Indirect imprints of primordial non-Gaussianity on cosmological observables

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Talk based on

Barnali Das and H. V. Ragavendra, arXiv:2304.05941 [astro-ph.CO];
H. V. Ragavendra, Phys. Rev. D **105**, *063533 (2022) [arXiv:2108.04193 [astro-ph.CO]].*

@ IIT Madras
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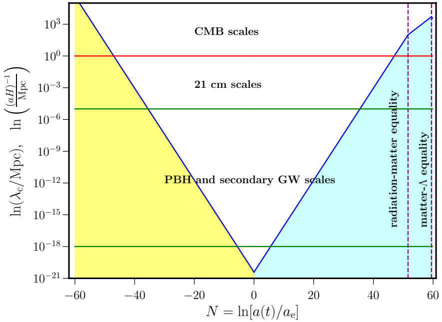
RRI

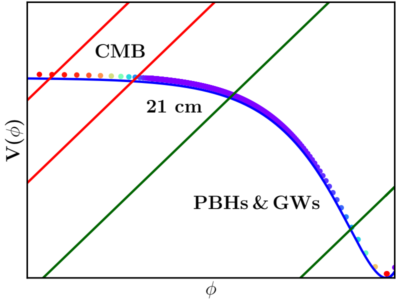
Overview

- ⇒ Introduction
  - → Primordial perturbations
  - → Gaussianity and beyond
- ⇒ Non-trivial non-Gaussianities
- ⇒ Non-Gaussian contributions to power spectrum
  - → Cosmic microwave background (CMB)
  - → Secondary gravitational waves (GWs)
- ⇒ Outlook

Primordial perturbations

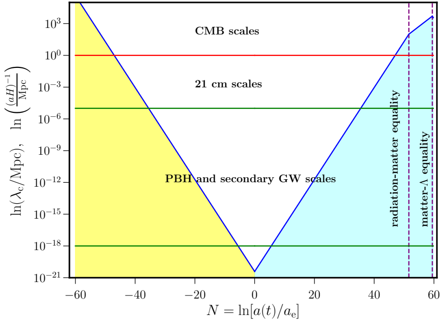
- Inflationary epoch solves the shortcomings of the hot Big-Bang model, namely the horizon problem and flatness problem.
- It also generates the primordial perturbations that relate to observational quantities at different scales today.

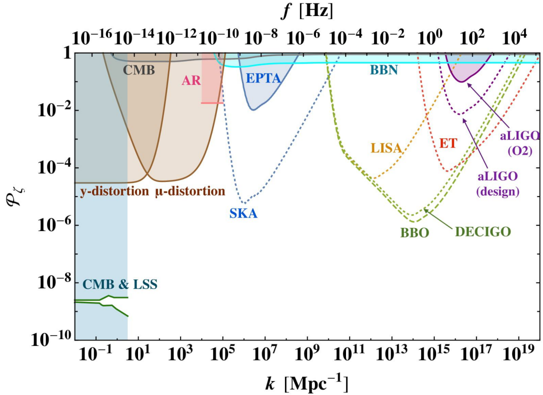




Primordial perturbations

- Inflationary epoch solves the shortcomings of the hot Big-Bang model, namely the horizon problem and flatness problem.
- It also generates the primordial perturbations that relate to observational quantities at different scales today[1].





[1] Figure on right from *K. Inomata and T. Nakama, Phys. Rev. D* **99**, *043511 (2019)*

Gaussianity and beyond

The primordial scalar perturbations $\mathcal{R}(t, \boldsymbol{x})$ are treated as Gaussian fields and the defining quantity, the power spectrum is computed as

$$\langle \hat{\mathcal{R}}_{\boldsymbol{k}_1} \hat{\mathcal{R}}_{\boldsymbol{k}_2} \rangle = \frac{2\pi^2}{k_1^3} \mathcal{P}_{\rm S}(k)\, \delta^{(3)}(\boldsymbol{k}_1 + \boldsymbol{k}_2)\,,$$

$$\mathcal{P}_{\rm S}(k) \simeq \frac{H^2}{8\pi^2 \epsilon_1}, \text{ under slow roll approximation}^{2}.$$

[2] H is the Hubble parameter during inflation and $\epsilon_1 = -\dot{H}/H^2$.

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A simple attempt to go beyond Gaussianity is by introducing non-linearity to $\mathcal{R}(t,\boldsymbol{x})$ as

$$\mathcal{R}(t,\boldsymbol{x}) = \mathcal{R}^{\rm G}(t,\boldsymbol{x}) - \frac{3}{5}f_{\rm NL}\left(\mathcal{R}^{\rm G}(t,\boldsymbol{x})\right)^2\,,$$

so that higher order correlations shall be non-zero.

[2] H is the Hubble parameter during inflation and $\epsilon_1 = -\dot{H}/H^2$.

Scalar bispectrum[3]

One can generalize the non-Gaussianity parameter $f_{_{\rm NL}}$ to be a function of k_1, k_2, k_3, as

$$\mathcal{R}_{\boldsymbol{k}}(\eta) \quad = \quad \mathcal{R}_{\boldsymbol{k}}^{\rm G}(\eta) - \frac{3}{5}\int \frac{{\rm d}^3\boldsymbol{k}_1}{(2\,\pi)^{3/2}}\mathcal{R}_{\boldsymbol{k_1}}^{\rm G}(\eta)\mathcal{R}_{\boldsymbol{k}-\boldsymbol{k_1}}^{\rm G}(\eta)\, f_{_{\rm NL}}[\boldsymbol{k}, (\boldsymbol{k}_1 - \boldsymbol{k}), -\boldsymbol{k}_1]\,.$$

[3] *J. Maldacena, JHEP* **0305**, *013 (2003); J. Martin and L. Sriramkumar, JCAP* **1201**, *008 (2012); F. Schmidt and M. Kamionkowski, Phys. Rev. D 82, 103002 (2010)*

Scalar bispectrum[3]

One can generalize the non-Gaussianity parameter f_{NL} to be a function of k_1, k_2, k_3, as

$$\mathcal{R}_{\boldsymbol{k}}(\eta) = \mathcal{R}^{\mathrm{G}}_{\boldsymbol{k}}(\eta) - \frac{3}{5}\int \frac{\mathrm{d}^3\boldsymbol{k}_1}{(2\,\pi)^{3/2}}\mathcal{R}^{\mathrm{G}}_{\boldsymbol{k_1}}(\eta)\mathcal{R}^{\mathrm{G}}_{\boldsymbol{k}-\boldsymbol{k_1}}(\eta)\, f_{\mathrm{NL}}[\boldsymbol{k}, (\boldsymbol{k}_1 - \boldsymbol{k}), -\boldsymbol{k}_1]\,.$$

We can relate the non-Gaussianity parameter to the scalar bispectrum $\mathcal{B}(k_1, k_2, k_3)$ in the following way.

$$\langle\hat{\mathcal{R}}_{\boldsymbol{k}_1}\hat{\mathcal{R}}_{\boldsymbol{k}_2}\hat{\mathcal{R}}_{\boldsymbol{k}_3}\rangle = (2\pi)^3\,\mathcal{B}(k_1, k_2, k_3)\,\delta^{(3)}(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3)\,,$$

$$f_{\mathrm{NL}}(k_1, k_2, k_3) = -\frac{10\sqrt{2\pi}}{3}\,(k_1 k_2 k_3)^3\,\mathcal{B}(k_1, k_2, k_3) \times \left[k_1^3\,\mathcal{P}_{\mathrm{S}}(k_2)\,\mathcal{P}_{\mathrm{S}}(k_3) + \text{two permutations}\right]^{-1}.$$

[3] *J. Maldacena, JHEP* **0305**, *013 (2003); J. Martin and L. Sriramkumar, JCAP* **1201**, *008 (2012); F. Schmidt and M. Kamionkowski, Phys. Rev. D 82, 103002 (2010)*

Scalar bispectrum[4]

To compute the bispectrum arising from an inflationary model and relate it to $f_{\rm NL}(k_1, k_2, k_3)$

$$\begin{aligned}\langle \hat{\mathcal{R}}_{\boldsymbol{k}_1} \hat{\mathcal{R}}_{\boldsymbol{k}_2} \hat{\mathcal{R}}_{\boldsymbol{k}_3} \rangle &= \left\langle \mathrm{e}^{i \int \mathrm{dt} \hat{\mathrm{H}}_{\rm int}} \left(\hat{\mathcal{R}}_{\boldsymbol{k}_1} \hat{\mathcal{R}}_{\boldsymbol{k}_2} \hat{\mathcal{R}}_{\boldsymbol{k}_3} \right) \mathrm{e}^{-i \int \mathrm{dt} \hat{\mathrm{H}}_{\rm int}} \right\rangle , \\ &\simeq -i \int \mathrm{d}\eta \, \langle [\hat{\mathcal{R}}_{\rm k_1} \hat{\mathcal{R}}_{\rm k_2} \hat{\mathcal{R}}_{\rm k_3}, \mathrm{H}_{\rm int}(\hat{\mathcal{R}}^3)] \rangle \, ,\end{aligned}$$

