Decoherence and entropy generation for Yukawa interaction at one loop in the inflationary de Sitter spacetime

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Introduction

- Observed high degree of spatial homogeneity and isotropy at large scales of universe $\rightarrow$ early universe underwent a phase of rapid accelerated expansion (the cosmic inflation). Requires exotic matter with negative pressure, called the dark energy/cosmological constant. Inflation also explains the flatness problem and the non-observed defects like magnetic monopoles (e.g. S. Weinberg, Cosmology (2009)).

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- Some open questions $\rightarrow$ Starting from the early inflationary high density DE, how did we reach its current tiny value? Can quantum effects explain this? How did the inflation end to begin the radiation dominated, thermalised era? How did the inflationary cosmological quantum perturbations become classical to develop into the large scale structures we observe today? Is Λ really vacuum energy density? How did the primordial magnetic field generate?

Introduction

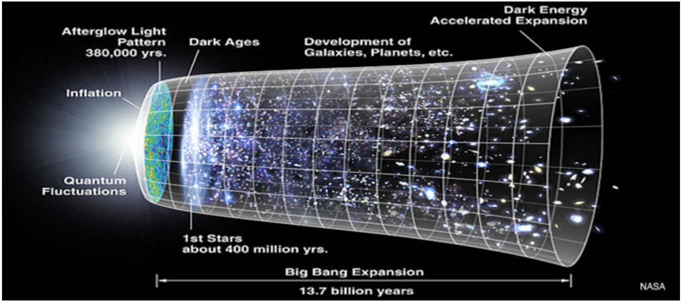

Figure: Our Universe – from history to present (source : NASA)

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- Cosmic coincidence problem $\rightarrow$ Only around $\pm 10\%$ mismatch of current Λ value compared to inflationary one would have changed the evolution history of the universe dramatically (T. N. Tomaras *et al*, PLB (1987)). A classical slow roll potential cannot fully address this, as it lacks any microscopic description, and there can be large quantum effects as well (e.g. T. Fujita *et al*, JCAP (2014)).

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- The de Sitter spacetime : A maximally symmetric spacetime with a constant positive curvature, $R = 2d\Lambda/(d-2)$ in d-dim. Simplest soln. with a positive Λ,

$$ds^2 = -dt^2 + a^2(t)d\vec{x}^2 = a^2(\eta)(-d\eta^2 + d\vec{x}^2)$$

where $a = e^{Ht}$, $H = \sqrt{\Lambda/3}$ and $\eta = -e^{-Ht}/H$, $0 \leq t < \infty$ and $-H^{-1} \leq \eta \leq 0^-$. An accelerated expansion with const. Hubble rate, $H = \dot{a}/a$, with H^{-1} being the Hubble or cosmological horizon size. Metric during inflation often taken to be dS.

Introduction

- Einstein eqn. $G_{\mu\nu} = 8\pi G T_{\mu\nu}$. General spatially homogeneous and isotropic cosmological background, $ds^2 = -dt^2 + a^2(t)\left(dx^2 + dy^2 + dz^2\right)$. Take matter field with energy density $\rho(t)$ and pressure $P(t)$. Friedman eqns.

$$\dot{a}^2/a^2 = 8\pi G\rho(t)/3 \qquad \ddot{a}/a = -4\pi G(\rho(t) + 3P(t))/3$$

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- $\rho > 0 \Rightarrow$ expansion spacetime. $P > 0 \Rightarrow \ddot{a} < 0 \rightarrow$ expansion rate decreases. Accelerated expansion, i.e. $\ddot{a} > 0$ only if $\rho + 3P$ is negative $\rightarrow$ negative pressure! Eqn. of state, $P = w\rho \rightarrow$ accelerated expansion possible only if $w < -1/3$. For $w = -1$, $\rho/8\pi G$ becomes a const., called Λ, the cosmological constant.

  Any matter field with $w < -1/3$ is called the dark energy. The simplest form of it is the cosmological constant, the simplest and phenomenologically very successful model of DE.

Massless quantum fields in dS

- Dynamical backgrounds such as dS $\to$ particle pair creation, save conformally invariant field theories prepared initially in the conformal vacuum. Massless but non-conformal fields (such as massless minimal scalar, gravitons) $\to$ a created such particle practically has infinite lifetime $\to$ indicates enhanced quantum effects as $Ht \gg 1$. Any process containing propagators of such fields as internal lines $\to$ large quantum effect at late times, created by long wavelength, super-Hubble or IR modes $\to$ the secular effect (T. Tanaka & Y. Urakawa, CQG (2013), for vast review).

