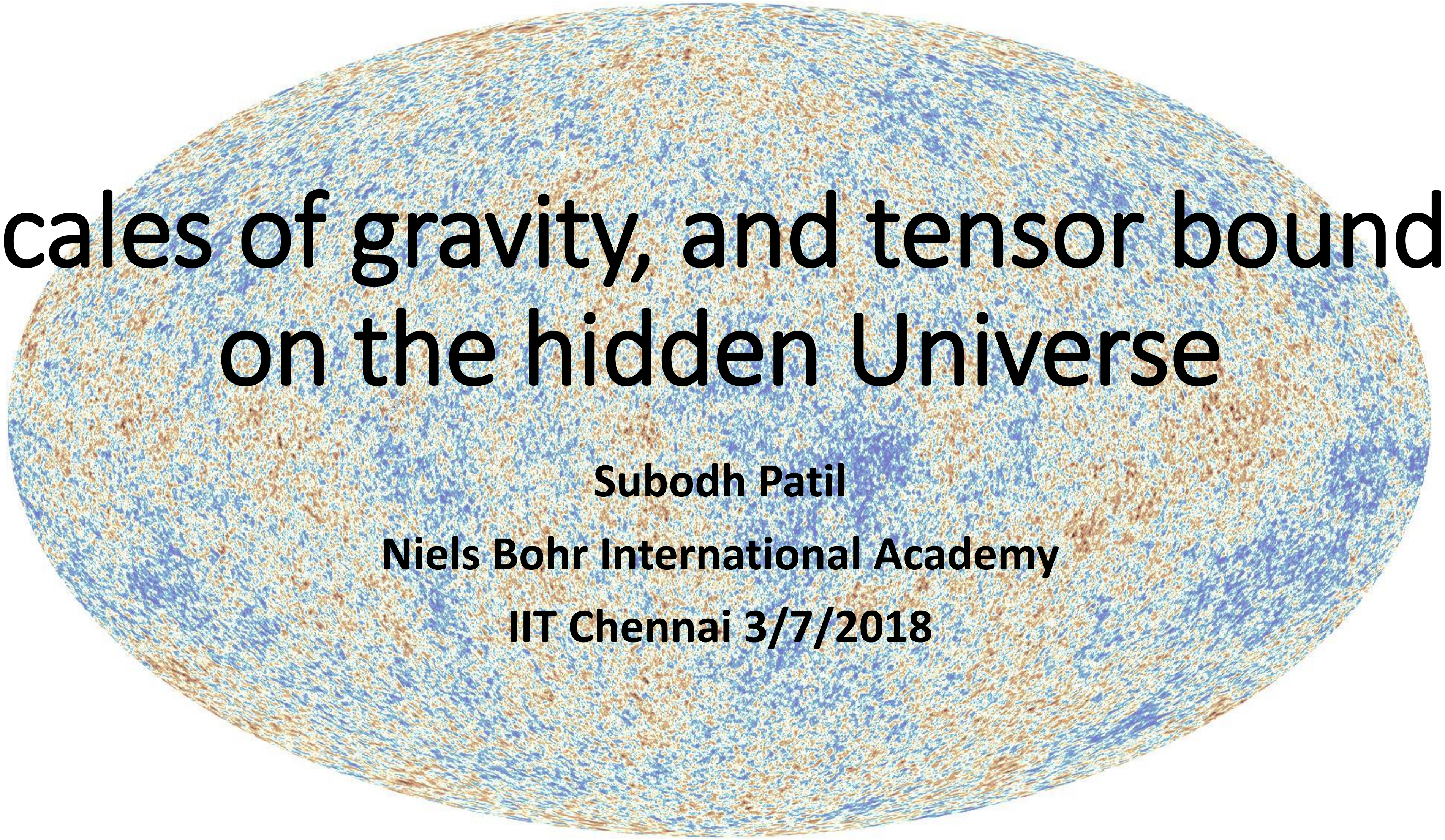


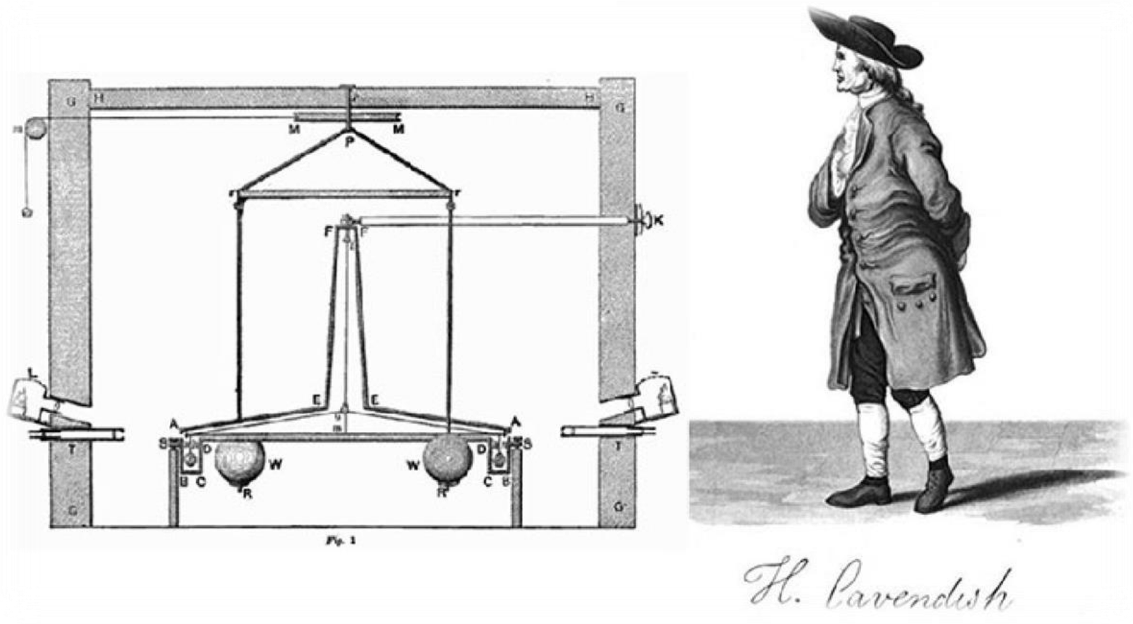
Scales of gravity, and tensor bounds on the hidden Universe

