Probing Primordial Magnetic Fields with Cosmic Microwave Background Radiation

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IIT Madras, May 17, 2012

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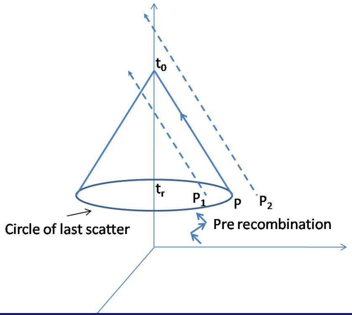
Some basic aspects about our Universe

- Homogeneous, isotropic Universe described by FRW metric.
- Characterized by the scale factor $a(t)$
- Energy density, pressure etc depend on $a(t)$
- Radiation energy density $\rho_r \propto a^{-4}$
- Radiation temperature $T_r \propto a^{-1}$
- Most models a increases with t.
- Temperature of rediation high in the past and cools down with expansion.

Origin of the Cosmic Microwave Background Radiation

- Relic Radiation of an era when the temperature of the constituents of the Universe was very high and matter was ionized
- Ionized matter undergoes significant interaction/scattering with photons
- With expansion the Universe cools
- ions $\longrightarrow$ neutral atoms $\longrightarrow$ photons decouple


t_0
t_r
P_1
P
P_2
Circle of last scatter
Pre recombination

Characteristic Features of the CMBR

- By-and-large preserves the information of the surface of last scatter.
- Small perturbations at the Surface of Last Scatter and later leave characteristic imprints on the CMBR
- Hence, CMBR could be a sensitive probe for the number of physical processes in the early universe.
- How well can CMBR probe the cosmic magnetic fields

Primordial Cosmic Magnetic Field - *Why care* ??

- $\vec{B}$ over galactic scales $\sim \mu$G
  - μGauss $\vec{B}$ observed in galaxies: both coherent & stochastic
  - $\vec{B}$ growth via either dynamo amplification or flux freezing
    $\longrightarrow$ a seed $\vec{B}$ field is required
  - These seed fields may be of primordial origin
- Evidence for equally strong $\vec{B}$ in high redshift ($z \sim 2$)
  [Bernet et al. 08, Kronberg et al. 08]
  - Enough time for dynamo to act?
- FERMI/*LAT* observations of γ-ray halos around AGN
  - ***Detection*** of intergalactic $\vec{B} \approx 10^{-15} G$ [Ando & Kusenko 10]
  - ***Lower*** limit: $\vec{B} \geq 3 \times 10^{-16}$ G on intergalactic $\vec{B}$ [Neronov & Vovk, *Science* 10]

No compelling mechanism yet for origin of strong primordial $\vec{B}$ fields
[e.g. Martin & Yokoyama 08]

Cosmic Magnetic Fields - CMBR connection

1. Arising due to vortical velosity field (in the photon-baryon fluid) due to Lorentz force.

   $\longrightarrow$ CMB Anisotropy spectrum

   $\longrightarrow$ CMB Polarization spectrum

2. Arising from 3-point and 4-point correlation function of density and anisotropic stress tensor of magnetic field.

   $\longrightarrow$ Induces Non-Gaussianity in CMB

Question addressed

Can CMB Polarization power-spectrum, CMB anisotropy power-spectrum and the statistics of CMB anisotropy be used as a probe to study the Cosmic Magnetic Fields?

Aim of the talk:

To show that not only is this possibilty, but it can be a very important probe.

Nature of the Magnetic Field Considered

1. Magnetic Field: Stochastic. Statistically homogeneous and isotropic.
2. Assumed to be a Gaussian Random Field. Statistical properties specified completely by 2-point correlation function.
3. Magnetic field $\longrightarrow$ velocity field
   On scales $> L_G$ (galactic scales) velocities small enough that the magnetic fields do not change.

$$\vec{B}(\vec{x}, t) = \frac{\vec{b}_0(\vec{x})}{a^2(t)}$$

Statistical specification of the Magnetic Field

Field: Gaussian and spectrum specified by
$\langle b_i(\vec{k}) b_j^*(\vec{q}) \rangle = (2\pi)^3 \delta(\vec{k} - \vec{q}) P_{ij}(\vec{k}) M(k)$
$\rightarrow$ Completely determined by $M(k)$