[4] For details of computation, refer *H. V. Ragavendra and L. Sriramkumar, Galaxies* **11**, *34 (2023).*

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$$\simeq -i\int {\rm d}\eta\,\langle[\hat{\mathcal{R}}_{\rm k_1}\hat{\mathcal{R}}_{\rm k_2}\hat{\mathcal{R}}_{\rm k_3}, {\rm H}_{\rm int}(\hat{\mathcal{R}}^3)]\rangle\,,$$

where $$H_{\rm int}(\hat{\mathcal{R}}^3) = -M_{_{\rm Pl}}^2\int {\rm d}^3\boldsymbol{x}\,\Big[a^2\,\epsilon_1^2\,\mathcal{R}\,\mathcal{R}'^2 + a^2\,\epsilon_1^2\,\mathcal{R}\,(\partial\mathcal{R})^2 - 2\,a\,\epsilon_1\,\mathcal{R}'\,(\partial\mathcal{R})\,(\partial\chi)$$

$$+\frac{a^2}{2}\,\epsilon_1\,\epsilon_2'\,\mathcal{R}^2\,\mathcal{R}' + \frac{\epsilon_1}{2}\,(\partial\mathcal{R})\,(\partial\chi)\,\partial^2\chi + \frac{\epsilon_1}{4}\,\partial^2\mathcal{R}\,(\partial\chi)^2 + 2\,\mathcal{F}(\mathcal{R})\,\frac{\delta\mathcal{L}_2}{\delta\mathcal{R}}\Big],$$

$$H_{\rm int}^{\rm B}(\mathcal{R}^3) = -M_{_{\rm Pl}}^2\int {\rm d}^3\boldsymbol{x}\,\frac{\rm d}{{\rm d}\eta}\Big\{-9\,a^3\,H\,\mathcal{R}^3 + \frac{a}{H}\,(1-\epsilon_1)\,\mathcal{R}\,(\partial\mathcal{R})^2 - \frac{1}{4\,a\,H^3}\,(\partial\mathcal{R})^2\,\partial^2\mathcal{R}$$

$$-\frac{a\,\epsilon_1}{H}\,\mathcal{R}\,\mathcal{R}'^2 - \frac{a\,\epsilon_2}{2}\,\mathcal{R}^2\,\partial^2\chi + \frac{1}{2\,a\,H^2}\,\mathcal{R}\,\left(\partial_i\partial_j\mathcal{R}\,\partial_i\partial_j\chi - \partial^2\mathcal{R}\,\partial^2\chi\right)$$

$$-\frac{1}{2\,aH}\,\mathcal{R}\,\left[\partial_i\partial_j\chi\,\partial_i\partial_j\chi - (\partial^2\chi)^2\right]\Big\}.$$

[4] For details of computation, refer *H. V. Ragavendra and L. Sriramkumar, Galaxies* **11**, *34 (2023)*.

Scalar bispectrum[6]

The bispectrum receives nine contributions from the cubic-order Hamiltonian.

$$\begin{aligned}\mathcal{B}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) &= (2\pi)^{-9/2} M_{_{\rm Pl}}^2 \sum_{C=1}^{6} \left[f_{k_1}(\eta_{\rm e})\, f_{k_2}(\eta_{\rm e})\, f_{k_3}(\eta_{\rm e})\, \mathcal{G}_{_C}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) + \text{complex conjugate}\right] \\ &\quad + \mathcal{B}_7^{\rm B}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) + \mathcal{B}_8^{\rm B}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) + \mathcal{B}_9^{\rm B}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3).\end{aligned}$$

where a typical $\mathcal{G}_{_C}$ shall look like

$$\mathcal{G}_1(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) = 2\,i \int_{\eta_{\rm i}}^{\eta_{\rm e}} {\rm d}\eta\; a^2\, \epsilon_1^2 \left(f_{k_1}^{*}\, f_{k_2}^{\prime *}\, f_{k_3}^{\prime *} + \text{two permutations}\right)^{5}.$$

[5] f_k are the mode functions of $\mathcal{R}(t,\boldsymbol{x})$ corresponding to the positive frequency part.

[6] For details of computation, refer *H. V. Ragavendra and L. Sriramkumar, Galaxies* **11**, *34 (2023).*

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$$+\,\mathcal{B}_7^{\rm B}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) + \mathcal{B}_8^{\rm B}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) + \mathcal{B}_9^{\rm B}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3).$$

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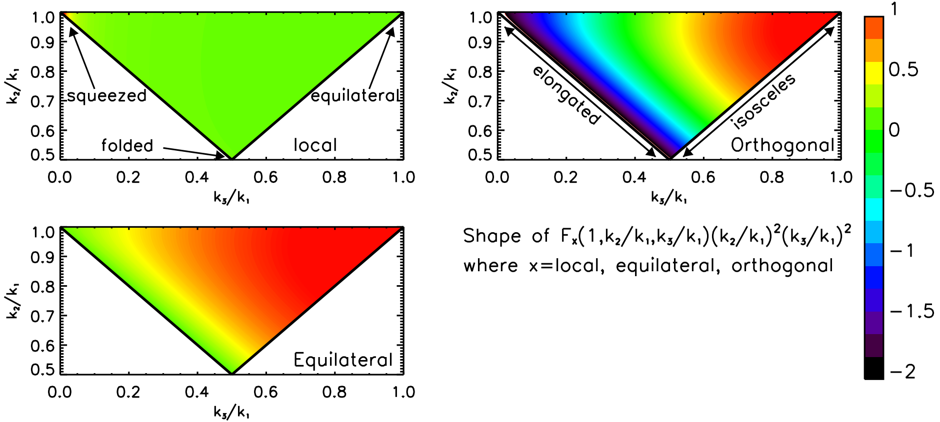
Under slow roll approximation, we can show that

$$f_{_{\rm NL}}(k_1,k_2,k_3) \simeq \mathcal{O}(\epsilon_1) \sim 10^{-2}\,.$$

[5] f_k are the mode functions of $\mathcal{R}(t,\boldsymbol{x})$ corresponding to the positive frequency part.
[6] For details of computation, refer *H. V. Ragavendra and L. Sriramkumar, Galaxies* **11**, *34 (2023).*

Shapes of $f_{\rm NL}(k_1, k_2, k_3)$[8]



Shape of $F_x(1, k_2/k_1, k_3/k_1)(k_2/k_1)^2(k_3/k_1)^2$
where x=local, equilateral, orthogonal

Current constraints: $f_{\rm NL}^{\rm loc} = -0.9 \pm 5.1$, $f_{\rm NL}^{\rm eq} = -26 \pm 47$ and $f_{\rm NL}^{\rm ortho} = -38 \pm 24$ at $1-\sigma$ level[7]

[7] *Planck Collaboration, Astron. Astrophys.* **641**, *A9 (2020) [arXiv:1905.05697 [astro-ph.CO]]*
[8] *E. Komatsu, Class. Quant. Grav.* **27**, *124010 (2010)*

Overview

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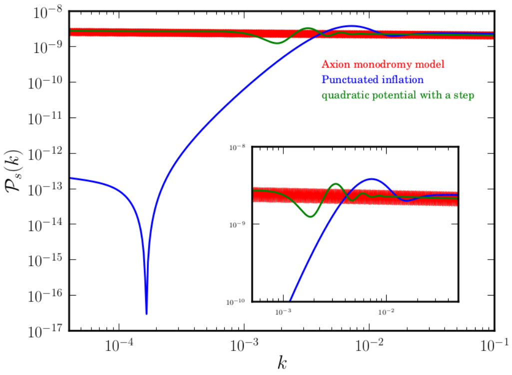
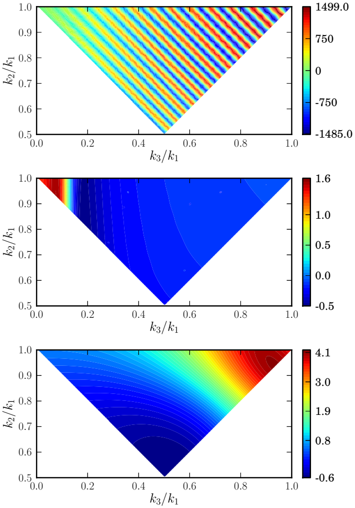
- ⇒ Non-Gaussian contributions to power spectrum
  - → CMB
  - → Secondary GWs
- ⇒ Outlook

