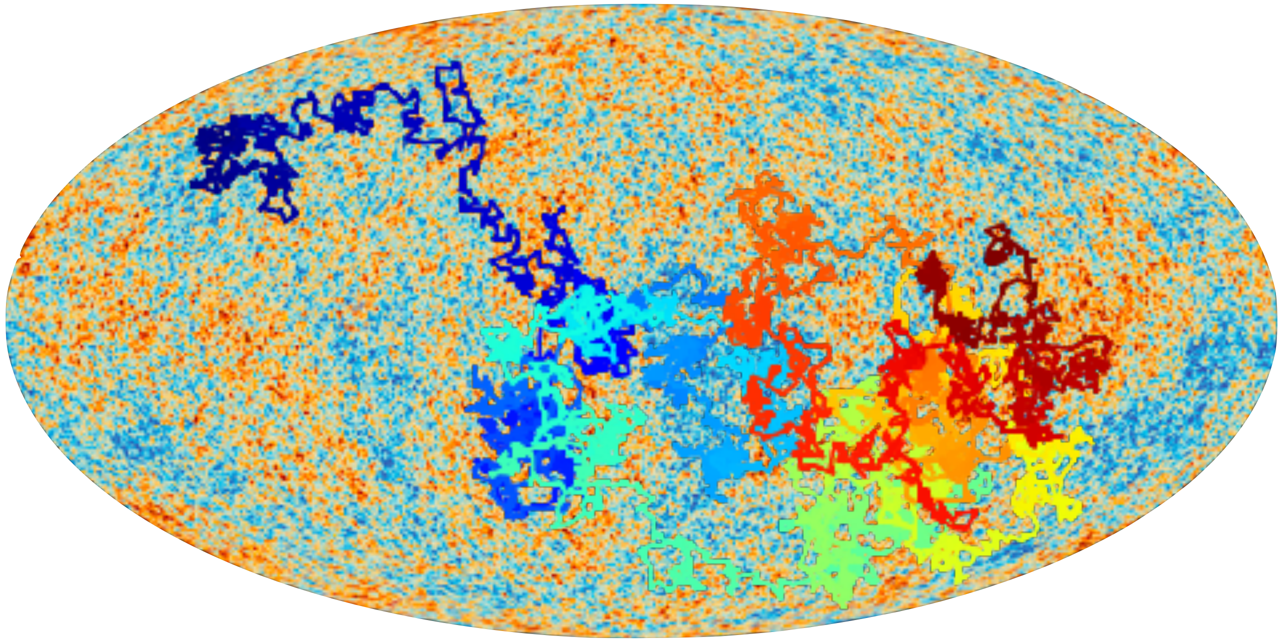
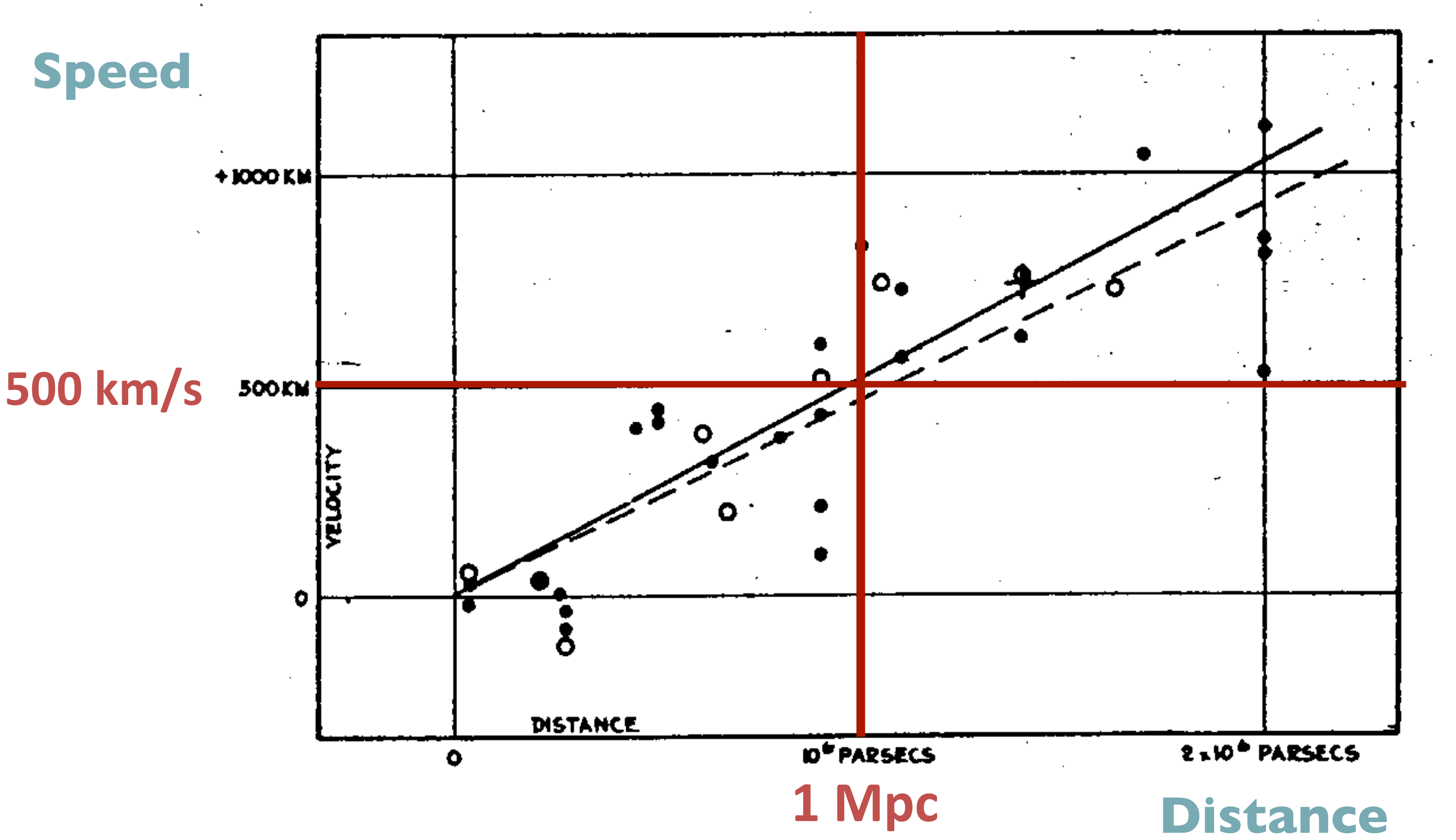
Quantum Diffusion During Cosmic Inflation



Vincent Vennin

Physics Colloquium, Indian Institute of Technology in Madras
8 November 2023

Hubble law (1929)



E. Hubble, Astrophysical Journal, vol. 74, 1929

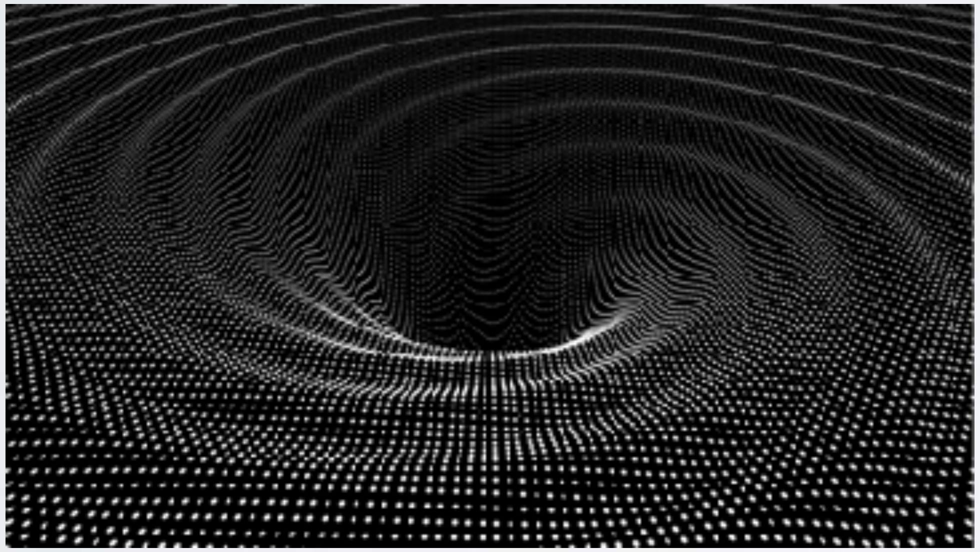
Edwin Hubble (1889-1953)

$$H_0 = 500\,\mathrm{km/s/Mpc}\ ?$$

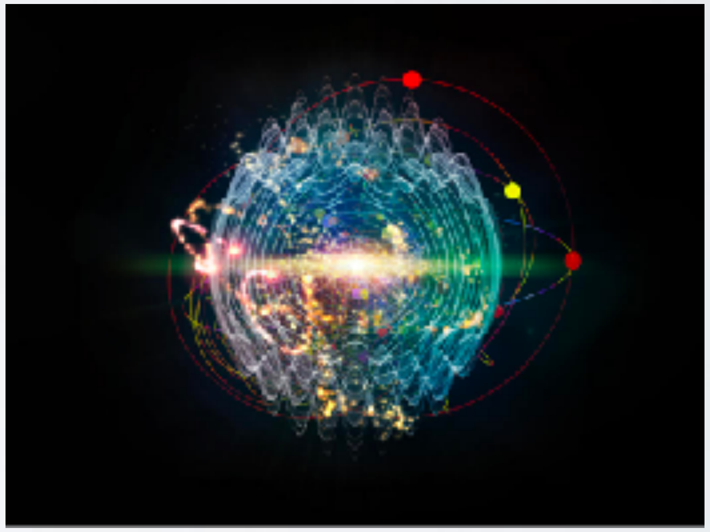
$$v = HD$$

GENERAL RELATIVITY

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

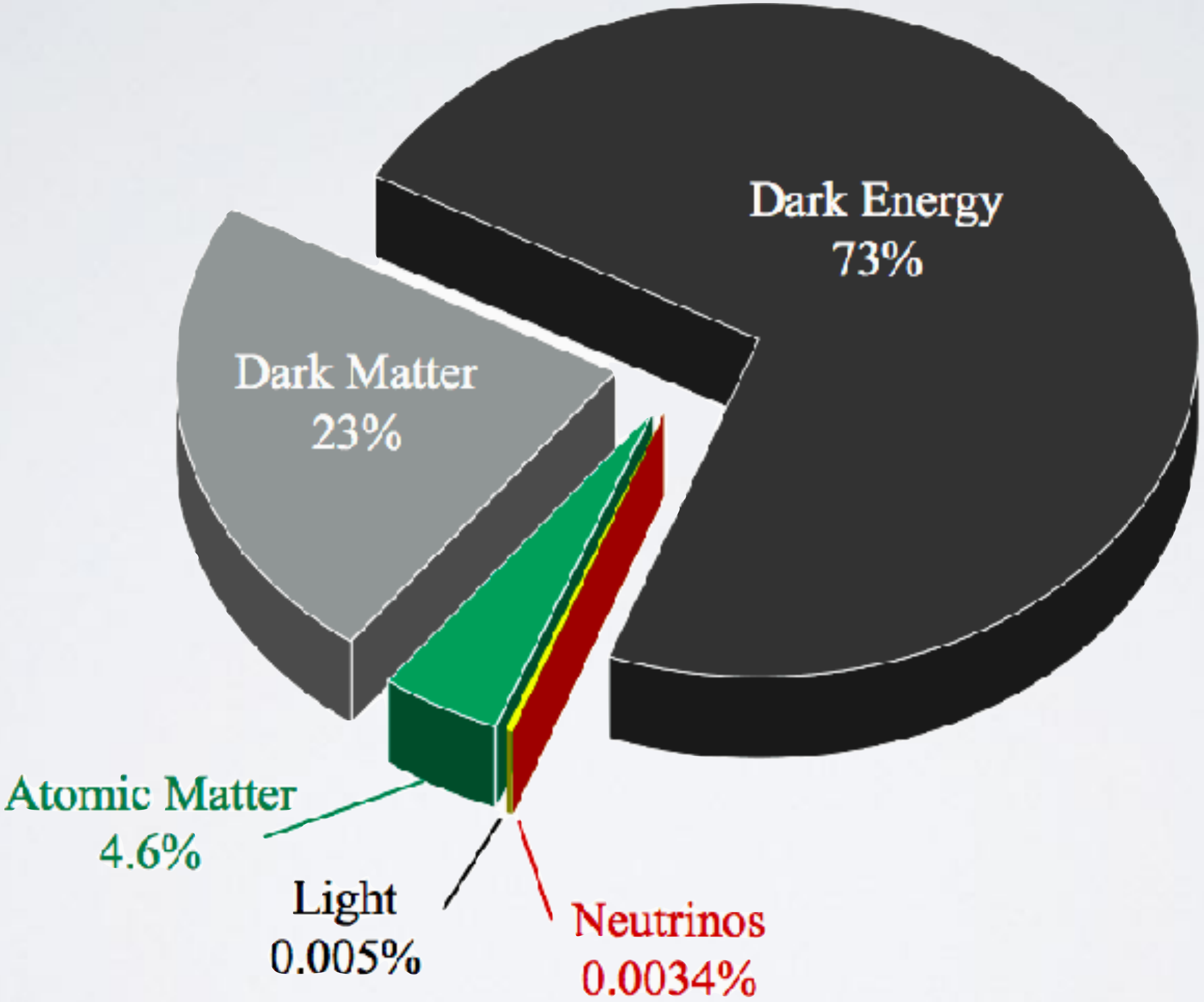


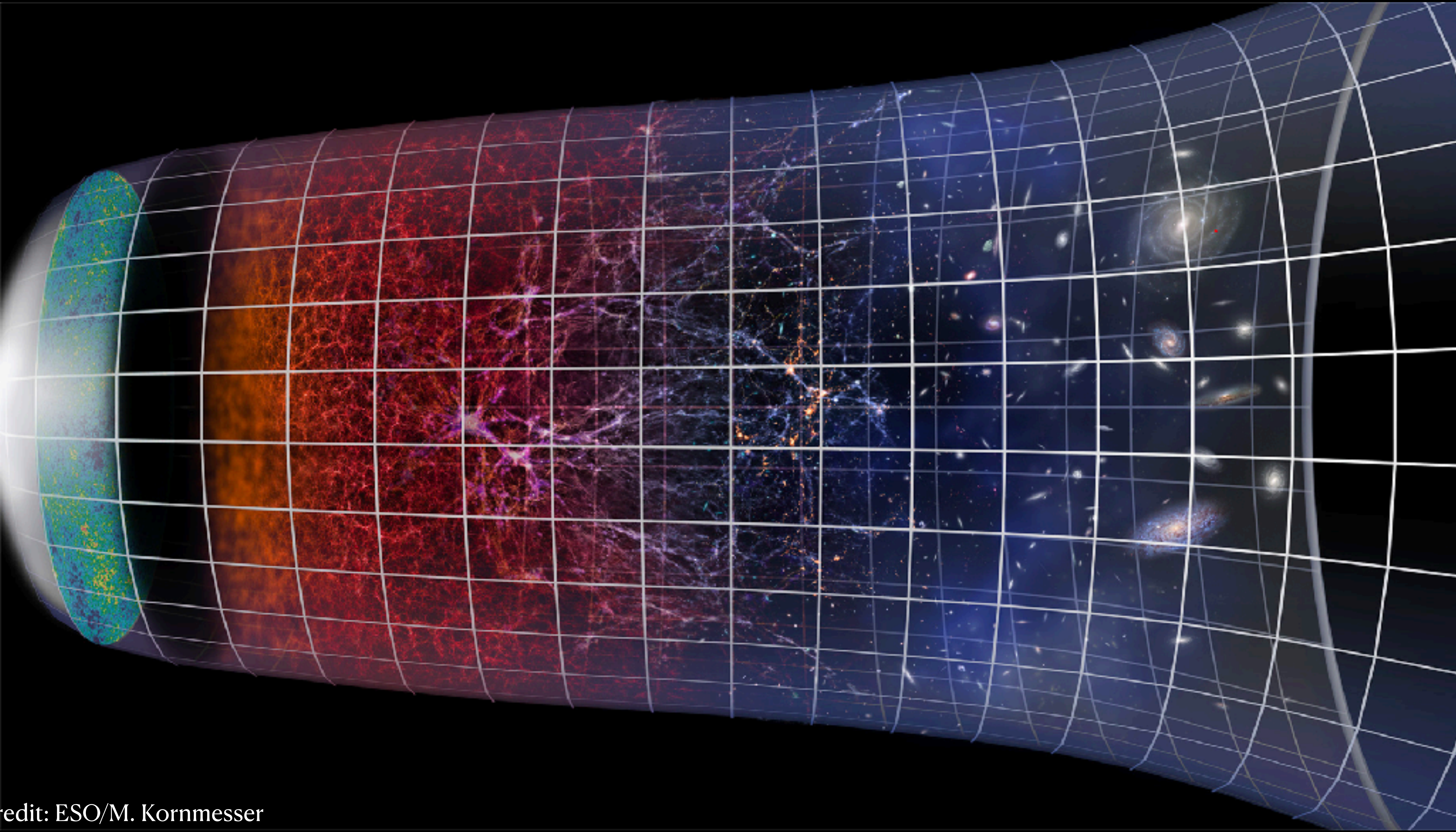
Space-time (curved) Geometry



Matter (particules, fields, fluids, etc)

