

Stochastic fluctuations in relativistic astrophysics

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The Langevin equation formalism I

Equation of motion (Brownian) with a stochastic source having a gaussian distribution.

$$\frac{dv}{dt} + \gamma v(t) = \Gamma(t) \quad (1)$$

$$\langle \Gamma(t) \rangle = 0, \langle \Gamma(t)\Gamma(t') \rangle = q\delta(t - t')$$

Equation of motion having a non-gaussian distribution

$$\frac{dv}{dt} + \int \gamma(t - t')v(t')dt' = \Gamma(t) \quad (2)$$

The Langevin equation formalism II

Formal solution given by

$$v(t) = v_0 e^{-\gamma t} + \int_0^t e^{-\gamma(t-t')} \Gamma(t') dt' \quad (3)$$

$$\langle v(t_1)v(t_2) \rangle = v_0^2 e^{-\gamma(t_1+t_2)} + \int_0^{t_1} \int_0^{t_2} e^{-\gamma(t_1+t_2-t'_1-t'_2)} q \delta(t'_1 - t'_2) dt'_1 dt'_2. \quad (4)$$

$$\langle E \rangle = \frac{1}{2} m v[t]^2 = \frac{1}{2} m \frac{q}{(2\gamma)} = \frac{1}{2} kT \quad (5)$$

Significance of modelling for and studying stochastic processes

- ▶ The theory of stochastic fluctuations accounts for randomness and irregularities in natural sciences, finance and mathematics. (Nobel Prize in 2021).
- ▶ Equilibrium and non-equilibrium statistical physics is based stochastic correlations of physical variables.
- ▶ Thermal, dynamical and kinetic theories at mesoscopic scales take into account the randomness and fluctuations for studying transport properties.
- ▶ Quantum physics inherently has quantum fluctuations to account for, while classical stochastic fluctuations are either thermal in nature, or due to complex mechanical and non-linear phenomena.
- ▶ Turbulences in fluids are characterized by fluctuations in fluid variables, mainly described in terms of velocity fluctuations.
- ▶ Early Universe cosmology based on quantum fluctuations.
- ▶ High energy physics in lab deals with relativistic fluids and fluctuations.

Stochastic processes in astrophysics and cosmology

- ▶ The regular stochastic processes formalism is based on fluctuations of physical variables w.r.t "time" Say a r.v defined by $X(t)$ with a probability distribution $P(X)$, is a random, w.r.t "time".
- ▶ Roughness can be defined with a variable r.v $X(x, y)$ with is random w.r.t x, y coordinates.
- ▶ Framework for stochastic processes on a spacetime background needed for applications in relativistic astrophysics. (i) Quantum field fluctuations on a spacetime background, are defined statistically for the fields $\hat{\phi}(x)$ as $\hat{\phi}(x) - \langle \hat{\phi}(x) \rangle$. (ii) In a 1 + 3 split in classical gravity, stochasticity can be defined w.r.t time in the evolution of the gravitating object.

Semiclassical stochastic gravity

The theory of semiclassical stochastic gravity is based on the semiclassical Einstein-Langevin equation.

- ▶ Can be obtained in an axiomatic way or from the influence functional method.

$$G_{ab}[g+h] + \Lambda(g_{ab} + h_{ab}) - 2(\alpha A_{ab} + \beta B_{ab})[g+h] = 8\pi G(\langle \hat{T}_{ab}^R[g+h] \rangle + \tau_{ab}[g]) \quad (6)$$

with

- ▶ $\langle \tau_{ab}[g; x] \rangle_s = 0$, $\langle \tau_{ab}[g; x] \tau_{cd}[g; y] \rangle_s = N_{abcd}[g; x, y]$
- ▶ $\nabla^a \tau_{ab}[g; x] = 0$, $\nabla_x^a N_{abcd}(x, y) = \nabla_x^b N_{abcd}(x, y) = \nabla_y^c N_{abcd}(x, y) = \nabla_y^d N_{abcd}(x, y) = 0$
- ▶ Is a first order extension to semiclassical Einstein's equation of semiclassical gravity and lowest level representation of stochastic gravity .
- ▶ From the statistical average of the E-L equation $g_{ab} + \langle h_{ab} \rangle_s$, must be a solution of the equation linearized around the background g_{ab} .

