}}{2}\right)\zeta\left(\vec{k'},t'-\frac{\vec{\theta^0}\cdot\vec{k}}{2}\right)|0\rangle \tag{33}$$

- In de Sitter space $\tau(t) = \frac{1}{aH}e^{-Ht}$ So in conformal time and in the limit $t' \to t$

$$\zeta\left(\vec{k},t+\frac{\overrightarrow{\theta^0}\cdot\vec{k'}}{2}\right) \quad\to\quad \zeta\left(\vec{k},\tau e^{-H\frac{\overrightarrow{\theta^0}\cdot\vec{k'}}{2}}\right) \tag{34}$$

$$\zeta\left(\vec{k},t'-\frac{\overrightarrow{\theta^0}\cdot\vec{k}}{2}\right) \quad\to\quad \zeta\left(\vec{k},\tau e^{H\frac{\overrightarrow{\theta^0}\cdot\vec{k}}{2}}\right) \tag{35}$$

- The two-point function will be

$$\begin{aligned}\langle\zeta_\theta(\vec{k},\tau)\zeta_\theta(\vec{k'},\tau)\rangle &= \langle 0|\zeta\left(\vec{k},\tau e^{-H\frac{\overrightarrow{\theta^0}\cdot\vec{k'}}{2}}\right)\zeta\left(\vec{k'},\tau e^{H\frac{\overrightarrow{\theta^0}\cdot\vec{k}}{2}}\right)|0\rangle e^{\frac{i}{2}\vec{k}\wedge\vec{k'}} \\ &= \left|u\left(\vec{k},\tau e^{H\frac{\overrightarrow{\theta^0}\cdot\vec{k}}{2}}\right)\right|^2 (2\pi)^3\delta^3(\vec{k}+\vec{k'})\end{aligned} \tag{36}$$

- In noncommutative spacetime $\zeta_\theta(\vec{x},t)\zeta_\theta(\vec{x'},t') \neq \zeta_\theta(\vec{x'},t')\zeta_\theta(\vec{x},t)$ so we use $\frac{1}{2}\left[\zeta_\theta(\vec{x},\tau),\zeta_\theta((\vec{x}),\tau)\right]_+$ for two-point correlation function. In Fourier space

$$\langle 0|\zeta_\theta(\vec{k},\tau)\zeta_\theta(\vec{k'},\tau)|0\rangle_M = \frac{1}{2}\left(\langle 0|\zeta_\theta(\vec{k},\tau)\zeta_\theta(\vec{k'},\tau)|0\rangle + \langle 0|\zeta_\theta(-\vec{k},\tau)\zeta_\theta(-\vec{k'},\tau)\right) \tag{37}$$

- The power spectrum is defined as

$$\langle 0|\zeta_\theta(\vec{k},\tau)\zeta_\theta(\vec{k'},\tau)|0\rangle_M = (2\pi)^3\delta^3\left(\vec{k}+\vec{k'}\right)P_{\zeta_\theta}(k) \tag{38}$$

which is

$$P_{\zeta_\theta}(k) = \frac{1}{2}\left(\left|u\left(\vec{k},\tau e^{H\frac{\vec{\theta^0}\cdot\vec{k}}{2}}\right)\right|^2 + \left|u\left(-\vec{k},\tau e^{-H\frac{\vec{\theta^0}\cdot\vec{k}}{2}}\right)\right|^2\right) \tag{39}$$

Since on super-horizon limit $v_{\vec{k}} = \frac{1}{\sqrt{2k}}\left(\frac{-i}{k\tau}e^{-H\frac{\vec{\theta^0}\cdot\vec{k}}{2}}\right)$ so

$$P_{\zeta_\theta}(k) = P_\zeta(k)\cosh\left(H\vec{\theta^0}\cdot\vec{k}\right) \tag{40}$$

Three point function

- For three-point function

$$S_3 = \int dt d^3x \Big[-a\epsilon\zeta(\partial\zeta)^2 - a^3\epsilon\dot{\zeta}^3 + 3a^3\epsilon\zeta\dot{\zeta}^2 + \frac{1}{2a}\left(3\zeta - \frac{\dot{\zeta}}{H}\right)\left(\partial_i\partial_j\psi\partial^i\partial^j\psi - \partial^2\psi\partial^2\psi\right) - 2a^{-1}\partial_i\psi\partial_i\zeta\partial^2\psi\Big]$$