Subodh Patil

Niels Bohr International Academy

IIT Chennai 3/7/2018

Effective field theory and the scales of gravity

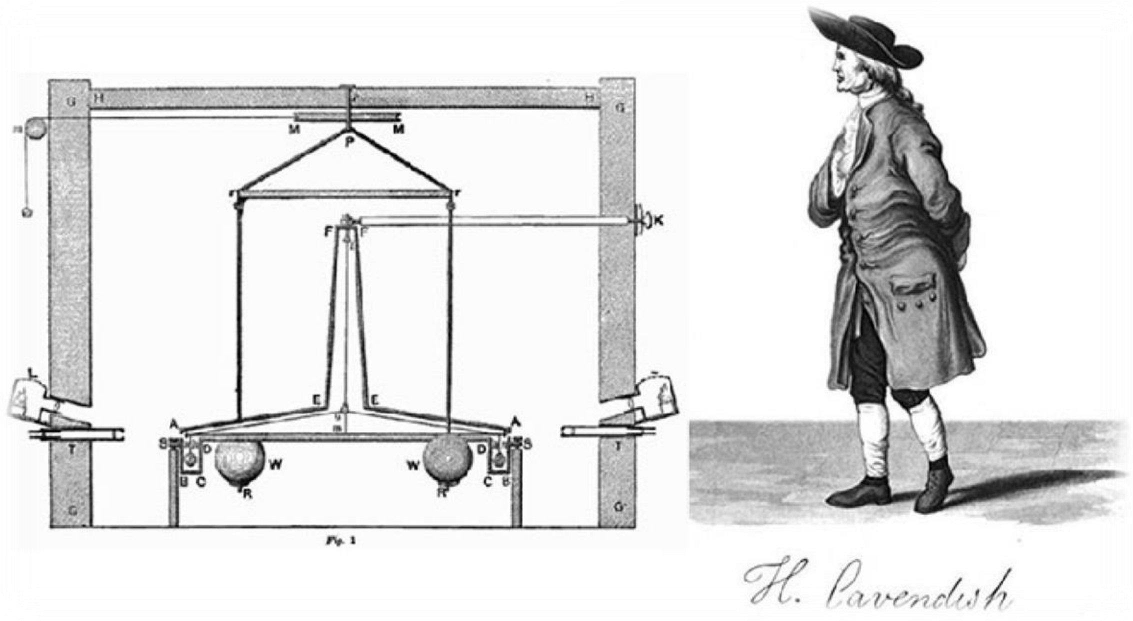


For any given momentum transfer, gravitational interactions have a strength set by a characteristic scale M_*

… inferred from amplitudes calculated in an effective theory with strong coupling scale M_{**}. In pure gravity:

$$M_* = M_{**} = M_{\rm pl} = 2.44 \times 10^{18}\ {\rm GeV}$$

Effective field theory and the scales of gravity



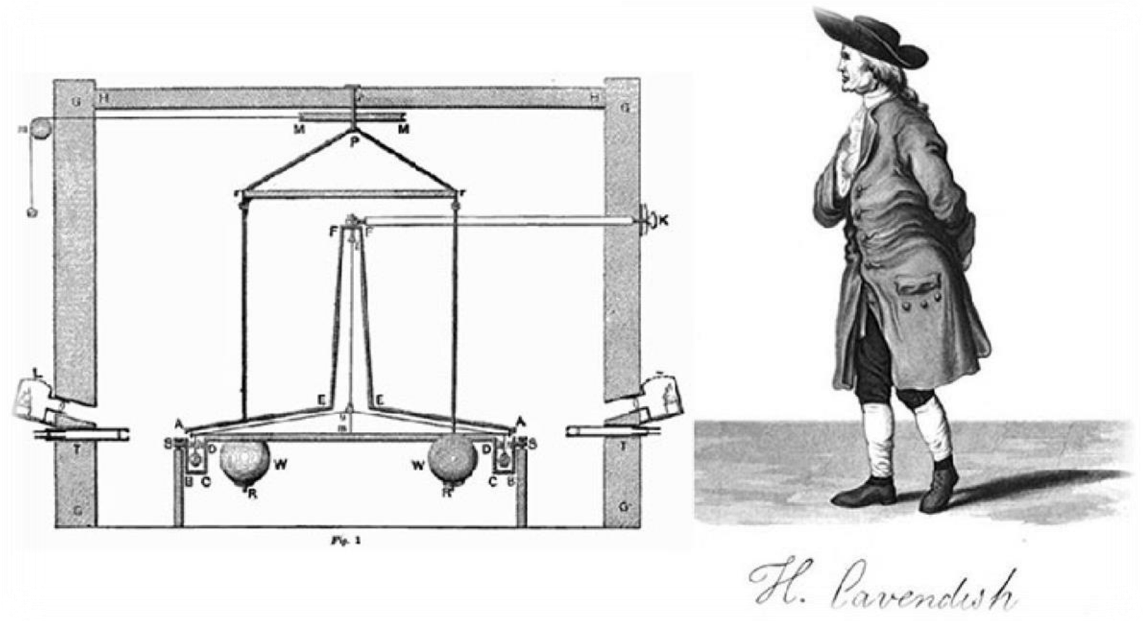
For any given momentum transfer, gravitational interactions have a strength set by a characteristic scale M_*