Connections with quantum gravity

- ▶ The two point correlation functions for the metric perturbations correspond exactly to the symmetrized two point correlation functions for quantum metric perturbations in large N expansion i.e the quantum theory describing the interaction of gravitational field with N arbitrary free fields and expanded in powers of $1/N$. To leading order for the graviton propagator one finds that

$$\langle \{ \hat{h}_{ab}(x), \hat{h}_{cd}(y) \} \rangle = 2 \langle h_{ab}(x) h_{cd}(y) \rangle_s \quad (7)$$

- ▶ One can relate correlations in quantum fields to coherence in quantum gravity. This stems from the self consistency in the backreaction equations for matter and gravity sectors. Stochastic gravity captures partial coherence in quantum gravity via the correlations in quantum fields.
- ▶ The Einstein Langevin equation is a low energy (partial) representation of the complete theory of quantum gravity and fields.

The semiclassical Noise Kernel

The Noise Kernel bitensor is defined as

$$4N_{abc'd'}(x, y) = \frac{1}{2} \langle \{\hat{t}_{ab}(x), \hat{t}_{c'd'}(y)\} \rangle \quad (8)$$

where $\hat{t}_{ab}(x) = \hat{T}_{ab}(x) - \langle \hat{T}_{ab}(x) \rangle$.

Which defines a real classical Gaussian stochastic symmetric tensor field τ_{ab} , such that

$$\langle \tau_{ab} \rangle_{\tau} = 0, \langle \tau_{ab}(x) \tau_{cd}(y) \rangle_{\tau} = N_{abc'd'}(x, y) \quad (9)$$

. Thus

$$\langle \dots \rangle_s = \langle \dots \rangle_q \quad (10)$$

Free classical scalar field action

$$S[\phi] = -\frac{1}{2} \int d^4x \sqrt{-g} [g^{ab} \nabla_a \phi \nabla_b \phi + (m^2 + \xi R) \phi^2] \quad (11)$$

The classical stress tensor is

$$T_{ab}(x) = \frac{2}{\sqrt{g}} \frac{\delta S[\phi]}{\delta g^{ab}(x)} \quad (12)$$

$$\begin{aligned} &= \nabla^a \phi \nabla^b \phi - \frac{1}{2} g^{ab} (\nabla^c \phi \nabla_c \phi + m^2 \phi^2) + \\ &\xi (g^{ab} \square - \nabla^a \nabla^b + G^{ab}) \phi^2 \end{aligned} \quad (13)$$

Raise $\phi \rightarrow \hat{\phi}$

$$\begin{aligned} 8N_{abc'd'}(x, y) &= \langle \{ \hat{T}_{ab}(x) - \langle T_{ab}(x) \rangle, \hat{T}_{c'd'}(y) - \langle \hat{T}_{c'd'}(y) \rangle \} \rangle \\ &= \langle \{ \hat{T}_{ab}(x), \hat{T}_{c'd'}(y) \} \rangle - 2 \langle \hat{T}_{ab}(x) \rangle \langle \hat{T}_{c'd'}(y) \rangle \end{aligned} \quad (14)$$

using point separation method for ill-defined operators like $\hat{\phi}^2(x) \rightarrow \hat{\phi}(x)\hat{\phi}(x')$ and Wightmann functions, defined by

$$G_+(x, y) = \langle \hat{\phi}(x)\hat{\phi}(y) \rangle \quad (15)$$

the noise kernel has the following explicit form

$$N_{abc'd'}(x, y) = \tilde{N}_{abcd}(x, y) + g_{ab}\tilde{N}'_{c'd'}(x, y) + g_{c'd'}\tilde{N}'_{ab} + g_{ab}g_{c'd'}\tilde{N}(x, y) \quad (16)$$