- Since $\psi = -\frac{\zeta}{H} + \chi$ and $\partial^2\chi = a^2\epsilon\dot{\zeta}$, so

$$S_3 = \int dt d^3x \Big[a^3\epsilon^2\zeta\dot{\zeta}^2 + a\epsilon^2\zeta(\partial\zeta)^2 - 2a\epsilon\dot{\zeta}(\partial\zeta)(\partial\chi) + \frac{a^3\epsilon}{2}\frac{d\eta}{dt}\zeta^2\dot{\zeta} + \frac{\epsilon}{2a}(\partial\zeta)(\partial\chi)(\partial^2\chi) + \frac{\epsilon}{4a}(\partial^2\zeta)(\partial\chi)^2 + \frac{1}{2}a\mathcal{F}\frac{\delta L}{\delta\zeta}\Big|_1\Big] \quad (41)$$

where $\mathcal{F} = (\eta\zeta^2 + terms\,with\,derivarives\ of\ \zeta)$ and $\frac{\delta L}{\delta\zeta}$ represents the terms propostional to the Gaussian EOM .

- Again integrate by parts the action to remove the terms involving $\partial\chi$ and using the Gaussian field equation

$$S_3 = \int dt d^3x \left[4a^5\epsilon^2 H\dot{\zeta}^2\partial^{-2}\dot{\zeta} + \frac{1}{2}a\mathcal{F}\frac{\delta L}{\delta\zeta}\bigg|_1\right] \tag{42}$$

Now $\mathcal{F} = (\eta - \epsilon)\zeta^2 + 2\epsilon\partial^{-2}(\zeta\partial^2\zeta)$ and ∂^{-2} is the inverse of ∂^2.

- To get rid of the last term in action we use field redefinition

$$\zeta \to \zeta_n + \frac{\mathcal{F}}{4}\zeta_n^2 \tag{43}$$

- $S = S_2 + \int dt d^3x 4a^5\epsilon^2 H\dot{\zeta}_n^2\partial^{-2}\dot{\zeta}_n$.
- So the three-point function becomes

$$\begin{aligned}\langle\zeta(x_1)\zeta(x_2)\zeta(x_3)\rangle &= \langle\zeta_n(x_1)\zeta_n(x_2)\zeta_n(x_3)\rangle \\ &+ \frac{(\eta-\epsilon)}{4}\left(\langle\zeta_n(x_1)\zeta_n(x_2)\rangle\langle\zeta_n(x_1)\zeta_n(x_3)\rangle + \text{permutations}\right) \\ &+ \frac{\epsilon}{2}\partial_{x_1}^{-2}\left(\langle\zeta(x_1)\zeta(x_2)\rangle\partial_{x_1}^2\langle\zeta_n(x_1)\zeta_n(x_3)\rangle + \text{permutations}\right).\end{aligned}$$

- So the interaction Hamiltonian in noncommutative spacetime is

$$\begin{aligned}\mathcal{H}(t') &= -\int d^3x 4a^5\epsilon^2 H\dot{\zeta}_\theta \star \dot{\zeta}_\theta \star \partial^{-2}\dot{\zeta}_\theta \\ &= -\int d^3x 4a^5\epsilon^2 H\dot{\zeta}^2\partial^{-2}\dot{\zeta}e^{\frac{1}{2}\overleftarrow{\partial_\mu}\wedge P_\nu}\end{aligned} \quad (44)$$

- Three-point using in-in formalism is

$$\begin{aligned}\langle\zeta_\theta(x_1)\zeta_\theta(x_2)\zeta_\theta(x_3)\rangle &= -i\int_{t_0}^{t} dt'\langle 0|\left[\zeta_\theta(x_1)\zeta_\theta(x_2)\zeta_\theta(x_3), \mathcal{H}(t')\right]|0\rangle \\ &= -i\int_{t_0}^{t} dt'\left(\langle 0|\zeta_\theta(x_1)\zeta_\theta(x_2)\zeta_\theta(x_3)\mathcal{H}(t')|0\rangle\right. \\ &- \left.\langle 0|\mathcal{H}(t')\zeta_\theta(x_1)\zeta_\theta(x_2)\zeta_\theta(x_3)|0\rangle\right)\end{aligned} \quad (45)$$