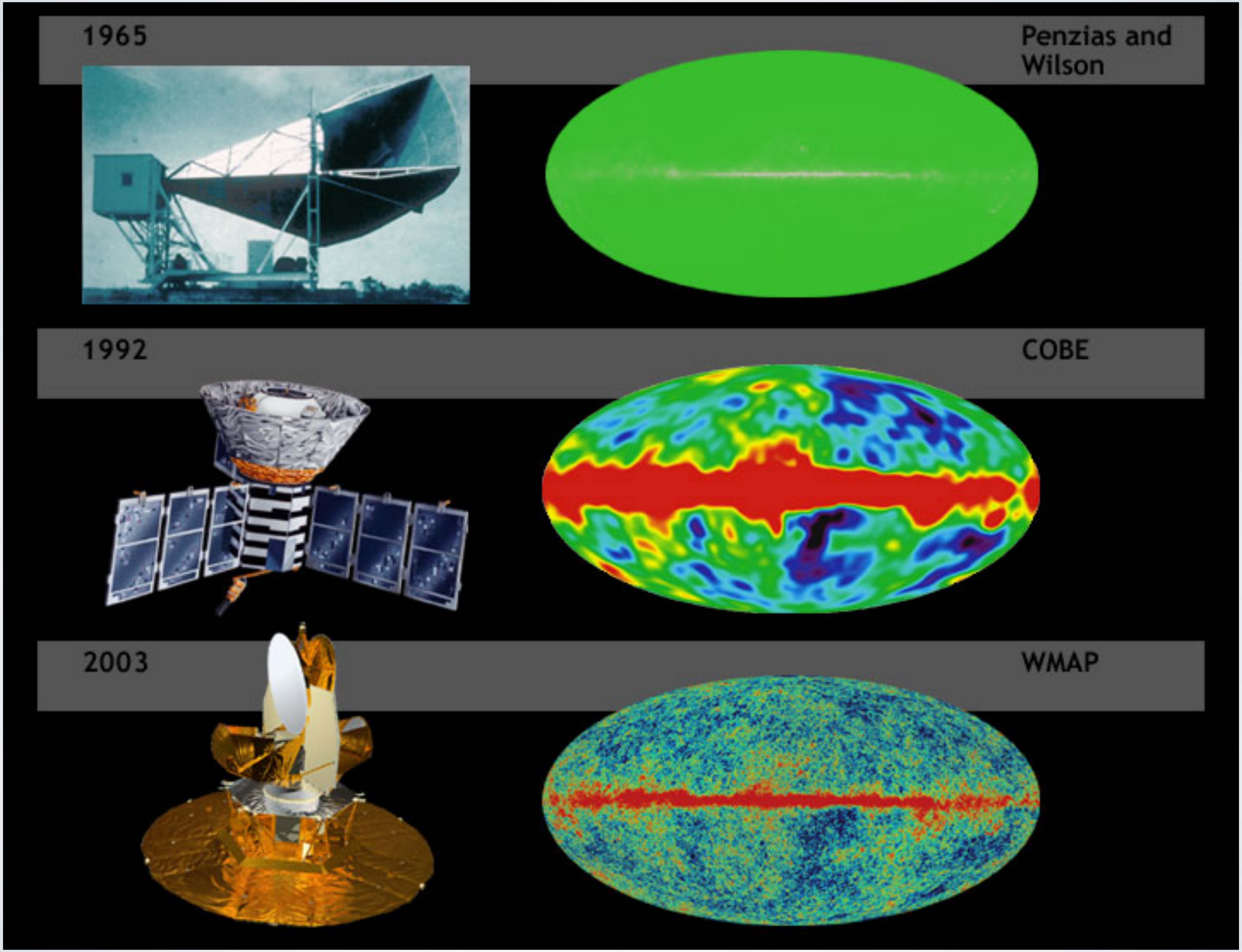
COSMIC PIE


Dark Energy
73%
Dark Matter
23%
Atomic Matter
4.6%
Light
0.005%
Neutrinos
0.0034%



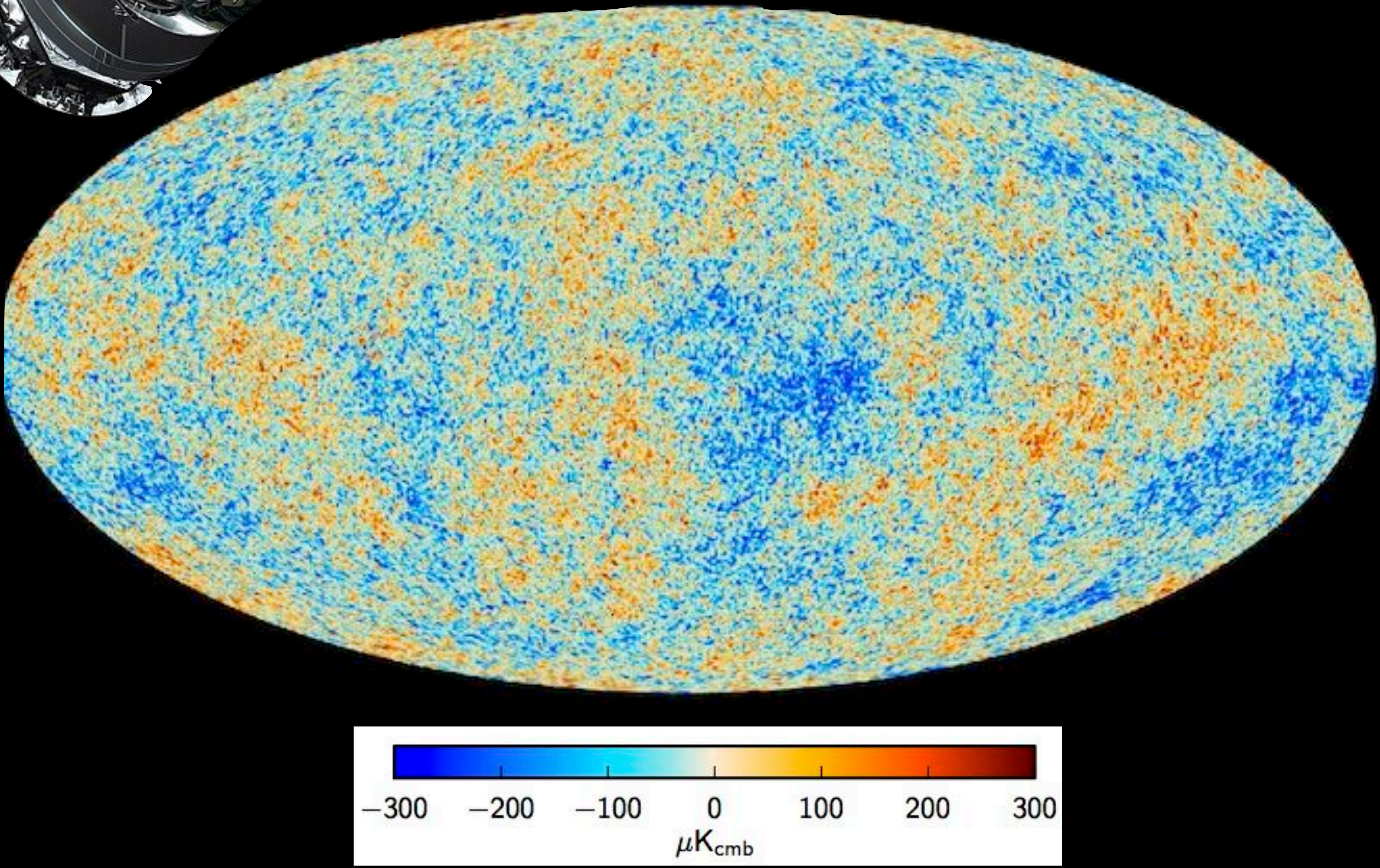
redit: ESO/M. Kornmesser

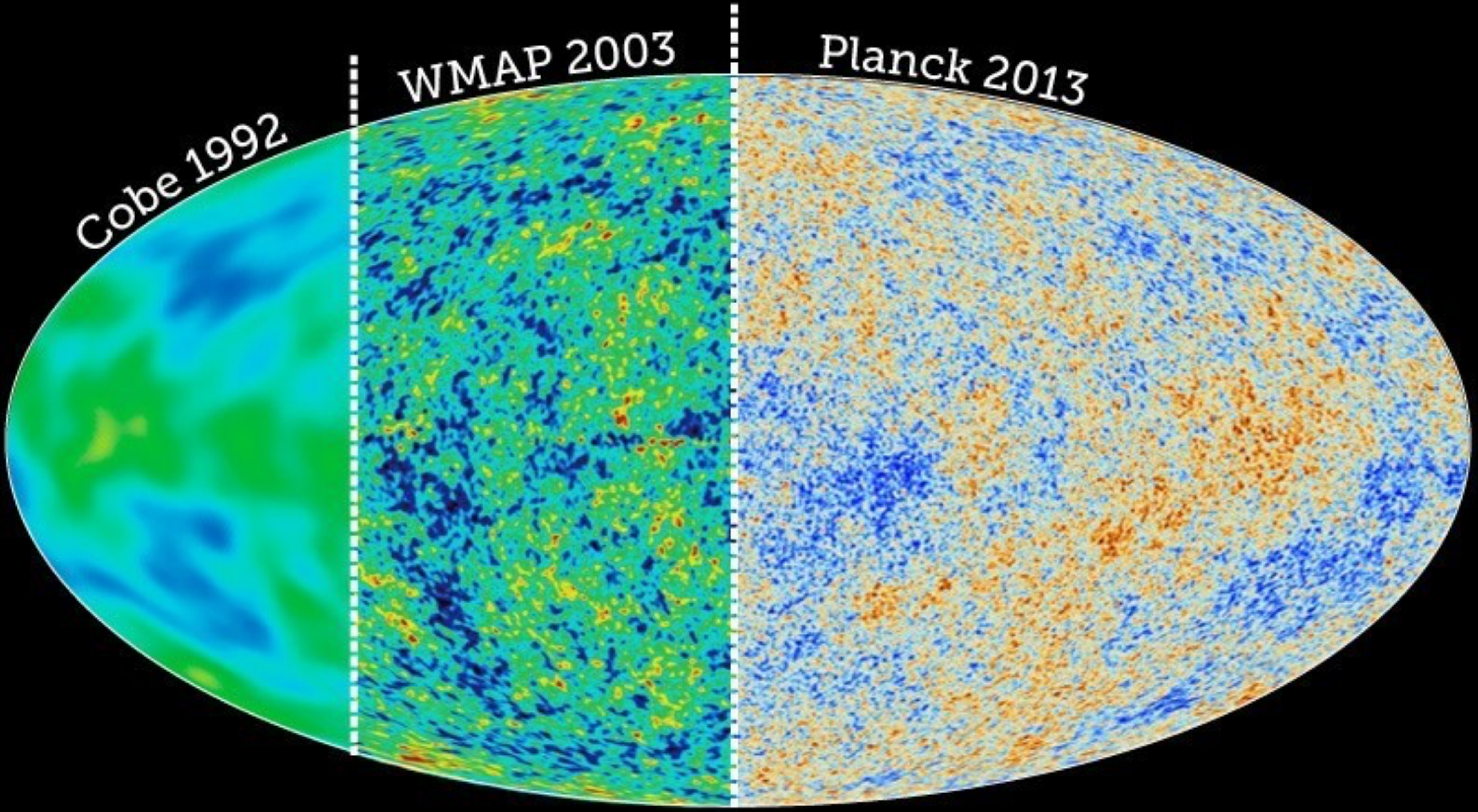
COSMIC MICROWAVE BACKGROUND



Planck satellite

$$\frac{\delta T}{T} \sim 10^{-5} \ll 1$$

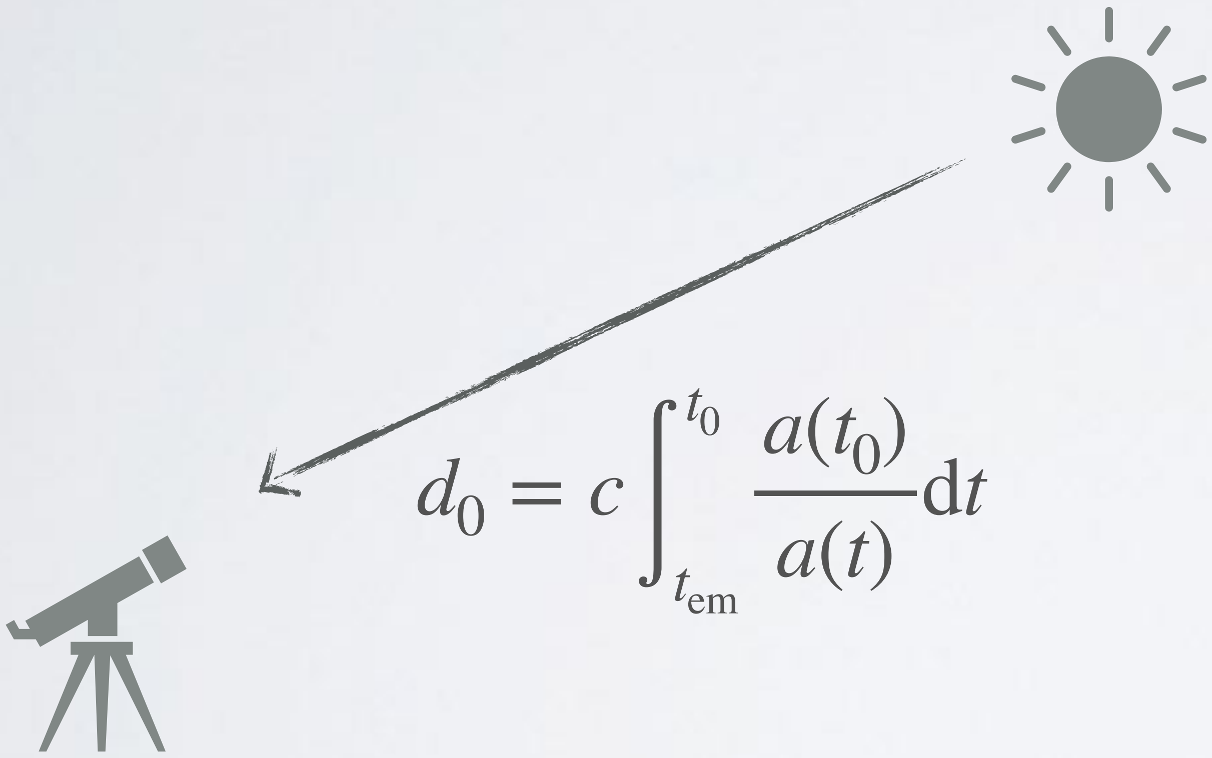



Cobe 1992
WMAP 2003
Planck 2013

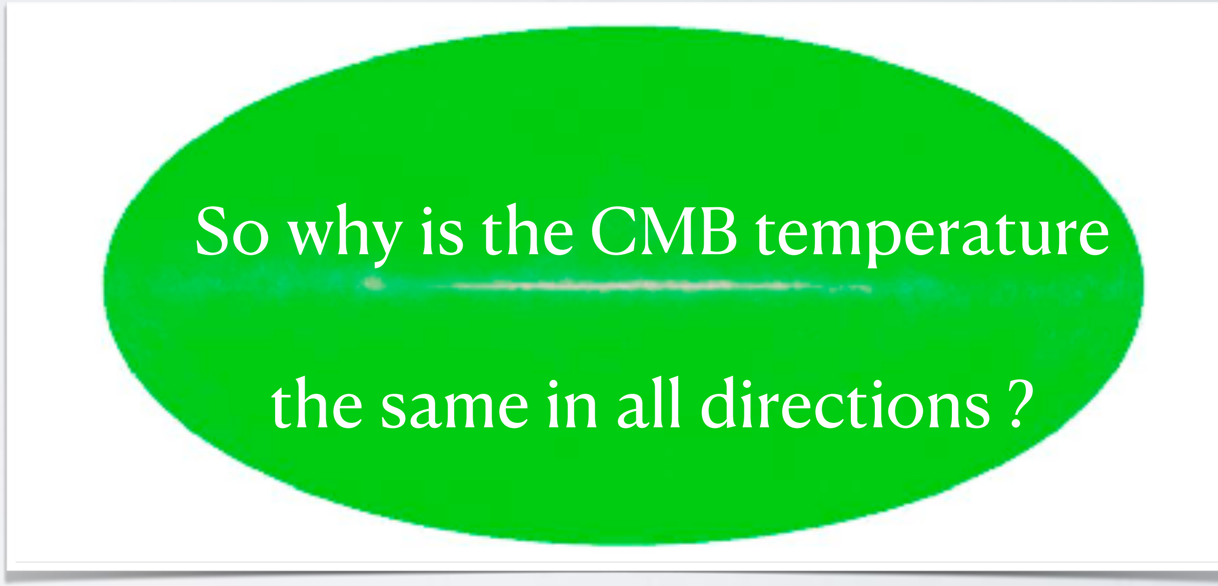
HORIZON PROBLEM

$ds^2 = -c^2 dt^2 + a^2(t) d\vec{x}^2$ (Friedmann-Lemaître-Robertson-Walker metric)

Light travels along null geodesics: $ds^2 = 0$



$$d_0 = c\int_{t_{\mathrm{em}}}^{t_0} \frac{a(t_0)}{a(t)} dt$$



So why is the CMB temperature the same in all directions ?

At the decoupling time, angular size of the cosmic horizon ~ 0.5 deg $\sim$ angular diameter of the moon

$$\Downarrow$$

There should be 450 000 causally disconnected patches!

Gravity is an attractive force: $\ddot{a} < 0$.

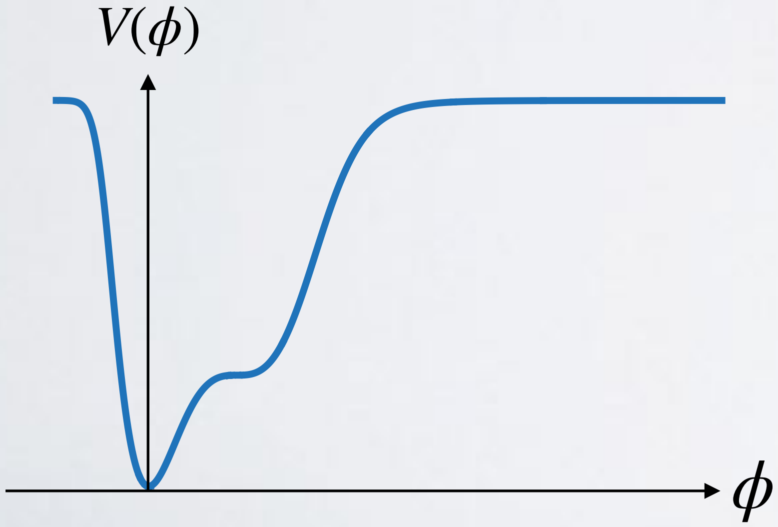
Around $t = 0$ ("big bang"), $a \propto t^{p<1} \implies d_0$ is finite.