Features in $f_{\rm NL}(k_1, k_2, k_3)$[9]





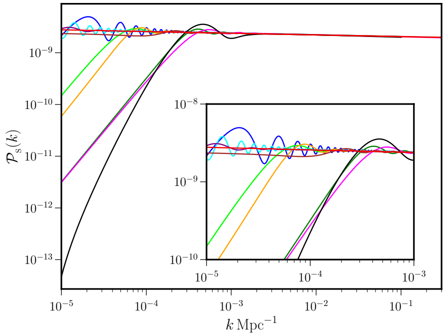
Axion monodromy

Punctuated inflation

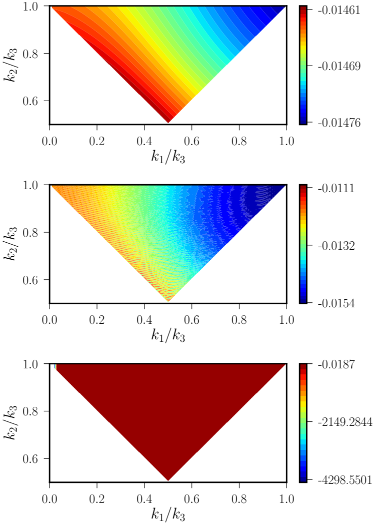
Quadratic potential with a step

[9] *V. Sreenath, D. K. Hazra and L. Sriramkumar, JCAP* **02**, *029 (2015)*

Features in $f_{\rm NL}(k_1, k_2, k_3)$[10]



Power spectra of models with kinetic dominated initial conditions (KDI) along with Starobinsky model (brown) and punctuated inflation (black)



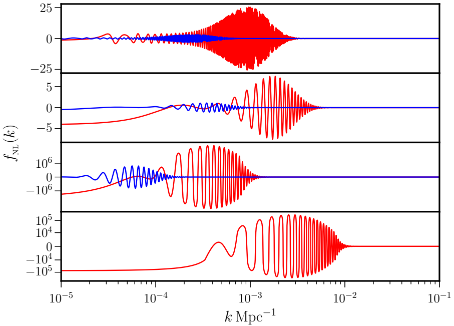
KDI

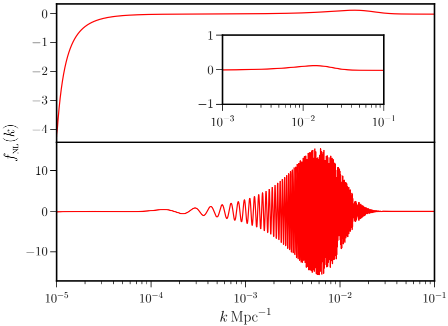
Starobinsky model

Punctuated inflation

[10] *H. V. Ragavendra, D. Chowdhury and L. Sriramkumar, Phys. Rev. D* **106**, *043535 (2022)*

$f_{\rm NL}$ with $k_1 = k_2 = k_3$



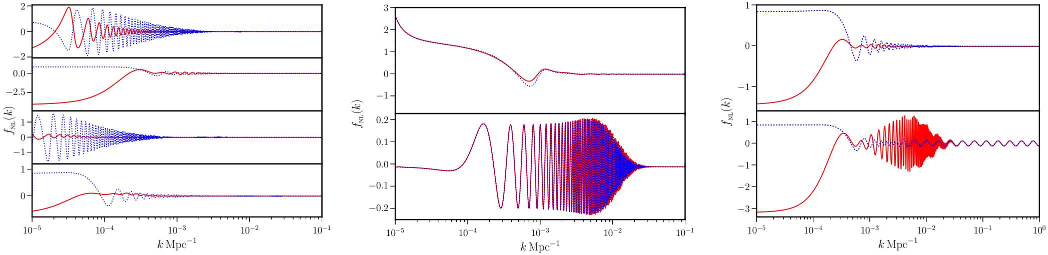


Behavior of $f_{\rm NL}$ is presented in equilateral limit for KDI models (on left) and Starobinsky model and punctuated inflation (on right)[11].

[11] *H. V. Ragavendra, D. Chowdhury and L. Sriramkumar, Phys. Rev. D* **106**, *043535 (2022)*

$f_{\rm NL}$ with $\boldsymbol{k}_1 = -\boldsymbol{k}_2;\ \boldsymbol{k}_3 \to \boldsymbol{0}$

Consistency relation : $f_{\rm NL}(k, k, k_3 \to 0) = \frac{5}{12}\frac{{\rm d}\ln \mathcal{P}_{\rm S}}{{\rm d}\ln k}$



Behavior of $f_{\rm NL}$ is presented in squeezed limit for KDI models (on left), Starobinsky model and punctuated inflation (in the middle), a small field and axion monodromy model (in right)[12].

[12] *H. V. Ragavendra, D. Chowdhury and L. Sriramkumar, Phys. Rev. D* **106**, *043535 (2022)*

Overview

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Modification to $\mathcal{P}_{_{\rm S}}(k)$ due to $f_{_{\rm NL}}(k_1, k_2, k_3)$[13]

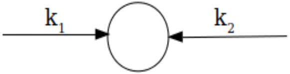
Recall that

$$\mathcal{R}_{\boldsymbol{k}}(\eta) = \mathcal{R}^{\rm G}_{\boldsymbol{k}}(\eta) - \frac{3}{5}\int \frac{{\rm d}^3\boldsymbol{k}_1}{(2\,\pi)^{3/2}}\mathcal{R}^{\rm G}_{\boldsymbol{k_1}}(\eta)\mathcal{R}^{\rm G}_{\boldsymbol{k}-\boldsymbol{k_1}}(\eta)\, f_{_{\rm NL}}[\boldsymbol{k}, (\boldsymbol{k}_1 - \boldsymbol{k}), -\boldsymbol{k}_1].$$

If we compute the two-point correlation of $\mathcal{R}_{\boldsymbol{k}}$ using this relation, we obtain

$$\mathcal{P}^{^{\rm M}}_{_{\rm S}}(k) = \mathcal{P}_{_{\rm S}}(k) + \underbrace{\frac{9}{25}\int_0^\infty {\rm d}x \int_{|1-x|}^{|1+x|} {\rm d}y \frac{\mathcal{P}_{_{\rm S}}(kx)}{x^2}\, \frac{\mathcal{P}_{_{\rm S}}(ky)}{y^2}\, f^2_{_{\rm NL}}[k, kx, ky]}_{\mathcal{P}_{_{\rm C}}(k)}\,.$$

We can represent them as the following Feynman diagrams

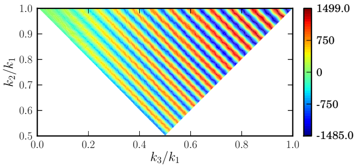


[13] *H. V. Ragavendra, Phys. Rev. D* **105***, 063533 (2022); B. Das and H. V. Ragavendra, arXiv:2304.05941 [astro-ph.CO]*

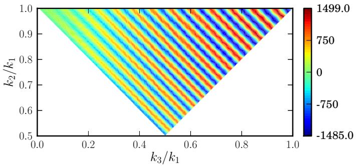
Oscillatory template

$$\mathcal{P}_{\rm S}^{\rm osc}(k) = A_{\rm S}\left(\frac{k}{k_*}\right)^{n_{\rm S}-1}\left\{1+b\,\sin\left[\omega\ln\left(\frac{k}{k_o}\right)\right]\right\}$$

$$\mathcal{B}^{\rm osc}(k_1,k_2,k_3) = \frac{6A_{\rm S}^2 f_{\rm NL}^{\rm osc}}{(k_1k_2k_3)^2}\sin\left[\omega\ln\left(\frac{k_1+k_2+k_3}{k_o}\right)\right]$$