- Let

$$(a) = 4i\epsilon^2\int dt' a^5 H\int d^3x\langle 0|\zeta_\theta(x_1)\zeta_\theta(x_2)\zeta_\theta(x_3)\,\dot{\zeta}^2\partial^{-2}\dot{\zeta}\Big|_{t',\vec{x}} e^{\frac{1}{2}\overleftarrow{\partial_{x\mu}}\wedge P_\nu}|0\rangle \quad (46)$$

- Now writing twisted quantum fields in terms of untwisted quantum fields $\zeta_\theta(\vec{x},t) = \zeta(\vec{x},t) e^{\frac{1}{2}\overleftarrow{\partial}_\mu \wedge P_\nu}$, we get

$$
\begin{aligned}
(a) &= 4i\epsilon^2 \int dt' a^5 H \int d^3x \langle 0|\zeta(x_1)\zeta(x_2)\zeta(x_3) \\
&\times\ e^{\frac{i}{2}\left(\overleftarrow{\partial_{x_1}}\wedge\overleftarrow{\partial_{x_2}}+\overleftarrow{\partial_{x_2}}\wedge\overleftarrow{\partial_{x_3}}+\overleftarrow{\partial_{x_1}}\wedge\overleftarrow{\partial_{x_3}}\right)} \\
&\times\ e^{\frac{1}{2}\overleftarrow{\partial_{x_1}}\wedge P} e^{\frac{1}{2}\overleftarrow{\partial_{x_2}}\wedge P} e^{\frac{1}{2}\overleftarrow{\partial_{x_3}}\wedge P}\, \dot{\zeta}^2 \partial^{-2}\dot{\zeta}\Big|_{t',\vec{x}} e^{\frac{1}{2}\overleftarrow{\partial_x}\wedge P}|0\rangle \qquad (47) \\
&= 4i\epsilon^2 \int dt' a^5 H \int d^3x \langle 0|\zeta(x_1)\zeta(x_2)\zeta(x_3) \\
&\times\ e^{-\frac{i}{2}\left(\overleftarrow{\partial_{x_1}}\wedge\overleftarrow{\partial_{x_2}}+\overleftarrow{\partial_{x_2}}\wedge\overleftarrow{\partial_{x_3}}+\overleftarrow{\partial_{x_1}}\wedge\overleftarrow{\partial_{x_3}}\right)} \\
&\times\ e^{-\frac{i}{2}\left(\overleftarrow{\partial_{x_1}}+\overleftarrow{\partial_{x_2}}+\overleftarrow{\partial_{x_3}}\right)\wedge\overrightarrow{\partial_x}}\dot{\zeta}\, e^{-\frac{i}{2}\left(\overleftarrow{\partial_{x_1}}+\overleftarrow{\partial_{x_2}}+\overleftarrow{\partial_{x_3}}\right)\wedge\overrightarrow{\partial_x}}\dot{\zeta} \\
&\times\ e^{-\frac{i}{2}\left(\overleftarrow{\partial_{x_1}}+\overleftarrow{\partial_{x_2}}+\overleftarrow{\partial_{x_3}}\right)\wedge\overrightarrow{\partial_x}}\partial^{-2}\dot{\zeta}|0\rangle
\end{aligned}
\qquad (48)
$$