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- Massless minimal scalar in dS, $\nabla_\mu \nabla^\mu \phi = 0 \Rightarrow$

$$\phi_{\vec{k}}(x) = \frac{H(1 + ik\eta)}{\sqrt{2k^3}} e^{-i(k\eta - \vec{k}\cdot\vec{x})}$$

Massless quantum fields in dS

- Initial time, $\eta \to -H^{-1}$, sub-Hubble modes ($k/H \gg 1$) $\Rightarrow$

$$\phi_{\vec{k}}(\eta \to -H^{-1}) \to \frac{1}{\sqrt{2k}} e^{-ik\eta + i\vec{k}\cdot\vec{x}}$$

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- However, the modes are *not* normalisable on hypersurfaces lying in the future of this initial hypersurface $\Rightarrow$ Wightman functions or the Feynman propagator would break de Sitter invariance. Take $\eta^{\mu\nu}\partial_\mu\left(a^{d-2}\partial_\nu i\Delta(x,x)\right) = i\delta^d(x-x')$, where (e.g. T. Brunier *et al*, CQG (2005)).

$$i\Delta(x,x') = A(x,x') + B(x,x') + C(x,x')$$

with (in $d = 4 - \epsilon$)

$$A(x,x') = \frac{H^{2-\epsilon}\,\Gamma\left(1-\frac{\epsilon}{2}\right)}{4\pi^{2-\frac{\epsilon}{2}}} \frac{1}{y^{1-\frac{\epsilon}{2}}}$$

$$B(x,x') = \frac{H^{2-\epsilon}}{(4\pi)^{2-\frac{\epsilon}{2}}}\left[-\frac{2\Gamma(3-\frac{\epsilon}{2})}{\epsilon}\left(\frac{y}{4}\right)^{\frac{\epsilon}{2}} + \frac{2}{\epsilon}\frac{\Gamma(3-\epsilon)}{\Gamma(2-\frac{\epsilon}{2})} + \frac{\Gamma(3-\epsilon)}{\Gamma(2-\frac{\epsilon}{2})}\ln(aa')\right]$$

$$C(x,x') = \frac{H^{2-\epsilon}}{(4\pi)^{2-\frac{\epsilon}{2}}}\sum_{n=1}^{\infty}\left[\frac{\Gamma(3-\epsilon+n)}{n\,\Gamma(2-\frac{\epsilon}{2}+n)}\left(\frac{y}{4}\right)^n - \frac{\Gamma\left(3-\frac{\epsilon}{2}+n\right)}{\left(n+\frac{\epsilon}{2}\right)\,\Gamma(n+2)}\left(\frac{y}{4}\right)^{n+\frac{\epsilon}{2}}\right]$$

Massless quantum fields in dS

- where

$$y(x,x') = aa'H^2\Delta x^2 = aa'H^2\left[-(|\eta-\eta'|-i\epsilon)^2 + |\vec{x}-\vec{x}'|^2\right]$$

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- A simple example of secular effect : compute $i\Delta(x,x')$ in the coincidence limit,

$$i\Delta(x,x) = \frac{H^{2-\epsilon}}{2^{2-\epsilon}\pi^{2-\epsilon/2}}\frac{\Gamma(2-\epsilon)}{\Gamma(1-\frac{\epsilon}{2})}\left(\frac{1}{\epsilon}+\ln a\right)$$

Imagine a quartic self interaction $\mathcal{L} = -\lambda\phi^4/4! - \delta m^2\phi^2/2$. One loop bubble self energy $\rightarrow$

$$-i\left[\frac{\lambda}{2}i\Delta(x,x) + \delta m^2\right]a^d\delta^d(x-x') \Rightarrow \delta m_\lambda^2 = -\frac{\lambda H^{2-\epsilon}}{(4\pi)^{2-\epsilon/2}}\frac{\Gamma(3-\epsilon)}{\Gamma(2-\epsilon/2)\epsilon}$$

However, the self energy grows monotonically as $\sim \ln a \rightarrow$ after sufficient e-foldings, becomes $\mathcal{O}(1) \rightarrow$ Non-perturbative effects.