$$\begin{aligned}
8\tilde{N}_{abc'd'} &= (1 - 2\xi)^2(G_{;c'b}G_{;d'a} + G_{;c'a}G_{;d'b}) + 4\xi^2(G_{;c'd'}G_{;ab} + GG_{;abc'd'}) \\
&\quad - 2\xi(1 - 2\xi)(G_{;b}G_{;c'd'} + G_{;a}G_{;c'd'} + G_{;d'}G_{;abc'} + G_{;c'}G_{;abd'}) \\
&\quad 2\xi(1 - 2\xi)(G_{;a}G_{;b}R_{c'd'} + G_{;c'}G_{;d'}R_{ab}) \\
&\quad - 4\xi^2(G_{;ab}R_{c'd'} + G_{;c'd'}R_{ab})G + 2\xi^2R_{c'd'}R_{ab}G^2
\end{aligned} \tag{17}$$

$$\begin{aligned}
8\tilde{N}'_{ab} &= 2(1 - 2\xi)[(2\xi - \frac{1}{2})G_{;p'b}G_{;p'a} + \xi(G_{;b}G_{;p'a})] \\
&\quad - 4\xi[(2\xi - \frac{1}{2})G_{;p'}G_{;abp'} + \xi(G_{;p'}G_{;ab} + GG_{;abp'})] \\
&\quad - (m^2 + \xi R')[(1 - 2\xi)G_{;a}G_{;b} - 2G\xi G_{;ab}] \\
&\quad 2\xi[(2\xi - \frac{1}{2})G_{;p'}G_{;p'} + 2G\xi G_{;p'}]R_{ab} - (m^2 + \xi R')\xi R_{ab}G^2
\end{aligned} \tag{18}$$

$$\begin{aligned}
8\tilde{N} &= 2(2\xi - \frac{1}{2})^2G_{;p'q}G_{;p'q} + 4\xi^2(G_{;p'}G_{;q} + GG_{;p'q}) \\
&\quad + 4\xi(2\xi - \frac{1}{2})(G_{;p}G_{;q'} + G_{;q}G_{;p'}) \\
&\quad - (2\xi - \frac{1}{2})[(m^2 + \xi R)G_{;p'}G_{;p'} + (m^2 + \xi R')G_{;p}G_{;p}] \\
&\quad - 2\xi[(m^2 + \xi R)G_{;p'} + (m^2 + \xi R')G_{;p}]G \\
&\quad + \frac{1}{2}(m^2 + \xi R)(m^2 + \xi R')G^2
\end{aligned} \tag{19}$$

The dissipation kernel

$$\langle \hat{T}^{ab}[g+h, x] \rangle = \langle \hat{T}^{ab}[g, x] \rangle + \langle \hat{T}^{(1)ab}[g, h; x] \rangle - \frac{1}{2} \int dy \sqrt{-g(y)} H^{abcd}[g; x, y] h_{cd}(y) + \mathcal{O}(h^2) \quad (20)$$

where $H^{abcd}(x, y) = H_S^{abcd}(x, y) + H_A^{abcd}(x, y)$. The antisymmetric part gives the dissipation kernel.

A fluctuation dissipation relation takes the form:

$$N^{abcd}(x, x') = \int K(x', y) H_A^{abcd}(y, y') d^4 y \quad (21)$$

where $K(x', y)$ is the fluctuation-dissipation kernel, which connects the two.

Methods of obtaining the semiclassical Einstein-Langevin equation

- ▶ Axiomatic
- ▶ Influence functional method
- ▶ Linear response theory (open to developments)

The three pillars on which this theory stands:

- ▶ General Relativity
- ▶ quantum fields
- ▶ non-equilibrium statistical physics

Valid and robust for sub-Planckian physics but expected to break down at Planck scale.

The idea of a classical analogue of Einstein-Langevin equation and its applications

- ▶ How to define classical fluctuations in gravitating systems in strong gravity regions.
- ▶ Classical fluctuations on a spacetime structure.
- ▶ Stochasticity or randomness defined w.r.t space + time variables in general a new conceptual idea. This can be used for coarse graining effects in strong gravity or relativistic regime.
- ▶ The formalism has to be based on specific applications.
- ▶ Origin of stochastic fluctuations can be thermal and non-thermal.
- ▶ This leads to scope for developing equilibrium and non-equilibrium (dynamical) statistical analysis based on first principles.
- ▶ New range of scales in relativistic astrophysics to be probed at mesoscopic range (namely sub-hydro mesoscopic scales).