… inferred from amplitudes calculated in an effective theory with strong coupling scale M_{**}. In the presence of matter:

$$M_* \neq M_{**} \neq M_{\rm pl}$$

Antoniadis, Patil ’14, ’15

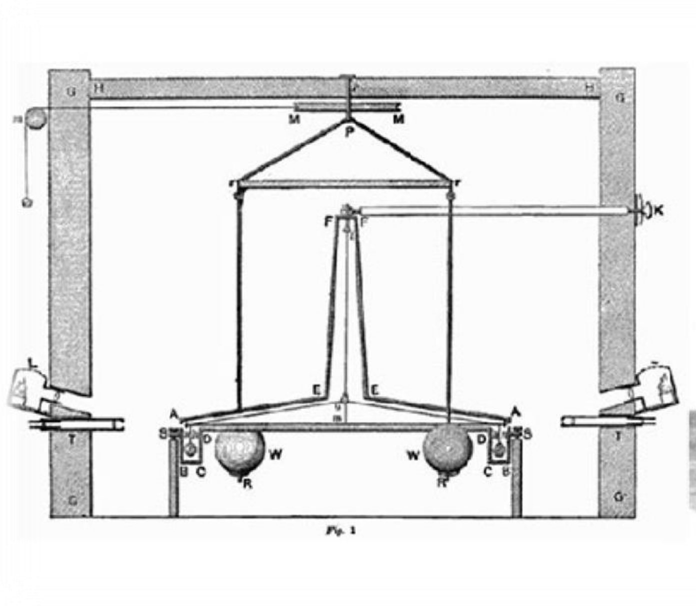
Effective field theory and the scales of gravity



$+\hbar$

- Consider some physical particle with mass m Dvali, Redi '07
- Scatter a test particle off of some very heavy point mass.
- When $\Delta x \sim \frac{h}{mc}$, virtual pairs of these particles are created.
- Positive/negative energy solutions attracted/ repulsed from the source, effectively anti-screening it – gravity *appears* to have gotten stronger.

Effective field theory and the scales of gravity



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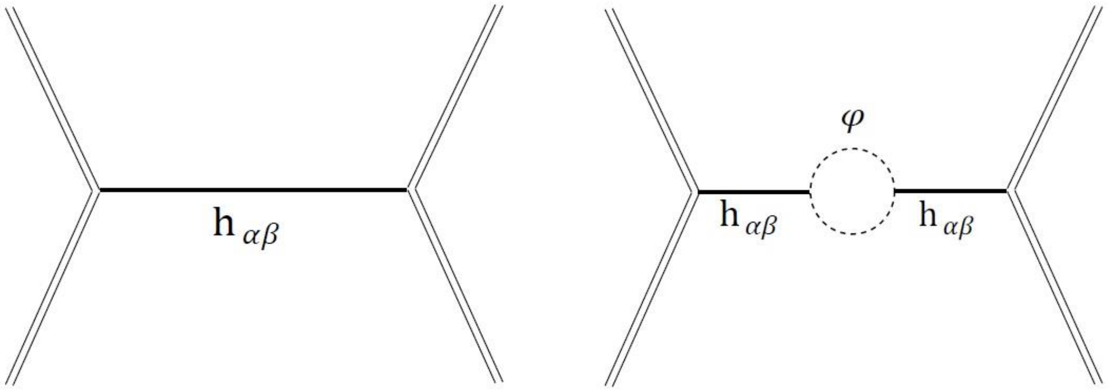
- Consider some physical particle with mass m
- Scatter a test particle off of some very heavy point mass.
- When $\Delta x \sim \frac{h}{mc}$, virtual pairs of these particles are created.
- Positive/negative energy solutions attracted/ repulsed from the source, effectively anti-screening it – gravity *appears* to have gotten stronger.
- What's actually happening: each massive species contributes to lowering the scale where strong gravitational effects become important.

Effective field theory and the scales of gravity



$$M_{**} \sim M_{\mathrm{pl}}/\sqrt{N}$$

- Consider the virtual effect of some massive particle φ with mass m
- On a Minkowski background Dvali, Redi '07

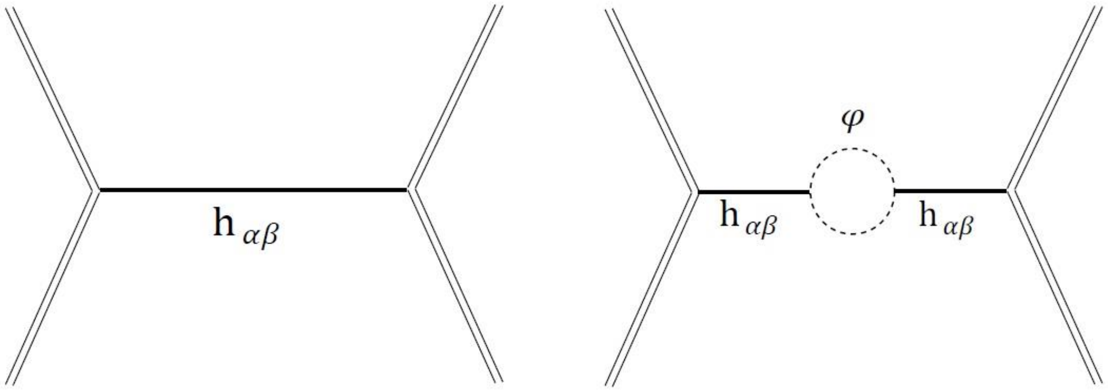
$$\sim \frac{1}{M_{\mathrm{pl}}^4} \frac{1}{p^2} \langle T(-p) T(p) \rangle \frac{1}{p^2}$$

