Effects of Nonstandard Cosmologies on PBH and induced GW

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IIT Madras

2101.02234 [astro-ph.CO]
2010.05071 [astro-ph.CO]
1912.01653 [astro-ph.CO]

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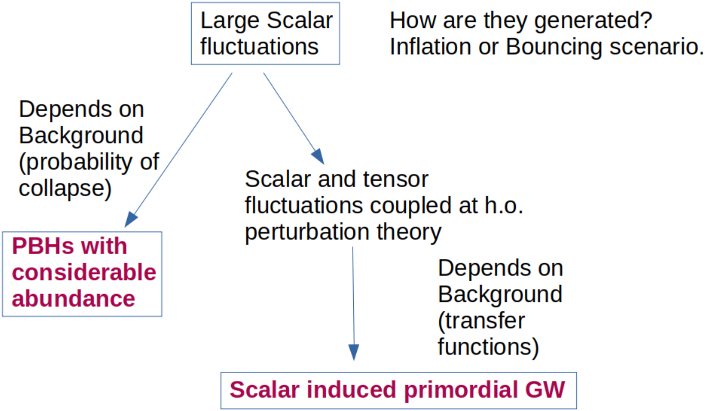
Why study them together?



Primordial Black Holes

- **What are PBHs?**
  Formed in the early universe when the density fluctuations of high amplitude ($\delta \equiv \frac{\rho-\bar{\rho}}{\bar{\rho}} > \delta_c$) re-enter the Hubble horizon at post-inflationary epochs and collapse gravitationally.

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- **Why PBHs?**
  • Nonrelativistic, massive and collisionless: Can be a candidate for DM: 100% DM can be constituted by PBH for a particular mass range.
  • GW detectors (LIGO/Virgo) observed binary black hole mergers: Can LIGO/Virgo events $M \sim M_\odot$ be described by PBH?
  • Ultra light PBHs $M < 10^{15}$ gm are radiating violently, relativistic particles are injected in the cosmos $\rightarrow$ explanation of an observed energetic particle.
  • A tool to probe smaller scales of inflation. $\rightarrow$ Accommodate $P_\zeta(k_{\rm CMB}) \sim 10^{-9}$ and $P_\zeta(k \gg k_{\rm CMB}) \sim 10^{-2}$ together in an inflation model.

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- **Basic quantities**

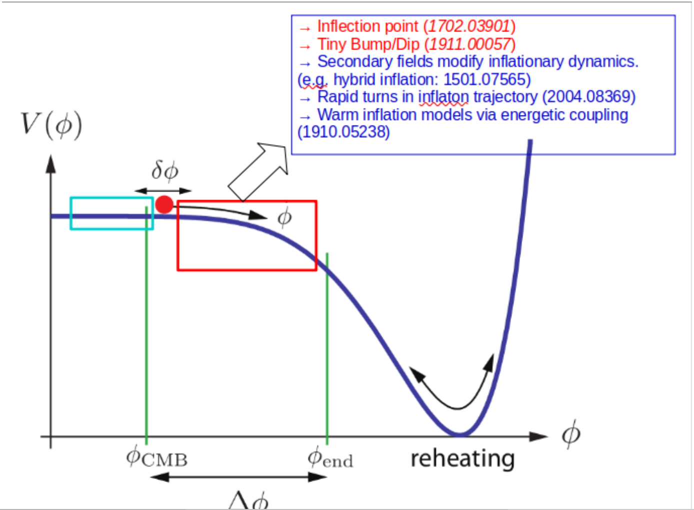
  • δ_c depends on background EoS w.

  • For $0 < w \leq 1$, the fraction of the energy density collapsing into a PBH is: $\beta\gamma$.

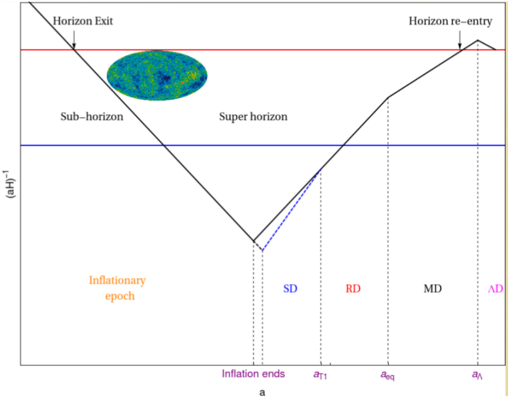
  • β: mass fraction $\rightarrow$ probability of collapse. β depends on δ_c and $P(\delta)$. • $P(\delta)$ depends on $P_\zeta(k)$ $\leftarrow$ links PBH to inflation.