P_{ij} is the projection operator that ensures $\vec{\nabla} \cdot \vec{b}_0 = 0$

$\langle \vec{b}_0 \cdot \vec{b}_0 \rangle = 2 \int \frac{dk}{k} \Delta_b^2(k)$ with $\Delta_b^2 = k^3 M(k)/2\pi^2$

Form of $M(k)$:
$M(k) \propto Ak^n$ with a cutoff at
Alfen wave damping scale

Fixing A: In terms of variance, B_0,
of Magnetic Field at $k_G = 1 h\mathrm{Mpc}^{-1}$

$$\Rightarrow \Delta_b^2(k) = \frac{B_0^2}{2}(n+3)\left(\frac{k}{k_g}\right)^{n+3}$$

Effect of Magnetic Field on the Baryon-Photon Fluid

Action of magnetic field $\longrightarrow$ Lorentz force on the baryon fluid.
$\mathbf{F}_L = (\nabla \times \mathbf{B}_0) \times \mathbf{B}_0/(4\pi a^5)$

$\downarrow$

Perturbations in the velocity field from Euler equations for the Baryon fluid

We consider scales > photon mean-free-path scales.

Viscosity effects due to the photons in diffusion approximation

$$\left(\frac{4}{3}\rho_\gamma + \rho_b\right)\frac{\partial v_i^B}{\partial t} + \left[\frac{\rho_b}{a}\frac{da}{dt} + \frac{k^2\eta}{a^2}\right] v_i^B = \frac{P_{ij}F_j}{4\pi a^5}.$$

'Small' and 'Large' scale limits

Larger than Silk length scales	Smaller than Silk length scales
$k \ll L_S^{-1}$ Damping due to photon diffusion is negligible $v_i^B = G_i D$, where $G_i = 3P_{ij}F_j/[16\pi\rho_0]$ and $D = \tau/(1+S_*)$	$k \gg L_S^{-1}$ Diffusion damping significant $\longrightarrow$ terminal velocity approximation $v_i^B = G_i(\mathbf{k})D$, where $D = (5/k^2 L_\gamma)$

Equating v_i^B in the two cases

$\downarrow$

Transition Scale $k_S \sim [5(1+S_*)/(\tau L_\gamma(\tau))]^{1/2}$.

$$
\begin{aligned}
C_l^{BB} &= 4\pi \frac{(l-1)(l+2)}{l(l+1)} \int_0^\infty \frac{k^2 dk}{2\pi^2} \quad \frac{l(l+1)}{2} \\
&\times < | \int_0^{\tau_0} d\tau g(\tau_0, \tau) (\frac{kL_\gamma(\tau)}{3}) v_B(k,\tau) \\
&\times \frac{j_l(k(\tau_0 - \tau))}{k(\tau_0 - \tau)} |^2 > . \qquad (1)
\end{aligned}
$$

We approximate the visibility function as a Gaussian:
$g(\tau_0, \tau) = (2\pi\sigma^2)^{-1/2} \exp[-(\tau - \tau_*)^2/(2\sigma^2)]$

τ_* is the conformal epoch of “last scattering”
σ measures the width of the LS.

$\Delta T_P^{BB}(l) \equiv [l(l+1)C_l^{BB}/2\pi]^{1/2} T_0$, where $T_0 = 2.728$

'Small' and 'Large' scale limits

Larger than Silk length scales $kL_s < 1$ and $k\sigma < 1$, the

$$\Delta T_P^{BB}(l) = T_0(\frac{\pi}{32})^{1/2} I(k) \frac{k^2 L_\gamma(\tau_*) V_A^2 \tau_*}{3(1+S_*)} \approx 0.4\mu K \left(\frac{B_{-9}}{3}\right)^2 \left(\frac{l}{1000}\right)^2 I(\frac{l}{R_*})$$