- When $p^2 \gg m^2$, theory becomes conformal:

$$\langle T(-p) T(p) \rangle \sim \frac{c}{16\pi^2} p^4 \log \frac{p^2}{\mu^2}$$

- Central charge $c := N = \frac{4}{3} N_\phi + 8 N_\psi + 16 N_V$ Duff '77

Effective field theory and the scales of gravity



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$$\langle T(-p)T(p)\rangle \sim \frac{c}{16\pi^2}p^4\log\frac{p^2}{\mu^2}$$

- Free propagator $1/(p^2 M_{\rm pl}^2)$; perturbative treatment fails at $p = M_{\rm pl}/\sqrt{N} \equiv M_{**}$

Effective field theory and the scales of gravity

- Consider generalization to curved backgrounds:

$$S = \frac{M_{\rm pl}^2}{2}\int d^4x\sqrt{-g}\,R + \int d^4x\sqrt{-g}\left[c_1 R^2 + c_2 R^{\mu\nu}R_{\mu\nu}\right] + ...$$

- c_1, c_2 indices that count a spin weighted sum of the particle content $\sim N$
- Expansion breaks down when $p^2 \sim M_{\rm pl}^2/N$ or when $R \sim M_{\rm pl}^2/N$
- e.g. during inflation, lets say we tried to calculate corrections to the graviton 2-pt function; $h_{\mu\nu} = g_{\mu\nu} - g^0_{\mu\nu}$
- Leading term – $S = \frac{M_{\rm pl}^2}{8}\int d^4x\sqrt{-g^0}\left[\dot{h}_{ij}\dot{h}_{ij} - \frac{1}{a^2}\partial_k h_{ij}\partial_k h_{ij}\right]$
- Higher curvature contributions s.t. $M_{\rm pl}^2 \to M_{\rm pl}^2\left(1 + c\frac{H^2}{M_{\rm pl}^2} + ...\right)$

Effective field theory and the scales of gravity

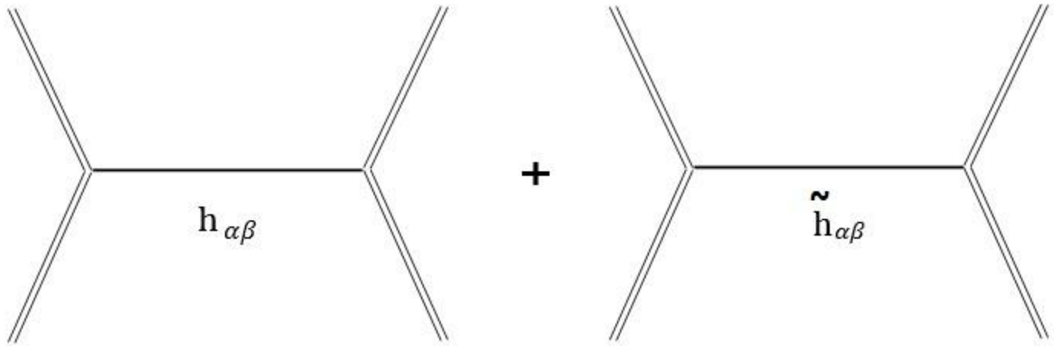
- Consider generalization to curved backgrounds:

$$S = \frac{M_{\rm pl}^2}{2}\int d^4x\sqrt{-g}\,R + \int d^4x\sqrt{-g}\left[c_1 R^2 + c_2 R^{\mu\nu}R_{\mu\nu}\right] + ...$$

- Corollary – it is not possible to consistently *infer* a scale of inflation higher than

$$H^2 \sim M_{\rm pl}^2/N$$

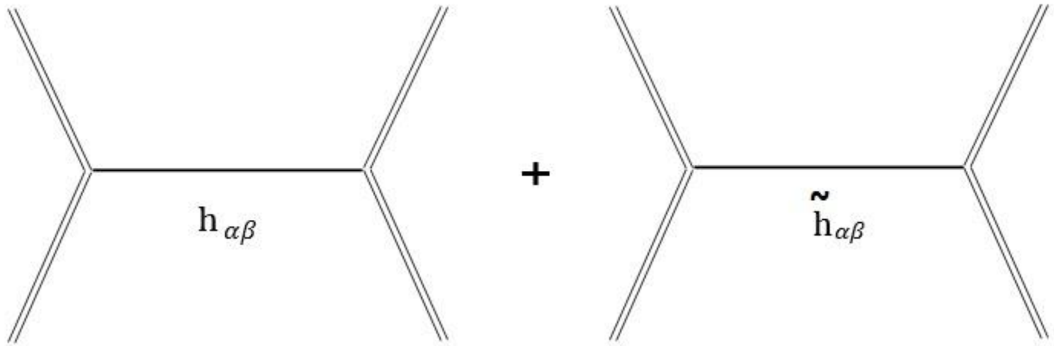
Effective field theory and the scales of gravity



$$M_* \sim M_{\rm pl}/\sqrt{N_*}$$

- The *strength* of gravity M_* (inferred e.g. from a Cavendish experiment) is an independent quantity.
- (Can be $M_{\rm pl}$ all the way up till M_{**}) Gasperini '15
- N_* counts the number of contributing species with masses below the momentum transfer of the process in question.