The mesoscopic scales in relativistic astrophysics I

- ▶ Interest in mesoscopic scales in asteroseismology is comparatively new.
- ▶ Pinning Effects, connections with pulsar glitches. ¹
The quantum vortex filaments store quantized angular momentum and are pinned to nuclear sites. In a rotating configuration these are suddenly released at instances and affect the rotation giving rise to glitches.
- ▶ Fluctuations in relativistic fluids. ²
- ▶ Turbulence in relativistic fluids as source of stochastic effects

The mesoscopic scales in relativistic astrophysics II

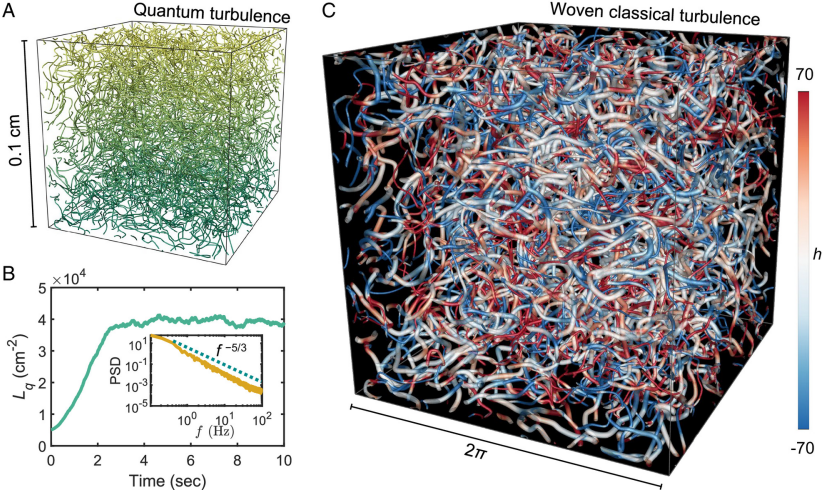


Figure: Turbulence: vortex formation

The mesoscopic scales in relativistic astrophysics III

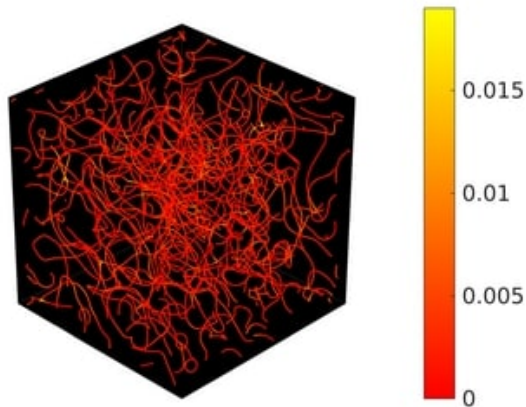


Figure: Quantum Turbulence: vortex tangle

¹S.Seveso., P.M.Pizzochero, F.Grill, B.Haskell, MNRAS, 455 (4) 2016

²Pavel Kovtun, J.Phys.A Math: Theor. 45 (2012) 473001

Perturbations in relativistic stars and Asteroseismology

- ▶ Perturbed Einstein's equations

$$\delta G_{ab}(x) = 8\pi\delta T_{ab}(x)$$

form the base of asteroseismology. Metric perturbation $h_{ab}(x)$, fluid Lagrangian displacement vector $\xi_a(x)$.

- ▶ These perturbations are deterministic, perturbing agency often considered to lie outside the relativistic star.
- ▶ The mode analysis of the frequency spectrum based on $\delta\nabla_a T^{ab} = 0$ gives information of the structure of the star. ³
- ▶ Equation of state for dense matter that the relativistic stars are composed of is an open and active area of interest.