- In Fourier space and

$$
\begin{aligned}
(a) \quad = \quad & -4i\epsilon^2 \int dt' a^5 H \int d^3x \int \prod_{i=1}^{6} \frac{d^3k_i}{k_6^2 (2\pi)^{18}} e^{i\left(\vec{k}_1\cdot\vec{x}_1+\vec{k}_2\cdot\vec{x}_2+\vec{k}_3\cdot\vec{x}_3\right)} \\
\times \quad & \langle 0|\zeta\left(\vec{k}_1, t_1 + \frac{\vec{\theta^0}\cdot\vec{k}_2 + \vec{\theta^0}\cdot\vec{k}_3 + \vec{\theta^0}\cdot\vec{k}_4 + \vec{\theta^0}\cdot\vec{k}_5 + \vec{\theta^0}\cdot\vec{k}_6}{2}\right) \\
\times \quad & \zeta\left(\vec{k}_1, t_2 + \frac{-\vec{\theta^0}\cdot\vec{k}_1 + \vec{\theta^0}\cdot\vec{k}_3 + \vec{\theta^0}\cdot\vec{k}_4 + \vec{\theta^0}\cdot\vec{k}_5 + \vec{\theta^0}\cdot\vec{k}_6}{2}\right) \\
\times \quad & \zeta\left(\vec{k}_1, t_3 + \frac{-\vec{\theta^0}\cdot\vec{k}_1 - \vec{\theta^0}\cdot\vec{k}_2 + \vec{\theta^0}\cdot\vec{k}_4 + \vec{\theta^0}\cdot\vec{k}_5 + \vec{\theta^0}\cdot\vec{k}_6}{2}\right) \\
\times \quad & \dot{\zeta}\left(\vec{k}_4, t' - \frac{\vec{\theta^0}\cdot\vec{k}_1 + \vec{\theta^0}\cdot\vec{k}_2 + \vec{\theta^0}\cdot\vec{k}_3}{2}\right) \\
\times \quad & \dot{\zeta}\left(\vec{k}_5, t' - \frac{\vec{\theta^0}\cdot\vec{k}_1 + \vec{\theta^0}\cdot\vec{k}_2 + \vec{\theta^0}\cdot\vec{k}_3}{2}\right) \\
\times \quad & \dot{\zeta}\left(\vec{k}_6, t' - \frac{\vec{\theta^0}\cdot\vec{k}_1 + \vec{\theta^0}\cdot\vec{k}_2 + \vec{\theta^0}\cdot\vec{k}_3}{2}\right) |0\rangle e^{i\left(\vec{k}_4\cdot\vec{x}+\vec{k}_5\cdot\vec{x}+\vec{k}_6\cdot\vec{x}\right)} e^{\frac{i}{2}\mathcal{P}}
\end{aligned}
$$

- Here

$$\mathcal{P} = \left(\vec{k}_1 \wedge \vec{k}_2 + \vec{k}_2 \wedge \vec{k}_3 + \vec{k}_1 \wedge \vec{k}_3 + \left(\vec{k}_1 + \vec{k}_2 + \vec{k}_3\right)\left(\vec{k}_4 + \vec{k}_5 + \vec{k}_6\right)\right) \tag{49}$$

- Since we need three-point function in Fourier space and $t_1 = t_2 = t_3 = t$ so

$$\begin{aligned} (a) &= -i\int_{t_0}^{t} dt' \left(\langle 0|\zeta_\theta(\vec{k}_1, t)\zeta_\theta(\vec{k}_2, t)\zeta_\theta(\vec{k}_3, t)\mathcal{H}(t')\right) \\ &= -4i\epsilon^2 \int dt' a^5 H \int d^3x \int \prod_{i=4}^{6} \frac{d^3k_i}{k_6^2(2\pi)^9} \\ &\times \langle 0|\zeta(\vec{k}_1, t_1)\zeta(\vec{k}_2, t_2)\zeta(\vec{k}_3, t_3)\dot{\zeta}(\vec{k}_4, t_4)\dot{\zeta}(\vec{k}_5, t_5)\dot{\zeta}(\vec{k}_6, t_6)|0\rangle \\ &\times e^{i\left(\vec{k}_4\cdot\vec{x}+\vec{k}_5\cdot\vec{x}+\vec{k}_6\cdot\vec{x}\right)} e^{\frac{i}{2}\mathcal{P}} \end{aligned} \tag{50}$$