Effective field theory and the scales of gravity



$$M_* \sim M_{\rm pl}/\sqrt{N_*}$$

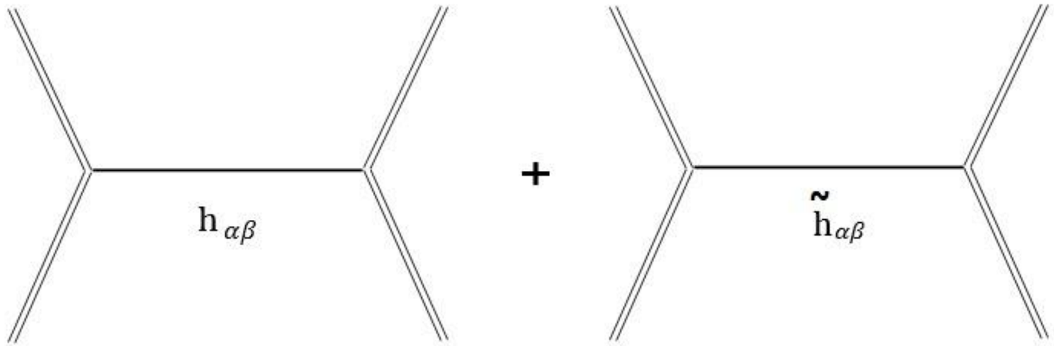
- If species in question is a KK mode with mass $m_{\rm KK}$, we have the additional *tree-level* exchange

$$\frac{1}{M_{\rm pl}^2 p^2} \to \frac{1}{M_{\rm pl}^2 p^2} + \frac{n}{M_{\rm pl}^2 (p^2 + m_{\rm KK}^2)}$$

- In the regime $m_{\rm KK}^2 \ll p^2 \ll M_{\rm pl}^2/N$, strength of gravity is given by:

$$\frac{1}{M_{\rm pl}^2 p^2} + \frac{n}{M_{\rm pl}^2 p^2 \left(1 + m_{\rm KK}^2/p^2\right)} \to \frac{n+1}{M_{\rm pl}^2 p^2}$$

Effective field theory and the scales of gravity



$$M_* \sim M_{\rm pl}/\sqrt{N_*}$$

- If species in question couples to the trace of the energy momentum tensor

$$\Delta\mathcal{L}_{\rm eff} \sim \xi\phi^2 R \sim \xi\frac{\phi^2}{M_{\rm pl}^2}T^\mu_\mu$$

- In the regime $m_\phi^2 \ll p^2 \ll M_{\rm pl}^2/N,$ expanding around $\langle\phi\rangle = v$

$$\frac{1}{M_{\rm pl}^2 p^2} \to \frac{1}{M_{\rm pl}^2 p^2} + \frac{g^2}{M_{\rm pl}^2(p^2+m_\phi^2)} \sim \frac{1+g^2}{M_{\rm pl}^2 p^2}; \qquad g^2 := \xi^2 v^2/M_{\rm pl}^2$$

- $M_* = M_{\rm pl}/\sqrt{N_*};\ N_*$ a (process dependent) weighted index.

Hidden fields in the CMB, or nothing is still something

Del Rio, Durrer, Patil *to appear*

Can one convert the *non-observation* of spectral running in to constraints on hidden field content?

- Fields with masses less than H will be QM'ly excited.
- Even if they do not couple directly to the inflaton (i.e. only interaction is via gravity), they still have an effect on the interactions (after renormalizing background quantities).
- If there are a large number of them, could they overcome Planck and slow roll suppression of interactions, generate a non-trivial running?

Nothing is still something

$$S = \frac{M_{\rm pl}^2}{2}\int d^4x\sqrt{-g}R[g] - \frac{1}{2}\int d^4x\sqrt{-g}\left[\partial_\mu\phi\partial^\mu\phi + 2V(\phi)\right]$$
$$- \sum_{n=1}^{n_{\rm max}}\frac{1}{2}\int d^4x\sqrt{-g}\left[\partial_\mu\chi_n\partial^\mu\chi_n - m_n^2\chi_n^2\right] + \ldots$$

$$ds^2 = -N^2dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

$$\begin{aligned}\phi(t,x) &= \phi_0(t),\\ h_{ij}(t,x) &= a^2(t)e^{2\zeta(t,x)}\hat{h}_{ij}, \quad \hat{h}_{ij} = \exp\left[\gamma_{ij}\right]\end{aligned}$$

$$\begin{aligned}N &= 1+\alpha_1\\ N^i &= \partial_i\theta + N_T^i\ ,\ w/\partial_i N_T^i \equiv 0\end{aligned} \qquad \alpha_1 = \frac{\dot{\mathcal{R}}}{H} \qquad \partial^2\theta = -\frac{\partial^2\mathcal{R}}{a^2H} + \epsilon\dot{\mathcal{R}}$$