³Schutz, Kip Thorne

A New Linear Response Relation and classical Einstein-Langevin Formalism

General form of LRT,

$$y(t) = \int_{-\infty}^t dt' \chi(t-t') j(t') + \dots \quad (22)$$

Proposed for strong gravity regions with fluid matter,

$$F[h; x] = \int K(x, x') f[\xi, x'] d^4 x' \quad (23)$$

Extended to the classical Einstein's equation,

$$\delta G_{ab}[h; x] - 8\pi \delta T_{ab}[h; x] = 8\pi \int K(x, x') \delta T_{ab}[\xi, x'] d^4 x' \quad (24)$$

for $K(x, x') = \delta(x, x')$ we assume the standard perturbed Einstein's equations and results in asteroseismology. Also it is necessary that $\nabla_x K(x, x') = 0$, i.e covariantly conserved w.r.t the background metric at x . One similar example is the Synge world function $\sigma(x, x')$.

A New Linear Response Relation and classical Einstein-Langevin Formalism II

While the classical Einstein-Langevin equation based on this linear response relation is given by

$$\delta G_{ab}[h; x] - 8\pi\delta T_{ab}[h; x] - 8\pi \int K(x, x')\delta T_{ab}[\xi, x']d^4x' = \tau_{ab}[g, x] \quad (25)$$

- ▶ This is a phenomenological proposal for the cases where perturbations are induced due to the noise term in the matter fields and the equation is mathematical inhomogeneous.
- ▶ Such a noise which is internal to the matter fields of the gravitating body is expected also to have non-thermal mechanical origin due to turbulences and eddies in the fluid which are naturally connected with dynamical fluctuating hydrodynamic or kinetic variables. (new concept altogether !)
- ▶ Thus it enables one to probe the newly defined sub-hydro mesoscopic scales in exotic matter of compact objects with this new theoretical formalism based on a first principles approach.
- ▶ The stochastic noise term, $\tau_{ab}(x) = \delta_s T_{ab}(x)$.

Applications: Relativistic stars

Spherically symmetric relativistic star,

$$ds^2 = -e^{2\nu(r)} dt^2 + e^{2\lambda(r)} dr^2 + r^2 d\Omega^2 \quad (26)$$

The stress-energy tensor for a perfect fluid is given by

$$T_{ab} = (\epsilon + p)u_a u_b + g_{ab}p \quad (27)$$

- ▶ We take perturbations around equilibrium configuration for mathematical simplicity of the first toy model in order to get qualitatively new closed form analytical results.
- ▶ Neutron stars are often modelled by perfect fluid. In order to explore the dense cold matter, perturbations to this model are considered important.
- ▶ The stochastic fluctuations that can be present in the cold dense matter can arise due to various non-thermal effects, like a coarse grained structure of the dense matter at mesoscopic scales and various mechanical effects which are averaged out at macro scales.

Model with velocity fluctuations I

⁴ Noise term due to 3-velocity fluctuations (radial dependence) ,

$$\tau_r^t(r, t) = 8\pi e^{(\lambda - \nu)} (\epsilon + p) \delta_s v(t, r) \quad (28)$$

A simple form of the response kernel chosen in specialized coordinates, can be taken, $K(x, x') = K_1(t - t') K_2(\delta(r, r') \delta(\theta, \theta') \delta(\phi, \phi'))$. Then, the solutions of the Einstein-Langevin equations for the toy model give,

$$\begin{aligned} \delta\lambda(r, t) = & h_1(r) \int \{ m_2(r') \int \delta_s v(r', t') dt' \} dr' - \\ & h_2(r) \int \delta_s v(r, t') dt' \end{aligned} \quad (29)$$

$$\delta\nu(r, t) = \int \int a_1(r') m_2(r'') \delta_s v(r'', t') dt' dr'' dr' + \int \int a_2(r') \delta v(r', t') dt' dr' \quad (30)$$

Two point correlations act as the building blocks for doing equilibrium and

Model with velocity fluctuations II

non-equilibrium statistical physics of the linearized perturbations (one can go to non-linear regime also with the Langevin formalism).