Smaller than Silk length scales $kL_S > 1, \; k\sigma > 1 \; kL_\gamma(\tau_*) < 1$

$$\Delta T_P^{BB}(l) = T_0 \frac{\pi^{1/4}}{\sqrt{32}} I(k) \frac{5V_A^2}{3(k\sigma)^{1/2}} \approx 1.2\mu K \left(\frac{B_{-9}}{3}\right)^2 \left(\frac{l}{2000}\right)^{-1/2} I(\frac{l}{R_*}).$$

The mode-coupling integral, $(n \longrightarrow -3, \, n > -3)$ $I^2(k) = \frac{8}{3}(n+3)(\frac{k}{k_G})^{6+2n}$

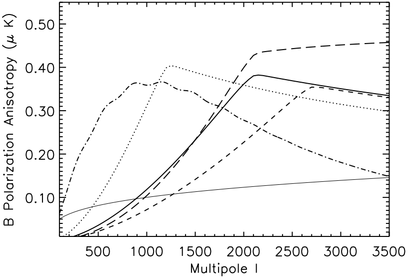
Larger than Silk length scales and $n = -2.9$:

$\Delta T_P^{BB}(l) \sim 0.16 \mu K (l/1000)^{2.1}$

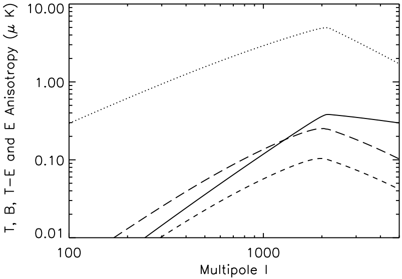
Smaller than Silk length scales and $n = -2.9$:

$\Delta T_P^{BB}(l) \sim 0.51 \mu K (l/2000)^{-0.4}$,
Larger signals possible for $n > -2.9$ at the higher l end.

Results for different models



$B_{-9} = 3$. Bold solid line is for a standard flat, Λ-dominated model, ($\Omega_\Lambda = 0.73$, $\Omega_m = 0.27$, $\Omega_b h^2 = 0.0224$, $h = 0.71$ $n = -2.9$).
The long dashed curve $n = -2.5$,
Short dashed curve $\Omega_b h^2 = 0.03$. The dotted curve : $\Omega_m = 1$ and $\Omega_\Lambda = 0$ $n = -2.9$.



The predicted anisotropy in temperature (dotted line), B-type polarization (solid line), E-type polarization (short dashed line) and T-E cross correlation (long dashed line) up to large $l \sim 5000$ for the standard Λ-CDM model, due to magnetic tangles with a nearly scale invariant spectrum.

Why is CMB-Nongaussianty of special significance for studying Cosmic Magnetic Fields

Inflationary models:
Small fluctuations in the field (and hence, linear order)

$\downarrow$

Gaussian statistics for Fluctuation

$\downarrow$

Gaussian statistics for CMB Temperature Anisotropy

CMB Non-gaussianity only from higher order effects

From Magnetic Fields:
Magnetic Stresses inherently quadratic in $\vec{B}$ field

$\downarrow$

Even for Gaussianity $\vec{B}$ field Magnetic stresses non-gaussian

$\downarrow$

Non-Gausianity in $\vec{B}$ field induced CMB anisotropy

CMB Non-gaussianity even from lowest order orders

Measures of Non-Gaussianity

- Bispectrum $\leftrightarrow$ 3-point correlation function
- Trispectrum $\leftrightarrow$ 4-point correlation function

Here we estimate the bispectrum and trispectrum of the CMBR temperature anisotropy statistics

3-point correlation function

$$\frac{\Delta T(\hat{n})}{T} = \sum_{lm} a_{lm} Y_{lm}(\hat{n})$$

Bispectrum $\leftrightarrow \langle \frac{\Delta T(\hat{n}_1)}{T} \frac{\Delta T(\hat{n}_2)}{T} \frac{\Delta T(\hat{n}_3)}{T} \rangle$

$$B^{m_1 m_2 m_3}_{l_1 l_2 l_3} = \langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} \rangle$$

$\Delta T/T$ from energy density of Magnetic Field

$\Omega_B = | b_0^2(\vec{x}) |^2 /(8\pi\rho_0)$ is the contribution of the Magnetic field towards the density parameter.