COSMIC INFLATION

$\mathrm{d}s^2 = -\,\mathrm{d}t^2 + a^2(t)\mathrm{d}\vec{x}^2$ (Friedmann-Lemaître-Robertson-Walker metric)

We thus need a primordial phase of accelerated expansion: $\ddot{a} > 0$. We call this “inflation”.

It cannot be realised with matter in the form of Newtonian fluids. In any case, it occurs at super-high energy where matter should rather be described by of fields.



In a cosmological background, a scalar field behaves “like” a perfect fluid with

$$\begin{cases} \rho = \dfrac{\dot{\phi}^2}{2} + V(\phi) \\ p = \dfrac{\dot{\phi}^2}{2} - V(\phi) \end{cases} \Longrightarrow \text{inflation takes place if } V(\phi) > \dot{\phi}^2$$

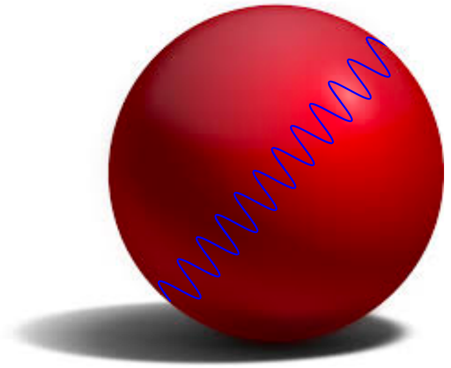
Cosmic Inflation: $\ddot{a} > 0$

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\,\mathrm{d}\vec{x}^2$$

Hubble parameter $H = \dot{a}/a$

→ H^{-1}: characteristic time scale, or length scale ($c = 1$), of the expansion

$\lambda \ll H^{-1}$

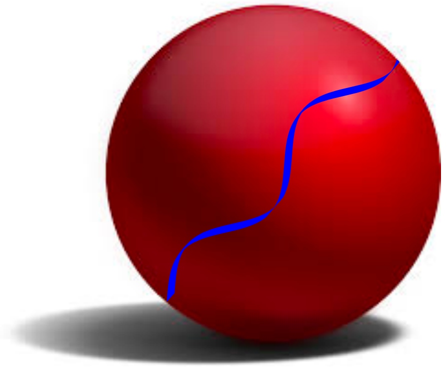


Insensitive to space-time curvature

↓

“unambiguous” vacuum state

$\lambda \gtrsim H^{-1}$



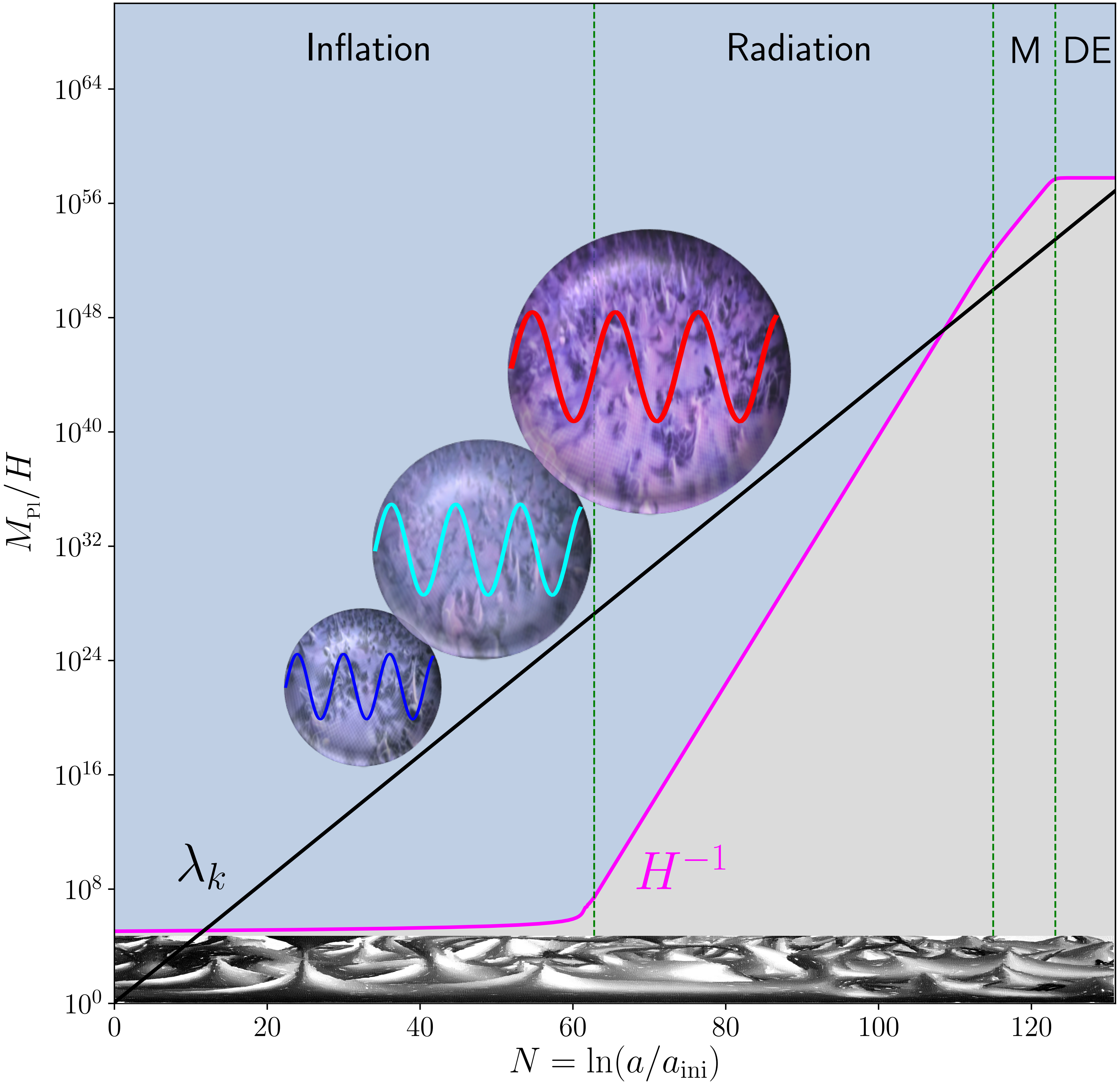
Feels space-time curvature

↓

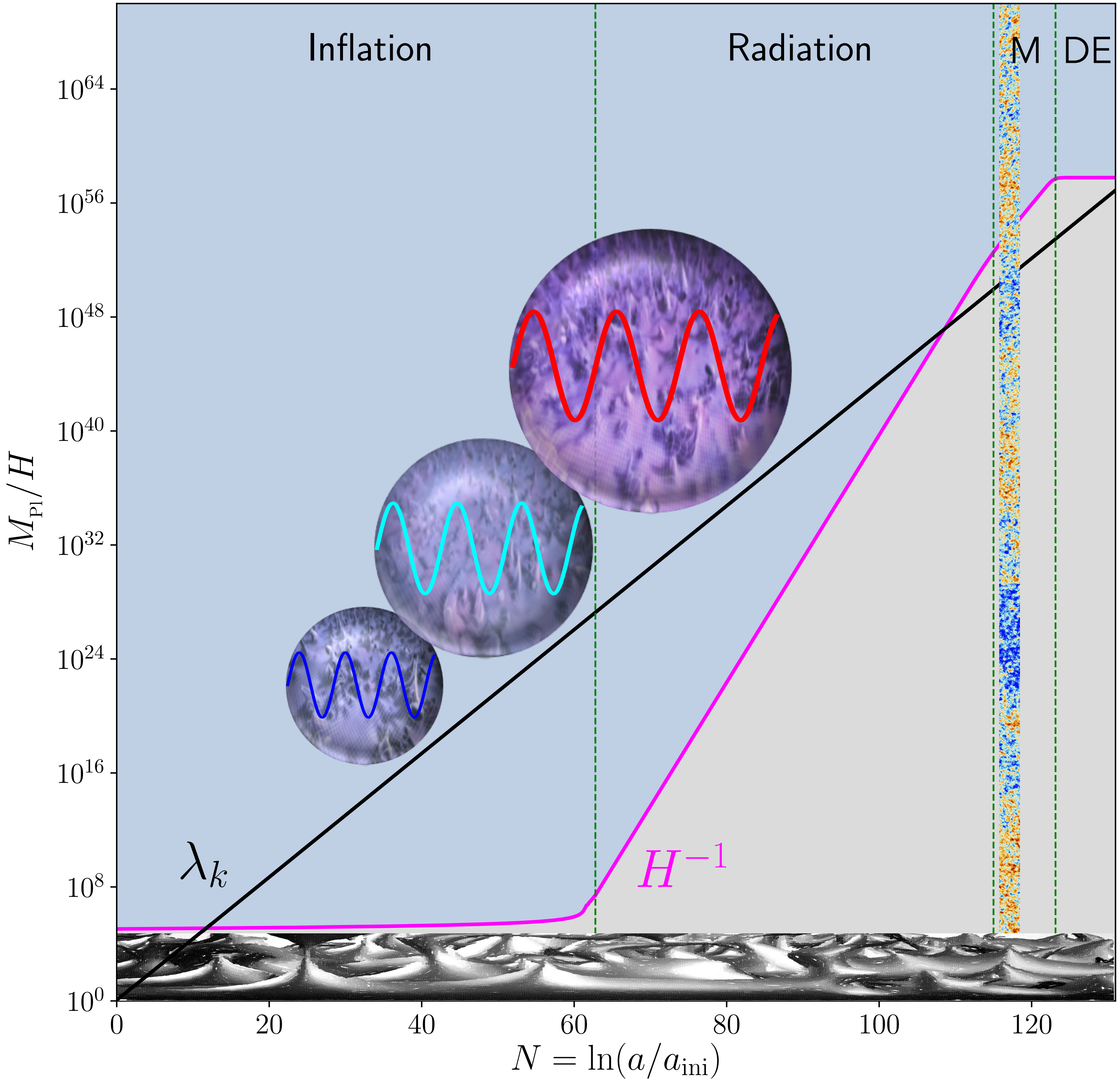
Quantum particule creation

(analogous to Schwinger effect, Hawking effect, etc)

Cosmic Inflation



Cosmic Inflation

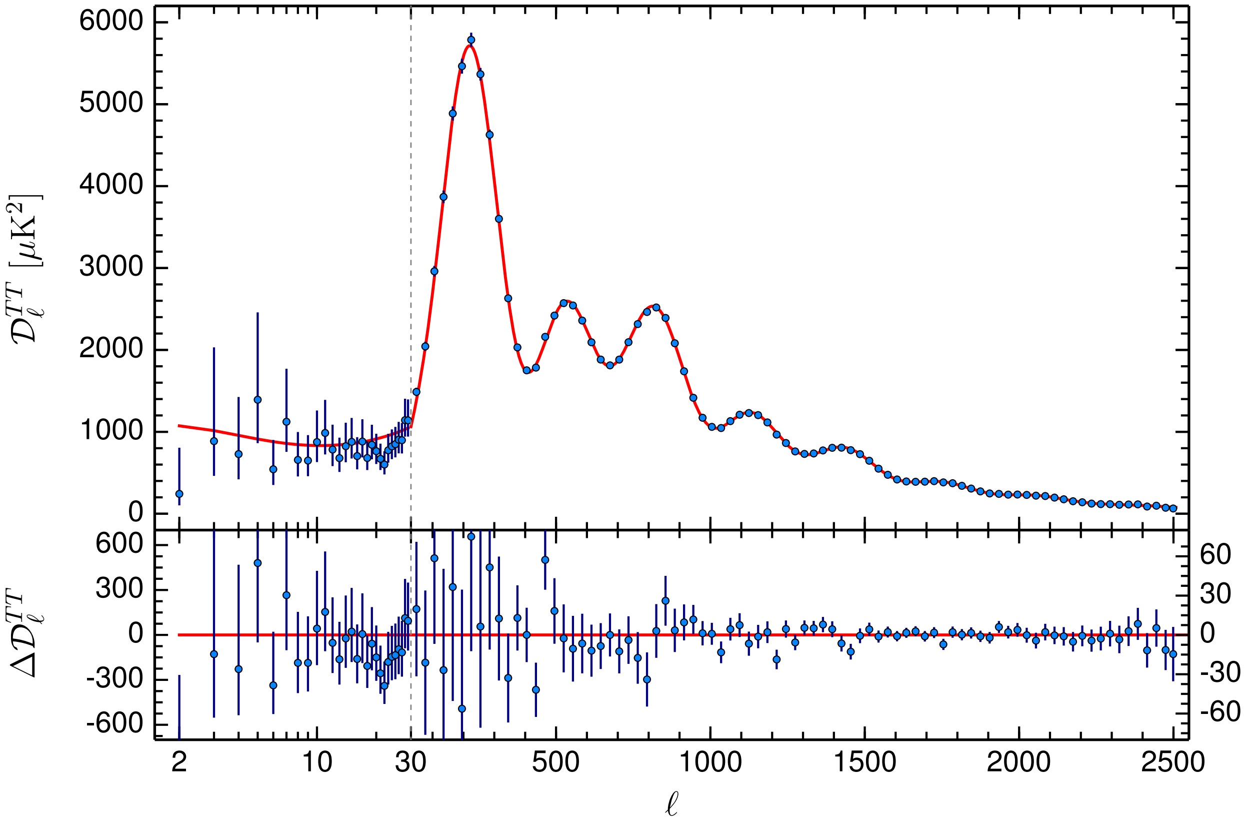


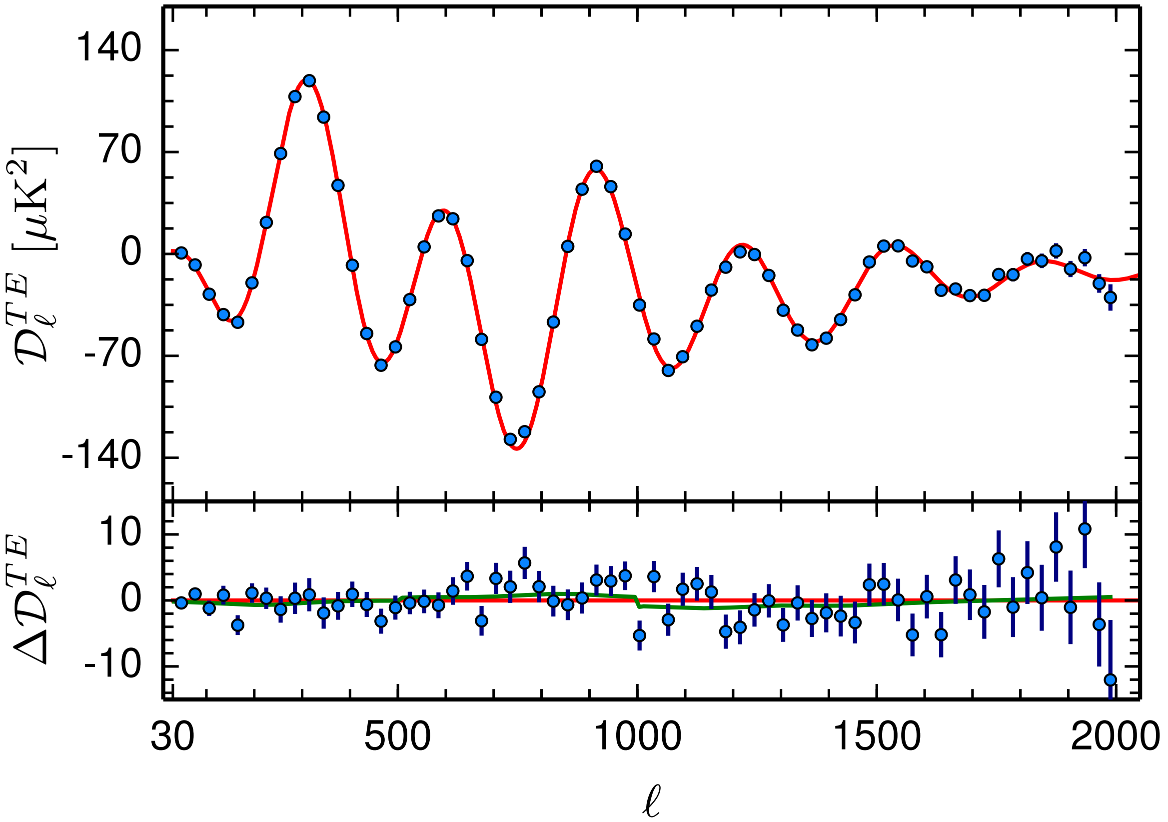
Cosmological perturbations

Single scalar gauge-invariant degree of freedom $v \ni \delta\phi, \delta g_{\mu\nu} \propto \delta T/T$