The secular effect

- Consider the 2-loop $\langle T_{\mu\nu} \rangle$: renormalised results (V. Onemli & R. Woodard, CQG (2002)) :

$$\langle \rho \rangle_{\lambda,\,\mathrm{ren.}} = \frac{\lambda H^4}{2^7 \pi^4} \left[\ln^2 a + \frac{4}{9a^3} - \sum_{n=1}^{\infty} \frac{(n+2)\, a^{-n-1}}{(n+1)^2} \right] \quad (\ln^2 a = (\ln a)^2)$$

$$\langle P \rangle_{\lambda,\,\mathrm{ren.}} = -\frac{\lambda H^4}{2^7 \pi^4} \left[\ln^2 a + \frac{2}{3} \ln a + \sum_{n=1}^{\infty} \frac{(n^2-4) a^{-n-1}}{3(n+1)^2} \right]$$

Large de Sitter breaking backreaction at late times $\rightarrow$ large screening of inflationary Λ? End of inflation? Similar loop effects computed for Yukawa coupling, QED with scalar and gravitons up to two loop (for a review, B. L. Hu, 1812.1185).

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- A massless minimal scalar with quartic self interaction – such late time secular logarithms can be resummed to obtain a bounded result, $\langle T_\mu{}^\nu\rangle \sim \mathcal{O}(1)$ and at late times the dS symmetry is retained via a dynamical generation of mass, $m^2_{\mathrm{dyn.}} \sim \lambda^{1/2}$ for $O(N)$ model (G. Moreau & J. Serreau, PRL (2019); M. Baugmart & R. Sundrum, 1912.09502).

The secular effect

- Consistent with the IR effective late time stochastic field theory (A. Starobinsky *et al*, PRD (2009)). The dynamically generated mass $\rightarrow$ interesting predictions in CMB, primordial gravitational waves and non-Gaussianity. Solving gravitons or gauge field couplings in dS however remains as an open issue (dS invariant gauge fixing +derivative interactions (e.g. N. Tsamis *et al*, 2110.08715)).

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- Quartic potential is positive $\rightarrow$ yields positive non-perturbative vev. $\Rightarrow$ *increase* in the inflationary Λ-value. For non-positive potentials we may find a decrease. Knowing the precise non-perturbative shift could be important (SB, JCAP (2022), SB & N. Joshi, JCAP (2023))

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- Yukawa interaction and a non-positive potential $\rightarrow$ non-perturbative $\langle\phi\rangle$ and the dynamical mass was computed recently (SB & M. Dutta Choudhury, 2308.11384).

The decoherence

- Decoherence $\rightarrow$ process in which the states of an open quantum system become entangled (via some interaction) with its surrounding which leads to the loss of coherence and correlation for the system. Particularly relevant in the context of interacting QFT's (e.g., I. Allali & M. Hertzberg, JCAP (2020) and Refs. therein.) Could be relevant to understand how the early inflationary perturbations became classical as well as decoherence of stochastic gravitational waves (e.g., T. Prokopec & G. Rigopoulos, JCAP (2007) and Refs. therein.).

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- QFT in time dependent background like dS → essentially a non-equilibrium phenomenon (e.g., J. Berges, hep-ph/0409233 for a vast review.). The theory is broken into two parts → system + environment. The measure of decoherence is usually the von Neumann entropy of the system generated at late times. Most popular approach to study this is the Feynman-Vernon influence functional or master equation approach → one integrates out the environment field to construct an effective action (e.g., D. Boyanovsky, PRD (2018)).

The correlator approach to decoherence

- Despite its popularity and successes, the influence functional approach has several shortcomings $\rightarrow$ once the surrounding is traced out, there is no unitary evolution. There is always problem with renormalisability if the surrounding has interactions. For interacting theories, it is very difficult to treat things non-perturbatively in this approach.

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- A rather recently proposed approach to compute the von Neumann entropy is based upon the correlation functions instead (J. Koksma *et al*, PRD (2011), B. L. Hu, 1812.11851). Precisely, if we know all the correlation functions of system and surroundings, we practically know everything about them and hence there can be no entropy. In practical scenarios however, an observer only measures a few correlation functions for the system. Such limitation leads to lack of information for the system and hence entropy. This approach has been applied to zero and finite temp. field theories, in gravitational wave backgrounds etc. In particular, for two self interacting scalar field theories in the inflationary dS (P. Friedrich and T. Prokopec, PRD (2019)).