Inflation models and PBH

- Requirement: Large $P_\zeta(k) \propto 1/\epsilon$. In RD, about 10% abundance of PBH requires $P_\zeta(k) \sim 10^{-2}$.
- ϵ has to be very small $\rightarrow$ ultra-slow roll regime.



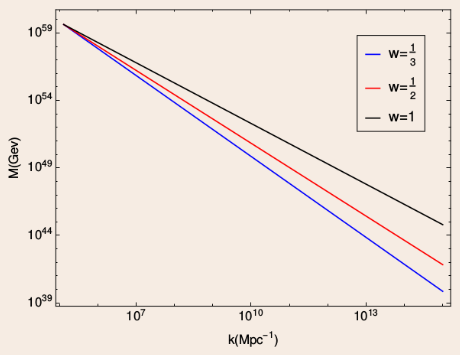
Alternate cosmological evolution and PBH mass



$$M(k) \propto k^{-2} \implies M(k) \propto k^{-2\pm\cdots}$$

- $w = 1$: kination epoch motivated in quintessential inflation mmodels.
- Moduli vacuum misalignment in String theory inspired models of inflation $\rightarrow$ post-inflationary moduli domination $w = 0$ + moduli reheating.

PBH in a stiff-domination



$$M(k) = \left(\frac{\gamma}{2G}\right)\left(2\times\frac{\pi^2 g_*^{eq}}{30}\right)^{\frac{1}{3w+1}}\left(\frac{8\pi G}{3}\right)^{\frac{1}{3w+1}}\left(\frac{g_s(T_{eq})}{g_s(T_1)}\right)^{\frac{3w-1}{3(3w+1)}} \times (a_{eq}T_{eq})^{\frac{3(1+w)}{3w+1}}\, T_1^{-\frac{3w-1}{3w+1}}\, k^{-\frac{3(1+w)}{3w+1}}$$

PBH Abundance: Relevant Quantities

- Critical density contrast: $\delta_c = \frac{3(1+w)}{(5+3w)} \sin^2\left(\frac{\pi\sqrt{w}}{1+3w}\right)$
- Fraction of the Horizon mass going into PBH: $\gamma = 0.2$
- Mass fraction: $\beta(M) \equiv \frac{1}{\rho_{\rm tot}} \frac{d\rho_{\rm PBH(M)}}{d\ln M} = 2\int_{\zeta_c}^{\infty} \frac{1}{\sqrt{2\pi}\sigma(M)} e^{-\frac{\zeta^2}{2\sigma(M)^2}} d\zeta = {\rm erfc}\left(\frac{\zeta_c}{\sqrt{2}\sigma(M)}\right)$

  $\zeta_c = \frac{(5+3w)}{2(1+w)}\delta_c$: critical value of curvature perturbation.
  $\sigma^2(M) \approx P_\zeta(k)$.
- Abundance: Fraction of PBH of a particular mass M as DM: $f_{\rm PBH}(M) \equiv \frac{\Omega_{\rm PBH}(M)}{\Omega_{cdm}}$
- Total abundance: Fraction of total PBH as DM: $f_{\rm PBH}^{\rm tot} \equiv \frac{\Omega_{\rm PBH}}{\Omega_{cdm}} = \int f_{\rm PBH}(M) d\ln M$.