$$\begin{aligned} \langle \xi(r_1, t_1) \xi(r_2, t_2) \rangle &= m_1(r_1) m_1(r_2) \int \{ m_2(r'_1) m_2(r'_2) \\ &\int \langle \delta_s v(r'_1, t'_1) \delta_s v(r'_2, t'_2) \rangle dt'_1 dt'_2 \} dr'_1 dr'_2 \end{aligned} \quad (31)$$

$$\begin{aligned} \langle \delta \lambda(r_1, t_1) \delta \lambda(r_2, t_2) \rangle &= h_1(r_1) h_1(r_2) \int \{ m_2(r'_1) m_2(r'_2) \int \int \langle \delta_s v(r'_1, t'_1) \delta_s v(r'_2, t'_2) \rangle dt'_1 dt'_2 \} \\ &dr'_1 dr'_2 + h_2(r_1) h_2(r_2) \int \langle \delta_s v(r_1, t'_1) \delta_s v(r_2, t'_2) \rangle dt'_1 dt'_2 - h_1(r_1) h_2(r_2) \\ &\int \{ m_2(r'_1) \int \langle \delta_s v(r'_1, t'_1) \delta_s v(r_2, t'_2) \rangle dt'_1 dt'_2 \} dr'_1 - h_2(r_1) h_1(r_2) \\ &\int \{ m_2(r'_2) \int \langle \delta_s v(r_1, t'_1) \delta_s v(r'_2, t'_2) \rangle dt'_1 dt'_2 \} dr'_2 \end{aligned} \quad (32)$$

Model with velocity fluctuations III

$$\begin{aligned}\langle \delta\nu(r_1, t_1)\delta\nu(r_2, t_2) \rangle &= \int a_1(r'_1)a(r'_2)m_2(r''_1)m_2(r''_2)\langle \delta_s\nu(r'_1, t'_1)\delta_s\nu(r'_2, t'_2) \rangle dt'_1 dt'_2 \\ &\quad dr''_1 dr''_2 dr'_1 dr'_2 + \int a_2(r'_1)a_2(r'_2)\langle \delta_s\nu(r'_1, t'_1)\delta_s\nu(r'_2, t'_2) \rangle dt'_1 dt'_2 dr'_1 dr'_2 \\ &\quad + \int a_1(r'_1)a_2(r'_1)m_2(r''_1)\langle \delta_s\nu(r''_1, t'_1)\delta_s\nu(r'_2, t'_2) \rangle dt'_1 dt'_2 dr''_1 dr'_1 dr'_2 \\ &\quad + \int a_2(r'_1)a_1(r'_1)m_2(r''_2)\langle \delta_s\nu(r'_1, t'_1)\delta_s\nu(r''_2, t'_2) \rangle dt'_1 dt'_2 dr'_1 dr''_2 dr'_2 \quad (33)\end{aligned}$$

Model of noise with pressure and energy density fluctuations I

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$$\begin{aligned}\tau_t^t(r)e^{i\omega_r t} &= \delta_s \epsilon(r)e^{i\omega_r t} \\ \tau_r^r(r)e^{i\omega_r t} &= \tau_\theta^\theta(r)e^{i\omega_r t} = \tau_\phi^\phi(r)e^{i\omega_r t} = \delta_s p(r)e^{i\omega_r t}\end{aligned}\quad (34)$$

Solution of the Einstein-Langevin equation gives,

$$\delta\lambda(r, t) = 4\pi e^{\nu-\lambda} r \int_{r_1}^r \frac{e^{3\lambda(r')-\nu(r')}}{[1 + \tilde{K}(\gamma_{r'})]} \delta_s \epsilon(r') e^{i\omega_{r'} t} dr' \quad (35)$$

$$\xi(r, t) = 4\pi(\nu'(r) + \lambda'(r))e^{\nu-\lambda} r \int_{r_1}^r \frac{e^{3\lambda(r')-\nu(r')}}{[1 + \tilde{K}(\gamma_{r'})]} \delta_s \epsilon(r') e^{i\omega_{r'} t} dr' \quad (36)$$