- Where

$$
\begin{aligned}
t_1 &= t + \frac{\overrightarrow{\theta^0}\cdot\vec{k}_2 + \overrightarrow{\theta^0}\cdot\vec{k}_3 + \overrightarrow{\theta^0}\cdot\vec{k}_4 + \overrightarrow{\theta^0}\cdot\vec{k}_5 + \overrightarrow{\theta^0}\cdot\vec{k}_6}{2} \\
t_2 &= t + \frac{\overrightarrow{-\theta^0}\cdot\vec{k}_1 + \overrightarrow{\theta^0}\cdot\vec{k}_3 + \overrightarrow{\theta^0}\cdot\vec{k}_4 + \overrightarrow{\theta^0}\cdot\vec{k}_5 + \overrightarrow{\theta^0}\cdot\vec{k}_6}{2} \\
t_3 &= t + \frac{\overrightarrow{-\theta^0}\cdot\vec{k}_1 - \overrightarrow{\theta^0}\cdot\vec{k}_2 + \overrightarrow{\theta^0}\cdot\vec{k}_4 + \overrightarrow{\theta^0}\cdot\vec{k}_5 + \overrightarrow{\theta^0}\cdot\vec{k}_6}{2} \\
t_4 &= t' - \frac{\overrightarrow{\theta^0}\cdot\vec{k}_1 + \overrightarrow{\theta^0}\cdot\vec{k}_2 + \overrightarrow{\theta^0}\cdot\vec{k}_3}{2} \\
t_5 &= t' - \frac{\overrightarrow{\theta^0}\cdot\vec{k}_1 + \overrightarrow{\theta^0}\cdot\vec{k}_2 + \overrightarrow{\theta^0}\cdot\vec{k}_3}{2} \\
t_6 &= t' - \frac{\overrightarrow{\theta^0}\cdot\vec{k}_1 + \overrightarrow{\theta^0}\cdot\vec{k}_2 + \overrightarrow{\theta^0}\cdot\vec{k}_3}{2} \qquad (51)
\end{aligned}
$$

$$(52)$$

- So finally we get

$$(a) = \epsilon(2\pi)^3\delta^3\left(\vec{k}_1+\vec{k}_2+\vec{k}_3\right)\frac{H^4}{16\epsilon^2}\prod_{i=1}^{3}\frac{1}{k_i^3} \times \frac{e^{\frac{5H\overrightarrow{\theta^0}\cdot\left(\vec{k}_1+\vec{k}_2+\vec{k}_3\right)}{2}}e^{\frac{i}{2}\left(\vec{k}_1\wedge\vec{k}_2+\vec{k}_2\wedge\vec{k}_3+\vec{k}_1\wedge\vec{k}_3\right)}}{K}\left(k_1^2k_2^2+\text{perm.}\right) \tag{53}$$

- Similarly for another term

$$\begin{aligned}(b) &= i\int_{t_0}^{t} dt'\langle 0|\mathcal{H}(t')\zeta_\theta(x_1)\zeta_\theta(x_2)\zeta_\theta(x_3)|0\rangle \\ &= \epsilon(2\pi)^3\delta^3\left(\vec{k}_1+\vec{k}_2+\vec{k}_3\right)\frac{H^4}{16\epsilon^2}\prod_{i=1}^{3}\frac{1}{k_i^3} \\ &\times \frac{e^{-\frac{5H\overrightarrow{\theta^0}\cdot\left(\vec{k}_1+\vec{k}_2+\vec{k}_3\right)}{2}}e^{\frac{i}{2}\left(\vec{k}_1\wedge\vec{k}_2+\vec{k}_2\wedge\vec{k}_3+\vec{k}_1\wedge\vec{k}_3\right)}}{K}\left(k_1^2k_2^2+\text{perm.}\right)\end{aligned} \tag{54}$$

- Remind the expression for three-point function

$$\begin{aligned}\langle\zeta(x_1)\zeta(x_2)\zeta(x_3)\rangle &= \langle\zeta_n(x_1)\zeta_n(x_2)\zeta_n(x_3)\rangle \\ &+ \frac{(\eta-\epsilon)}{4}\left(\langle\zeta_n(x_1)\zeta_n(x_2)\rangle\langle\zeta_n(x_1)\zeta_n(x_3)\rangle + \text{permutations}\right) \\ &+ \frac{\epsilon}{2}\partial_{x_1}^{-2}\left(\langle\zeta(x_1)\zeta(x_2)\rangle\partial_{x_1}^2\langle\zeta_n(x_1)\zeta_n(x_3)\rangle + \text{permutations}\right).\end{aligned}$$