Oscillatory template



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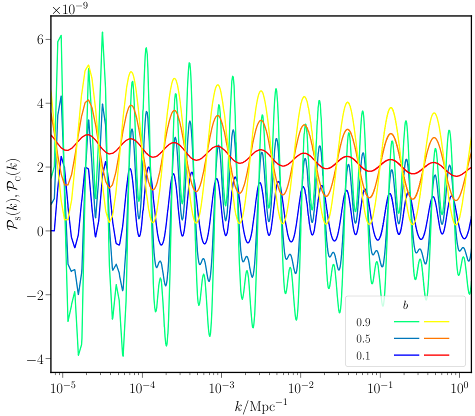
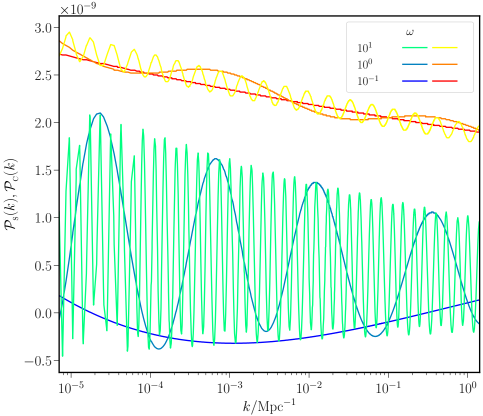
$$\mathcal{B}^{\rm osc}(k_1,k_2,k_3) = \frac{6A_{\rm S}^2 f_{\rm NL}^{\rm osc}}{(k_1k_2k_3)^2}\sin\left[\omega\ln\left(\frac{k_1+k_2+k_3}{k_o}\right)\right]$$

$$\begin{aligned}\mathcal{P}_{\rm C}^{\rm osc}(k) = {} & 288\,\pi\left[A_{\rm S}\left(\frac{k}{k_*}\right)^{n_{\rm S}-1} f_{\rm NL}^{\rm osc}\right]^2\int_0^\infty {\rm d}x\int_{|1-x|}^{1+x}{\rm d}y\,xy\sin^2\left[\omega\ln\left(\frac{k}{k_o}(1+x+y)\right)\right] \\ & \times\left[1+b\sin\left(\omega\ln\frac{kx}{k_o}\right)\right]\left[1+b\sin\left(\omega\ln\frac{ky}{k_o}\right)\right] \\ & \times\left\{\left[1+b\sin\left(\omega\ln\frac{kx}{k_o}\right)\right]\left[1+b\sin\left(\omega\ln\frac{ky}{k_o}\right)\right]\right. \\ & +x^3\left[1+b\sin\left(\omega\ln\frac{k}{k_o}\right)\right]\left[1+b\sin\left(\omega\ln\frac{ky}{k_o}\right)\right] \\ & \left.+y^3\left[1+b\sin\left(\omega\ln\frac{k}{k_o}\right)\right]\left[1+b\sin\left(\omega\ln\frac{kx}{k_o}\right)\right]\right\}^{-1}\end{aligned}$$

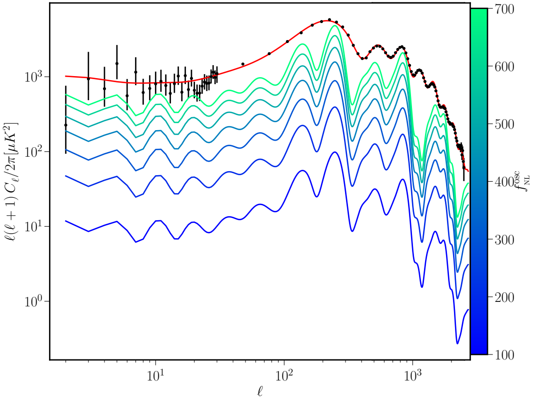
Oscillatory template





$\mathcal{P}_{\rm S}^{\rm osc}(k)$ (in shades of red to yellow) and $\mathcal{P}_{\rm C}^{\rm osc}(k)$ (in shades of blue to green) are presented for different values of b (on left) and ω (on right). We set $b = 0.05$, $\omega = 5$ (unless varied), $f_{\rm NL}^{\rm osc} = 500$, and $k_o/\,{\rm Mpc}^{-1} = 10^{-1}$ in obtaining these plots.

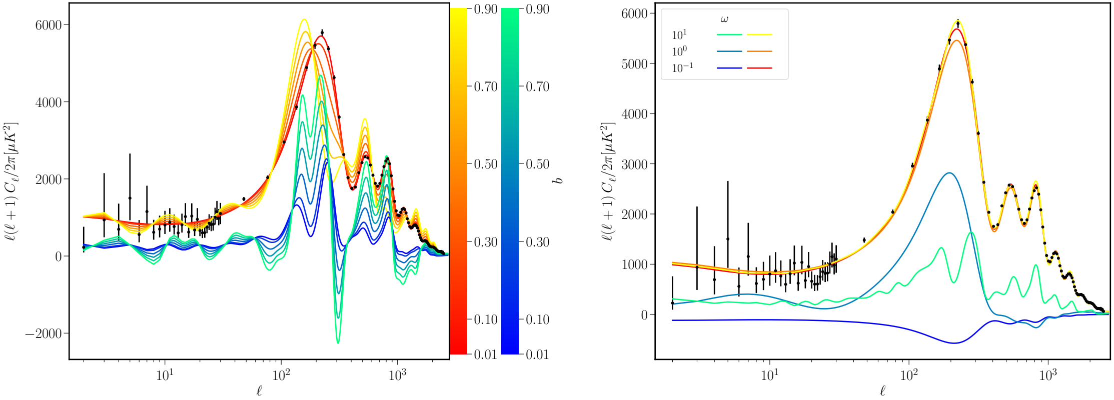
Oscillatory template



C_ℓs due to $\mathcal{P}_{\rm C}^{\rm osc}(k)$ are presented for different values of $f_{\rm NL}^{\rm osc}$. We set $b = 0.05$, $\omega = 5$ and $k_o/\,{\rm Mpc}^{-1} = 10^{-1}$ in this plot[14].

[14] Angular spectra are computed using the publicly available package called `CAMB`.

Oscillatory template

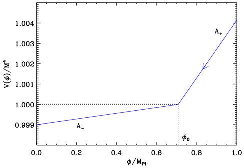


C_ℓs due to $\mathcal{P}_{\rm S}^{\rm osc}(k)$ (in shades of red to yellow) and $\mathcal{P}_{\rm C}^{\rm osc}(k)$ (in shades of blue to green) are presented for different values of b (on left) and ω (on right). We set $f_{\rm NL}^{\rm osc} = 500$, $k_o/\,{\rm Mpc}^{-1} = 10^{-1}$, $b = 0.05$ and $\omega = 5$ (unless varied) in these plots.

Starobinsky model

This model has been well studied in the literature for its interesting feature of suppression and oscillations in the power and bispectrum[15].

$$V(\phi) = \begin{cases} V_0 + A_+(\phi - \phi_0), & \text{for } \phi > \phi_0\,, \\ V_0 + A_-(\phi - \phi_0), & \text{for } \phi < \phi_0\,, \end{cases}$$



$$\mathcal{P}_{\rm S}(k) = \frac{1}{12\pi^2}\left(\frac{V_0}{M_{\rm Pl}^4}\right)\left(\frac{V_0}{A_- M_{\rm Pl}}\right)^2 \left\{1 - \frac{3\Delta A}{A_+}\frac{k_0}{k}\left[\left(1-\frac{k_0^2}{k^2}\right)\sin\left(\frac{2k}{k_0}\right) + \frac{2k_0}{k}\cos\left(\frac{2k}{k_0}\right)\right] + \frac{9\Delta A^2}{2A_+^2}\frac{k_0^2}{k^2}\left(1+\frac{k_0^2}{k^2}\right)\left[1+\frac{k_0^2}{k^2} - \frac{2k_0}{k}\sin\left(\frac{2k}{k_0}\right) + \left(1-\frac{k_0^2}{k^2}\right)\cos\left(\frac{2k}{k_0}\right)\right]\right\}^{16}.$$

[15] *A. A. Starobinsky, JETP Lett.* **55**, *489 (1992); J. Martin and L. Sriramkumar, JCAP* **01**, *008 [arXiv:1109.5838 [astro-ph.CO]]; J. Martin, L. Sriramkumar, and D. K. Hazra, JCAP* **09**, *039 [arXiv:1404.6093 [astro-ph.CO]].*

[16] $\Delta A = A_- - A_+$

Starobinsky model

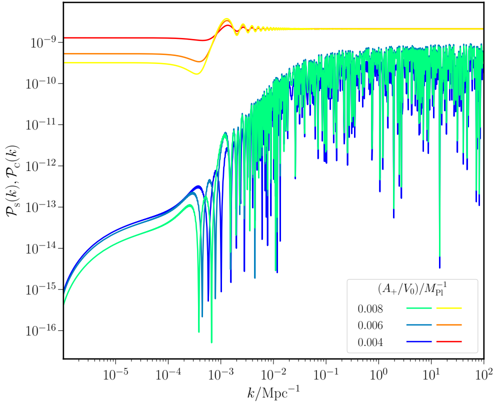
We obtain $\mathcal{P}_{_{\rm C}}(k)$ for this model to be