Entropy and phase space area

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- The ground state of a theory is the most symmetric one $\rightarrow$ corresponds to minimum uncertainty $\rightarrow$ take a Gaussian wave function (e.g. a HO) $\Rightarrow$ $\Delta x \Delta p = \hbar/2$. However, if the ground state is not *pure*, such minimum uncertainty will not be satisfied. In general,

$$\langle x^2\rangle\langle p^2\rangle - \frac{1}{4}\langle [x,p]_+\rangle^2 = \frac{\hbar^2}{4} \quad (\text{pure}, \ \text{Gaussian})$$

$$\langle x^2\rangle\langle p^2\rangle - \frac{1}{4}\langle [x,p]_+\rangle^2 > \frac{\hbar^2}{4} \quad (\text{mixed}, \ \text{non}-\text{Gaussian})$$

Put together

$$\langle x^2\rangle\langle p^2\rangle - \frac{1}{4}\langle [x,p]_+\rangle^2 = \frac{\hbar^2 \Xi^2}{4}$$

The dimensionless quantity $\Xi \geq 1$ is related to the phase space area. Increase in Ξ opening up of new phase space area due to to interaction. One then *defines* the von Neumann entropy,

$$S = \frac{\Xi + 1}{2} \ln \frac{\Xi + 1}{2} - \frac{\Xi - 1}{2} \ln \frac{\Xi - 1}{2}$$

The in-in formalism

- Dynamical background like dS $\rightarrow$ the ‘in’ vacuum evolves to an out vacuum $\rightarrow$ computing expectation values using the standard in-out S-matrix elements not meaningful $\rightarrow$ one needs the in-in or closed time path formalism $\rightarrow$

$$\langle\psi|\overline{T}(B[\phi])T(A[\phi])|\psi\rangle = \int \mathcal{D}\phi_+\mathcal{D}\phi_-\,\delta(\phi_+(t_f) - \phi_-(t_f))e^{i\int_{t_i}^{t_f}\sqrt{-g}d^dx\left(\mathcal{L}[\phi_+]-\mathcal{L}[\phi_-]\right)}$$
$$\times\Psi^\star[\phi_-(t_i)]B[\phi_-]A[\phi_+]\Psi[\phi_+(t_i)]$$

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- The two kind of fields $\phi_\pm$ introduces four 2-pt functions to deal with : a) $i\Delta_{-+}(x,x') = \langle\phi_-(x)\phi_+(x')\rangle = (i\Delta_{+-}(x,x'))^\star$ and b) $i\Delta_{++}(x,x') = \theta(t-t')i\Delta_{-+}(x,x') + \theta(t'-t)i\Delta_{+-}(x,x') = (i\Delta_{--}(x,x'))^\star$.

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- For example, the one loop tadpole with a cubic self interaction :

$$\langle\phi(x)\rangle = -\frac{i\beta}{2}\int d^dx' a'^d i\Delta(x',x')\left(i\Delta_{+-}(x,x') - i\Delta_{-+}(x,x')\right)$$

$\Rightarrow$ expectation values are causal.

The basic setup

- The action $\rightarrow$

$$S = \int d^d x a^d \left(-\frac{1}{2}(\nabla_\mu \phi)(\nabla^\mu \phi) - i\bar{\psi}\gamma^\mu \nabla_\mu \psi - g\bar{\psi}\psi\phi \right)$$

In the mostly positive signature of the metric, the anti-commutation : $[\gamma^\mu, \gamma^\nu]_+ = -2g^{\mu\nu}\mathbf{I}_{d\times d}$.

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- The scalar Wightman functions in 3-momentum space :

$$i\Delta_\phi^{\mp\pm}(\eta, \eta', k) = \frac{H^2}{2k^3}(1 \pm ik\eta)(1 \mp ik\eta')e^{\mp ik(\eta - \eta')}$$

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- We also need the statistical propagator,

$$F_\phi(x, x') = \frac{1}{2}\langle[\phi(x), \phi(x')]_+\rangle = \frac{1}{2}\left(i\Delta_\phi^{-+}(x, x') + i\Delta_\phi^{+-}(x, x')\right)$$

In 3-momentum space,

$$F_\phi(\eta, \eta', k) = \frac{H^2}{2k^3}\left[(1 + k^2\eta\eta')\cos k(\eta - \eta') + k(\eta - \eta')\sin k(\eta - \eta')\right]$$

The basic setup

- In a scalar QFT, make an analogy $x \equiv \phi$ and $p \equiv \dot{\phi}$. In dS then (P. Friedrich and T. Prokopec, PRD (2019))

$$\frac{\Xi_\phi^2(\eta, k)}{4a^4} = \left[F_\phi(\eta, \eta', k)\partial_\eta \partial_{\eta'} F_\phi(\eta, \eta', k) - \left(\partial_{\eta'} F_\phi(\eta, \eta', k)\right)^2\right]_{\eta=\eta'}$$

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- For a free theory, it is easy to check that the von Neuman entropy vanishes. Once the interaction turned on, the various correlations would change due to radiative processes. Implies opening up of previously unaccessed phase space area owing to the interaction $\rightarrow$ might give rise to generation of entropy.