$$\begin{aligned}\delta\nu(r, t) &= \int Y_1(r', \gamma_{r'}) \left(\int \frac{e^{3\lambda(r'')-\nu(r'')}}{[1 + \tilde{K}(\gamma_{r''])]} \delta_s \epsilon(r'') e^{i\omega_{r''} t} dr'' \right) dr' + \\ &\quad \int Y_2(r', \gamma_{r'}) \delta_s \epsilon(r') e^{i\omega_{r'} t} dr' + \int 4\pi r' e^{2\lambda(r')} \delta_s p(r') e^{i\omega_{r'} t} dr' \quad (37)\end{aligned}$$

where

Model of noise with pressure and energy density fluctuations II

$$\begin{aligned}
 Y_1(r, \gamma_r) &= -4\pi[\tilde{K}(\gamma_r)C_s^2\{2e^{\nu-\lambda} + (1 + e^{\nu-\lambda})r(\nu' - \lambda')\} \\
 &\quad + (\lambda' + \nu')C_s^2r + 2\nu'r + e^{\nu-\lambda}] \\
 Y_2(r, \gamma_r) &= -4\pi r \frac{e^{2\lambda(r)}\tilde{K}(\gamma_r)}{(1 + \tilde{K}(\gamma_r))} C_s^2
 \end{aligned} \tag{38}$$

with $C_s = \sqrt{\frac{dp}{d\epsilon}}$, show the dependence on the speed of sound in dense matter.

Two point correlations

$$\begin{aligned}
 \langle \delta_s \lambda(r, t) \delta_s \lambda^*(\bar{r}, \bar{t}) \rangle_s &= 8\pi^2 e^{\nu(r) + \nu(\bar{r} - \lambda(r) - \lambda(\bar{r}))} \int_{r_1}^r \int_{\bar{r}_1}^{\bar{r}} \frac{e^{3(\lambda(r') + \lambda(\bar{r}')) - \nu(r') - \nu(\bar{r}')}}{[1 + \tilde{K}(\gamma')][1 + \tilde{K}(\bar{\gamma}')]}} \\
 &\quad \langle \delta_s \epsilon(r') \delta_s \epsilon(\bar{r}') \rangle_s \langle e^{i(\omega_{r'} t - \omega_{\bar{r}'} \bar{t})} \rangle_s dr' d\bar{r}' \tag{39}
 \end{aligned}$$

$$\begin{aligned}
 \langle \xi(r, t) \xi^*(\bar{r}, \bar{t}) \rangle_s &= 8\pi^2 (\nu'(r) + \lambda'(r)) (\nu'(\bar{r}) + \lambda'(\bar{r})) e^{\nu(r) + \nu(\bar{r} - \lambda(r) - \lambda(\bar{r}))} \\
 &\quad \int_{r_1}^r \int_{\bar{r}_1}^{\bar{r}} \frac{e^{3(\lambda(r') + \lambda(\bar{r}')) - \nu(r') - \nu(\bar{r}')}}{[1 + \tilde{K}(\gamma')][1 + \tilde{K}(\bar{\gamma}')]}} \tag{40}
 \end{aligned}$$

$$\langle \delta_s \epsilon(r') \delta_s \epsilon(\bar{r}') \rangle_s \langle e^{i(\omega_{r'} t - \omega_{\bar{r}'} \bar{t})} \rangle_s dr' d\bar{r}' \tag{41}$$

Perturbed TOV equations I

A perturbed TOV relation is due to the contributions from the generalized stochastic pressure perturbations which are induced due to Langevin noise.