Nothing is still something

$$S_{2,\mathcal{R}} = M_{\rm pl}^2 \int d^4x\, a^3\, \epsilon \left[\dot{\mathcal{R}}^2 - \frac{1}{a^2}(\partial\mathcal{R})^2\right] \qquad \epsilon := \frac{\dot{\phi}_0^2}{2H^2M_{\rm pl}^2}$$

$$S_{2,\chi} = \frac{1}{2}\int d^4x\, a^3 \left[\dot{\chi}_n\dot{\chi}_n - \frac{1}{a^2}\partial_i\chi_n\partial_i\chi_n - m_n^2\chi_n^2\right]$$

$$S_{2,\gamma} = \frac{M_{\rm pl}^2}{8}\int d^4x\, a^3 \left[\dot{\gamma}_{ij}\dot{\gamma}_{ij} - \frac{1}{a^2}\partial_k\gamma_{ij}\partial_k\gamma_{ij}\right]$$

$$S_{3,\mathcal{R}\chi} = \frac{1}{2}\int d^4x \Bigg\{ a^3\dot{\chi}_n\dot{\chi}_n\left(3\mathcal{R} - \frac{\dot{\mathcal{R}}}{H}\right) - 2a^3\dot{\chi}_n\partial_i\theta\partial_i\chi_n - a^3\left(\mathcal{R} + \frac{\dot{\mathcal{R}}}{H}\right)\frac{1}{a^2}\partial_i\chi_n\partial_i\chi_n - a^3\left(3\mathcal{R} + \frac{\dot{\mathcal{R}}}{H}\right)m_n^2\chi_n^2\Bigg\}$$

$$S_{3,\gamma\chi} = \frac{1}{2}\int d^4x\, a\left[\gamma_{ij}\partial_i\chi_n\partial_j\chi_n\right]$$

Nothing is still something

$$S_{3,\mathcal{R}\chi} = \frac{1}{2}\int d^4x \Bigg\{ \; a^3 \dot{\chi}_n \dot{\chi}_n \left(3\mathcal{R} - \frac{\dot{\mathcal{R}}}{H}\right) - 2a^3 \dot{\chi}_n \partial_i \theta \partial_i \chi_n$$

$$- \; a^3 \left(\mathcal{R} + \frac{\dot{\mathcal{R}}}{H}\right) \frac{1}{a^2} \partial_i \chi_n \partial_i \chi_n - a^3 \left(3\mathcal{R} + \frac{\dot{\mathcal{R}}}{H}\right) m_n^2 \chi_n^2 \Bigg\}$$

$$S_{3,\mathcal{R}\chi} = \int d^4x \, a^3 \epsilon \left[\frac{\mathcal{R}}{2}\left(\dot{\chi}_n \dot{\chi}^n + \frac{1}{a^2}\partial_i \chi_n \partial_i \chi_n + m_n^2 \chi_n^2\right) - \dot{\chi}_n \partial_i \chi_n \partial_i \partial^{-2} \dot{\mathcal{R}}\right]$$

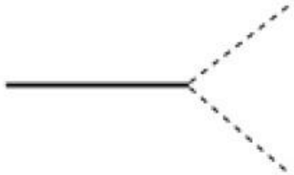
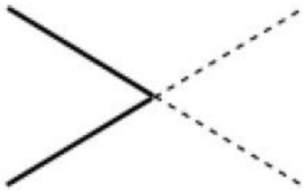
ϵ is an order parameter – it book keeps the expansion

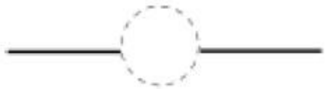
Nothing is still something

$$S_{3,\mathcal{R}\chi} = \int d^4x\, a^3 \epsilon \left[\frac{\mathcal{R}}{2} \left(\dot{\chi}_n \dot{\chi}^n + \frac{1}{a^2} \partial_i \chi_n \partial_i \chi_n + m_n^2 \chi_n^2 \right) - \dot{\chi}_n \partial_i \chi_n \partial_i \partial^{-2} \dot{\mathcal{R}} \right]$$

$$S_{3,\gamma\chi} = \frac{1}{2} \int d^4x\, a \left[\gamma_{ij} \partial_i \chi_n \partial_j \chi_n \right] .$$

$\langle \mathcal{R}\mathcal{R} \rangle$ $\propto \frac{1}{\epsilon M_{\rm pl}^2}$

ϵ  ϵ^2 

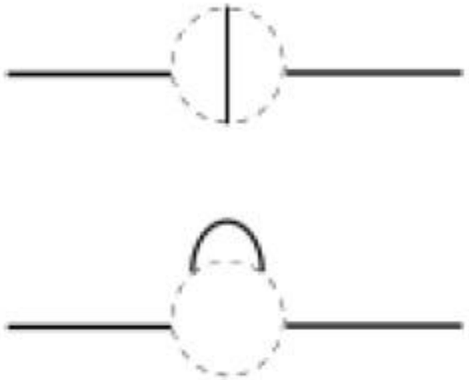
  $\sim \frac{N}{16\pi^2} \frac{H^2}{M_{\rm pl}^4}$

Nothing is still something

$$S_{3,\mathcal{R}\chi} = \int d^4x\, a^3 \epsilon \left[\frac{\mathcal{R}}{2} \left(\dot{\chi}_n \dot{\chi}^n + \frac{1}{a^2} \partial_i \chi_n \partial_i \chi_n + m_n^2 \chi_n^2 \right) - \dot{\chi}_n \partial_i \chi_n \partial_i \partial^{-2} \dot{\mathcal{R}} \right]$$

$$S_{3,\gamma\chi} = \frac{1}{2} \int d^4x\, a \left[\gamma_{ij} \partial_i \chi_n \partial_j \chi_n \right] .$$

$$\sim \frac{\epsilon\, N^2}{(16\pi^2)^2} \frac{H^4}{M_{\rm pl}^6}$$



$$\sim \frac{\epsilon N}{(16\pi^2)^2} \frac{H^4}{M_{\rm pl}^6}$$

Nothing is still something

$$S_{3,\mathcal{R}\chi} = \int d^4x\, a^3 \epsilon \left[\frac{\mathcal{R}}{2} \left(\dot{\chi}_n \dot{\chi}^n + \frac{1}{a^2} \partial_i \chi_n \partial_i \chi_n + m_n^2 \chi_n^2 \right) - \dot{\chi}_n \partial_i \chi_n \partial_i \partial^{-2} \dot{\mathcal{R}} \right]$$