The 2-loop 2PI effective action

- The task is now to compute the correlation functions in the presence of interactions. We assume that we observe the scalar field correlations and as the simplest realistic scenario, only the two point correlation functions are measured $\rightarrow$ basically 1PI self energy diagrams with external points attached.

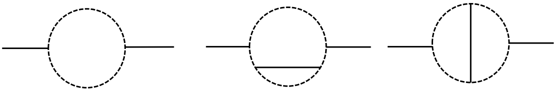

Figure: One and two loop self energy diagrams for the scalar with Yukawa interaction.

The 2-loop 2PI effective action

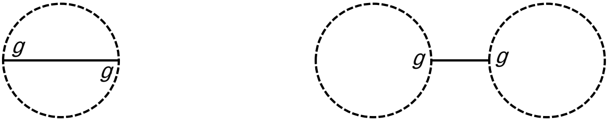

Figure: *Contributions to the 2-loop 2PI effective action due to the Yukawa interaction.*

We shall restrict our computations to $\mathcal{O}(g^2)$ only. The second diagram contains tadpoles $\rightarrow$ no contribution, as they can be renormalised away completely.

The effective action reads

$$\Gamma_{2PI}^{2-\mathrm{loop}} = \Gamma^{(0)} + \Gamma^{(1)} + \Gamma^{(2)}$$

The 2-loop 2PI effective action

- Explicitly reads

$$\Gamma^{(0)}[i\Delta_\phi^{ss'}, iS_\psi^{ss'}] = \int d^dx d^dx' a^d \Big(\sum_{s,s'=\pm} \Box_x \delta^d(x-x') \frac{s\,\delta^{ss'}}{2} i\Delta_\phi^{s's}(x',x) - \sum_{s,s'=\pm} i\nabla\!\!\!/_x \delta^d(x-x') s\delta^{ss'} iS_\psi^{s's}(x,x') \Big)$$

$$\Gamma^{(1)}[i\Delta_\phi^{ss'}, iS_\psi^{ss'}] = -\frac{i}{2}\mathrm{Tr}\ln\left[i\Delta_\phi^{ss}(x;x')\right] + i\mathrm{Tr}\ln\left[iS_\psi^{ss}(x,x)\right]$$

$$\Gamma^{(2)}[i\Delta_\phi^{ss'}, iS_\psi^{ss'}] = -\sum_{s,s'=\pm} \frac{iss'g^2}{2} \int d^dx d^dx' a^d a'^d \mathrm{Tr}\left[iS_\psi^{ss'}(x,x') iS_\psi^{s's}(x',x)\right] i\Delta_\phi^{ss'}(x,x')$$

$s, s' = \pm$, corresponding to the in-in formalism. Fermion propagators :

$$iS_\psi^{ss'}(x,x') = -\frac{i(aa')^{\frac{1-d}{2}}\Gamma\left(\frac{d}{2}\right)}{2\pi^{\frac{d}{2}}} \frac{\Delta\!\!\!/ x}{\left[\Delta x^2_{ss'}(x,x')\right]^{\frac{d}{2}}}$$

$$\Delta x^2_{\pm\pm} = -(|\eta-\eta'| \mp i\varepsilon)^2 + |\vec{x}-\vec{x}'|^2$$

$$\Delta x^2_{\pm\mp} = -(\eta-\eta' \pm i\varepsilon)^2 + |\vec{x}-\vec{x}'|^2 \qquad (\varepsilon = 0^+)$$

dS invariant interval :

$$y^2_{ss'} = aa'H^2\Delta x^2_{ss'}$$

The Kadanoff-Baym equation

Kadanoff-Baym equation is the equation of motion satisfied by the correlator (for a review, J. Berges, hep-ph/0409233). The effective action is varied to obtain the EoM :

$$\Box_x i\Delta_\phi^{ss'}(x, x'') = \frac{is\,\delta^{ss'}\,\delta^d(x - x'')}{a^d} + \sum_{s''=\pm} \int d^dx'\, a'^d s''\, iM_\phi^{ss''}(x, x')\, i\Delta_\phi^{s''s'}(x', x'')$$

where the one loop scalar self energy :

$$(aa')^d iM_\phi^{ss'}(x, x') = i(aa')^d g^2 \mathrm{Tr}\left[iS_\psi^{ss'}(x, x')\, iS_\psi^{s's}(x', x)\right] \qquad (\text{no sum on } s \text{ or } s')$$