Quantised starting from Bunch-Davies vacuum $v \longrightarrow \hat{v}$

Quantum mean values compared with statistical averages in the sky





Cosmological Perturbation Theory

Density fluctuations are small at CMB scales → Perturbation Theory

$$g_{\mu\nu}(\vec{x},t) = \bar{g}_{\mu\nu}(t) + \widehat{\delta g}_{\mu\nu}(\vec{x},t)$$

$$\phi(\vec{x},t) = \bar{\phi}(t) + \widehat{\delta\phi}\,(\vec{x},t)$$

Homogeneous and isotropic solution of the classical problem

Quantised fluctuation

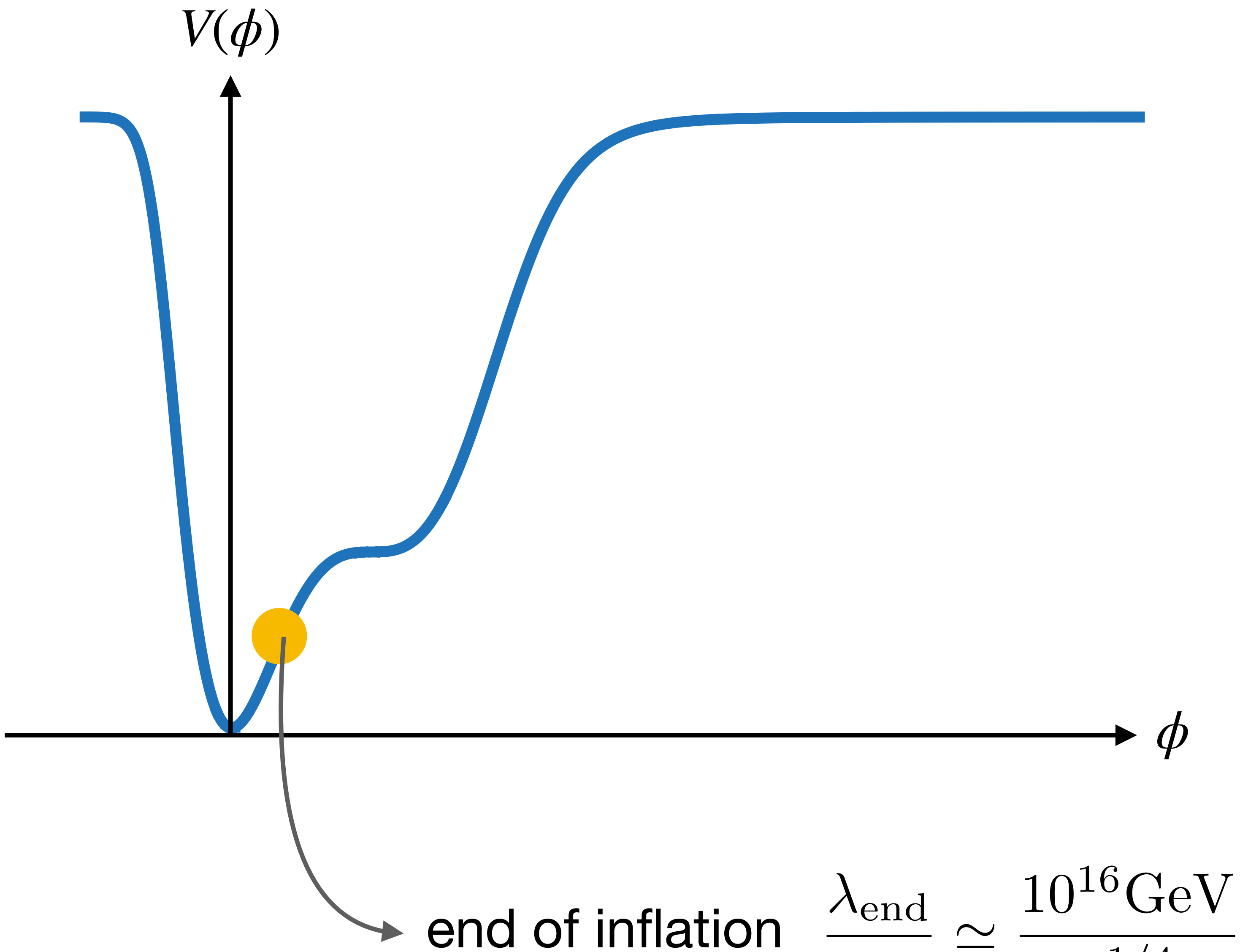
—> Quantum-field-theory on curved space-time

Strong assumption: universe is quasi homogeneous and isotropic at all scales

This may be broken at:

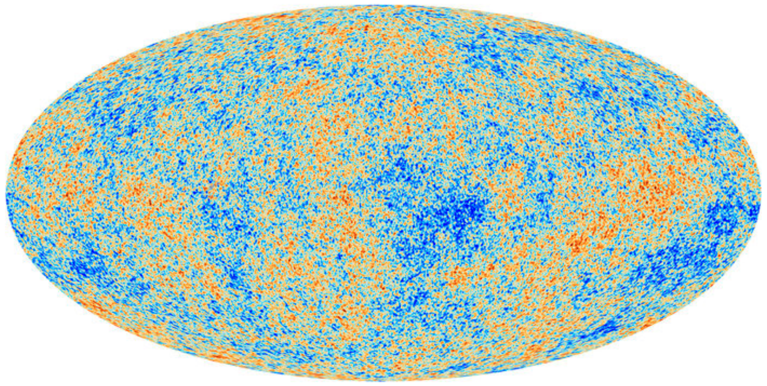
- Larger scales: space-time structure beyond the observable universe
- Smaller scales: formation of extreme objects such as primordial black holes, heavy clusters, large voids etc

Probing the end of inflation



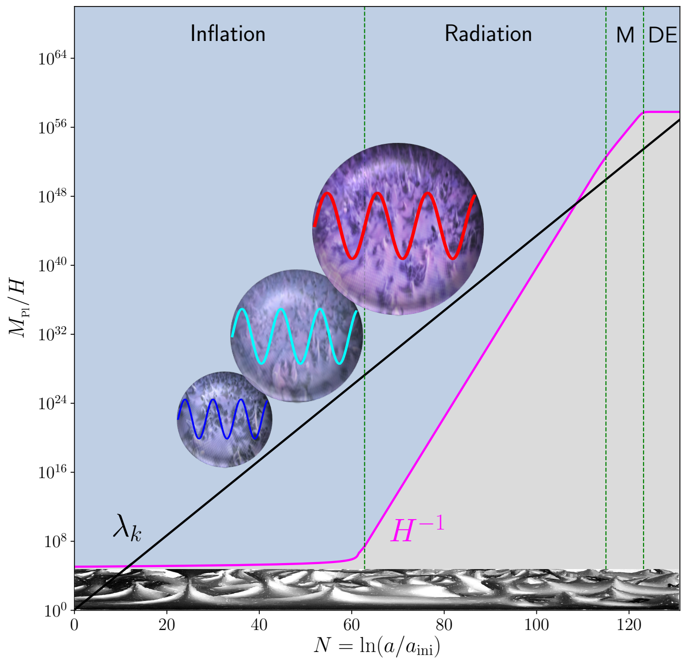
end of inflation $\frac{\lambda_{\rm end}}{\rm m} \simeq \frac{10^{16}\,{\rm GeV}}{\rho_{\rm end}^{1/4}} R_{\rm rad}^{-1}$

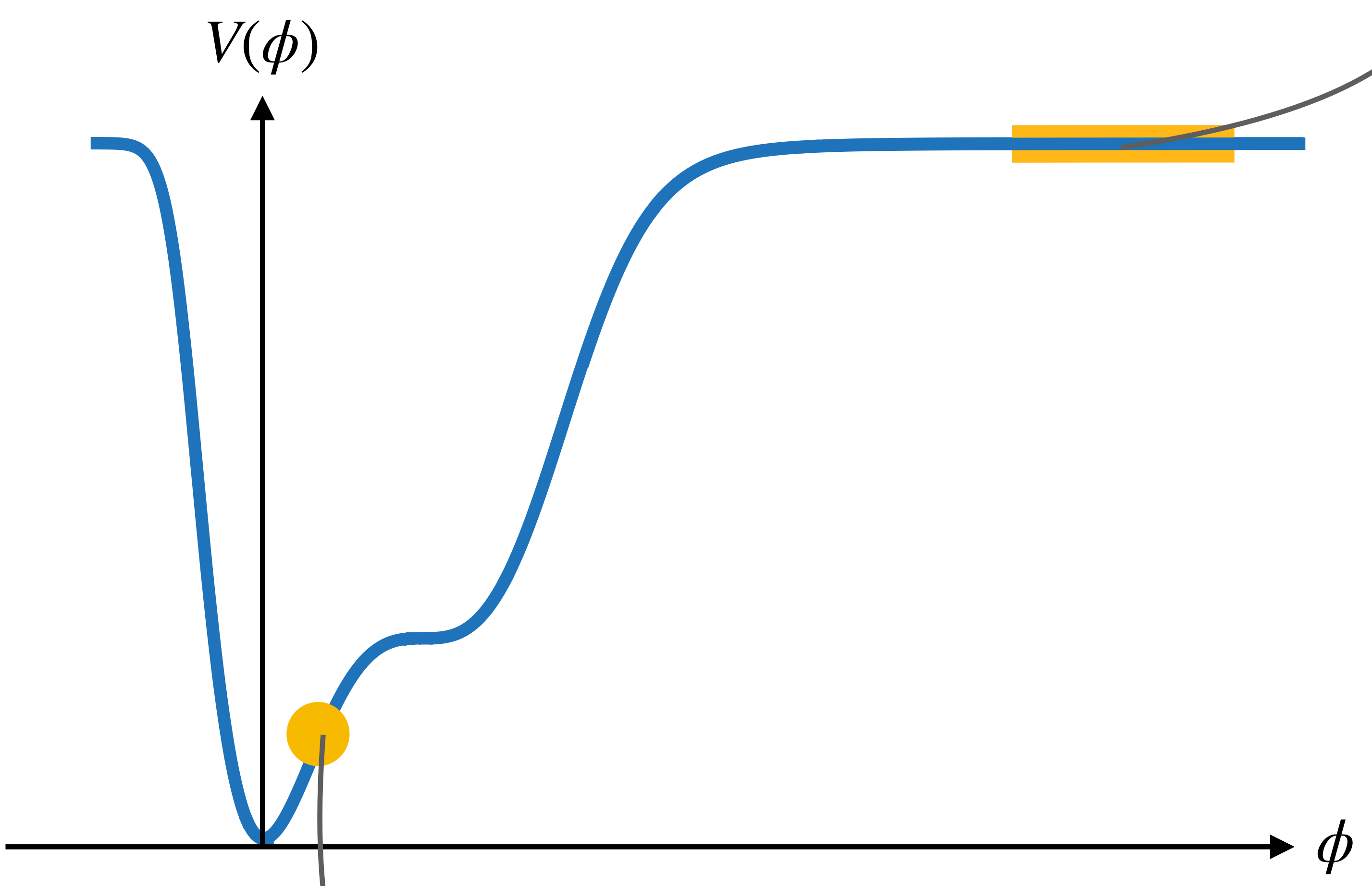
Probing the end of inflation



CMB observed scales

$\lambda \simeq \mathrm{Mpc}$





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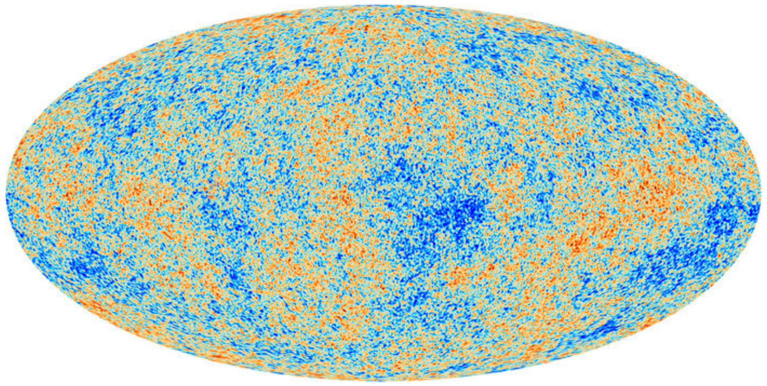
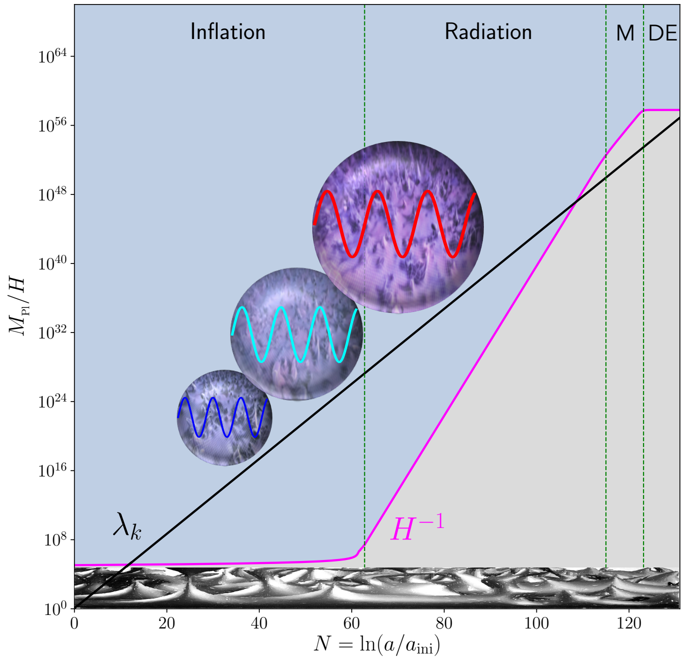
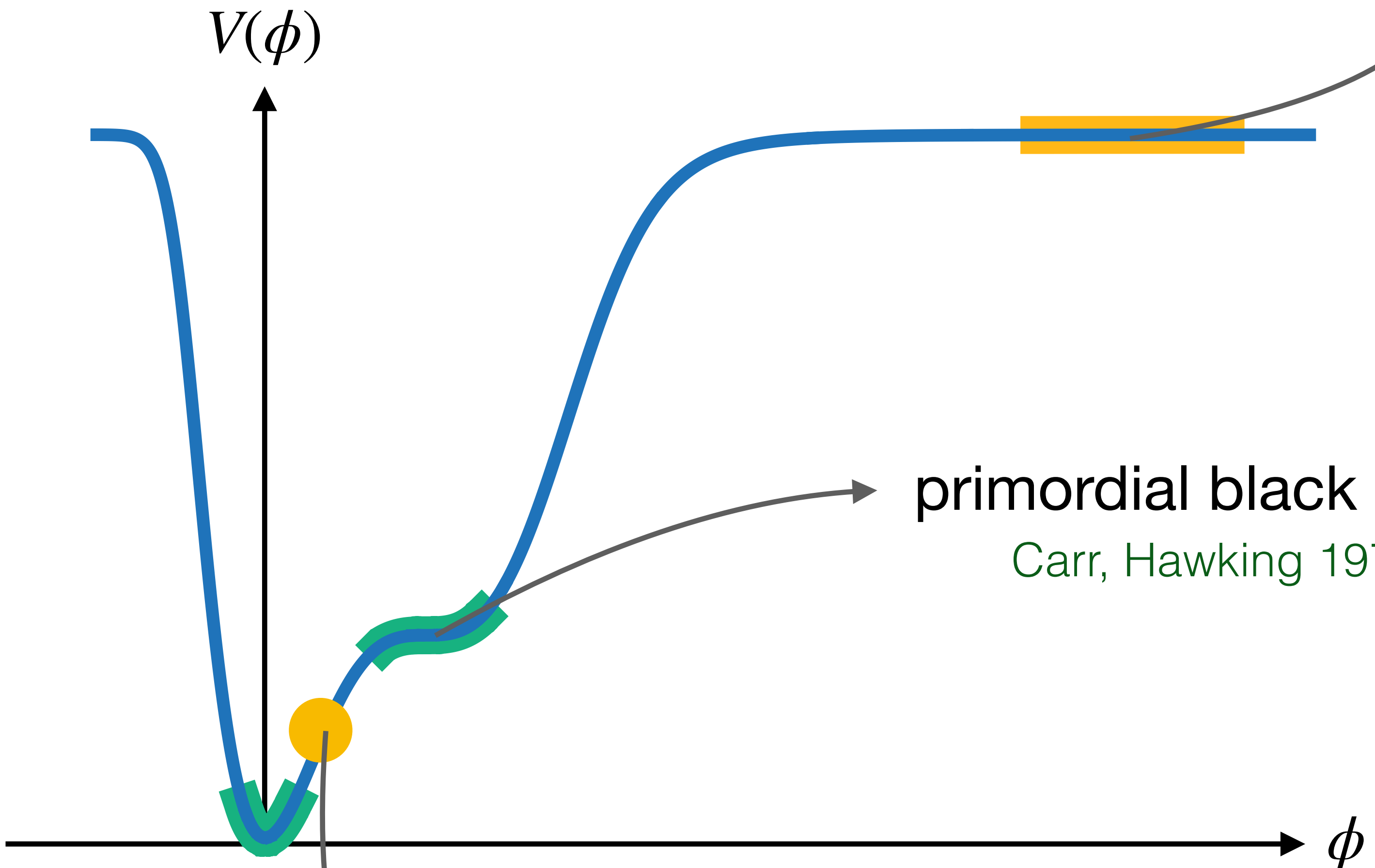




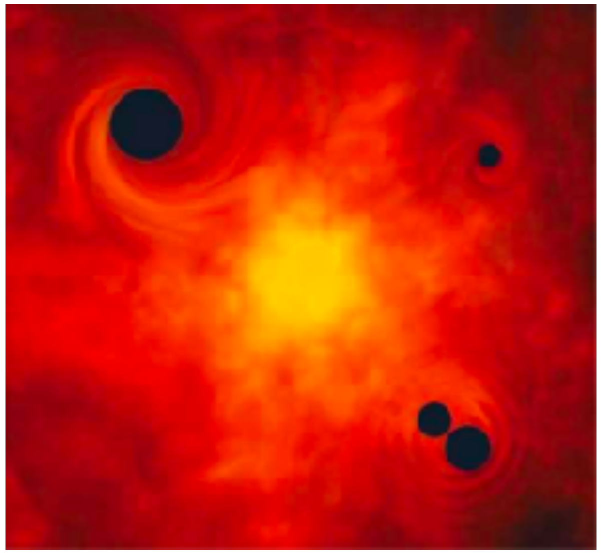


CMB observed scales

$\lambda \simeq \text{Mpc}$

primordial black holes?