The mode analysis for stochastic perturbations of relativistic stars is based on,

$\nabla_a \delta T_b^a(x) = 0$, for which the component

$\nabla_a \delta T_1^a = 0$ gives,

$$\delta p' = -\nu'(\delta p + \delta \epsilon) - e^{2(\lambda - \nu)}(\epsilon + p)\ddot{\xi}(r, t) \quad (42)$$

The above equation can be written as

$$\delta p(r, t) = - \int \{ \nu'(r')(\delta p(r') + \delta \epsilon(r')) + e^{2(\lambda(r') - \nu(r'))}(\epsilon(r') + p(r'))\ddot{\xi}(r', t) \} dr' \quad (43)$$

From the standard TOV equations (unperturbed)

Perturbed TOV equations II

$$\rho(r) = - \int \nu'(r')(\epsilon(r') + \rho(r')) dr' \quad (44)$$

The root mean square then can be easily obtained and put in as an additive part in the regular TOV equation to get,

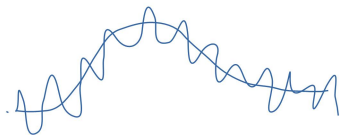
$$\rho(r) + (\delta\rho(r, t))_{rms} = - \int \nu'(r')(\epsilon(r') + \rho(r')) dr' + \sqrt{\lim_{r_1, t_1 \rightarrow r_2, t_2} < \delta\rho(r_1, t_1) \delta\rho(r_2, t_2) >} \quad (45)$$

To solve this perturbed TOV equations one needs to consider an equations of state like $\rho(r) = K_2 \epsilon^\Gamma(r)$ and its perturbation $\delta\rho(r, t) = K_2 \Gamma (\delta\epsilon(r, t))^{\Gamma-1}(r, t)$. The quantity on the lhs of equation (45) gives the new near equilibrium dynamical pressure in the fluid .

Resulting Views I

- ▶ The two point and higher correlations of the perturbations form the base for a statistical analysis of perturbations.
- ▶ Equilibrium and non-equilibrium properties can be analysed.
- ▶ Extended non-local structure inside the dense matter is possible to probe with this new formalism.
- ▶ Turbulence and vortex filament structure inside the dense matter superfluid can be investigated and modelled using the velocity fluctuations in a statistical physics description.

Resulting Views II



Challenges and scope of the research program

- ▶ To build up a sub-hydro mesoscopic theory for dense compact matter of relativistic stars.
- ▶ To investigate in detail the different velocity, pressure and energy density fluctuations and their correlations in the dense matter fluid of neutron star.
- ▶ Numerical modelling of stochasticity and generalized stochasticity is a big challenge but with potential for vast scope in research in applications of general relativity.
- ▶ Theoretical and foundational level mathematical developments using basic new ideas from complex systems, statistical physics, fluid dynamics need to be extended for a spacetime structure, with focus on dense matter star applications.

Further directions

- ▶ Obtain formally a classical analogue of the Einstein-Langevin equation starting from a variational principle in general relativity.
- ▶ Formalism for relativistic fluids with stochastic fluctuations (thermal and non-thermal turbulent flows) from the first principles.
- ▶ Incorporating a source term in a perturbed Reynolds' Navier-Stokes equations for non-relativistic fluids and two and three fluid systems for applications in fluid mechanics.

Concluding Remarks

- ▶ A source of perturbations for relativistic stars can be introduced in the perturbed Einstein's equations, if it lies in the interiors of the star.
- ▶ Such a framework would allow us to probe through the structural properties and matter characteristics at the meso scale for the dense matter fluids, which is yet an unexplored regime.
- ▶ The seeds of generalized fluctuations can be present due to thermal effects, quantum effects retained in the bulk (superfluid) properties in fluids, dynamical or other mechanical effects in the compact dense matter star.
- ▶ Induced perturbations are a cumulative effect of the background stochastic fluctuations. Growth and decay of the perturbations can happen as the stochastic perturbations evolve inside the structure of the relativistic star.
- ▶ With this enhanced framework for mesoscopic scales, dynamical equilibrium and non-equilibrium statistical physics and transport properties can be addressed in the cold dense matter and hot massive stars for the relativistic fluids.
- ▶ The research directions further carry scope for fluctuations in relativistic fluids and turbulence as that of including the large eddies.
- ▶ A fully general relativistic solution of the classical Einstein-Langevin equation is an enormously challenging task, while one can get reasonable results expected for post-newtonian approximations and for relativistic hydrodynamics in the flat spacetime.
- ▶ Of particular interests and further work are the rotating relativistic fluids and compact objects and stochastic modes of oscillations.

Thank You .