The KB eqn can be solved perturbatively or non-perturbatively. A non-perturbative analysis requires a proper resummation technique. (for flat space Yukawa theory, SB, N. Joshi & S. Kaushal, EPJC (2022)). Will attempt this perturbatively for now.

The scalar self energy

$$iM_\phi^{++}(x,x') = \frac{ig^2\Gamma^2(\frac{d}{2})H^{2d-2}}{2^{2d-2}\pi^d}\left[\frac{2}{(d-2)^2}\frac{\Box}{H^2} - \frac{2}{(d-2)}\right]\left[\frac{\Box}{H^2}\left(\frac{4}{y_{++}}\ln\frac{\mu^2 y_{++}}{H^2}\right) - \frac{4}{y_{++}}\left(2\ln\frac{\mu^2 y_{++}}{H^2} - 1\right)\right] + \mathcal{O}(d-4)$$

μ is some renormalisation scale. $iM_\phi^{--}(x,x')$ is just the complex conjugation of the above. The divergent part:

$$-\frac{i(aa')^d g^2\Gamma^2(\frac{d}{2})H^{2d-2}}{2^{2d-2}\pi^d}\left[\frac{2}{(d-2)^2}\frac{\Box}{H^2} - \frac{2}{(d-2)}\right]\left[\frac{2(4\pi)^{d/2}}{(d-3)(d-4)\Gamma[\frac{d}{2}-1]}\left(\frac{\mu}{H}\right)^{d-4}\frac{i\delta^d(x-x')}{(Ha)^d}\right]s\delta^{ss'}$$

The mixed propagators (Wightman functions) do not contain any divergence. Can be renormalised using a scalar field strength plus a scalar-curvature non-minimal coupling counterterms. Renormalised result for one loop scalar self energy :

$$\frac{ig^2H^6(aa')^d}{2^6\pi^4}\left[\frac{\Box}{2H^2} - 1\right]\left[\frac{\Box}{H^2}\left(\frac{4}{y_{ss'}}\ln\frac{\mu^2 y_{ss'}}{H^2}\right) - \frac{4}{y_{ss'}}\left(2\ln\frac{\mu^2 y_{ss'}}{H^2} - 1\right)\right]$$

Self energy in momentum space

- $iM_\phi^{++}(x,x')$ in the spatial momentum space :

$$-\frac{ig^2H^6}{2^5\pi^2k^3}\Bigg\{\frac{1}{4}\left(\frac{\Box_k}{H^2}\right)^3\Bigg(\left[2+\left[1+ik|\Delta\eta|\right]\left(\ln\frac{2|\Delta\eta|\mu^2}{ek\eta\eta' H^2}+\frac{i\pi}{2}-\gamma_E\right)\right]e^{-ik|\Delta\eta|}$$

$$-(1-ik|\Delta\eta|)\Big[\mathrm{ci}\left[2k|\Delta\eta|\right]-i\,\mathrm{si}\left[2k|\Delta\eta|\right]\Big]e^{+ik|\Delta\eta|}\Bigg)$$

$$-\left(\frac{\Box_k}{H^2}\right)^2\Bigg(\left[2+\left[1+ik|\Delta\eta|\right]\left(\ln\frac{|\Delta\eta|H^2}{e^2k\eta\eta'\mu^2}+\frac{i\pi}{2}-\gamma_E\right)\right]e^{-ik|\Delta\eta|}$$

$$-(1-ik|\Delta\eta|)\Big[\mathrm{ci}\left[2k|\Delta\eta|\right]-i\,\mathrm{si}\left[2k|\Delta\eta|\right]\Big]e^{+ik|\Delta\eta|}\Bigg)$$