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$$k \rightarrow P_\zeta(k)$$
$$M \rightarrow f_{\text{PBH}}(M).$$

Dynamics of PBH formation

- Energy density during a single additional pre-BBN epoch:

$$\rho(T) = \left(\frac{3}{8\pi G}\right)\left(\frac{\gamma}{2G}\right)^2 M^{-2} = \frac{\pi^2}{30} g_*(T_1)\left(\frac{g_s(T)}{g_s(T_1)}\right)^{1+w}\left(\frac{T}{T_1}\right)^{3(1+w)} T_1^4$$

- $\rho(T_1) = \rho^{\rm rad}(T_1)$.
- PBH of mass M is formed at temperature T. At formation, $\frac{\rho_{\rm PBH}(M)}{\rho_T} = \gamma\beta(M)$.

$$f_{\rm PBH}(M) = \gamma\beta(M)\left(\frac{g_s(T)}{g_s(T_1)}\right)^{w}\left(\frac{g_s(T_1)}{g_s(T_{\rm eq})}\right)\left(\frac{T}{T_1}\right)^{3w}\left(\frac{T_1}{T_{\rm eq}}\right)\left(\frac{\Omega_m h^2}{\Omega_c h^2}\right)$$

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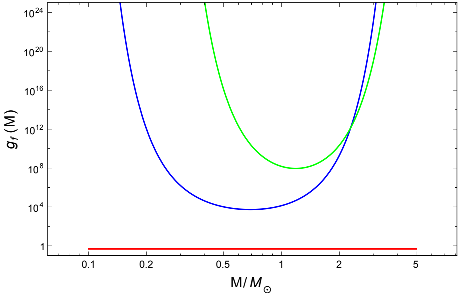
Dynamics of PBH formation

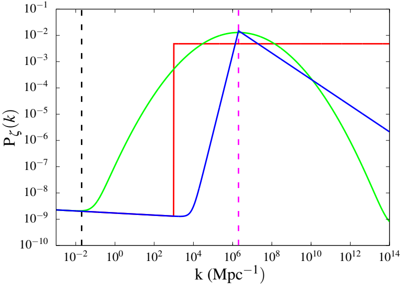
- Gain over PBH formation at radiation domination:

$$g_f \equiv \frac{f_{\rm PBH}(M)}{f_{\rm PBH}^{\rm rad}(M)} \simeq \frac{\beta(M)}{\beta^{\rm rad}(M)} \left(\frac{T}{T_1}\right)^{3w-1} > 1.$$



Results: Analysis with different power spectra

- $P_\zeta(k) = A_s \left(\frac{k}{k_*}\right)^{n_s-1} + f(P_p, k_p, ...)$.
- 1. Scale-independent power spectrum; 2. Broken Power Law; 3. Gaussian Power Spectrum.
- 2 and 3 are theoretically motivated.



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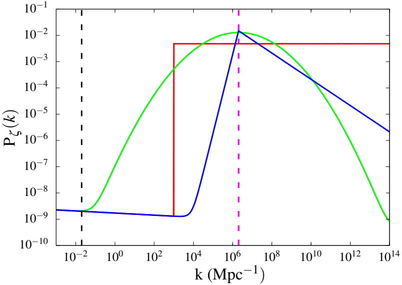
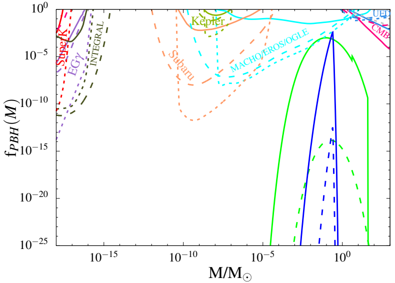


- 3. Gaussian Power Spectrum $P_\zeta(k) = A_s \left(\frac{k}{k_*}\right)^{n_s-1} + P_p \exp\left[-\frac{(N_k - N_p)^2}{2\sigma_p^2}\right]$.
- Analysis done for $k_p \sim 10^6 Mpc^{-1}$ (∼solar mass PBH) and $k_p \sim 10^{12} Mpc^{-1}$ (LISA)