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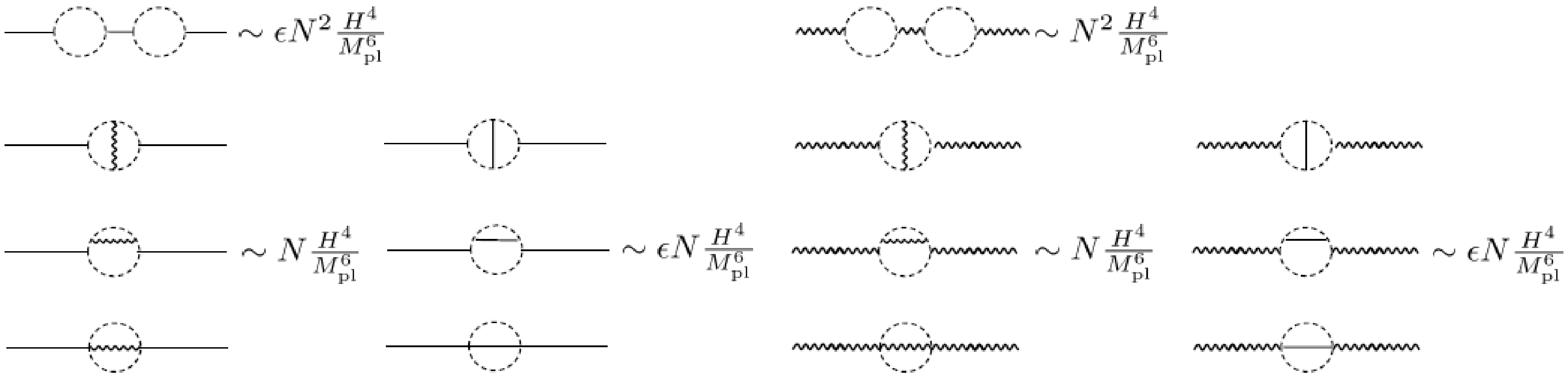


FIG. 3: Two loop corrections to $\langle \zeta\zeta \rangle$. Wavy lines denote the graviton propagator. The double sunset graphs dominate when $N \gg 1/\epsilon$.

FIG. 4: Two loop corrections to $\langle \gamma\gamma \rangle$, where here we only require $N \gg 1$ for the double sunset graphs to dominate.

… calculating the running of these quantities turns out to be rather non-trivial!

Nothing is still something

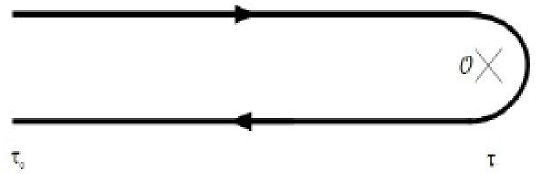
$$S_{3,\mathcal{R}\chi} = \int d^4x\, a^3\epsilon \left[\frac{\mathcal{R}}{2}\left(\dot{\chi}_n\dot{\chi}^n + \frac{1}{a^2}\partial_i\chi_n\partial_i\chi_n + m_n^2\chi_n^2\right) - \dot{\chi}_n\partial_i\chi_n\partial_i\partial^{-2}\dot{\mathcal{R}}\right]$$

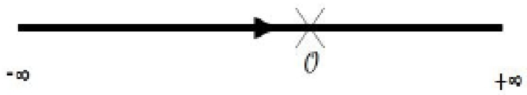
$$S_{3,\gamma\chi} = \frac{1}{2}\int d^4x\, a\left[\gamma_{ij}\partial_i\chi_n\partial_j\chi_n\right]$$

$$\langle\mathcal{O}(\tau)\rangle = \sum_{n=0}^{\infty} i^n \int_{\tau_0}^{\tau} d\tau_n \int_{\tau_0}^{\tau_n} d\tau_{n-1}\ldots\int_{\tau_0}^{\tau_2} d\tau_1 \langle[H_I(\tau_1),[H_I(\tau_2),\ldots[H_I(\tau_n),\mathcal{O}(\tau)]\ldots]]\rangle$$

⇕

$$\langle\mathcal{O}(\tau)\rangle = \langle 0_{in}|T_C\left[\exp\left(-i\oint H_I(\tau')d\tau'\right)\mathcal{O}(\tau)\right]|0_{in}\rangle$$





(Interlude on loops in Inflation)

Weinberg in ‘05 calculated the one loop correction from a hidden field:

$$P_\zeta = \frac{H^2}{8\pi^2 M_{\rm pl}^2 \epsilon} \left[1 - \epsilon \frac{4\pi}{15} \frac{H^2}{M_{\rm pl}^2} \log\left(k/\mu\right)\right]$$

… which was subsequently verified by a host of authors.

Senatore and Zaldarriaga ‘09 – cannot be! Corrections must go like $\log\left(H/\mu\right)$ (seen from putting a hard cut-off in frequency).

Terms omitted in dimensional regularizing integrals (!)