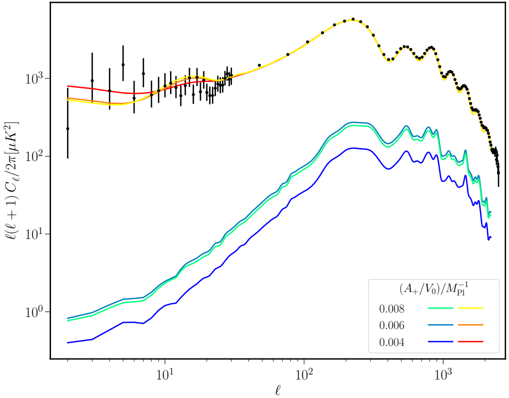
$$\mathcal{P}_{_{\rm C}}(k) \simeq \frac{9}{16}\left(\frac{k_0}{k}\right)^2\left(\frac{A_-}{A_+}\right)^2\left(1-\frac{A_-}{A_+}\right)^2\left(\mathcal{P}_{_{\rm S}}^0\right)^2\int_0^\infty {\rm d}x\int_{|1-x|}^{1+x}{\rm d}y\,\frac{|\alpha_{kx}-\beta_{kx}|^2}{x^2}\,\frac{|\alpha_{ky}-\beta_{ky}|^2}{y^2}$$
$$\times\left(\frac{Z(k,x,y)}{|\alpha_{kx}-\beta_{kx}|^2|\alpha_{ky}-\beta_{ky}|^2+y^3|\alpha_k-\beta_k|^2|\alpha_{kx}-\beta_{kx}|^2+x^3|\alpha_k-\beta_k|^2|\alpha_{ky}-\beta_{ky}|^2}\right)^2,$$

where $\mathcal{P}_{_{\rm S}}^0=1/(12\pi^2)\left(V_0/M_{\rm Pl}^4\right)[V_0/(A_-M_{\rm Pl})]^2$, α_k and β_k are the Bogoliubov coefficients of the mode functions and $Z(k,x,y)$ is the function that captures the dependence of the dominant component of the bispectrum $G_4(k,kx,ky)$ on these coefficients[17].

[17] For details of computation, see *B. Das and H. V. Ragavendra, arXiv:2304.05941 [astro-ph.CO].*

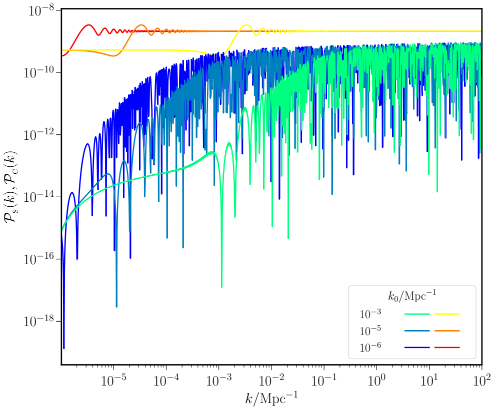
Starobinsky model

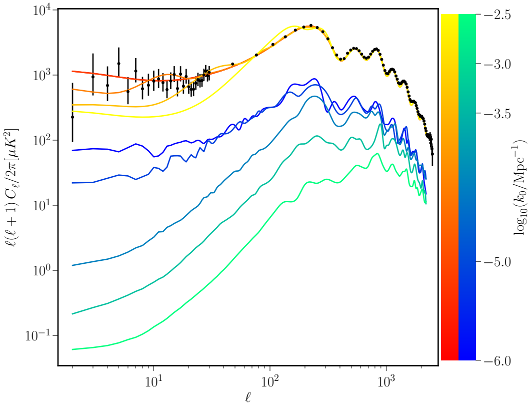




The non-Gaussian correction $\mathcal{P}_{\rm C}(k)$ (in blue to green) can reach up to $1-10\%$ of $\mathcal{P}_{\rm S}(k)$ (in red to yellow) and hence leave imprints on the corresponding CMB angular spectra in Starobinsky model.

Starobinsky model





Decreasing k_0, reduces the feature in Gaussian spectrum but increases the amplitude of $\mathcal{P}_{\rm C}(k)$ over large scales[18].

[18] *B. Das and H. V. Ragavendra, arXiv:2304.05941 [astro-ph.CO]*

Overview

Secondary GWs

The scalar perturbations source the tensors at the second order. If enhanced sufficiently, they lead to detectable strengths of secondary gravitational waves[19]. The relation between such secondary tensor perturbations $h_{\boldsymbol{k}}$ and scalar perturbations $\mathcal{R}_{\boldsymbol{k}}$ is given by

$$\begin{aligned}\langle h_{\boldsymbol{k}}^{\lambda}(\eta)h_{\boldsymbol{k}'}^{\lambda'}(\eta)\rangle &= \frac{16}{81}\frac{1}{kk'\eta^2}\int\frac{{\rm d}^3\boldsymbol{p}}{(2\pi)^{3/2}}\int\frac{{\rm d}^3\boldsymbol{p}'}{(2\pi)^{3/2}}\,Q^{\lambda}(k,p)Q^{\lambda'}(k',p')\\ &\quad\times\mathcal{I}(k,p)\mathcal{I}(k',p')\,\langle\mathcal{R}_{\boldsymbol{p}}\mathcal{R}_{\boldsymbol{k}-\boldsymbol{p}}\mathcal{R}_{\boldsymbol{p}'}\mathcal{R}_{\boldsymbol{k}'-\boldsymbol{p}'}\rangle,\end{aligned}$$

where the quantity $\mathcal{I}(k,p)$ arises from the transfer function relating the Bardeen potential to the curvature perturbation $\mathcal{R}_{\boldsymbol{k}}$ and the function $Q(k,p)$ arises from the polarization tensor associated with $h_{\boldsymbol{k}}$.

[19] See, for instance, *K. Kohri and T. Terada, Phys. Rev. D* **97**, *123532 (2018); N. Bartolo et al, Phys. Rev. D* **99**, *103521 (2019)*.

Secondary GWs

The power spectrum of such secondary tensor perturbations $\mathcal{P}_h(k,\eta)$, is defined through the relation

$$\langle h^{\lambda}_{\boldsymbol{k}}(\eta) h^{\lambda'}_{\boldsymbol{k}'}(\eta)\rangle = \frac{2\pi^2}{k^3}\mathcal{P}_h(k,\eta)\delta^{(3)}(\boldsymbol{k}+\boldsymbol{k}')\delta^{\lambda\lambda'}.$$

The dimensionless energy density of corresponding secondary GWs in the current universe can then be estimated as

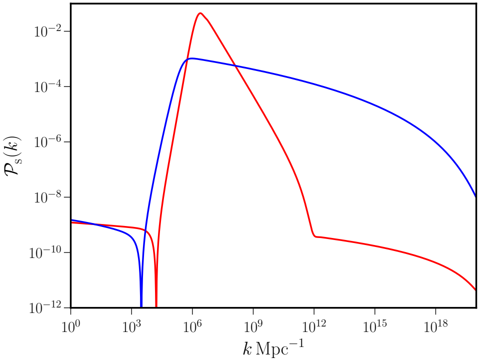
$$\mathrm{h}^2\,\Omega_{_{\rm GW}}(k) = \frac{1}{24}\left(\frac{g_{*,k}}{g_{*,0}}\right)^{-1/3}\Omega_{\rm r}\,\mathrm{h}^2\,(k^2\eta^2)\,\overline{\mathcal{P}_h(k,\eta)}\,,$$

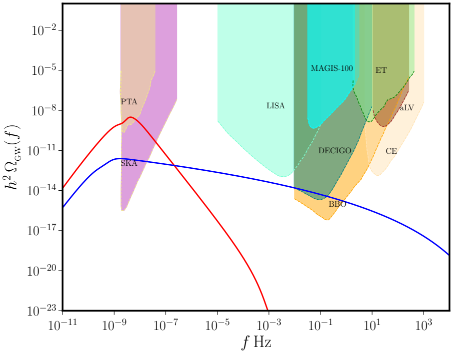
where $\Omega_{\rm r}$ denotes the fraction of energy density of radiation today,
$g_{*,k}$ denotes the relativistic degrees of freedom when k re-enters the Hubble radius and $g_{*,0}$ is the quantity evaluated today.
The overline about the secondary tensor power spectrum denotes averaging over oscillations of small time scales.