In Fourier space,

$$\vec{\Omega}_B(\vec{k}) = \frac{1}{(2\pi)^3} \int d^3s \;\; b_i(\vec{k}+\vec{s}) b_i^*(\vec{s})/(8\pi\rho_0)$$

$$\frac{\Delta T(\hat{n})}{T} \sim 0.03 \; \Omega_B(\vec{x}_0 - \hat{n}D^*)$$

$\hat{n} \longrightarrow$ direction of observation
$D^* \longrightarrow$ angular diameter distance to SLS.
$\vec{x}_0 \longrightarrow$ position vector of the observer.

3-point correlation function

$$\frac{\Delta T(\hat{n})}{T} = \sum_{lm} a_{lm} Y_{lm}(\hat{n})$$

Bispectrum $\leftrightarrow \langle \frac{\Delta T(\hat{n}_1)}{T} \frac{\Delta T(\hat{n}_2)}{T} \frac{\Delta T(\hat{n}_3)}{T} \rangle$

$$B^{m_1 m_2 m_3}_{l_1 l_2 l_3} = \langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} \rangle$$

$$= \mathcal{R}^3 \int \left[\prod_{i=1}^{3} (-i)^{l_i} \frac{d^3 k_i}{2\pi^2} j_{l_i}(k_i D^*) Y^*_{l_i m_i}(\hat{k}_i) \right] \zeta_{123}$$

$$\zeta_{123} = < \hat{\Omega}_B(\vec{k}_1) \hat{\Omega}_B(\vec{k}_2) \hat{\Omega}_B(\vec{k}_2) > .$$

Sachs Wolf contribution in the two Limits: Equilateral Case and Isosceles Case

Equilateral Case

$$l_1(l_1+1)l_2(l_2+1)l_3(l_3+1)b_{l_1 l_2 l_3} \sim 2.3 \times 10^{-23} \left(\frac{n+3}{0.2}\right)^2 \left(\frac{B_{-9}}{3}\right)^6$$

Isosceles Case

$$l_1(l_1+1)l_2(l_2+1)l_3(l_3+1)b_{l_1 l_2 l_3} \sim 1.5 \times 10^{-22} \left(\frac{n+3}{0.2}\right)^2 \left(\frac{B_{-9}}{3}\right)^6$$

with $B_{-9} \equiv (B_0/10^{-9}\mathrm{Gauss})$.

What can we conclude from the Bispectrum Calculated above

- For $B_0 \sim 3$nG, $l_1(l_1+1)l_2(l_2+1)l_3(l_3+1)b_{l_1 l_2 l_3} \sim 10^{-22}$ for a scale invariant magnetic field spectrum.
- This is a new probe of primordial magnetic fields. But only scalar modes included.
- Present limits on bispectrum $\rightarrow$upper limits on $B_0 \sim 35$nG. Limits expected to improve significantly when vector and tensor modes also to be included.

Now Consider Scalar Anisotropic Stress from $\vec{B}$

- Magnetic stress tensor

$$T^i_j(\mathbf{x}) = \frac{1}{4\pi a^4}\left(\frac{1}{2}b_0^2(\mathbf{x})\delta^i_j - b_0^i(\mathbf{x})b_{0j}(\mathbf{x})\right)$$

- in Fourier space

$$S^i_j(\mathbf{k}) = \frac{1}{(2\pi)^3}\int b^i(\mathbf{q})b_j(\mathbf{k}-\mathbf{q})d^3\mathbf{q}$$

$$T^i_j(\mathbf{k}) = \frac{1}{4\pi a^4}\left(\frac{1}{2}S^\alpha_\alpha(\mathbf{k})\delta^i_j - S^i_j(\mathbf{k})\right).$$