Carr, Hawking 1974



end of inflation $\frac{\lambda_{\rm end}}{\rm m} \simeq \frac{10^{16}\,\text{GeV}}{\rho_{\rm end}^{1/4}} R_{\rm rad}^{-1}$

Primordial black holes

- Could constitute part or all of dark matter Chapline 1975

  $M = 10^{16} - 10^{17}\mathrm{g},\ 10^{20} - 10^{24}\mathrm{g},\ 10 - 10^{3} M_{\odot}$

- Could provide progenitors for the LIGO/VIRGO events

  $M = 10 - 100\ M_{\odot}$

- Could provide seeds for cosmological structures

  $M > 10^{3} M_{\odot}$

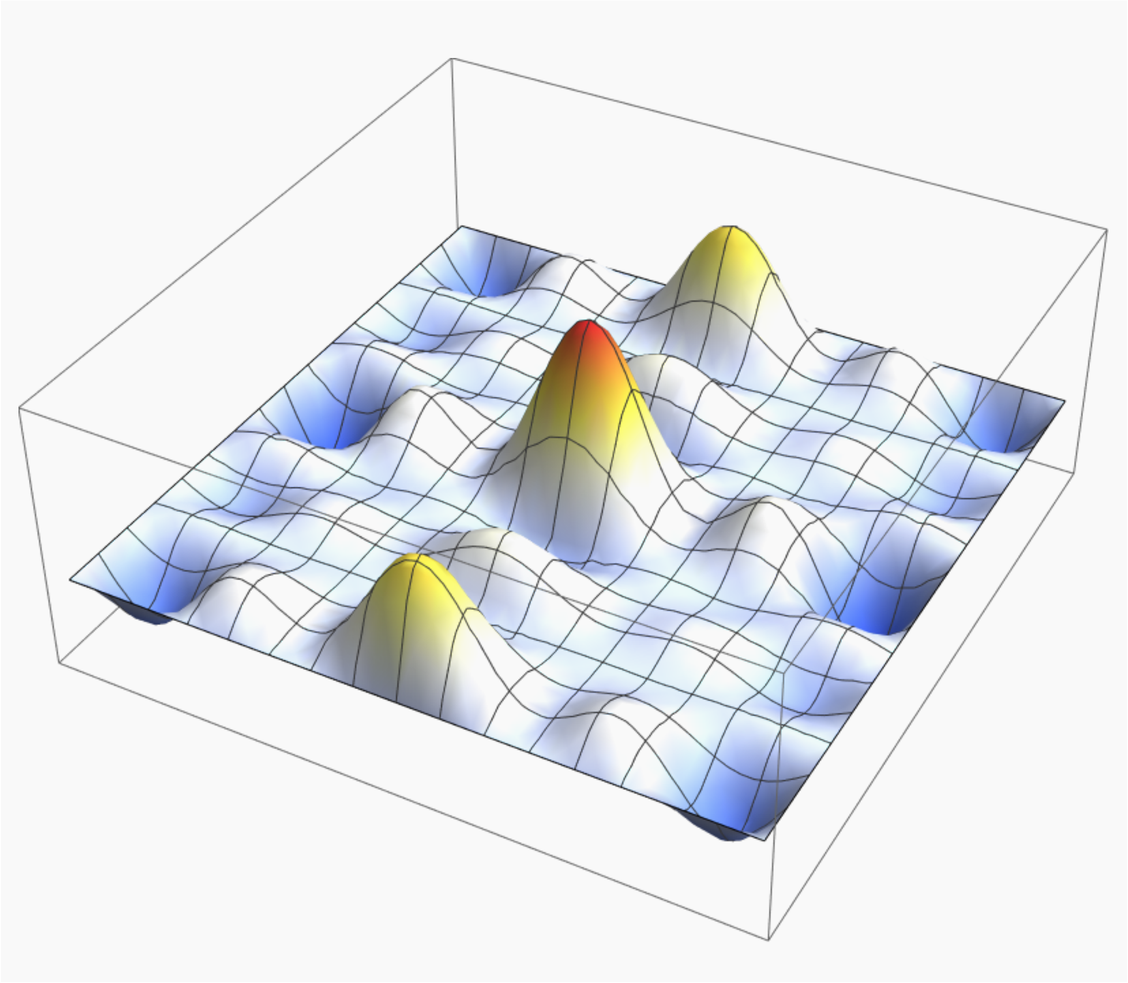
  Mészáros 1975
  Afshordi, McDonald, Spergel, 2003

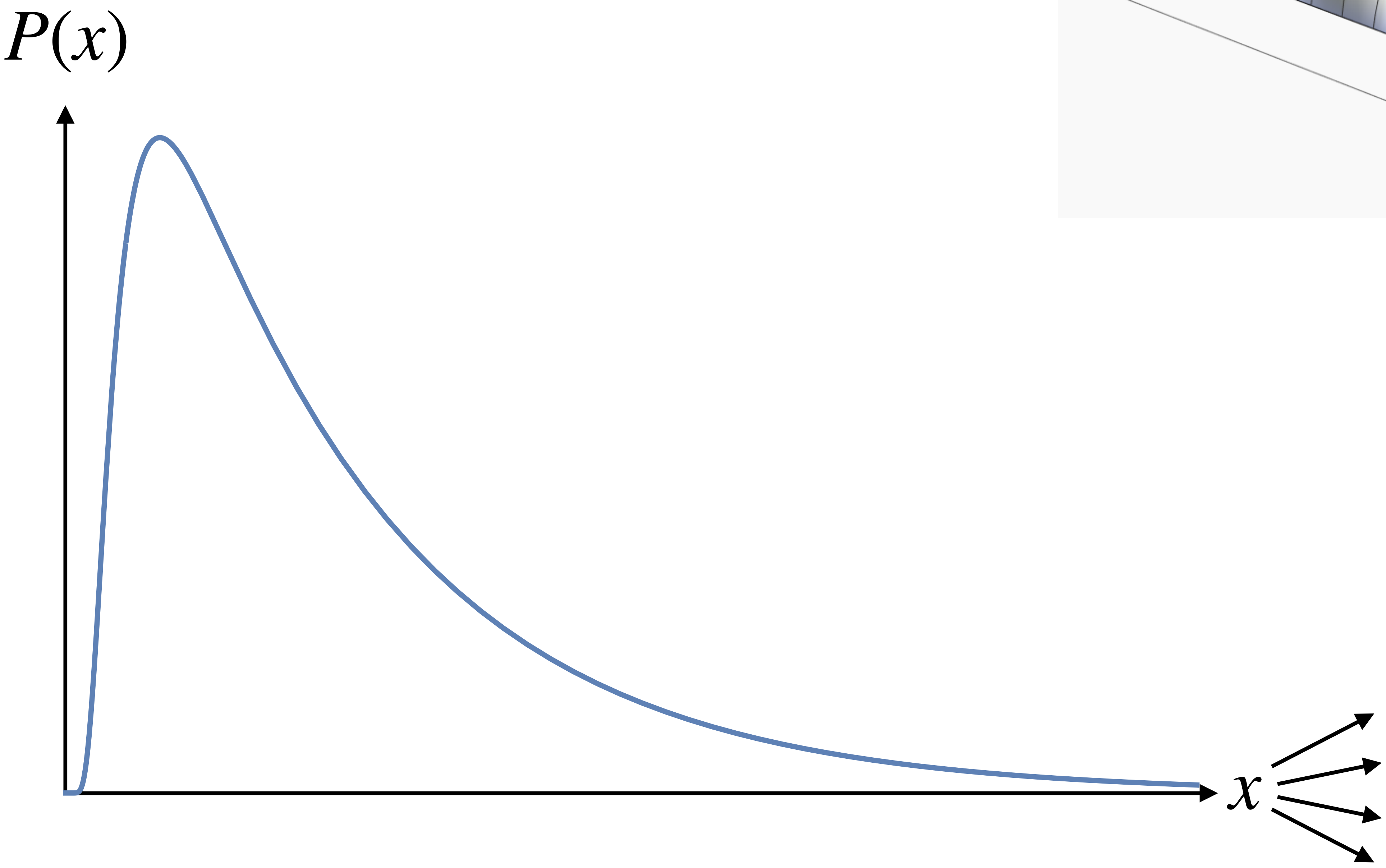
- Could provide seeds for supermassive black holes in galactic nuclei

  $M > 10^{3} M_{\odot}$

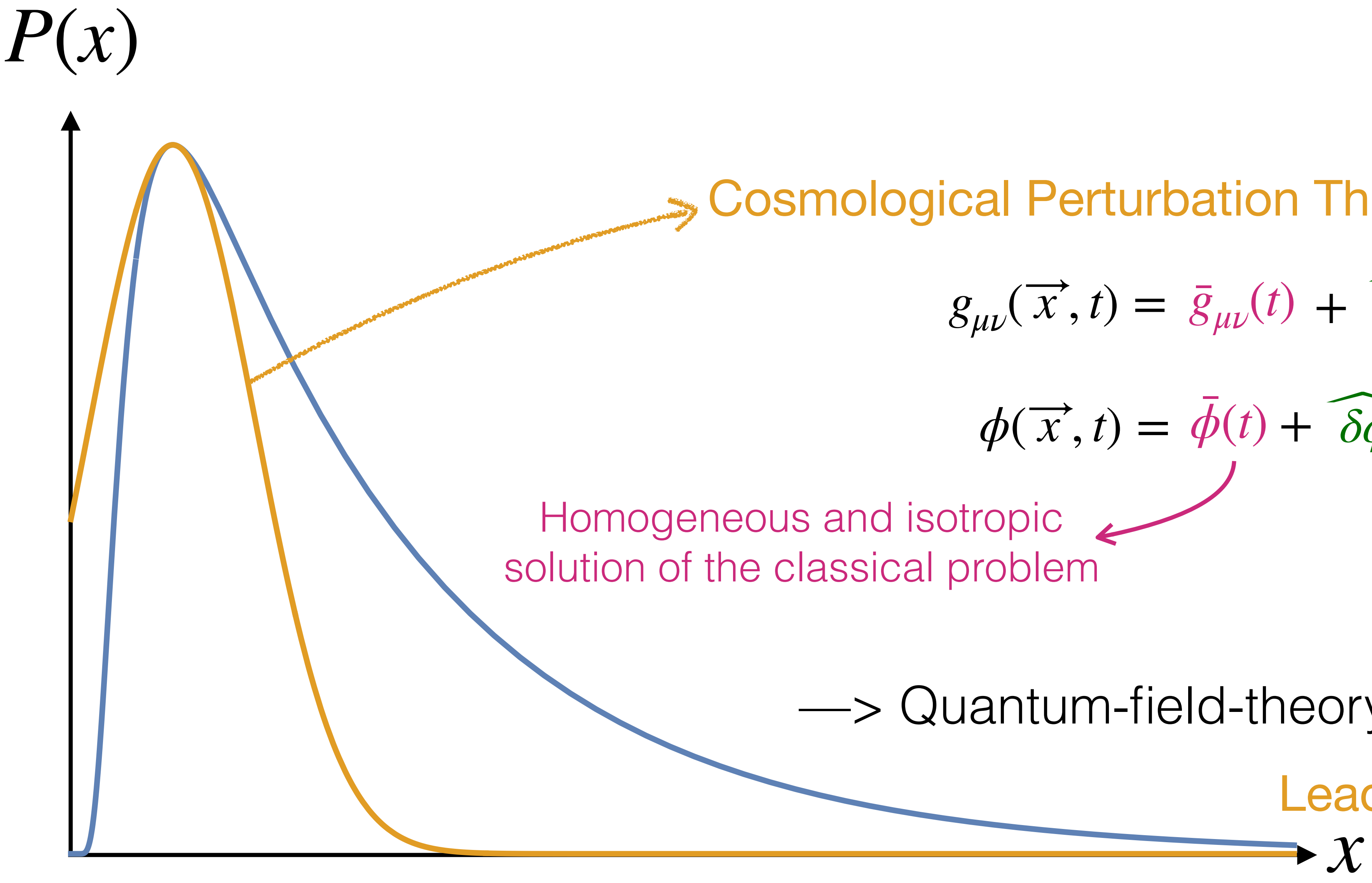
  Carr, Rees 1984
  Bean, Magueijo 2002

How likely is it to form a given cosmological structure?





- Local curvature
- energy density
- maximum compaction
- etc



Cosmological Perturbation Theory, leading order

$$g_{\mu\nu}(\vec{x},t) = \bar{g}_{\mu\nu}(t) + \widehat{\delta g}_{\mu\nu}(\vec{x},t)$$

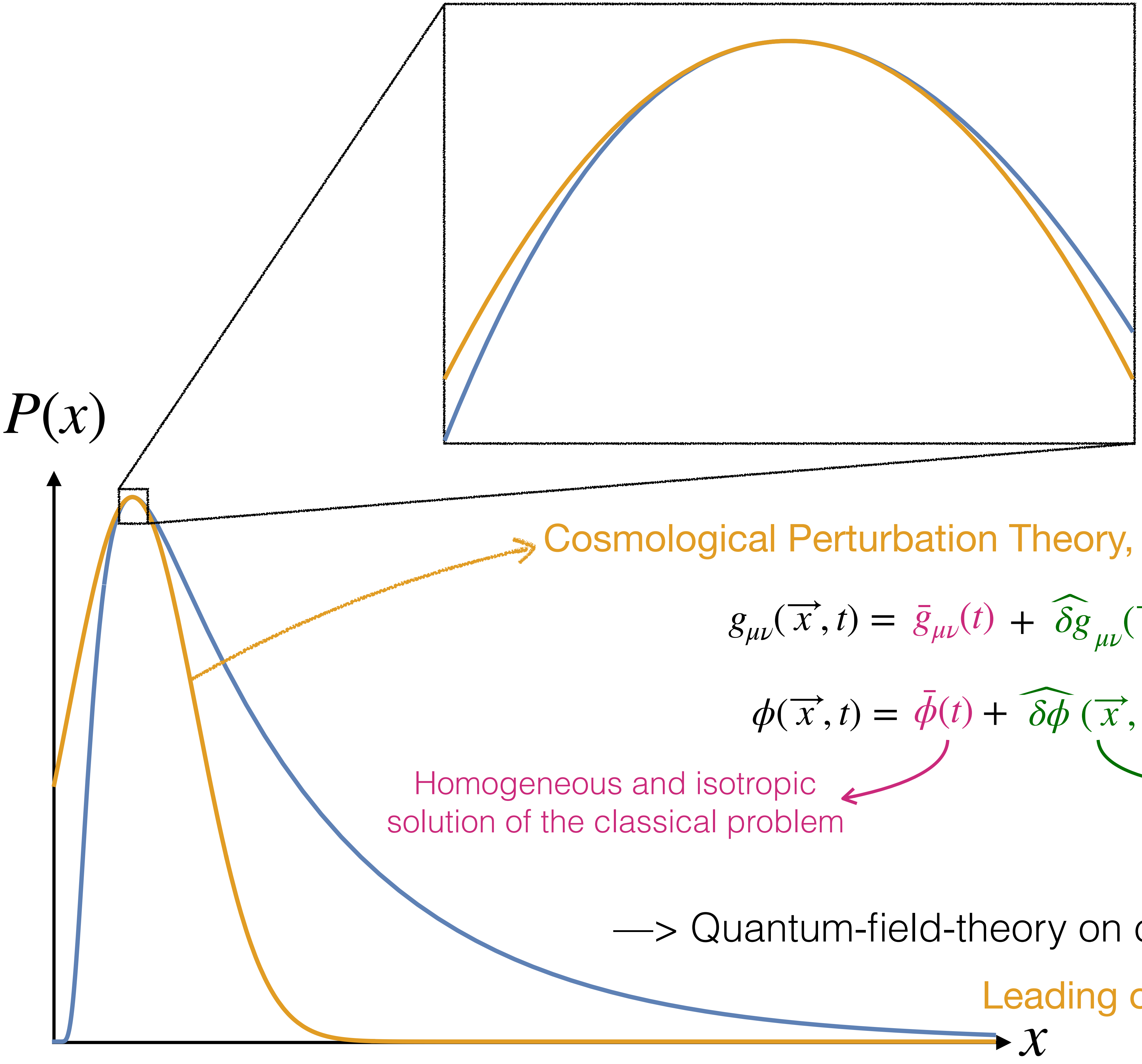
$$\phi(\vec{x},t) = \bar{\phi}(t) + \widehat{\delta\phi}(\vec{x},t)$$