Secondary GWs from ultra slow roll (USR) models[20]

Starobinsky model with a dip (SMD): $V(\phi) = V_0 \left[1-\exp\left(-\sqrt{\frac{2}{3}}\frac{\phi}{M_{\rm Pl}}\right)\right]^2 \left\{1-\lambda\exp\left[-\frac{1}{2}\left(\frac{\phi-\phi_0}{\Delta\phi}\right)^2\right]\right\}$

Critical-Higgs model (CHI): $V(\phi) = V_0\, \frac{\left[1+a\,\ln^2\left(\frac{\phi}{\mu}\right)\right]\left(\frac{\phi}{\mu}\right)^4}{\left\{1+c\left[1+b\ln\left(\frac{\phi}{\mu}\right)\right]\left(\frac{\phi}{\mu}\right)^2\right\}^2}$

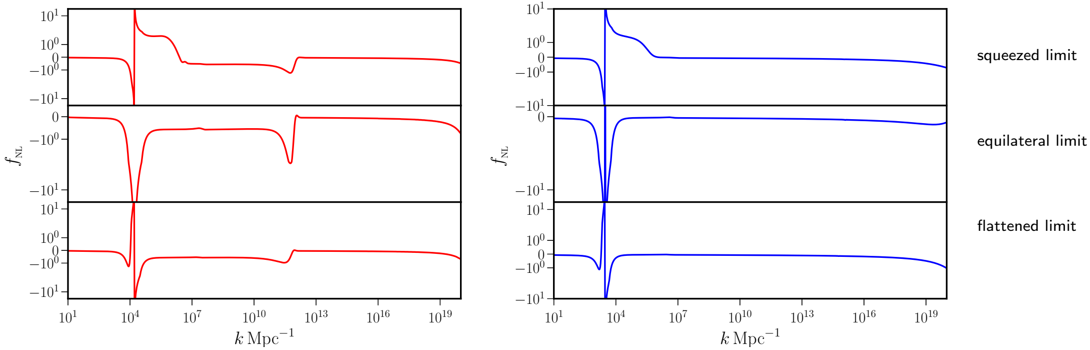




[20]For a review on USR models, refer *H. V. Ragavendra and L. Sriramkumar, Galaxies* **11**, *34 (2023)*.

Non-Gaussianity parameter in USR

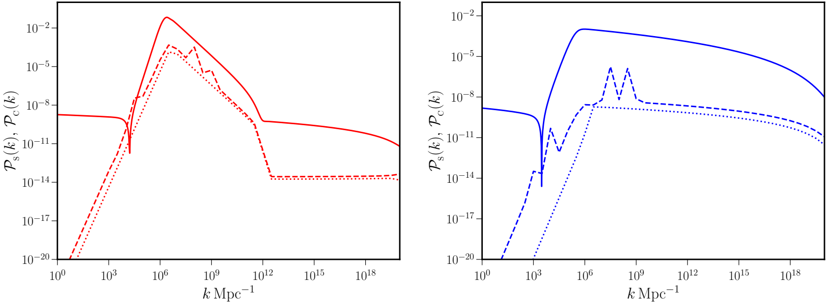
$$f_{_{\rm NL}}(k_1, k_2, k_3) = -\frac{10}{3}\,\sqrt{2\pi}\;k_1^3\,k_2^3\,k_3^3\,\mathcal{B}(k_1, k_2, k_3)\,\left[k_1^3\,\mathcal{P}_{_{\rm S}}(k_2)\,\mathcal{P}_{_{\rm S}}(k_3) + \text{two permutations}\right]^{-1}$$



$f_{_{\rm NL}}(k_1, k_2, k_3)$ in USR models have highly non-trivial scale dependence.

$\mathcal{P}_{\rm C}(k)$ from USR

$$\mathcal{P}_{\rm C}(k) = \begin{cases} \frac{1}{8}\left(\frac{k}{k_{\rm f}}\right)^3 \{\mathcal{P}_{\rm S}(k_{\rm f})[n_{\rm S}(k_{\rm f})-1]\}^2\,, & \text{for } k < k_{\rm f}\,, \\ \frac{1}{4}\,\mathcal{P}_{\rm S}(k_{\rm f})\mathcal{P}_{\rm S}(k)\,[n_{\rm S}(k)-1]^2\,, & \text{for } k > k_{\rm f}\,. \end{cases}$$



$\mathcal{P}_{\rm C}(k)$ (dashed lines) against $\mathcal{P}_{\rm S}(k)$ (solid lines)[21]

[21] *H. V. Ragavendra, Phys. Rev. D* **105**, *063533 (2022)*; for implications of the dip in $\mathcal{P}_{\rm S}(k)$, refer, *S. Balaji, H. V. Ragavendra, S. K. Sethi, J. Silk, L. Sriramkumar, Phys. Rev. Lett.* **129**, *261301 (2022)*

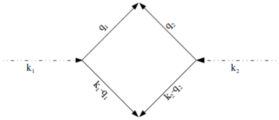
Non-Gaussian imprints on secondary GWs

We once again resort to Feynman diagrams to track the non-Gaussian contributions to GWs[22].

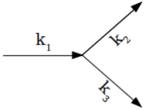
$$\Omega_{\rm GW} = \Omega_{\rm GW}^{(0)} + \Omega_{\rm GW}^{(1)} + \Omega_{\rm GW}^{(2)}(f_{\rm NL}^2)$$

$\Omega_{\rm GW}^{(0)}$

$\Omega_{\rm GW}^{(1)}$



$+ \quad f_{\rm NL}$



$\Omega_{\rm GW}^{(2)}$

[22] See for instance, *C. Unal, Phys. Rev. D* **99**, *041301 (2019); P. Adshead, K. D. Lozanov and Z. J. Weiner, JCAP* **10**, *080 (2021).*

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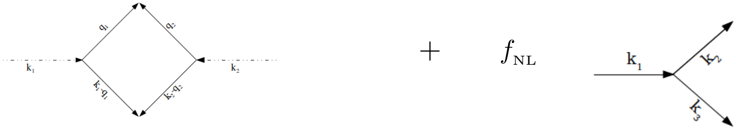
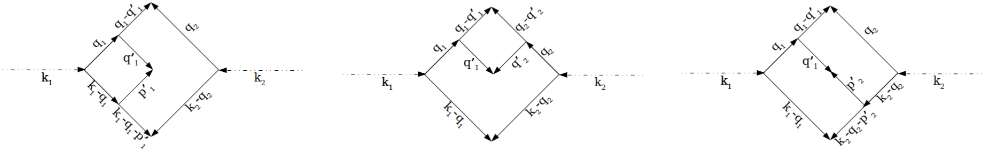
$\Omega_{\rm GW}^{(1)}$



$\Omega_{\rm GW}^{(2)}$



[22] See for instance, *C. Unal, Phys. Rev. D* **99**, *041301 (2019); P. Adshead, K. D. Lozanov and Z. J. Weiner, JCAP* **10**, *080 (2021).*

Non-Gaussian contributions to $\Omega_{\rm GW}$

A typical non-Gaussian contribution to $\Omega_{\rm GW}$ looks like

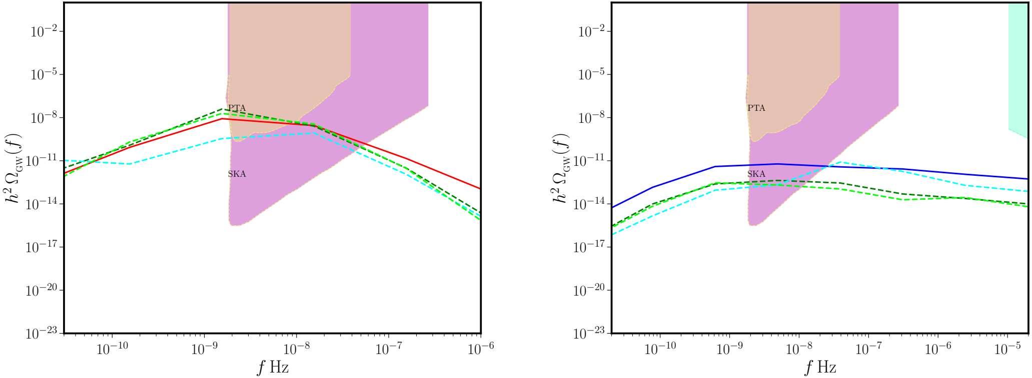
$$\Omega_{\rm GW}^{(2)}(k) \sim \int {\rm d}^3\boldsymbol{k}_1 \int {\rm d}^3\boldsymbol{k}_2\, \mathcal{P}_{\rm S}(k_1+k_2)\mathcal{P}_{\rm S}(k-k_2)\mathcal{P}_{\rm S}(k+k_1) \times \left[f_{\rm NL}(k,k_1,k_2)f_{\rm NL}(k,k_1+k_2,k-k_2)\right].$$

→ The non-trivial arguments of $\mathcal{P}_{\rm S}(k)$ and $f_{\rm NL}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)$ do not easily permit an analytical estimate.