PBH Mass Spectra

$T_1 = 10$ MeV



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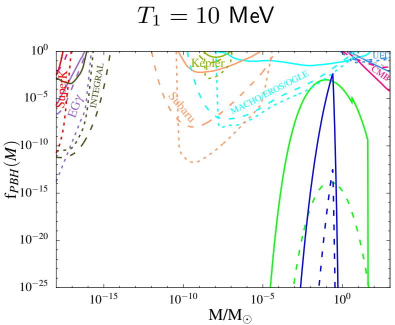
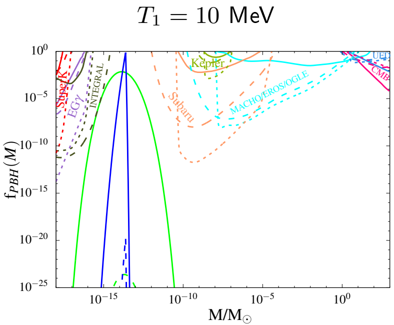
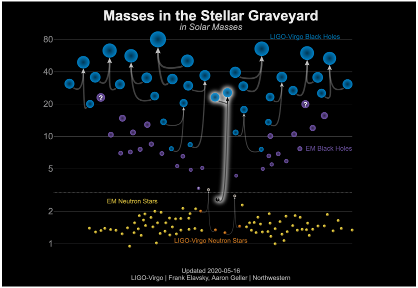


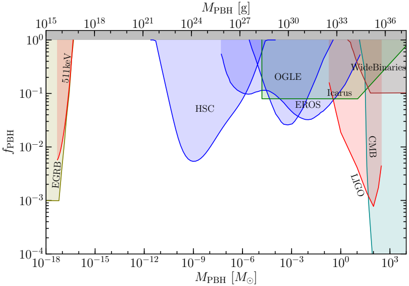


k_p		Scale-inv P_p	Broken Power Law P_p	Gaussian P_p
2×10^6 Mpc^{-1}	$w = 1/3$	0.021	0.0275	0.025
2×10^6 Mpc^{-1}	$w = 1$	0.0048	0.0113	0.0105
6×10^{12} Mpc^{-1}	$w = 1/3$	0.013	0.016	0.0163
6×10^{12} Mpc^{-1}	$w = 1$	0.0048	0.0067	0.006

for comparison, check *1812.11011*; for exact experimental bounds, check *1705.05567,1912.01014*

Moduli dominated PBH production: mass range of interest





LIGO Collaboration and B.Carr et. al., 2006.02838

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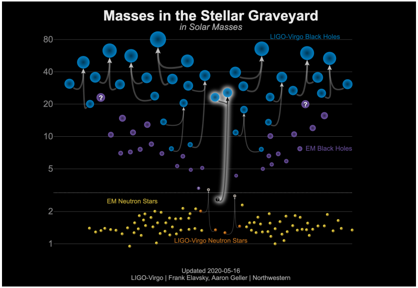

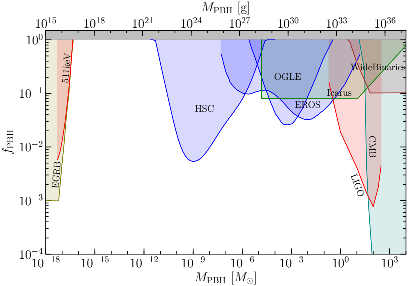

LIGO Collaboration and B.Carr et. al., 2006.02838

- Mass range of interest: $M \sim 30 M_\odot$.
- $\Gamma = \frac{m_\phi^3}{2m_P^2}$ and $T_{\rm rh} = \left(\frac{90}{8\pi^3 g_*(T_{\rm rh})}\right)^{1/4} \sqrt{\Gamma m_P} \simeq 2.75 MeV \left(\frac{10.66}{g_*(T_{\rm rh})}\right)^{1/4} \left(\frac{m_\phi}{100 TeV}\right)^{3/2}$.

$$M \propto \frac{1}{H} \propto a^{3(1+w)/2}$$

PBH formation: RD vs MD

RD

- Standard deviation of fluctuations is determined in the general relativistic perturbation theory.
- $\delta_c = \dfrac{3(1+w)}{5+3w} \sin^2\left(\dfrac{\pi\sqrt{w}}{1+3w}\right).$
- $\beta(M) = \int_{\delta_c}^{\infty} d\delta P(\delta) = \mathrm{erfc}\left(\frac{\delta_c}{\sqrt{2}\sigma(M)}\right).$
- $\psi(M) = 5\times 10^8 \dfrac{\beta(M)}{M}\left(\dfrac{M_\odot}{M}\right)^{1/2}$

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EMD

- Standard deviation of fluctuations is determined in the Newtonian cosmology.
- $\beta(M) \simeq 0.056\sigma^5(M)$.
- $\psi(M) \simeq 8.2 \times 10^{27} \left(\frac{\Gamma}{M_p} \right)^{1/2} \frac{\beta(M)}{M}$.
- $M_{\rm max} \simeq M_{\rm rh}\sigma_{\rm max}^{3/2}$
  $M_{\rm min} = M_{\rm max} \left(\frac{a_{\rm md}}{a_{\rm rh}} \right)^{3/2}$.