Homogeneous and isotropic solution of the classical problem

Quantised fluctuation

—> Quantum-field-theory on curved space-time

Leading order: x is Gaussian



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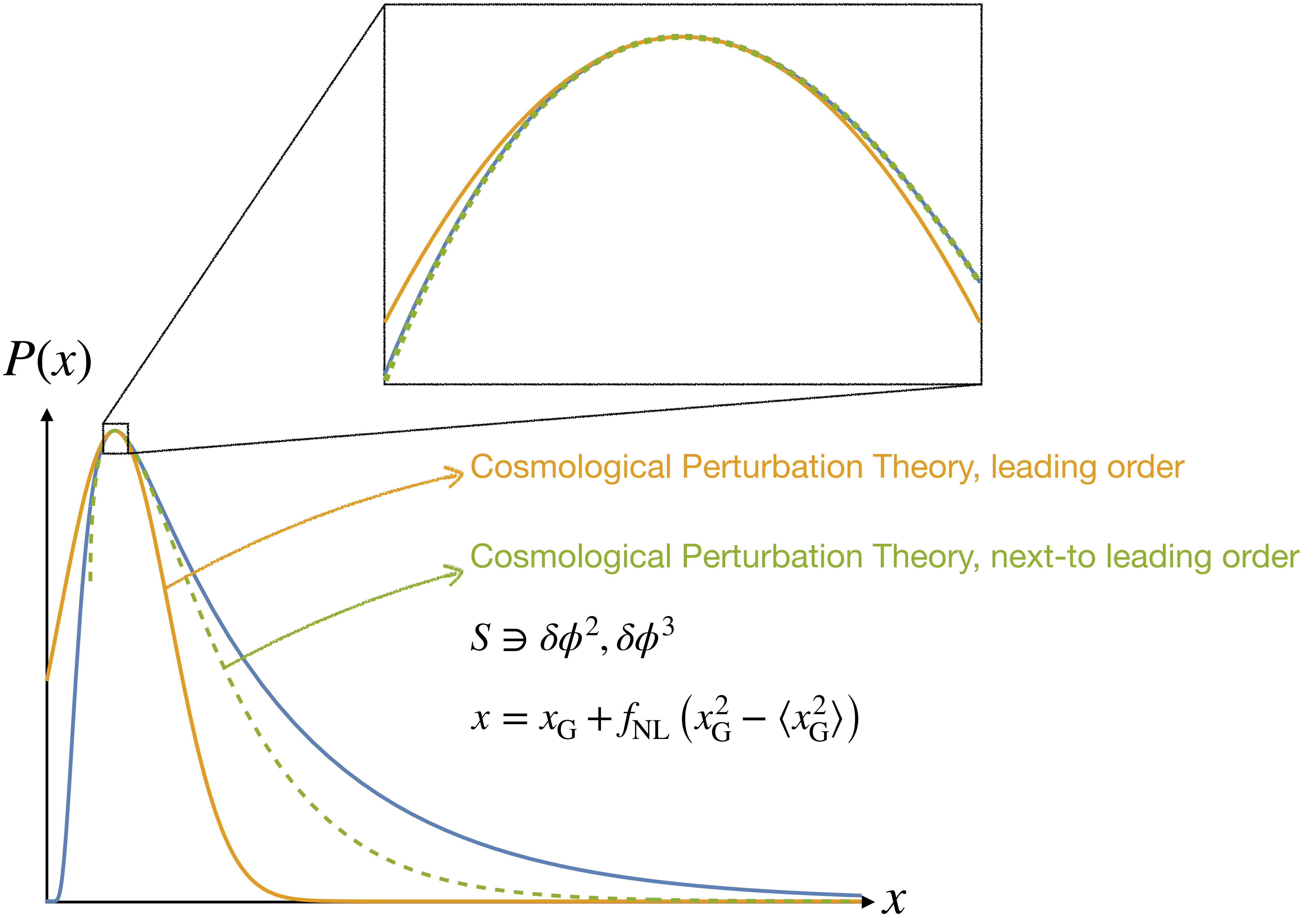
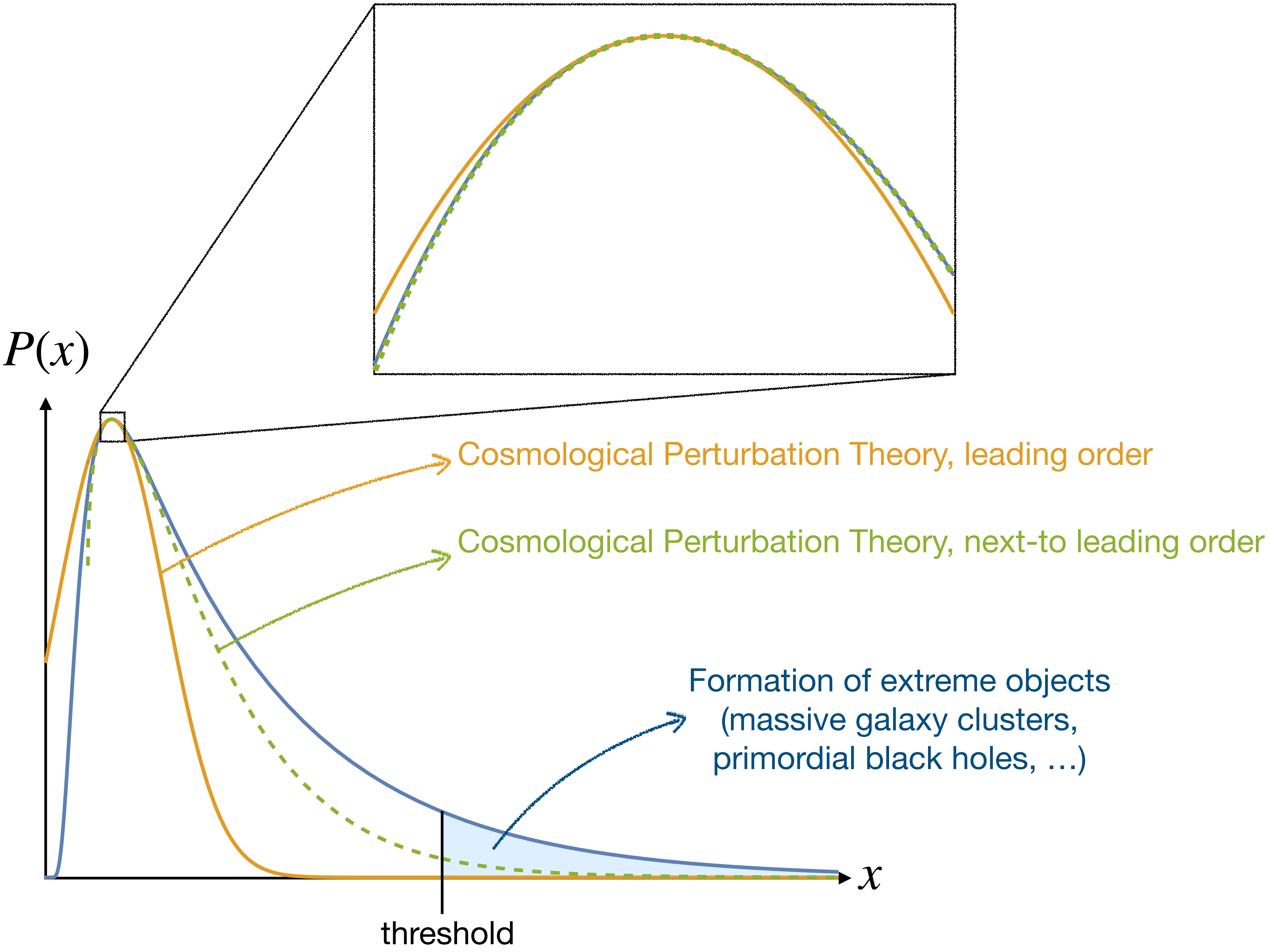


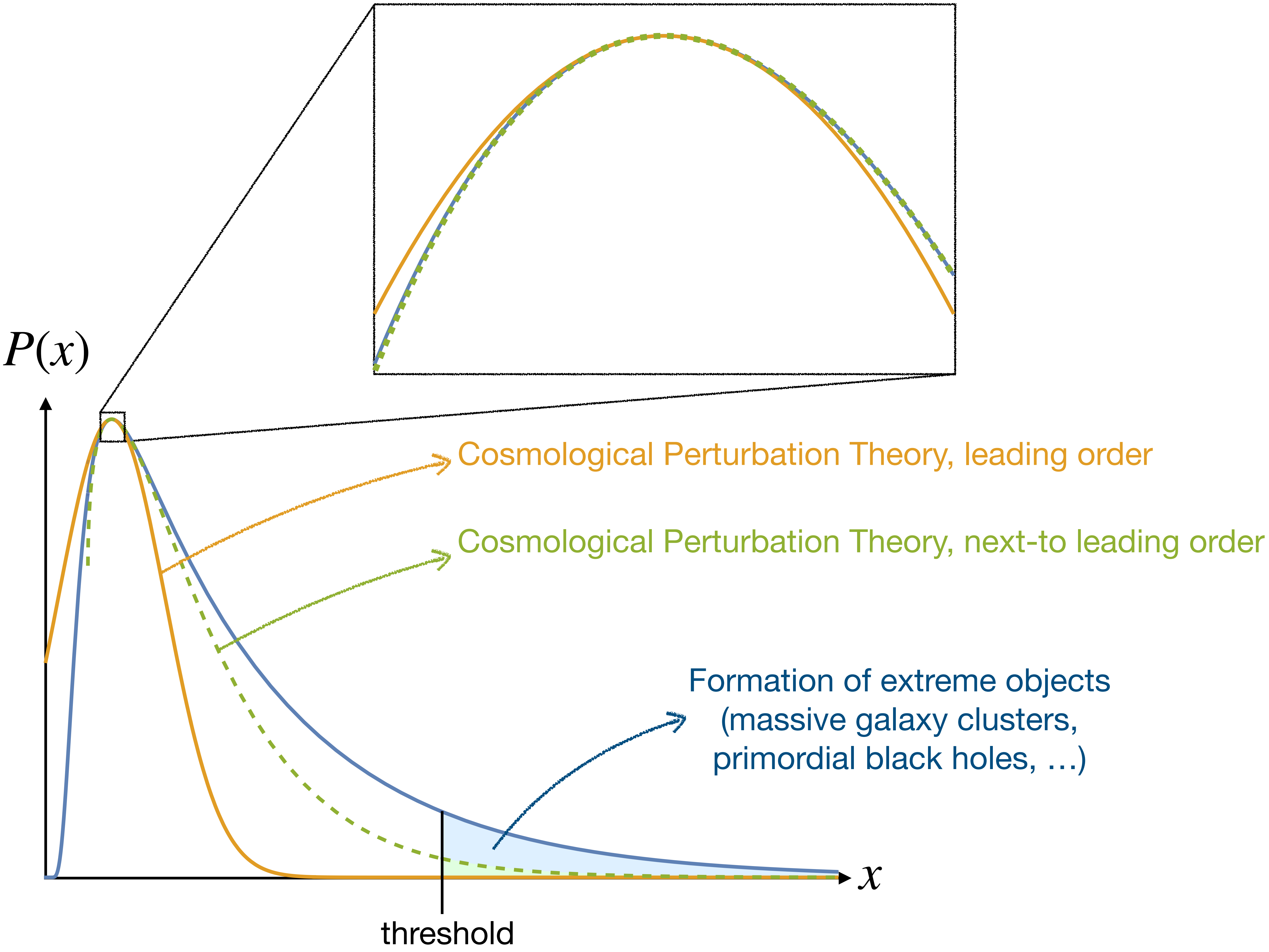
Cosmological Perturbation Theory, leading order

Cosmological Perturbation Theory, next-to leading order

$$S \ni \delta\phi^2, \delta\phi^3$$

$$x = x_{\mathrm{G}} + f_{\mathrm{NL}} \left(x_{\mathrm{G}}^2 - \langle x_{\mathrm{G}}^2 \rangle \right)$$

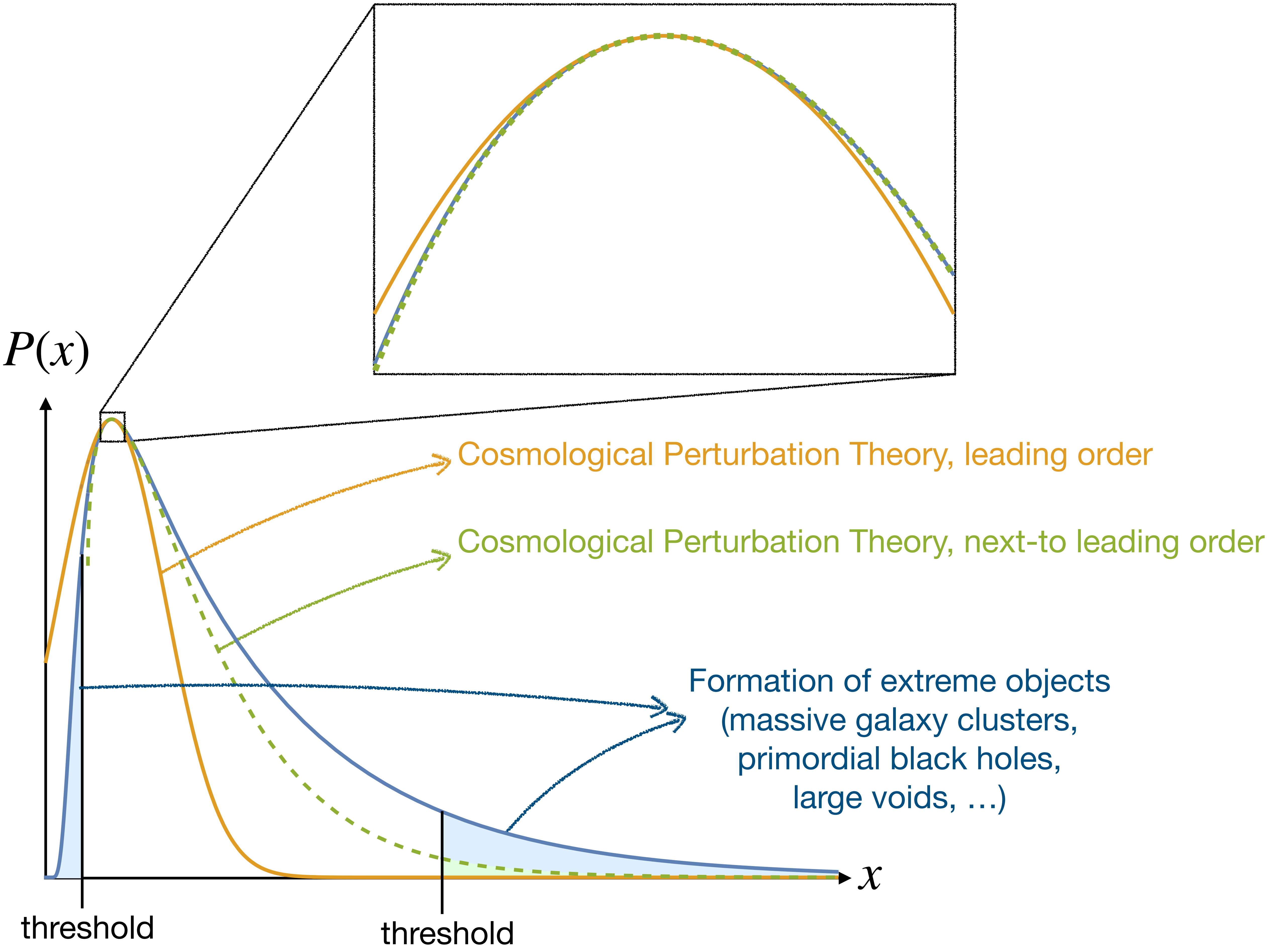

$P(x)$
Cosmological Perturbation Theory, leading order
Cosmological Perturbation Theory, next-to leading order
Formation of extreme objects
(massive galaxy clusters,
primordial black holes, …)
x
threshold



Cosmological Perturbation Theory, leading order

Cosmological Perturbation Theory, next-to leading order

Formation of extreme objects
(massive galaxy clusters,
primordial black holes, …)



Cosmological Perturbation Theory, leading order

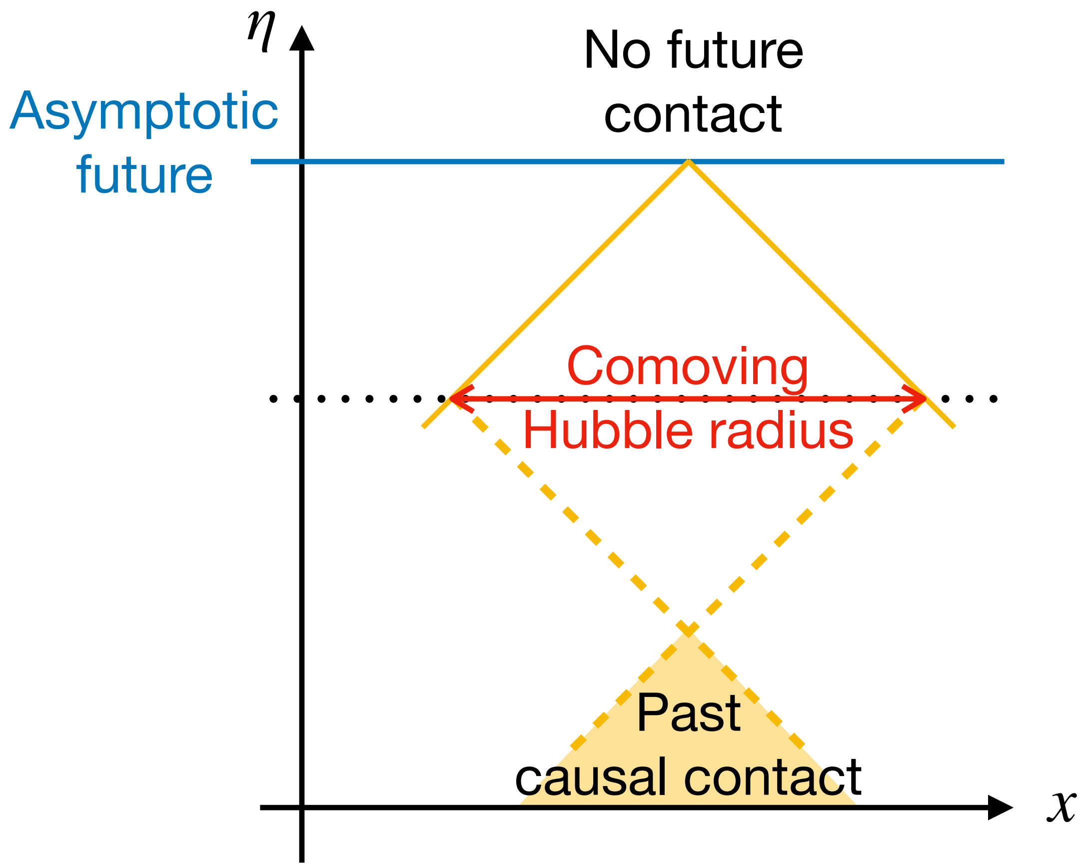
Cosmological Perturbation Theory, next-to leading order