(Interlude on loops in Inflation)

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Furthermore, Adshead, Easther and Lim pointed out vacuum selection prescription doesn’t always allow for the equivalence

$$\langle \mathcal{O}(\tau)\rangle = \sum_{n=0}^{\infty} i^n \int_{\tau_0}^{\tau} d\tau_n \int_{\tau_0}^{\tau_n} d\tau_{n-1} \ldots \int_{\tau_0}^{\tau_2} d\tau_1 \langle [H_I(\tau_1), [H_I(\tau_2), \ldots [H_I(\tau_n), \mathcal{O}(\tau)] \ldots]]\rangle$$

$$\Updownarrow$$

$$\langle \mathcal{O}(\tau)\rangle = \langle 0_{in}| T_C \left[\exp\left(-i\oint H_I(\tau')d\tau'\right)\mathcal{O}(\tau)\right]|0_{in}\rangle$$

(Interlude on loops in Inflation)

Therefore:

$$P_\zeta = \frac{H^2}{8\pi^2 M_{\rm pl}^2 \epsilon}\left[1 - \epsilon\frac{4\pi}{15}\frac{H^2}{M_{\rm pl}^2}\log\left(H/\mu\right)\right]$$

SZ: correlation functions *do not run* as log k…*

However $H \equiv H_k$ – the Hubble rate when k'th mode `exits the horizon'.

Fixing the above at some pivot scale $k_* \to \log\left(H_k/H_*\right)$

$$\log\left(H_k/H_*\right) \sim -\int_0^{\mathcal{N}_k}\epsilon(\mathcal{N}')d\mathcal{N}'; \quad k = H_* e^{-\int_0^{\mathcal{N}_k}(1+\epsilon)}$$

So that $\log\left(H_k/H_*\right) = -\epsilon\log\left(k/k_*\right)$

*otherwise no model of inflation would be eternal Creminelli, Dubovski, Nicolis, Senatore, Zaldarriaga '08

Correlation functions do run, but much more weakly…

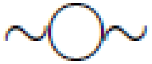
$$P_\zeta = \frac{H^2}{8\pi^2 M_{\rm pl}^2 \epsilon}\left[1 + N\epsilon^2 \frac{4\pi}{15}\frac{H^2}{M_{\rm pl}^2}\log\left(k/k_*\right)\right]\left(\frac{k}{k_*}\right)^{n_s - 1 + \frac{1}{2}\frac{d\,n_s}{d\log k}\log(k/k_*)}$$

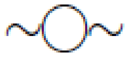
$$P_\gamma = \frac{2H^2}{\pi^2 M_{\rm pl}^2}\left[1 - \epsilon N \frac{3\pi}{10}\frac{H^2}{M_{\rm pl}^2}\log\left(k/k_*\right)\right]\left(\frac{k}{k_*}\right)^{n_t + \frac{1}{2}\frac{d\,n_t}{d\log k}\log(k/k_*)}$$

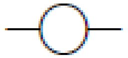
Extra ϵ suppression, but *with opposite sign**

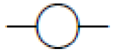
* By criterion of CDNSZ, every model of inflation still eternal in spite of log *k* running…

Can in principle resum in the large N limit…

$$P_\gamma = \frac{\Delta_\gamma \left(\frac{k}{k_*}\right)^{n_t(\epsilon_*,\dot{\epsilon}_*,\ldots)}}{1+}$$



$$= \epsilon_* N \frac{3\pi H_*^2}{10 M_{\rm pl}^2} \log \frac{k}{k_*} + \ldots,$$

$$P_\zeta = \frac{\Delta_\zeta \left(\frac{k}{k_*}\right)^{-1+n_s(\epsilon_*,\dot{\epsilon}_*,\ldots)}}{1+}$$



$$= -\, c\, \epsilon_*^2 N \frac{3\pi H_*^2}{10 M_{\rm pl}^2} \log \frac{k}{k_*} + \ldots$$

Nothing is still something

$$P_\gamma = \Delta_\gamma \left(\frac{k}{k_*}\right)^{-2\epsilon_* + \mathcal{O}(\epsilon^2)} \left[1 - \epsilon_* N \frac{3\pi H^2}{10 M_{\rm pl}^2} \log\left(k/k_*\right) + \mathcal{O}(\epsilon^2)\right]$$

$$n_t = -2\epsilon_1 - \epsilon_1 \lambda \qquad n_t = -\frac{r_*}{8}\left(1 + \frac{\lambda}{2}\right) \qquad \lambda = \frac{12\pi}{5}\frac{N}{8}\frac{H^2}{M_{\rm pl}^2} = \frac{3\pi^3}{20} N r_* \Delta_\zeta$$

$$\frac{1.41 \times 10^9}{r_*^2}\left(n_T + \frac{r_*}{8}\right) \approx N,$$

Therefore, if we can bound the quantity in the parenthesis from above to some significance by some amount ξ … then

$$N \lesssim \frac{1.44}{r_*^2} 10^9 \xi$$

Implications –

In the most *optimistic* case, if we detected $r_* \sim 0.1$ then if we could bound $10^{-4} \lesssim \xi \lesssim 10^{-2}$, then

$$N \lesssim \xi \cdot 10^{11} \sim 10^7 - 10^9$$

N.B. This is more competitive than the naïve strong coupling bound at $r_* \sim 0.1$ of $N \leq 10^9$

SKA: nHz peak sensitivity – $(k \sim 10^8 k_* \sim 10^5 \text{ Mpc}^{-1})$

If we detect tensors right at cosmic variance limit, then the bound > 10^{13} ...

Cf. \`N-Naturalness', Arkani-Hamed et al arXiv:1607.06821'
Earlier solutions to the Hierarchy problem by invoking many copies of the Standard Model (up to $N \sim 10^{32}$)