

Matters of Gravity

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NEWTON'S GRAVITY



ional field around it



agnetism

PRINCIPLE OF EQUIVALENCE



Galileo Galilei 1564-1642



Apollo 15 Astronaut David Scott dropping feather and hammer at same time in vacuum of Moon, August of 1971.

Principle of equivalence: Einstein's vision



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Application of Equivalence Principle: Gravity Bends Light



A S



Going straight is tricky!



How matter curves spacetime







EINSTEIN'S GRAVITY

Not a force but curvature of spacetime

Verified experimentally in several contexts



Lanczos-Lovelock models of gravity

 Field equation in a general theory of gravity:

 $\left\{ \begin{array}{l} \textbf{A geometrical object} \\ \textbf{depending on two} \\ \textbf{tensors } R^{ab}_{cd}, P^{ab}_{cd} \end{array} \right\} = \left\{ \begin{array}{l} \textbf{Energy and momentum} \\ \textbf{densities of matter} \\ \textbf{generating curvature} \end{array} \right\}$

 These equations will be of degree greater than 2. This is avoided if

$$\nabla_a P_{cd}^{ab} = 0$$

QUANTUM THEORY AND GRAVITY

Einstein's gravity works well in classical regime

But nature is quantum mechanical!

* Theoretically we need quantum theory of gravity

THE TROUBLE IS

Attempts to combine principles of gravity and quantum theory have repeatedly failed!

This is in sharp contrast with other forces



Recent work suggests that we need another paradigm shift!



What is an emergent phenomenon?

- Simple examples: Elasticity, gas dynamics ...
- Laws are expressible in terms of macroscopic variables; e.g.

$P V = N k_B T$

 Could be studied without knowing the existence of atoms etc.

Quantizing elasticity will not help in understanding atomic structure!

IF GRAVITY IS EMERGENT

Field equations ⇔ laws of gas dynamics

Quantizing gravity will not help in understanding quantum structure of spacetime

THERMODYNAMICS

Describes macroscopic systems using certain laws; for example,

T dS = dE + P dV

Not a fundamental description

The formalism survived for centuries through relativistic and quantum revolutions !

BOLTZMANN'S INSIGHT

If you can heat it, it must have microstructure

Microscopic degrees of freedom are needed for the rmal phenomena

Spacetimes, like matter, can be hot

 Observers with horizon assign to spacetime a temperature:

$$T = \frac{\hbar a}{2\pi ck}$$

(Davies, 75; Unruh, 76)

 Examples: Black holes, accelerated observers

SPACETIME THERMODYNAMICS







ENTROPY OF HORIZONS







Wheeler (\sim 1971): Can one violate second law of thermodynamics by hiding entropy behind a horizon ?



- Wheeler (\sim 1971): Can one violate second law of thermodynamics by hiding entropy behind a horizon ?
- Bekenstein (1972): No! Horizons have entropy $S \propto (Area)$ in Einstein's theory, which goes up when you try this.

THERMODYNAMICS OF HORIZON

- Temperature of the horizon is independent of the theory of gravity.
- But the entropy depends/determines the theory of gravity !
- Remember that horizons are everywhere !

ENTROPY OF HORIZONS

- ★ The invariance under $x^a \rightarrow x^a + q^a$ (x) leads to a conserved current J^a which depends on P^{ab}_{cd} of the theory.
- ★ The entropy of the horizon is given by the (Noether) charge:

$$S = (1/4) \int_{H} (32\pi P^{ab}_{cd}) \varepsilon_{ab} \varepsilon^{dc} d\sigma$$

Thus the entropy depends crucially on the theory and viceversa through the 'entropy tensor' *P^{ab}cd*.

- ★ Entropy knows about spacetime dynamics; temperature does not.
- * The connection between $x^a \rightarrow x^a + q^a$ (x) and entropy is a mystery in the conventional approach.