$$+\frac{\Box_k}{H^2}\Bigg(\left[2+\left[1+ik|\Delta\eta|\right]\left(\ln\frac{|\Delta\eta|H^2}{e^{\frac{9}{2}}k\eta\eta'\mu^2}+\frac{i\pi}{2}-\gamma_E\right)\right]e^{-ik|\Delta\eta|}$$

$$-(1-ik|\Delta\eta|)\Big[\mathrm{ci}\left[2k|\Delta\eta|\right]-i\,\mathrm{si}\left[2k|\Delta\eta|\right]\Big]e^{+ik|\Delta\eta|}\Bigg)+3\Big[\left[1+ik|\Delta\eta|\right]e^{-ik|\Delta\eta|}\Big]\Bigg\}$$

where $k=|\vec{k}|$, $\Delta\eta=\eta-\eta'$.

The IR limit of the self energy

- We are interested in the late time, super-Hubble limit of the self energy. Will look for long wavelength, or IR effective correlations when $k|\Delta\eta| \ll 1$,

$$iM^{++}_{\phi,\mathrm{ren}}(\eta,\eta',k)_{k|\Delta\eta|\ll 1} \approx \frac{ig^2H^6}{2^5\pi^2k^3}\left(\frac{1}{4}\left(\frac{\Box_k}{H^2}\right)^3 - \left(\frac{\Box_k}{H^2}\right)^2 + \frac{\Box_k}{H^2}\right)\left(\ln\frac{H^2k^2\eta\eta'}{\mu^2} + 2ik|\Delta\eta|\right)$$

The IR limit of the other self energies, iM^{++}, iM^{+-} and iM^{-+} can be computed in a similarly. Also note that

$$F_\phi(x,x') = \langle[\phi(x),\phi(x')]_+\rangle = \frac{1}{2}\left(i\Delta^{-+}_\phi(x,x') + i\Delta^{+-}_\phi(x,x')\right)$$

Make appropriate linear combinations of KB eqn + plug in the self energies $\rightarrow$

The von Neuman entropy

- The leading and next to the leading behaviour of the statistical propagator in the late time IR :

$$
\begin{aligned}
&F_\phi(\eta,\eta',k)_{k|\Delta\eta|\ll 1}|_{\text{1 loop}} \\
&\quad\approx \frac{g^2H^2}{768\pi^2k^3}\Bigg[\ln\frac{\eta H^2k^2\eta'}{\mu^2}\left(\left(\eta^4k^4+2\eta^2k^2+4\right)\ln\frac{\eta H^2k^2\eta'}{\mu^2}-4\left(\eta^2k^2+6\right)\right) \\
&\quad+4\left(\eta^2k^2+6\right)\ln\left(-\frac{\eta Hk^2}{\mu^2}\right)-\left(\eta^4k^4+2\eta^2k^2+4\right)\ln^2\left(-\frac{\eta Hk^2}{\mu^2}\right)\Bigg]+F_{\text{free}}(\eta,\eta',k)
\end{aligned}
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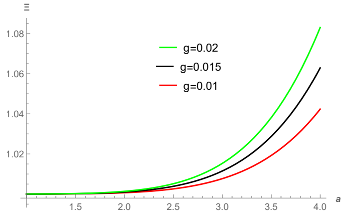
$$+4\left(\eta^2k^2+6\right)\ln\left(-\frac{\eta Hk^2}{\mu^2}\right)-\left(\eta^4k^4+2\eta^2k^2+4\right)\ln^2\left(-\frac{\eta Hk^2}{\mu^2}\right)\Bigg]+F_{\text{free}}(\eta,\eta',k)$$

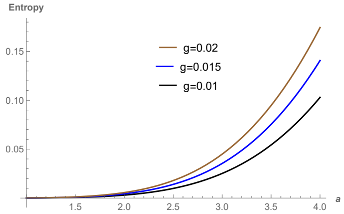
- Yields the change in the phase space area at late times :

$$\delta\left(\frac{\Xi_\phi^2}{4a^4}\right)\approx\frac{g^2H^4\left(\eta^2k^2\ln\frac{\eta^2H^2k^2}{\mu^2}\left(\eta^2k^2\ln\frac{\eta^2H^2k^2}{\mu^2}-4\right)+2\eta^2k^2+2\right)}{384\pi^2\eta^2k^6}\approx\frac{g^2H^6a^6}{48\pi^2k^6}$$

The von Neuman entropy

The von-Neumann entropy generated can be computed using this one loop change in the phase space area. At tree level it is vanishing.





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- Inclusion of gravitons?