- Magnetic perturbations to $T^i_j(\mathbf{k})$

$$T^i_j(\mathbf{k}) = p_\gamma\left(\Delta_B(\mathbf{k})\delta^i_j + \Pi_{B}{}^i_j(\mathbf{k})\right)$$

Scalar Anisotropic Stress $\rightarrow$ Passive Mode

- Assume $\vec{B}$ stresses small compared to total ρ, Π of photons + baryons
- linear perturbations
- scalar, vector, tensor evolve independently
- we focus on the scalar part of $\Pi_{B}{}^{i}_{j}$ as a source of CMB non-Gaussianity
- Scalar Anisotropic perturbations $\Pi_B(\mathbf{k})$ given by projection operator

$$\Pi_B(\mathbf{k}) = -\frac{3}{2}\left(\hat{k}_i\hat{k}_j - \frac{1}{3}\delta_{ij}\right)\Pi_B^{ij}$$

- Neutrinos: also develop scalar anisotropic stress after decoupling
- Prior to neutrino decoupling, $\Pi_B(\mathbf{k})$ only source
- After neutrino decoupling, $\Pi_\nu(\mathbf{k})$ also contributes with equal magnitude and opposite sign: rapid compensation [Lewis 04]

Magnetic anisotropic stress $\Pi_B(\mathbf{k})$ has effect only till neutrino decoupling

Magnetic CMB Anisotropy

$$\frac{\Delta T}{T}(\hat{n}) \simeq -(0.04)\ln\left(\frac{\tau_\nu}{\tau_B}\right)\Pi_B$$

- Spherical harmonic expansion

$$\frac{\Delta T(\hat{n})}{T} = \sum_{lm} a_{lm} Y_{lm}(\hat{n})$$

$$a_{lm} = 4\pi(-i)^l \int \frac{d^3k}{(2\pi)^3}\, \mathcal{R}_p\, \Pi_B(\hat{k})\, j_l(kD^*)\, Y^*_{lm}(\hat{\hat{k}})$$

CMB Bispectrum **Results** for Magnetic Passive Mode

- General configuration approximate evaluation: $l_1(l_1+1)l_3(l_3+1)b_{l_1 l_2 l_3} \approx 6-9\times 10^{-16}$
- using WMAP7 $f_{NL} < 74$ get upper limit $B_0 < 3nG$
- Inflationary bispectrum with $f_{NL} \sim 1$ is $l_1(l_1+1)l_3(l_3+1)b_{l_1 l_2 l_3} \approx 10^{-18}$
- CAVEAT: only Sachs-Wolfe
- CAVEAT: τ_B dependence: But little change

Results from CMB Trispectrum

- More stringent than bispectrum
- Best limits from 4-point correlation function of the magnetic anisotropic stress
- B_0 is less than about 0.7 nG

Conclusions

- Cosmological magnetic fields (CMF) are an interesting possibility: **CMB non-Gaussianity a unique probe of them**
- CMF leaves characteristic imprints on the CMB both in the form of power-spectrum as well as Non-Gaussianity. **We get much stronger B_0 upper limit** from non-gaussianity in the anisotropic stress as compared to Power-spectrum.
- The trispectrum of anisotropic stress gives by far the most stringent limits. (0.7 nG)

This talk based on the following references:

- TRS and K Subramanian
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  Phys. Rev. Lett **87** (2001) 101301
- K Subramanian, TRS and J D Barrow
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  Cosmic Microwave Background Bispectrum from Primordial Magnetic Fields on Large Angular Scales
  Phys Rev Lett, **103** (2009) 081303
- Pranjal Trivedi, K. Subramanian and TRS
  Primordial magnetic field limits from cosmic microwave background bispectrum of magnetic passive scalar modes
  Phys Rev **D82** (2010) 123006
- Pranjal Trivedi, TRS and K. Subramanian.
  Cosmic Microwave Background Trispectrum and Primordial Magnetic Field Limits
  Phys Rev Lett. accepted for publication

Future Work

- Magnetic Tensor and Vector Mode Bispectrum
- NG in CMB Polarization
- Numerical estimates with $\vec{B}$ realizations
- Scale-dependence of NG and estimators cf. PLANCK data