Possible strategy

Study gravity the way physicists studied matter before knowing atomic structure

(TP, 2002-2011)

RELEVANT LENGTH SCALES

For matter atomic structure is relevant ≈ 10⁻⁷ cm

For gravity the corresponding scale is
≈ 10⁻³³ cm





Thermodynamics of Gravitational Equations



TdS = dE + PdV

FIELD EQUATIONS \Rightarrow THERMODYNAMIC IDENTITY

Spherically symmetric spacetime with horizon at r = a; surface gravity g:

Femperature:
$$k_BT = \left(\frac{\hbar}{c}\right) \frac{g}{2\pi}$$

• Einstein's equation at r = a is (textbook result!)

$$\frac{c^4}{G} \left[\frac{ga}{c^2} - \frac{1}{2} \right] = 4\pi P a^2$$

• Multiply *da* to write:

$$\frac{\hbar}{c} \left(\frac{g}{2\pi}\right) \underbrace{\frac{c^3}{G\hbar} d\left(\frac{1}{4}4\pi a^2\right)}_{k_BT} - \underbrace{\frac{1}{2}\frac{c^4da}{G}}_{k_BT} = Pd\left(\frac{4\pi}{3}a^3\right)}_{k_BT} \underbrace{\frac{1}{k_B}\frac{1}{2}dS}_{k_BT} - dE \underbrace{PdV}_{k_B}$$

• Field equations become TdS = dE + PdV; with :

$$S = \frac{1}{4L_P^2} (4\pi a^2) = \frac{1}{4} \frac{A_H}{L_P^2}; \quad E = \frac{c^4}{2G} a = \frac{c^4}{G} \left(\frac{A_H}{16\pi}\right)^{1/2}$$

HOLDS TRUE FOR A LARGE CLASS OF MODELS

- * Stationary axisymmetric horizons and evolving spherically symmetric horizons in Einstein gravity, [gr-qc/0701002]
- * Static spherically symmetric horizons in Lanczos-Lovelock gravity, [hep-th/0607240]
- * Dynamical apparent horizons in Lanczos-Lovelock gravity [arXiv: 0810.2610]
- * Generic, static horizon in Lanczos-Lovelock gravity [arXiv: 0904.0215]
- * Three dimensional BTZ black hole horizons [arXiv:0911.2556]; [hep-th/0702029]
- FRW and other solutions in various gravity theories [hep-th/0501055]; [arXiv:0807.1232]; [hep-th/0609128]; [hep-th/0612144]; [hep-th/0701198]; [hep-th/0701261]; [arXiv:0712.2142]; [hep-th/0703253]; [hep-th/0602156]; [gr-qc/0612089]; [arXiv:0704.0793]; [arXiv:0710.5394]; [arXiv:0711.1209]; [arXiv:0801.2688]; [arXiv:0805.1162]; [arXiv:0808.0169]; [arXiv:0809.1554]; [gr-qc/0611071].
- * Horova-Lifshiftz gravity [arXiv:0910.2307]

IN ALL THESE CASES FIELD EQUATIONS REDUCE TO TdS = dE + PdV WITH CORRECT S !

The Avogadro number of matter

 The equipartition law determines the density of microscopic degrees of freedom

$$N = \frac{E}{\left[(1/2) \, k_B T \right]}$$

★ For matter this was determined even before we knew what it was counting!

THE AVOGADRO NUMBER OF SPACETIME

(TP, 04, 09)

★ We can do the same thing for spacetime

★ Gravity turns out to be "holographic"

* For Einstein's theory, N = A / L_P^2
A NEWTONIAN ANALOGY





A NEWTONIAN ANALOGY



A NEWTONIAN ANALOGY



System

Can the system be hot? Can it transfer heat

How could the heat energy be stored in the system?

Number of degrees of freedom required to store energy *dE* at temperature *T*

Can we read off *dn*?

Expression for entropy

Does this entropy match with expressions obtained by other methods?

How does one close the loop on dynamics?

Macroscopic body

Yes

Yes; for e.g., hot gas can be used to heat up water

The body must have microscopic degrees of freedom

Equipartition law $dn = dE / (1/2) k_B T$

Yes; when thermal equilibrium holds; depends on the body

 $\Delta S \propto \Delta n$

Yes

Use an extremum principle for a thermodynamical potential (*S, F*, ...)