- So the first term in Fourier space is

$$\begin{aligned}\langle\zeta_\theta(\vec{k}_1,t)\zeta_\theta(\vec{k}_2,t)\zeta_\theta(\vec{k}_3,t)\rangle &= 2\epsilon(2\pi)^3\delta^3\left(\vec{k}_1+\vec{k}_2+\vec{k}_3\right)\frac{H^4}{16\epsilon^2}\prod_{i=1}^{3}\frac{1}{k_i^3} \\ &\times \frac{\cosh\frac{5H\overrightarrow{\theta^0}\cdot(\vec{k}_1+\vec{k}_2+\vec{k}_3)}{2}\times e^{\frac{i}{2}(\vec{k}_1\wedge\vec{k}_2+\vec{k}_2\wedge\vec{k}_3+\vec{k}_1\wedge\vec{k}_3)}}{K} \\ &\times \left(k_1^2k_2^2+\text{perm.}\right)\end{aligned}$$

- The contribution due to first field redefinition term is

$$\langle\zeta_\theta(x_1)\zeta_\theta(x_2)\zeta_\theta(x_3) = \frac{\eta-\epsilon}{4}\left(\langle\zeta_\theta(x_1)\zeta_\theta(x_2)\rangle\langle\zeta_\theta(x_1)\zeta_\theta(x_3)\rangle + \text{perm.}\right) \tag{55}$$

- Now

$$\langle\zeta_\theta(x_1)\zeta_\theta(x_2)\rangle = \int \frac{d^3k_2}{(2\pi)^3}\frac{H^2}{4\epsilon}\frac{1}{k_2^3}e^{-H\overrightarrow{\theta^0}\cdot\vec{k}_2}e^{i\vec{k}_2\cdot(\vec{x}_1-\vec{x}_2)} \tag{56}$$

- So

$$\begin{aligned}\langle\zeta_\theta(x_1)\zeta_\theta(x_2)\rangle\langle\zeta_\theta(x_1)\zeta_\theta(x_3)\rangle &= \int \frac{d^3k_2d^3k_3}{(2\pi)^9}\frac{H^4}{16\epsilon^2}\frac{1}{k_2^3k_3^3} \\ &\times\; e^{-H\overrightarrow{\theta^0}\cdot\left(\vec{k}_2+k_3\right)}e^{i\left(\vec{k}_2+\vec{k}_3\right)\cdot\vec{x}_1-i\vec{k}_2\cdot\vec{x}_2-i\vec{k}_3\cdot\vec{x}_3} \\ &= (2\pi)^3\int\frac{d^3k_1d^3k_2d^3k_3}{(2\pi)^9}\delta^3\left(\vec{k}_1+\vec{k}_2+\vec{k}_3\right) \\ &\times\; \frac{H^4}{16\epsilon^2}\frac{1}{k_2^3k_3^3} \\ &\times\; e^{H\overrightarrow{\theta^0}\cdot\vec{k}_1}e^{-i\vec{k}_1\cdot\vec{x}_1-i\vec{k}_2\cdot\vec{x}_2-i\vec{k}_3\cdot\vec{x}_3}\end{aligned} \tag{57}$$

- So in Fourier space

$$\langle \zeta_\theta(\vec{k}_1, t)\zeta_\theta(\vec{k}_2, t)\zeta_\theta(\vec{k}_3, t)\rangle = \frac{\eta - \epsilon}{2}(2\pi)^3\delta^3\left(\vec{k}_1 + \vec{k}_2 + \vec{k}_3\right) \times \frac{H^4}{16\epsilon^2}\prod_{i=1}^{3}\frac{1}{k_i^3}\left(\sum_i k_i^3 e^{H\overrightarrow{\theta^0}\cdot\vec{k}_i}\right) \quad (58)$$

- The contribution due to second field redefinition term is

$$\langle \zeta_\theta(\vec{k}_1, t)\zeta_\theta(\vec{k}_2, t)\zeta_\theta(\vec{k}_3, t)\rangle = \frac{\epsilon}{2}(2\pi)^3\delta^3\left(\vec{k}_1 + \vec{k}_2 + \vec{k}_3\right) \times \frac{H^4}{16\epsilon^2}\prod_{i=1}^{3}\frac{1}{k_i^3}\left(\sum_{i\neq j} k_i k_j^2 e^{H\overrightarrow{\theta^0}\cdot\vec{k}_i}\right) \quad (59)$$