→ We resort to Monte-Carlo method of numerical integration to deal with the issue of dimensionality.

→ Further, at each point of the integral, we need to evaluate the power and bi-spectra numerically.

Non-Gaussian imprints on secondary GWs



In USR models, SMD (on left) and CHI (on right), the non-Gaussian contribution from $f_{\rm NL}(k_1, k_2, k_3)$ to secondary $\Omega_{\rm GW}$ (dashed lines) becomes comparable to Gaussian contribution[23].

[23] *H. V. Ragavendra, Phys. Rev. D* **105**, *063533 (2022)*

Overview

⇒ Introduction

⇒ Non-trivial non-Gaussianities

⇒ Non-Gaussian contributions to power spectrum

→ CMB

→ Secondary GWs

⇒ Outlook

Conclusions

- Inflationary models with features in potential generate non-Gaussianities of significant amplitudes and non-trivial shapes. They have to be consistently accounted for in the computation of observational predictions.
- $\mathcal{P}_{\rm C}(k)$ due to $f_{\rm NL}(k_1, k_2, k_3)$ from models with features in their potential give rise to non-negligible corrections to the angular spectrum of CMB. We are currently working on constraining them against data.
- $f_{\rm NL}(k_1, k_2, k_3)$ from USR models lead to significant non-Gaussian contribution to their predictions of $\Omega_{\rm GW}$.
- The significance of non-Gaussian contributions treated as loop corrections to the Gaussian estimates has generated quite an interest in recent literature[24].

[24] Refer, for an active debate, *J. Kristiano and J. Yokoyama, arXiv:2211.03395 [hep-th]; criticism, A. Riotto, arXiv:2301.00599 [astro-ph.CO]; and response to criticism, J. Kristiano, J. Yokoyama, arXiv:2303.00341 [hep-th].*

Outlook

- Similar computation of $\mathcal{P}_{_{\rm C}}(k)$ arising from cross-correlations such as $\mathcal{R}\gamma\gamma$ and $\mathcal{R}\mathcal{R}\gamma$ may further our understanding of tensor perturbations through scalar power spectrum[25].
- Non-Gaussian contributions due to interaction of inflaton with spectator fields shall be interesting to explore and constrain using relevant observables[26].
- Calculation of $\mathcal{P}_{_{\rm C}}(k)$ due to gauge fields may be interesting and can complement the existing bounds on primordial magnetic field[27].

[25] *D. Chowdhury, V. Sreenath, and L. Sriramkumar, JCAP* **11**, *041 (2016) [arXiv:1605.05292 [astro-ph.CO]]*

[26] *L.-T. Wang, Z.-Z. Xianyu, and Y.-M. Zhong, JHEP* **02**, *085, [arXiv:2109.14635 [hep-ph]]*

[27] *S. Tripathy, D. Chowdhury, H. V. Ragavendra, R. K. Jain, and L. Sriramkumar, Phys. Rev. D* **107**, *043501 (2023), [arXiv:2211.05834 [astro-ph.CO]]*

Conclusions

Thanks for your attention.

The talk was based on

1. *Barnali Das and H. V. Ragavendra, arXiv:2304.05941 [astro-ph.CO],*
2. *H. V. Ragavendra, Phys. Rev. D* **105**, *063533 (2022) [arXiv:2108.04193 [astro-ph.CO]].*

Structure of the scalar bispectrum

$$\mathcal{G}_1(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) = 2\,i\int_{\eta_{\rm i}}^{\eta_{\rm e}} {\rm d}\eta\; a^2\,\epsilon_1^2\,\left(f_{k_1}^{\ast}\,f_{k_2}^{\prime\ast}\,f_{k_3}^{\prime\ast} + \text{two permutations}\right), \tag{1a}$$

$$\mathcal{G}_2(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) = -2\,i\,\left(\boldsymbol{k}_1\cdot\boldsymbol{k}_2 + \text{two permutations}\right)\int_{\eta_{\rm i}}^{\eta_{\rm e}} {\rm d}\eta\; a^2\,\epsilon_1^2\,f_{k_1}^{\ast}\,f_{k_2}^{\ast}\,f_{k_3}^{\ast}, \tag{1b}$$

$$\mathcal{G}_3(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) = -2\,i\int_{\eta_{\rm i}}^{\eta_{\rm e}} {\rm d}\eta\; a^2\,\epsilon_1^2\,\left(\frac{\boldsymbol{k}_1\cdot\boldsymbol{k}_2}{k_2^2}\,f_{k_1}^{\ast}\,f_{k_2}^{\prime\ast}\,f_{k_3}^{\prime\ast} + \text{five permutations}\right), \tag{1c}$$

$$\mathcal{G}_4(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) = i\int_{\eta_{\rm i}}^{\eta_{\rm e}} {\rm d}\eta\; a^2\,\epsilon_1\,\epsilon_2^{\prime}\,\left(f_{k_1}^{\ast}\,f_{k_2}^{\ast}\,f_{k_3}^{\prime\ast} + \text{two permutations}\right), \tag{1d}$$

$$\mathcal{G}_5(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) = \frac{i}{2}\int_{\eta_{\rm i}}^{\eta_{\rm e}} {\rm d}\eta\; a^2\,\epsilon_1^3\,\left(\frac{\boldsymbol{k}_1\cdot\boldsymbol{k}_2}{k_2^2}\,f_{k_1}^{\ast}\,f_{k_2}^{\prime\ast}\,f_{k_3}^{\prime\ast} + \text{five permutations}\right), \tag{1e}$$

$$\mathcal{G}_6(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) = \frac{i}{2}\int_{\eta_{\rm i}}^{\eta_{\rm e}} {\rm d}\eta\, a^2\,\epsilon_1^3\,\left(\frac{k_1^2\,\left(\boldsymbol{k}_2\cdot\boldsymbol{k}_3\right)}{k_2^2\,k_3^2}\,f_{k_1}^{\ast}\,f_{k_2}^{\prime\ast}\,f_{k_3}^{\prime\ast} + \text{two permutations}\right). \tag{1f}$$

Structure of the scalar bispectrum

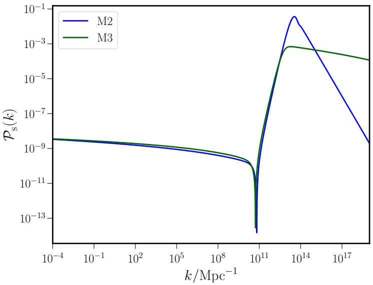
$$G_7(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) = -i\, M_{\rm Pl}^2\, \left(f_{k_1}(\eta_{\rm e})\, f_{k_2}(\eta_{\rm e})\, f_{k_3}(\eta_{\rm e})\right) \left[a^2 \epsilon_1 \epsilon_2\, f_{k_1}^{\ast}(\eta)\, f_{k_2}^{\ast}(\eta)\, f_{k_3}^{\prime\ast}(\eta) + \text{two permutations}\right]_{\eta_i}^{\eta_{\rm e}} + \text{c.c.},$$

$$\begin{aligned}
G_8(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) &= i\, f_{k_1}(\eta_{\rm e})\, f_{k_2}(\eta_{\rm e})\, f_{k_3}(\eta_{\rm e}) \left[\frac{a}{H}\, f_{k_1}^{\ast}(\eta)\, f_{k_2}^{\ast}(\eta)\, f_{k_3}^{\ast}(\eta)\right]_{\eta_{\rm i}} \\
&\times \Biggl\{54\,(a\,H)^2 + 2\,(1-\epsilon_1)(\boldsymbol{k}_1\cdot\boldsymbol{k}_2 + \boldsymbol{k}_1\cdot\boldsymbol{k}_3 + \boldsymbol{k}_2\cdot\boldsymbol{k}_3) \\
&+ \frac{1}{2\,(a\,H)^2}\left[(\boldsymbol{k}_1\cdot\boldsymbol{k}_2)\,{k_3}^2 + (\boldsymbol{k}_1\cdot\boldsymbol{k}_3)\,{k_2}^2 + (\boldsymbol{k}_2\cdot\boldsymbol{k}_3)\,{k_1}^2\right]\Biggr\}_{\eta_{\rm i}} + \text{c.c.},
\end{aligned}$$