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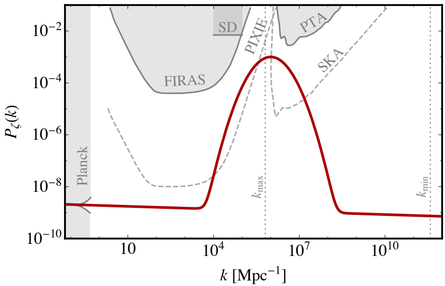
- $H_{\rm dom} \simeq m_\phi(\phi_0/M_P)^2$.
- Parameters entering from moduli domination: m_ϕ and $\phi_0 = M_P/100$.

Mass functions

$$\psi(M) = \begin{cases} 2.6 \times 10^8 \left(\frac{M_\odot}{M}\right)^{1/2} \left(\frac{m_\phi M_{\rm Pl}}{\phi_0^2}\right)^{1/3} \frac{\beta_{\rm RD}(M)}{M}\,, & M < M_{\sf min}\,, \\ 5.2 \times 10^{26} \left(\frac{m_\phi}{M_{\rm Pl}}\right)^{3/2} \frac{\beta_{\rm MD}(M)}{M}\,, & M_{\sf min} \leq M \leq M_{\sf max}\,, \\ 5 \times 10^8 \left(\frac{M_\odot}{M}\right)^{1/2} \frac{\beta_{\rm RD}(M)}{M}\,, & M > M_{\sf max}\,. \end{cases}$$

Primordial Power spectrum

$$P_\zeta(k) = A_s \left(\frac{k}{k_*}\right)^{n_s - 1} + A_p \exp\left[-\frac{(N_k - N_p)^2}{2\sigma_p^2}\right]$$



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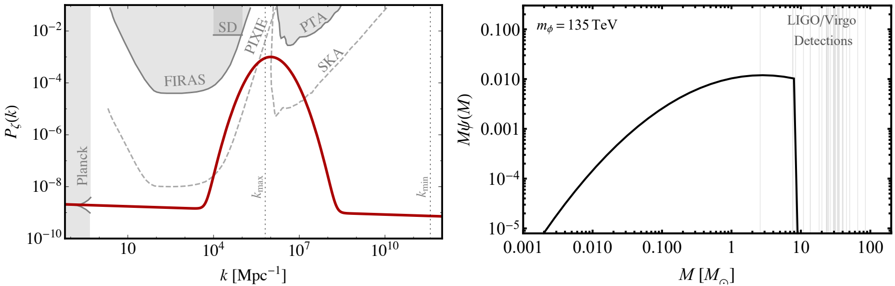

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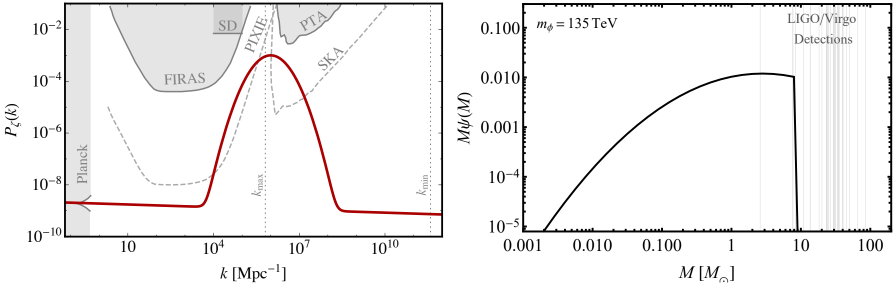
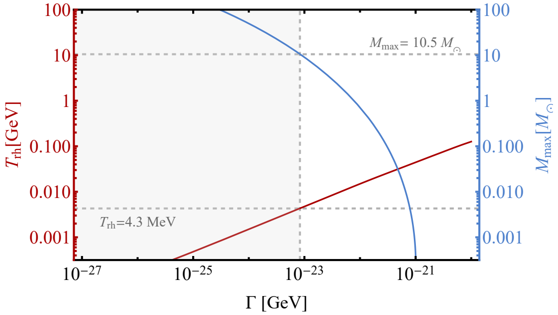