- Combining all terms

$$\langle \zeta_\theta(\vec{k}_1, t)\zeta_\theta(\vec{k}_2, t)\zeta_\theta(\vec{k}_3, t)\rangle = (2\pi)^3\delta^3\left(\vec{k}_1 + \vec{k}_2 + \vec{k}_3\right)\frac{H^4}{16\epsilon^2}\prod_{i=1}^{3}\frac{1}{k_i^3}\mathcal{A} \quad (60)$$

Where

$$\begin{aligned}\mathcal{A} &= 4\epsilon\frac{\cosh\frac{5H\overrightarrow{\theta^0}\cdot\left(\vec{k}_1+\vec{k}_2+\vec{k}_3\right)}{2}\times e^{\frac{i}{2}\left(\vec{k}_1\wedge\vec{k}_2+\vec{k}_2\wedge\vec{k}_3+\vec{k}_1\wedge\vec{k}_3\right)}}{K}\left(\sum_{i<j}k_i^2k_j^2\right)\\ &+ \frac{\eta-\epsilon}{2}\left(\sum_i k_i^3 e^{H\overrightarrow{\theta^0}\cdot\vec{k}_i}\right) + \frac{\epsilon}{2}\left(\sum_{i\neq j}k_ik_j^2e^{H\overrightarrow{\theta^0}\cdot\vec{k}_i}\right) \quad (61)\end{aligned}$$

- Using translational invariance $\vec{k}_1 + \vec{k}_2 + \vec{k}_3 = 0$ and taking self-adjoint we get

$$\begin{aligned}\langle \zeta_\theta(\vec{k}_1, t)\zeta_\theta(\vec{k}_2, t)\zeta_\theta(\vec{k}_3, t)\rangle_M &= \frac{1}{2}\Big(\langle \zeta_\theta(\vec{k}_1, t)\zeta_\theta(\vec{k}_2, t)\zeta_\theta(\vec{k}_3, t)\rangle\\ &+ \langle \zeta_\theta(-\vec{k}_1, t)\zeta_\theta(-\vec{k}_2, t)\zeta_\theta(-\vec{k}_3, t)\rangle\Big)\end{aligned}$$

- Finally

$$
\begin{aligned}
\langle \zeta_\theta(\vec{k}_1,t)\zeta_\theta(\vec{k}_2,t)\zeta_\theta(\vec{k}_3,t)\rangle_M &= (2\pi)^3\delta^3\left(\vec{k}_1+\vec{k}_2+\vec{k}_3\right)\frac{H^4}{16\epsilon^2}\prod_{i=1}^{3}\frac{1}{k_i^3} \\
&\times \left[\frac{4\epsilon\cos\left(\frac{\vec{k}_1\wedge\vec{k}_2}{2}\right)}{K}\left(\sum_{i<j}k_i^2k_j^2\right)\right. \\
&+ \frac{\eta-\epsilon}{2}\left(\sum_i k_i^3\cosh\left(H\overrightarrow{\theta^0}\cdot\vec{k}_i\right)\right) \\
&+ \left.\frac{\epsilon}{2}\left(\sum_{i\neq j}k_ik_j^2\cosh\left(H\overrightarrow{\theta^0}\cdot\vec{k}_i\right)\right)\right]
\end{aligned}
\tag{62}
$$

- $\vec{k}_1\wedge\vec{k}_2 = k^i\theta_{ij}k^j$ and and it goes to standard case (See Maldacena JHEP 0305 (2003) 013) for $\theta = 0$.