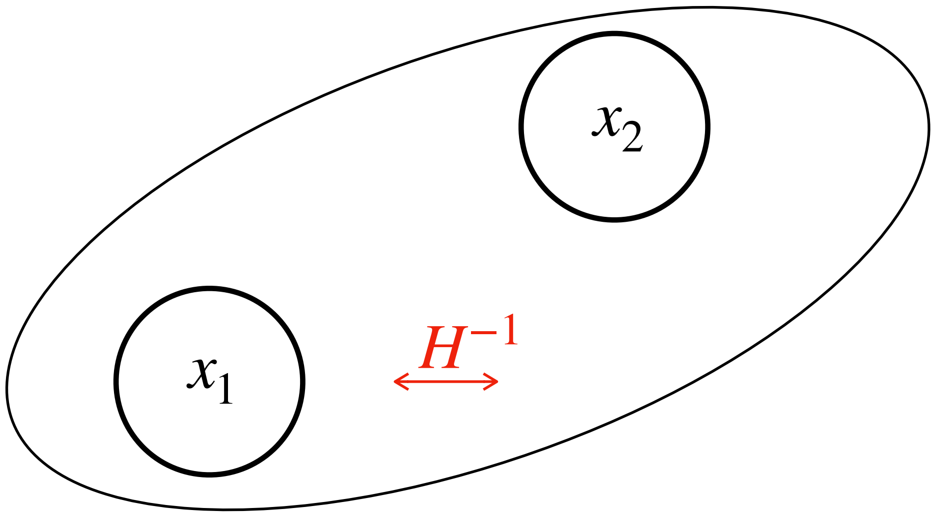
Formation of extreme objects
(massive galaxy clusters,
primordial black holes,
large voids, …)

Separate Universe

$$\mathrm{d}s^2 = a^2\left(-\mathrm{d}\eta^2 + \mathrm{d}\vec{x}^2\right)$$

de-Sitter universe: $a = -1/(H\eta), \quad -\infty < \eta < 0$



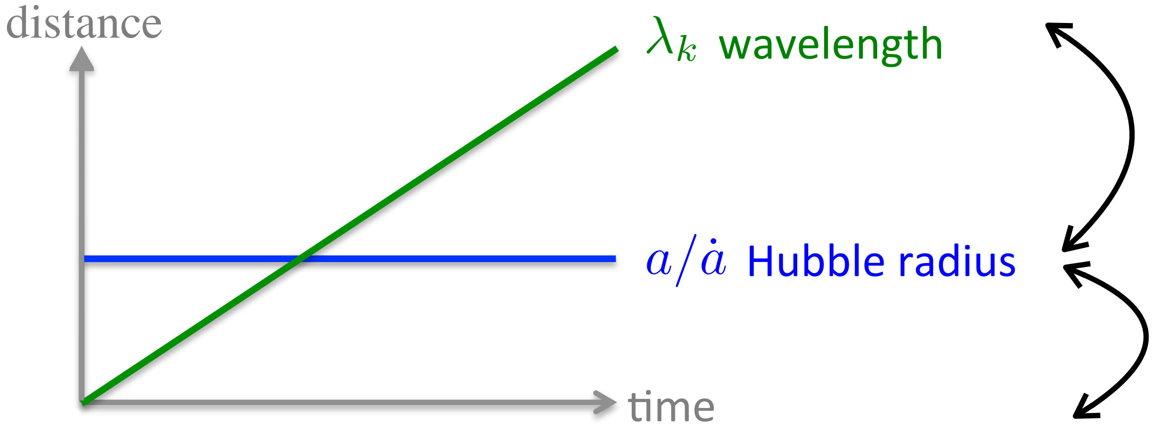


If a large fluctuation develops at $\boldsymbol{x}_1$, this cannot affect the local geometry at $\boldsymbol{x}_2$

Separate universe: On large scales, the universe can be described by an ensemble of independent, locally homogeneous and isotropic patches

Salopek & Bond; Sasaki & Stewart; Wands, Malik, Lyth & Liddle

Stochastic Inflation



super-Hubble scales —> stochastic inflation

sub-Hubble scales —> CPT

Coarse-grained field $\hat{\mathbf{\Phi}}_{\rm cg}(N,\vec{x}) = \int_{k<\sigma H a(N)} \mathrm{d}\vec{k}\left[\mathbf{\Phi}_{\vec{k}}(N)e^{-i\vec{k}\cdot\vec{x}}\hat{a}_{\vec{k}} + \mathbf{\Phi}^{\star}_{\vec{k}}(N)e^{i\vec{k}\cdot\vec{x}}\hat{a}^{\dagger}_{\vec{k}}\right]$

$N = \ln(a)$

Quantum fluctuations
source the **background**

Equation of motion $\frac{\mathrm{d}}{\mathrm{d}N}\mathbf{\Phi}_{\rm cg} = \mathscr{F}_{\rm background}(\mathbf{\Phi}_{\rm cg}) + \xi$

Starobinsky, (1982) 1986

Why does inflation look single field to us?

Koki Tokeshi, VV, 2023

Most high-energy constructions that allow for a phase of inflation contain many additional degrees of freedom

Still, all cosmological observations are compatible with inflation being driven by a single field.

WHY?

Volume selection effet: The universe is dominated by the regions that inflate most, hence that contribute the largest volume

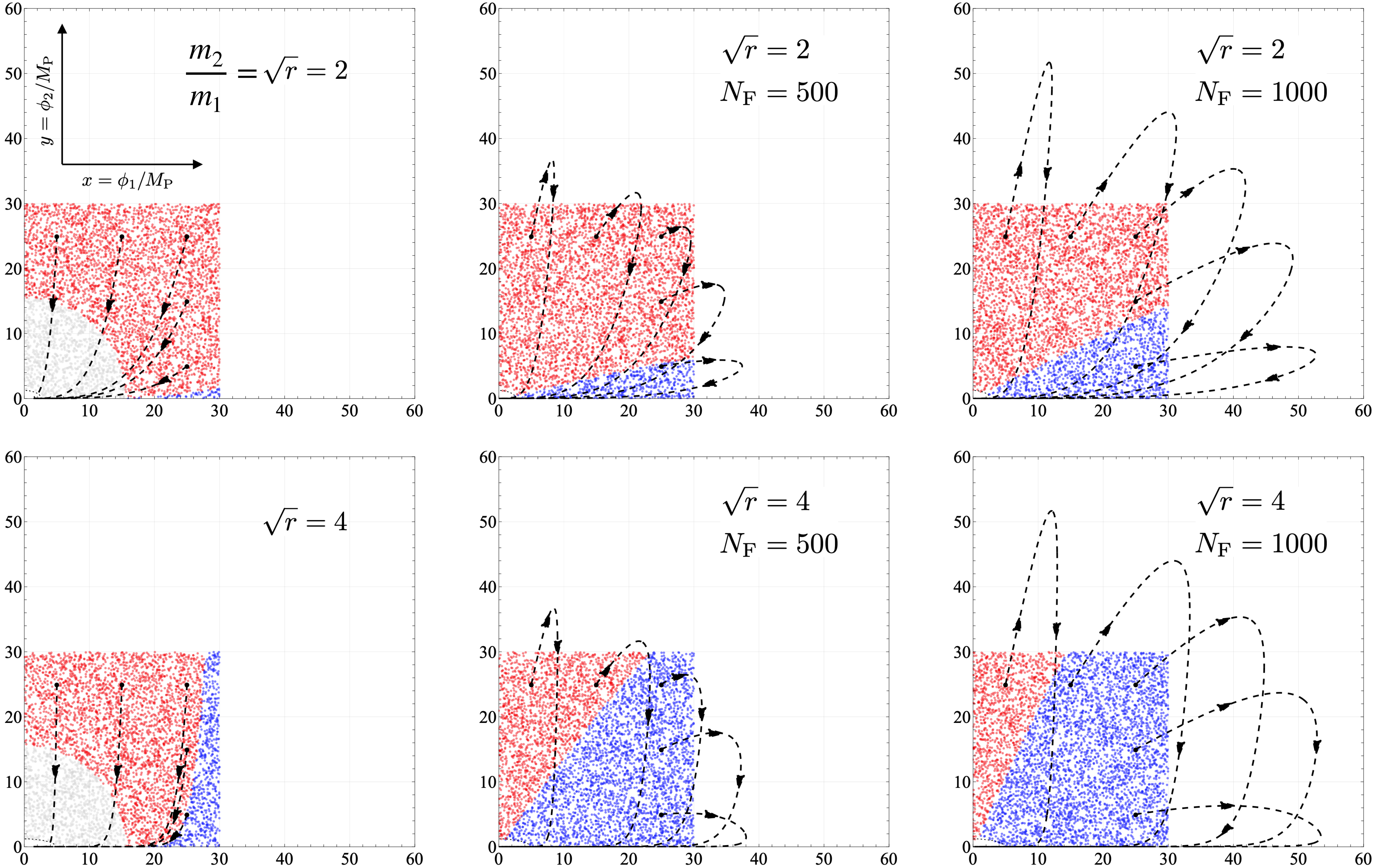
Theory of constrained stochastic processes: one can derive a modified Langevin equation, that only samples realisations of (or above) a given duration.

One can thus restrict the analysis to those regions of the universe where observers are most likely to end up in!

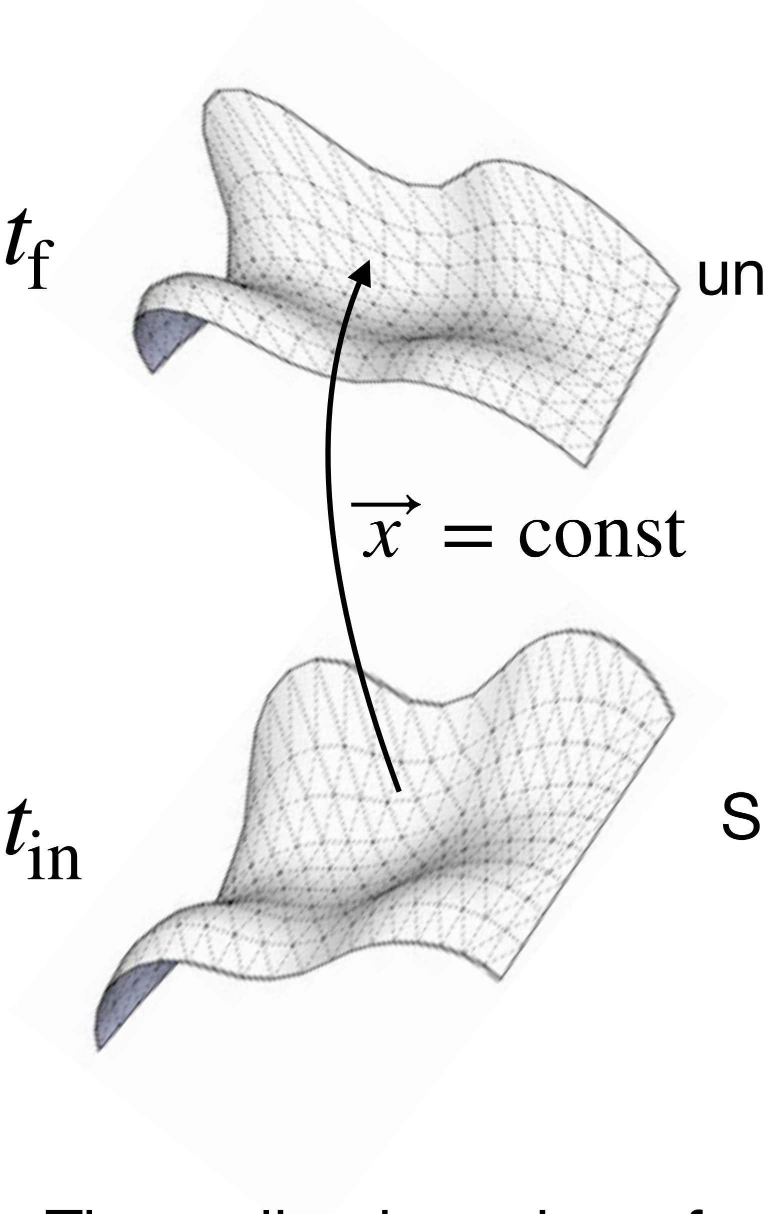
Why does inflation look single field to us?

Double quadratic inflation: $V = \frac{m_1^2}{2}\phi_1^2 + \frac{m_2^2}{2}\phi_2^2$

Koki Tokeshi, VV, 2023



Stochastic-δ*N* formalism



t_f

$\overrightarrow{x} = \mathrm{const}$

t_in

uniform density slice

$\delta\rho = 0$

Spatially flat slice

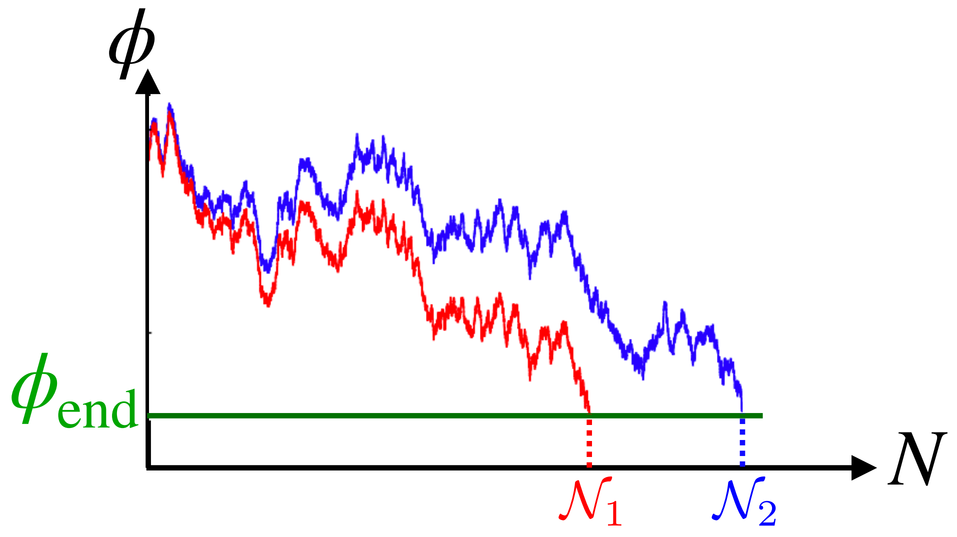
$\psi = 0$

$$\zeta(t, x) = N(t, x) - N_0(t) \equiv \delta N$$

Lifshitz, Khalatnikov (1960)
Starobinsky (1983)
Wands, Malik, Lyth, Liddle (2000)

The realised number of e-folds
is a stochastic quantity:

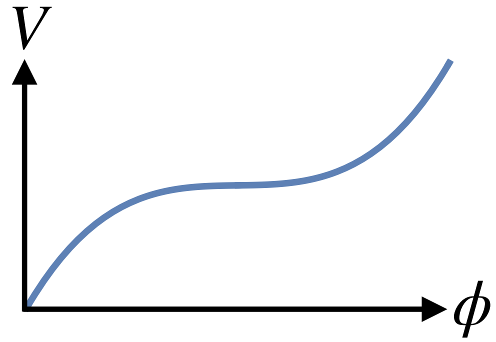
$$\zeta_{\text{coarse grained}} = \mathcal{N} - \langle \mathcal{N} \rangle$$

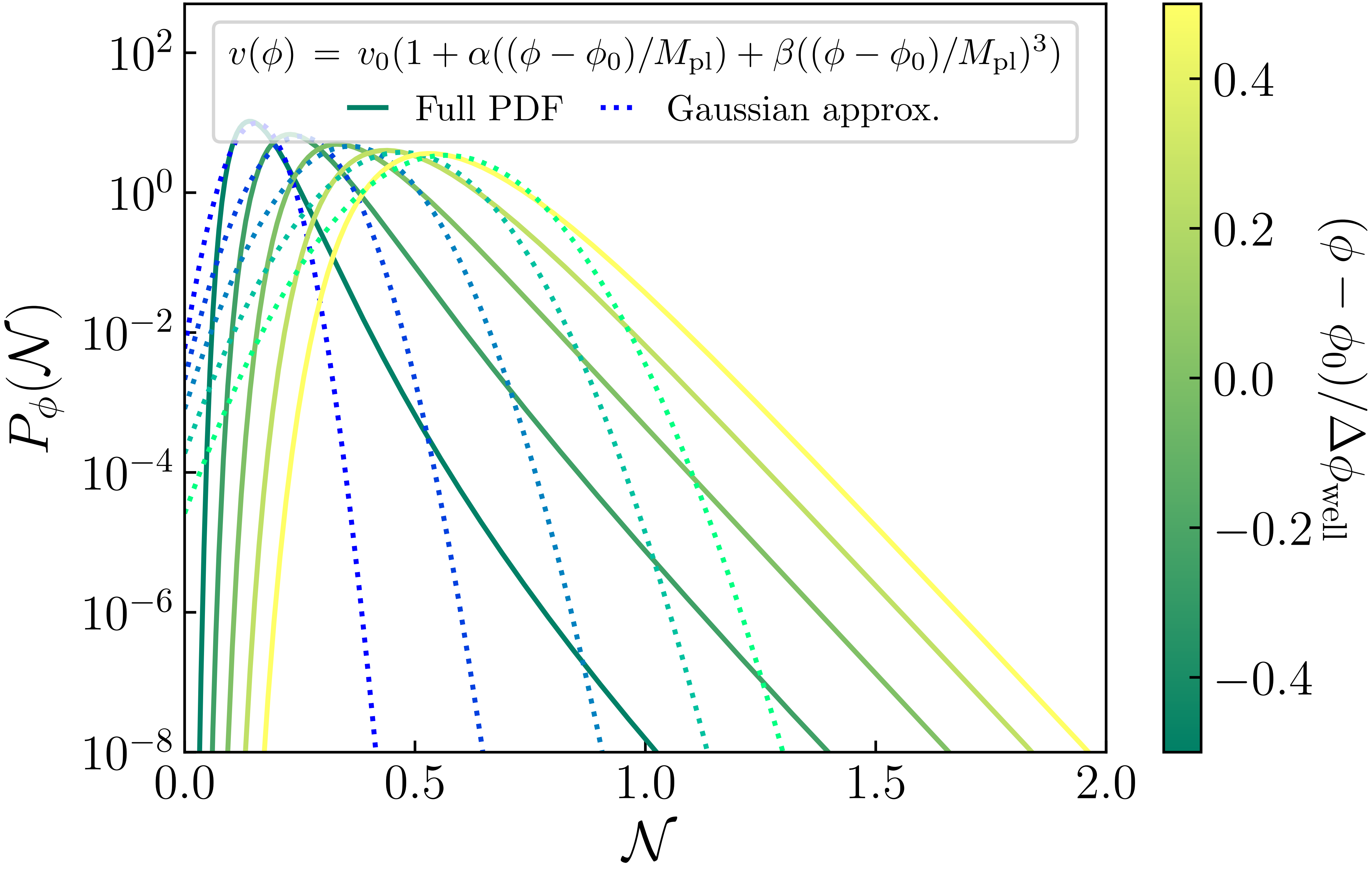


Enqvist, Nurmi, Podolsky, Rigopoulos (2008) ; Fujita, Kawasaki, Tada, Takesako (2014); VV, Starobinsky (2015)