Spacetime

Yes

Yes; water at rest in Rindler spacetime will get heated up

Spacetime must have microscopic degrees of freedom

Equipartition law

 $dn = dE / (1/2) k_B T$

Yes; when static field eqns hold; depends on the theory of gravity

```
\Delta S \propto \Delta n
Yes
```

Use an extremum principle for a thermodynamical

potential (S, F, ...)

THERMODYNAMIC ROUTE TO GRAVITY

(TP, A. Paranjape, 07: TP, 08)

★For matter, we have a maximum entropy principle

*Same principle works for gravity!

Maximizing the entropy of horizons for all observers leads to the field equations

SPACETIME IN ARBITRARY COORDINATES



FREE – FALL OBSERVERS



Validity of laws of SR \Rightarrow kinematics of gravity

LOCAL RINDLER OBSERVERS



Thermodynamic extremum principle \Rightarrow dynamics of gravity







ASSOCIATE THERMODYNAMIC POTENTIALS WITH NULL VECTORS

DEFORMING A SOLID



DEFORMING A SOLID



$$\mathbf{x} \rightarrow \mathbf{x} + \mathbf{q}(\mathbf{x})$$

DEFORMING A SOLID





$$\Im \sim A(\nabla q)^2 + Bq^2$$

DEFORMING A NULL SURFACE



DEFORMING A NULL SURFACE



 $x^i \to \bar{x}^i = x^i + \epsilon \xi^i$

DEFORMING A NULL SURFACE



$$x^i \to \bar{x}^i = x^i + \epsilon \xi^i$$

 $\Im \sim A(\nabla \xi)^2 + B\xi^2$

• Associate with the virtual displacements of null vectors ξ^a a potential $\Im(\xi^a)$ which is quadratic in deformation field:

 $\Im[\xi] \sim [A(\nabla\xi)^2 + B\xi^2] = -\left[4P^{abcd}\nabla_c\xi_a\nabla_d\xi_b - T^{ab}\xi_a\xi_b\right]$

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• Demand that $\delta \Im / \delta \xi^a = 0$ for all null vectors ξ^a should lead to second order field equations. [T.P., 08; T.P., A.Paranjape, 07]

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The resulting field equations are those of Lanczos-Lovelock theory of gravity which reduces to Einstein's theory in D=4!

A NEW APPROACH TO COSMOLOGY

Emergence of cosmic space

COSMIC SECRETS

- Observations show that, universe singles out a prefered Lorentz frame – we try not to draw attention to it!
- We have actually measured the *absolute velocity* of our motion wrt this `cosmic ether' (aka CMBR!).
- Universe exhibits larger symmetry (general covariance) at smaller scales!
- At cosmic scales we can think of **space** as emergent as cosmic time evolves.

HOLOGRAPHIC EQUIPARTITION

• For a dS universe with Hubble radius H^{-1} , let:

$$N_{\rm sur} = 4\pi \frac{H^{-2}}{L_P^2};$$
 $N_{\rm bulk} = \frac{|E|}{(1/2)k_B T} = -\frac{E}{(1/2)k_B T}$

with $k_B T = (H/2\pi)$ and $E = (\rho + 3P)V$ being the Komar energy.

- Holographic equipartition is the demand: $N_{sur} = N_{bulk}$
- For pure deSitter universe with $P = -\rho$ this gives $H^2 = 8\pi L_P^2 \rho/3$ which is the standard result.
- Pure deSitter universe maintains holographic equipartition with constant *V*.
- The holographic discrepancy $(N_{sur} N_{bulk})$ drives the expansion of the universe, interpreted as emergence of cosmic space.

EMERGENCE OF SPACE AS A QUEST FOR HOLOGRAPHIC EQUIPARTITION

Raise this to the status of a postulate:

$$\frac{dV}{dt} = L_P^2 (N_{\rm sur} - \varepsilon N_{\rm bulk}) \qquad \varepsilon = \pm 1$$

with

$$V = \frac{4\pi}{3} H^{-3} \qquad N_{\rm sur} = 4\pi \frac{H^{-2}}{L_P^2}; \qquad N_{\rm bulk} = -\varepsilon \frac{E}{(1/2)k_B T}$$

- Remarkably enough, this leads to the standard FRW dynamics!
- In Planck units, this has a discrete version:

$$V_{n+1} = V_n + (N_{sur} - \varepsilon N_{bulk})$$

This provides an alternative way of studying cosmology.