Implications for observations

- The non-gaussianity in CMB is described by angular three-point correlation functions in harmonic space called as "angular bispectrum", which is related to the three-dimensional bispectrum of the primordial curvature perturbations defined as

$$\langle\zeta(\vec{k}_1,t)\zeta(\vec{k}_2,t)\zeta(\vec{k}_3,t)\rangle = (2\pi)^3\delta^3\left(\vec{k}_1+\vec{k}_2+\vec{k}_3\right)B_\zeta\left(k_1,k_2,k_3\right) \quad (63)$$

- For twisted quantum fields in noncommutative spacetime

$$\begin{aligned} B_{\zeta_\theta}\left(\vec{k}_1,\vec{k}_2,\vec{k}_3\right) &= \frac{H^4}{16\epsilon^2}\prod_{i=1}^{3}\frac{1}{k_i^3}\left[\frac{4\epsilon\cos\left(\frac{\vec{k}_1\wedge\vec{k}_2}{2}\right)}{K}\left(\sum_{i<j}k_i^2k_j^2\right)\right.\\ &+ \frac{\eta-\epsilon}{2}\left(\sum_i k_i^3\cosh\left(H\vec{\theta^0}\cdot\vec{k}_i\right)\right)\\ &+ \left.\frac{\epsilon}{2}\left(\sum_{i\neq j}k_ik_j^2\cosh\left(H\vec{\theta^0}\cdot\vec{k}_i\right)\right)\right] \end{aligned} \quad (64)$$

- Bispectrum also breaks statistical isotropy.
- We define f_{NL} as

$$f_{NL} = \frac{5}{6}\frac{B_{\zeta_\theta}\left(\vec{k}_1, \vec{k}_2, \vec{k}_3\right)}{P_\zeta(k_1)P_\zeta(k_2) + P_\zeta(k_2)P_\zeta(k_3) + P_\zeta(k_1)P_\zeta(k_3)}$$

$$\begin{aligned} f_{NL} &= \frac{5}{6}\frac{1}{\sum_i k_i^3}\left[4\epsilon\frac{\cos\left(\frac{\vec{k}_1\wedge\vec{k}_2}{2}\right)}{K}\sum_{i<j}\left(k_i^2k_j^2\right) + \frac{\eta-\epsilon}{2}\left(\sum_i k_i^3\cosh\left(H\overrightarrow{\theta^0}\cdot\vec{k}_i\right)\right)\right. \\ &+ \left.\frac{\epsilon}{2}\left(\sum_{i\neq j} k_i k_j^2\cosh\left(H\overrightarrow{\theta^0}\cdot\vec{k}_i\right)\right)\right] \end{aligned}$$

- This kind of f_{NL} arises where the curvature perturbation is expressed as $\zeta_g = \zeta_g + \frac{3}{5}\zeta_g^2$ and f_{NL} peaks at the squeezed triangle limit defined as $|\vec{k}_1| = |\vec{k}_2| = k$ and $|\vec{k}_3| << k$.

- So the f_{NL} for noncommutative case is

$$f_{NL} = \frac{5}{12}\left[2\epsilon \cos\left(\frac{\vec{k}_1 \wedge \vec{k}_2}{2}\right) + \frac{\eta}{2}\left(\cosh\left(H\overrightarrow{\theta^0}\cdot\vec{k}_1\right) + \cosh\left(H\overrightarrow{\theta^0}\cdot\vec{k}_2\right)\right)\right]$$

- The amplitude of f_{NL} is very small and of the order of slow-roll parameters for small statistical anisotropy.
- f_{NL} has scale dependence and direction dependence.
- The current limits on the amplitude of f_{NL} for squeezed triangle limit are $f_{NL} = 2.7 \pm 5.8$ from the recently released Planck data.
- Ongoing and future observations will be able to constraint running of f_{NL}, n_{NG}, with a $1-\sigma$ uncertainty of $\Delta n_{NG} \sim 0.1$.
- It may be possible to see the effects of scale dependence of f_{Nl} due to noncommutativity in future observations.

Conclusions

- We have computed two-point and three-point correlation functions for comoving curvature perturbations in noncommutative spacetime using ADM formalism.
- Both the power spectrum and the bispectrum for this model are direction dependent and breaks the statistical isotropy due to the preferred direction of $\hat{\theta}$.
- The amplitude of the non-linearity parameter f_{NL} is very small for small statistical anisotropy but it has a high scale dependence which can be tested in ongoing and future observations.
- We are studying direction dependent power spectrum in the light of Planck data.
- With the help of these observational signatures of noncommutative inflation we will, in future, be able to determine the scale of noncommutativity.

THANK YOU