- $A_p = 10^{-3}, \sigma_p = 1$, and $k_p = 10^6$ Mpc^{-1}.
- The Planck experiment restricts any deviation from scale-invariant spectrum at large scales.
- FIRAS data excludes the upper gray-shaded region between $k = 10^2 - 10^4$ Mpc^{-1}.
- Future experiment PIXIE (dashed gray) could potentially rule out such feature in the primordial power spectrum at small scale.
- Parameters from the primordial sector: A_p and σ_p.

Maximum PBH mass



- $\Gamma \gtrsim 8 \times 10^{-24}$ GeV, $m_\phi \sim 135$ TeV.
- This is value of $M_{\rm max}$ falls in the parameters space of BH mass that is being probed by the LIGO/Virgo.
- PBHs form only about 4.3% of the DM population, and about 95% of these black holes are above $0.1 M_\odot$.

Stochastic GW Background

- 1st order scalar perturbations are source for 2nd order tensor perturbations.
- Large amplitude of fluctuations required for PBH production can source large tensor perturbations.
- First order scalar transfer functions $\Phi(p,\eta) = \frac{1}{2+(k/p)^{3/2}}$;
- Second order tensor transfer functions $t(k,\eta) = \left(\frac{k(T_1)}{k}\right)^{1/2}\left(\frac{k_{\rm eq}}{k(T_1)}\right)a_{\rm eq}$.

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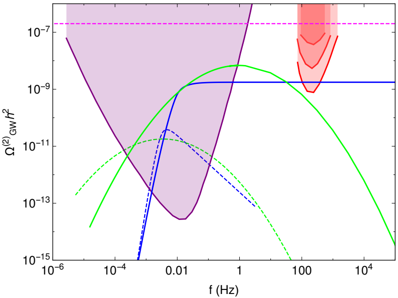
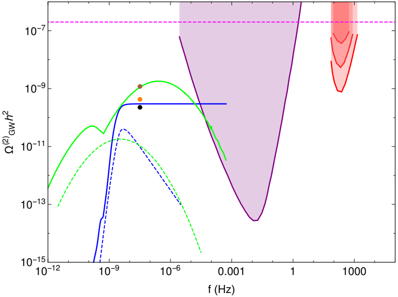


Future LISA probe is important.



Present and future PTA probes are important. TtoB: EPTA, NanoGrav, PPTA.

Scenarios consistent with NANOGrav 12.5 yr results

- The recent 12.5 year pulsar timing array (PTA) data released by NANOGrav has reported the discovery of a stochastic common-spectrum process with amplitude $A_{\rm CP}$ and slope $\gamma_{\rm CP}$, which can be fitted into a power law in a narrow range of frequencies.[Arzoumanian et. al., 2009,04496][1]

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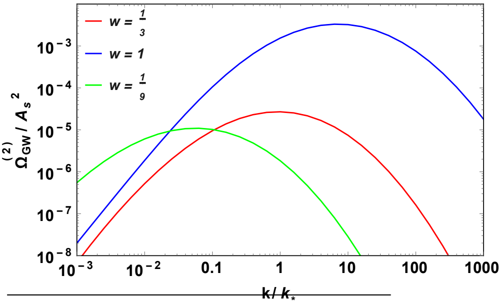
- Assuming the NANOGrav signal to be stochastic GW background, we explored the implications for PBH abundance in nonstandard epochs.
- $A_s(w=1/3) = 6\times10^{-3}$, $A_s(w=1) = 5.5\times10^{-4}$ and $A_s(w=1/9) = 5.1\times10^{-3}$ to have $\Omega_{GW,\rm peak} = 10^{-9}$.



$$y = \Omega_{GW,\rm NANOGrav}$$
$$A_s = \sqrt{\frac{\Omega_{\rm GW,peak}}{\Omega_{\rm GW,NANOGrav}}}.$$