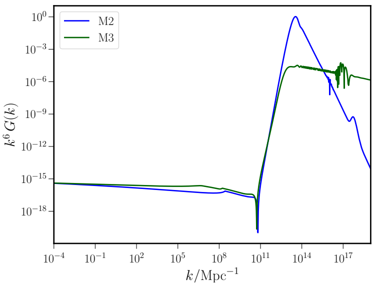
$$\begin{aligned}
G_9(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) &= i\, f_{k_1}(\eta_{\rm e})\, f_{k_2}(\eta_{\rm e})\, f_{k_3}(\eta_{\rm e}) \Biggl\{\frac{\epsilon_1}{2\,H^2}\, f_{k_1}^{\ast}(\eta)\, f_{k_2}^{\ast}(\eta)\, f_{k_3}^{\prime\ast}(\eta) \\
&\times \left[k_1^2 + k_2^2 - \left(\frac{\boldsymbol{k}_1\cdot\boldsymbol{k}_3}{k_3}\right)^2 - \left(\frac{\boldsymbol{k}_2\cdot\boldsymbol{k}_3}{k_3}\right)^2\right] - \frac{a\,\epsilon_1}{H}\, f_{k_1}^{\ast}(\eta)\, f_{k_2}^{\prime\ast}(\eta)\, f_{k_3}^{\prime\ast}(\eta) \\
&\times \left[2 - \epsilon_1 + \epsilon_1\left(\frac{\boldsymbol{k}_2\cdot\boldsymbol{k}_3}{k_2\, k_3}\right)^2\right]\Biggr\}_{\eta_{\rm i}}^{\eta_{\rm e}} + \text{two permutations} + \text{c.c.}.
\end{aligned}$$

Scalar bispectrum in USR

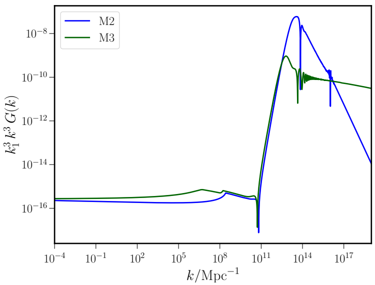
$$\langle\hat{\mathcal{R}}_{\boldsymbol{k}_1}(\eta_{\rm e})\,\hat{\mathcal{R}}_{\boldsymbol{k}_2}(\eta_{\rm e})\,\hat{\mathcal{R}}_{\boldsymbol{k}_3}(\eta_{\rm e})\rangle = (2\,\pi)^{-3/2}\,G(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)\,\delta^{(3)}(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)$$



Power spectrum



Bispectrum in equilateral limit



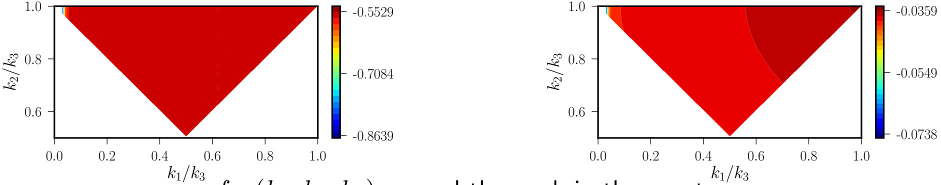
Bispectrum in squeezed limit

The scalar bispectrum closely mimics the shape of the power spectrum in USR models[28].

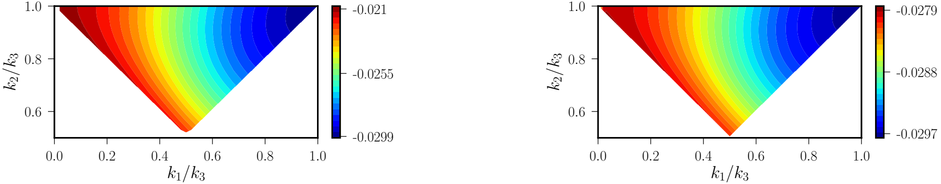
[28] *H. V. Ragavendra and L. Sriramkumar, Galaxies* **11**, *34 (2023)*

Non-Gaussianity parameter in USR

In USR models, the shape of $f_{\rm NL}(k_1, k_2, k_3)$ varies widely over the range of wavenumbers[29]



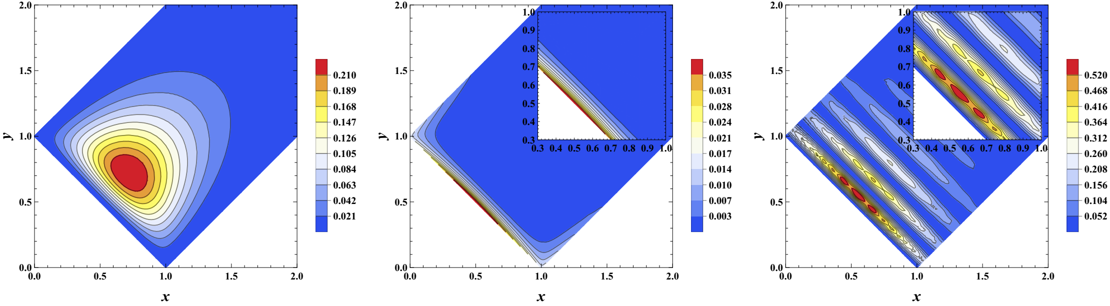
$f_{\rm NL}(k_1, k_2, k_3)$ around the peak in the spectra



$f_{\rm NL}(k_1, k_2, k_3)$ around the pivot scale of CMB

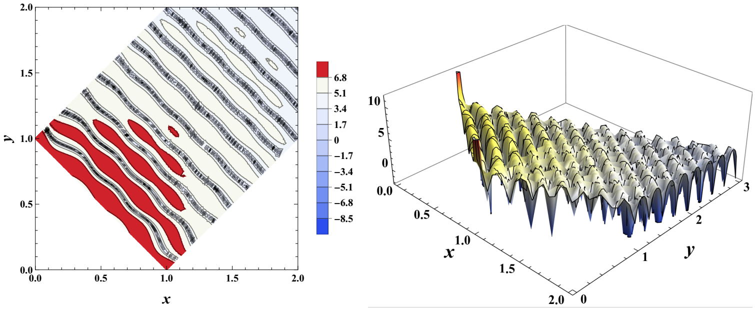
[29] *H. V. Ragavendra and L. Sriramkumar, Galaxies* **11**, *34 (2023)*

Shapes of integrands of $\mathcal{P}_{\rm C}(k)$



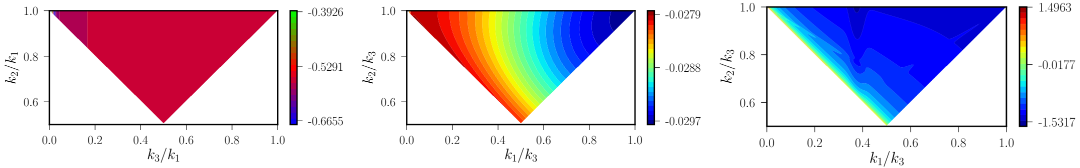
Shapes of integrand involved in computing $\mathcal{P}_{\rm C}(k)$ for equilateral (left), orthogonal (middle) and oscillatory (right) templates.

Shapes of integrands of $\mathcal{P}_{\rm C}(k)$

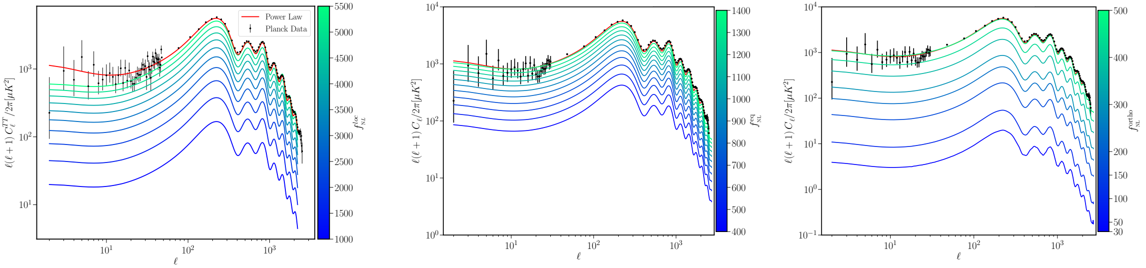


Shapes of integrand involved in computing $\mathcal{P}_{\rm C}(k)$ for the Starobinsky model.

Conventional templates and corrections



Templates of $f_{\rm NL}$



CMB angular spectra due to respective $\mathcal{P}_{\rm C}(k)$