Exponential tails

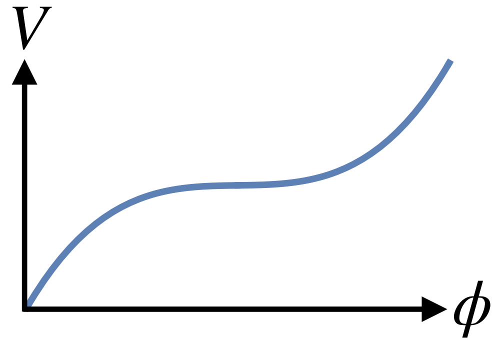
Pattison, VV, Assadullahi, Wands (2017)
Ezquiaga, Garcia-Bellido, VV (2020)

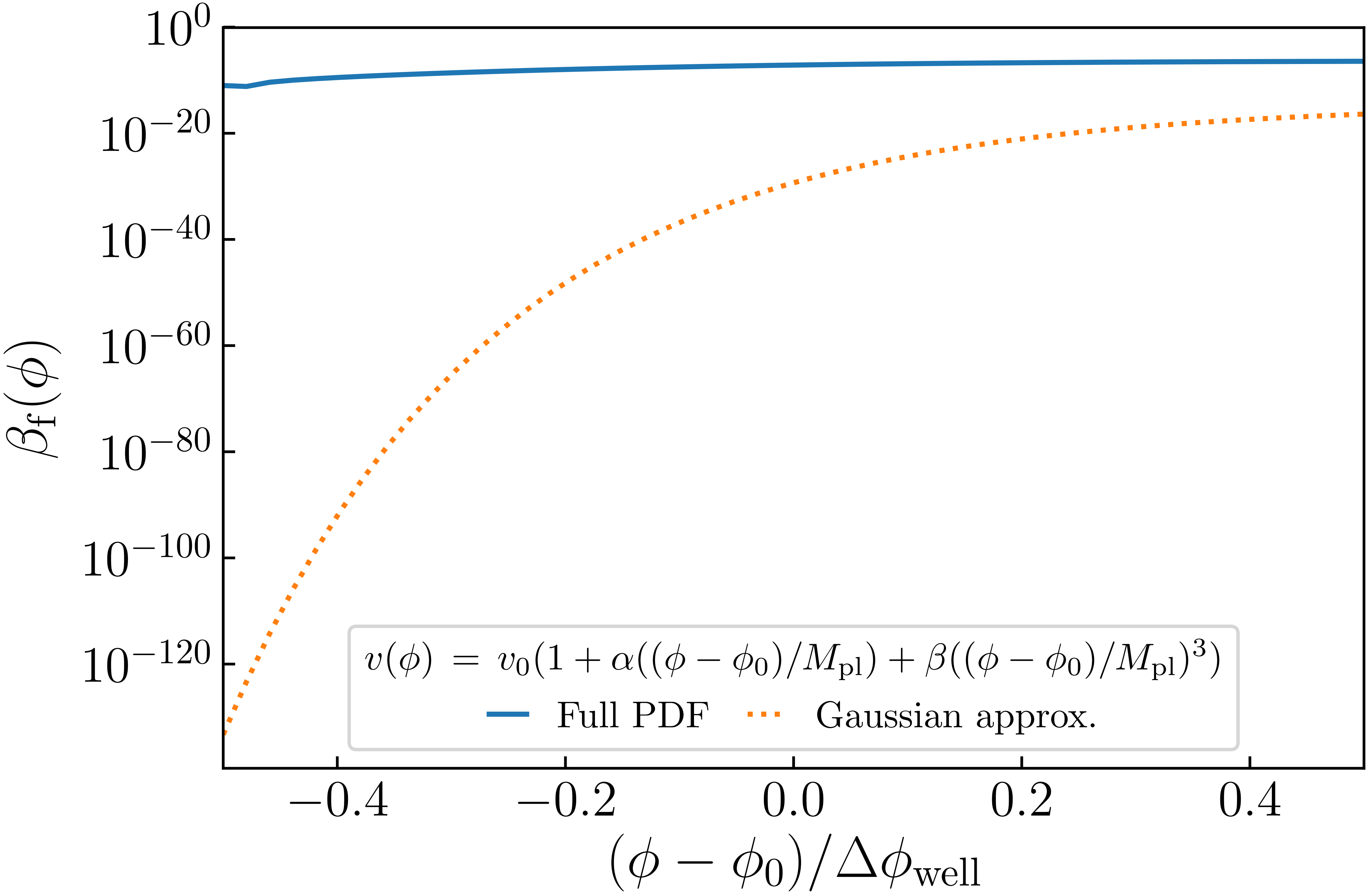




Impact on PBHs

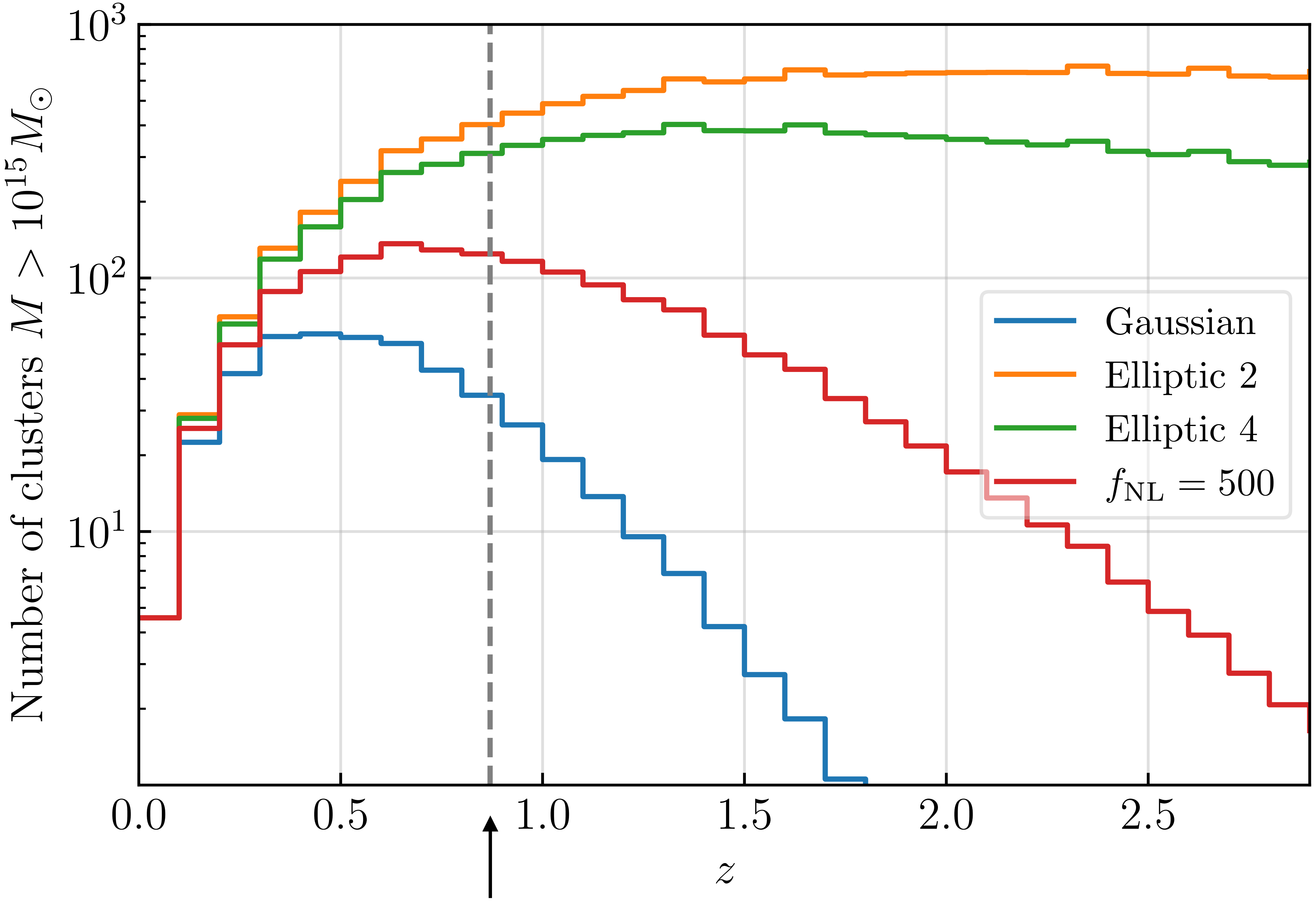
Pattison, VV, Assadullahi, Wands (2017)
Ezquiaga, Garcia-Bellido, VV (2020)





Impact on LSS

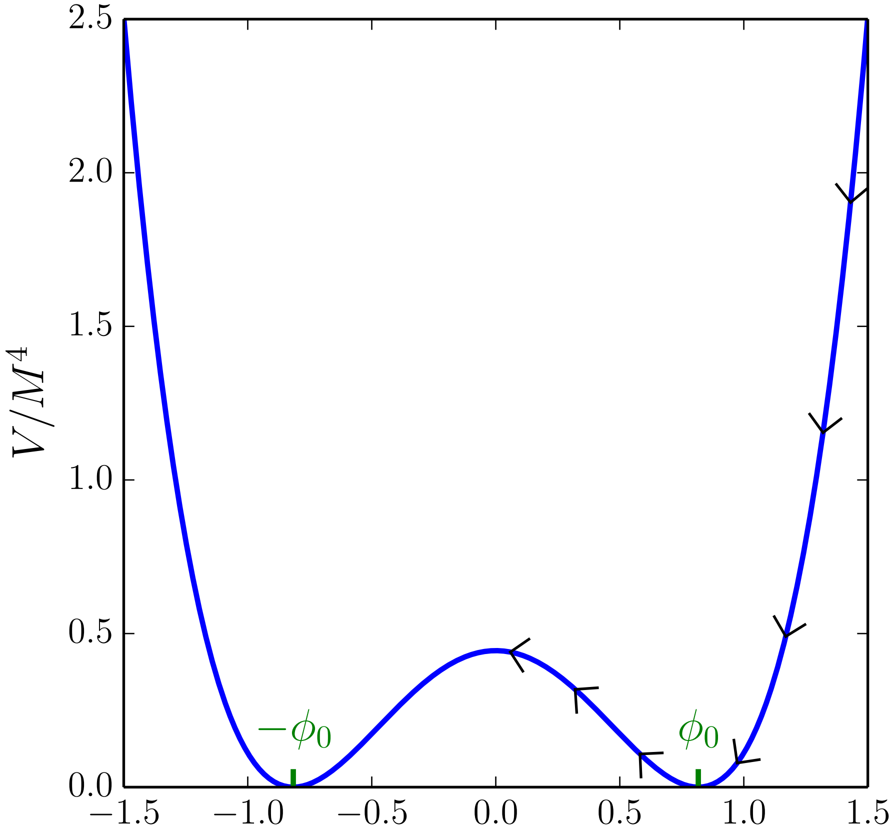
Ezquiaga, Garcia-Bellido, VV (2022)

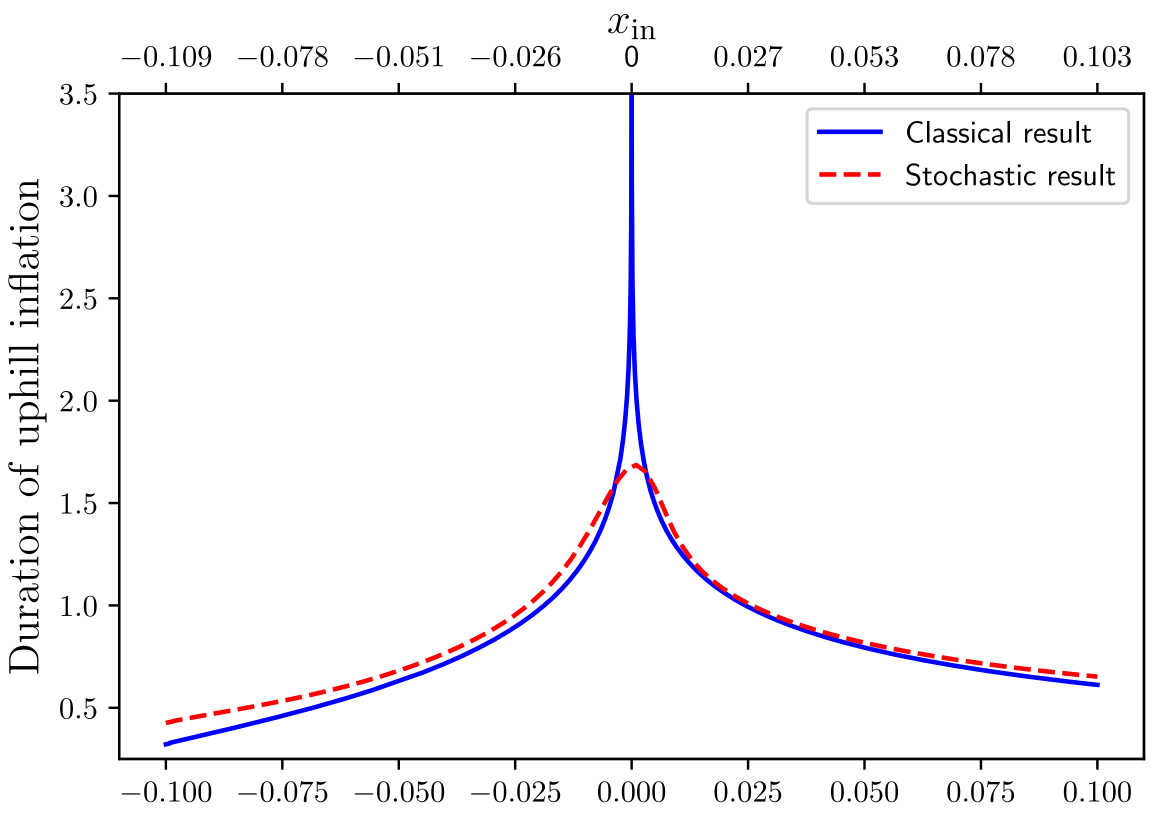


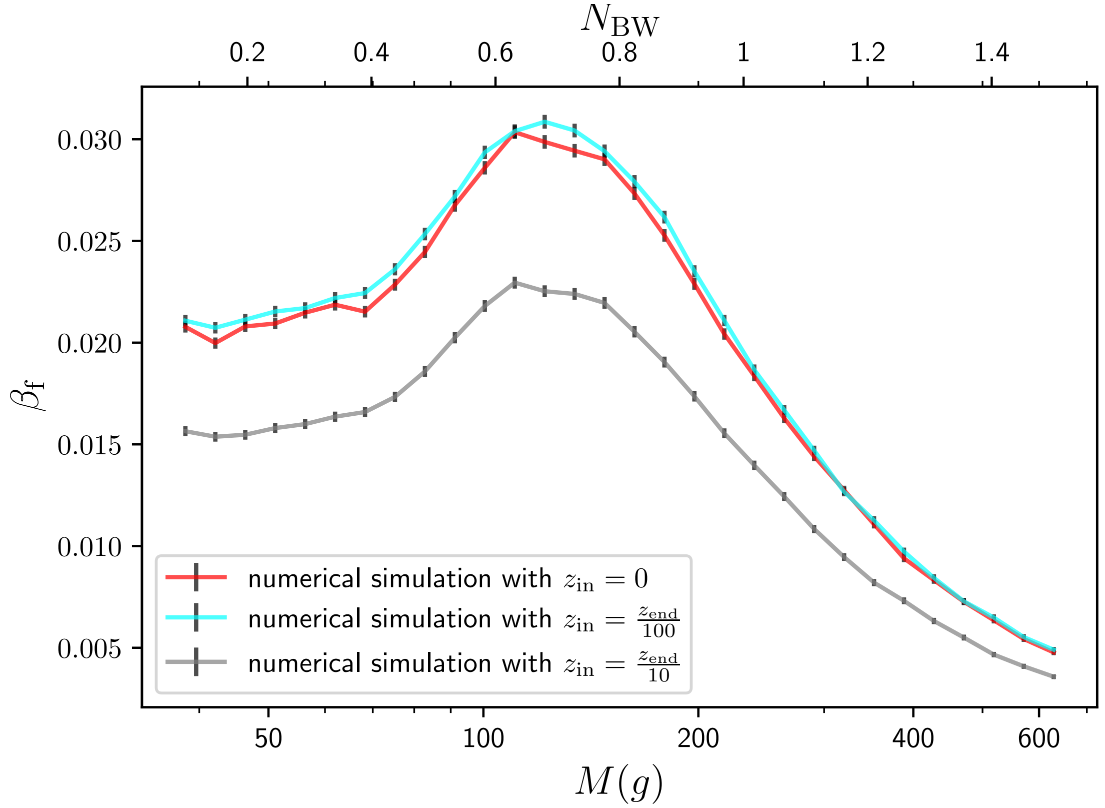
El gordo

Exponential tails

in non-slow-roll models







Vadim Briaud, VV (2023)

Conclusions

- The back-reaction of vacuum quantum fluctuations on the background dynamics can be incorporated within the formalism of stochastic inflation

- This is necessary to describe regimes leading to large fluctuations, such as those yielding primordial black holes

- Quantum diffusion leads to exponential tails: non-perturbative breakdown of Gaussian statistics

- Most cosmological observables can be reconstructed from first-passage time analysis (power spectrum, mass functions, clustering, n-point functions)

- Quantum diffusion makes the CMB probe the whole potential: models leading to PBHs are constrained by the CMB, even if those two sets of scales are well separated

- **What is the best strategy to look for exponential tails in the data?**