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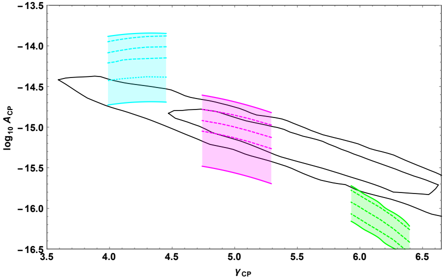
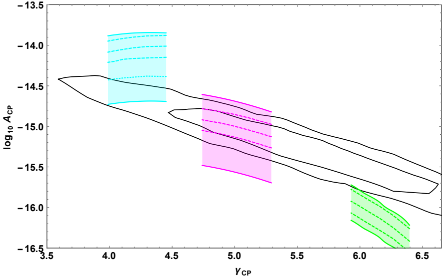
Result: NANOGrav analysis

	$w=1/3$	$w=1$	$w=1/9$
$A_s^{\max}$	0.015	0.007	0.0082
$A_s^{\min}$	0.002	0.001	0.003

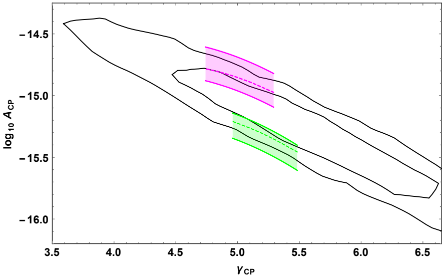


Result: NANOGrav analysis

	$w=1/3$	$w=1$	$w=1/9$
$A_s^{\max}$	0.015	0.007	0.0082
$A_s^{\min}$	0.002	0.001	0.003



- Both abundant PBH and NANOGrav consistency are reached with $A_s \simeq 0.007$ in the $w = 1/9$ case and $A_s \simeq 0.01$ required in the RD case.



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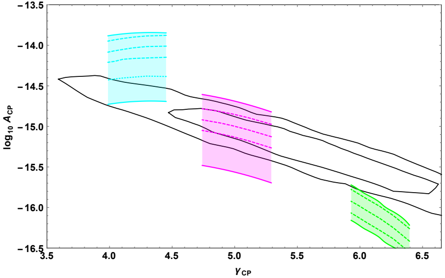

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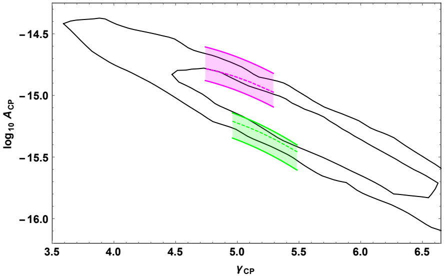
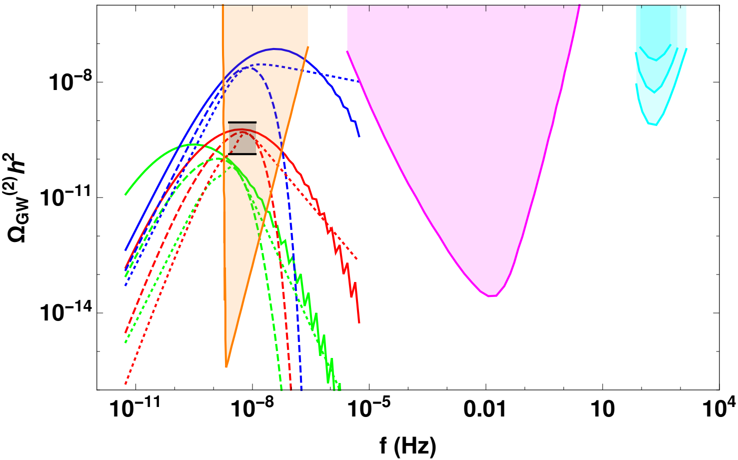


	$w = 1/3$	$w = 1$	$w = 1/9$
Range of k_* in Mpc^{-1}	2×10^6 - 7×10^6	2×10^6 - 7×10^6	2×10^7 - 7×10^7
Range of $M/M_\odot$ at $k_{*,\min}$	0.2-2	1-10	10^{-4} - 5×10^{-3}
Range of $M/M_\odot$ at $k_{*,\max}$	0.01 - 0.33	0.08-2	3×10^{-6} - 3×10^{-4}

GW spectrum for Different power spectra



$A_s = 0.007.$