• The results generalise to Lanczos-Lovelock models

[Rong-Gen Cai,1207.0622; Ke Yang,1207.3515]







LINKING INFLATION TO DARK ENERGY

• The degrees of freedom in the modes crossing the horizon (k = Ha) during $a_1 < a < a_2$ will be:

$$\mathcal{N}(a_1, a_2) = \int \frac{V_{com} d^3 k}{(2\pi)^3} = \pm \frac{2}{3\pi} \ln(Ha) \Big|_{a_1}^{a_2}$$

• During dS phase, $H_a \propto a$; during radiation dominated phase, $H_a \propto a^{-1}$ so

$$\frac{a_2}{a_1} = \exp\left[\frac{3\pi}{2}\mathcal{N}(a_1, a_2)\right]$$

• For our universe, we have the result:

$$\mathcal{N}(a_I, a_F) = \mathcal{N}(a_\Lambda, a_F) = \mathcal{N}(a_\Lambda, a_{\mathrm{vac}}) \approx 15$$

which implies

$$\Lambda L_P^2 \simeq 3 \exp(-6\pi \mathscr{N}) \simeq 10^{-122}$$

• So the problem of determining the cosmological constant reduces to understanding $\mathcal{N} \approx 15!$

Summary

- There is sufficient 'internal evidence' to conclude dynamics of gravity is like fluid mechanics, elasticity
- The deep connection between gravity and thermodynamics goes well beyond Einstein's theory.
- ➤ Deformations of 'spacetime medium' xⁱ → xⁱ + qⁱ(x), applied to null surfaces, affects accessibility of information. Extremisation of relevant thermodynamic potential ℑ[q] gives field equations.
- > Expansion of the universe can be thought of as emergence of cosmic space, governed by $(dV/dt) = N_{sur} \varepsilon N_{bulk}$ in Planck units.
- The universe has three equal phases of expansion by factor $\exp[(3\pi/2)\mathcal{N}]$ with $\mathcal{N}\approx 15$.

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Thank you for your time





- There is 'internal evidence' to suggest that dynamics of gravity is like thermodynamic description of macroscopic body in e.g., field equations, action functionals ...
- One can determine the Avogadro number corresponding to microscopic degrees of freedom of spacetime. Shows gravity is 'holographic'!
- Null surfaces acting as local Rindler horizons capture the thermodynamics of these degrees of freedom. Dynamical equations are equivalent to Navier-Stokes equations.
- ➤ Deformations of 'spacetime medium' xⁱ → xⁱ + qⁱ(x), applied to null surfaces, affects accessibility of information. Extremisation of relevant thermodynamic potential ℑ[q] gives field equations.
- The deep connection between gravity and thermodynamics goes well beyond Einstein's theory.

OPEN QUESTIONS, FUTURE DIRECTIONS ... OK, but so what ...?

- What are the atoms of spacetime ? [Asking Boltzmann to get Schrodinger equation from thermodynamics of hydrogen gas ?!]
- How come horizons act as a 'magnifying glass' for microscopic degrees of freedom that 'come alive' only near null surfaces?
- New level of observer dependence in thermodynamic variables like temperature, entropy etc. What are the broader implications ?
- 'Equilibrium' and fluctuations around equilibrium, Brownian motion, L_P² as zero-point-area of spacetime
- Can one do better than a host of other 'QG candidate models'? E.g., cosmological constant problem, singularity problem ...
- > Where does matter come from? Esp. Fermions

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- Uniquely fixes P_{cd}^{ab} as to be the entropy tensor of a Lanczos-Lovelock Lagrangian with $\nabla_a P_{cd}^{ab} = 0$; $\nabla_a T^{ab} = 0$ ('elastic *constants*' !).
- Resulting equations are the field equations of Lanczos-Lovelock theory with an arbitrary cosmological constant arising as integration constant.
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