Magnetic Sky in an ab-Normal Universe

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AB & Supratik Pal, "Primordial magnetic non-Gaussianity with generic vacua and detection prospects in CMB spectral distortions", *arXiv:2310.10342*

A very brief intro to cosmic magnetic fields...

Growing evidence since the 1950s...



Where did the seed magnetic fields come from?

Inflationary vacuum fluctuations?

Simplest option (Ratra '92) ⇒ Break conformal invariance of Maxwell action!

$$-rac{1}{4}\int d^4x\sqrt{-g}F_{\mu
u}F^{\mu
u}
ightarrow -rac{1}{4}\int d^4x\sqrt{-g}\,\lambda(\phi)F_{\mu
u}F^{\mu
u}$$

Can generate nearly scale-invariant non-helical PMFs of required amplitude...

(High conductivity during reheating \Rightarrow PMF frozen, PEF shorted)

Only one of many, many proposed theoretical scenarios, leading to a plethora of possible observational predictions...





Using the master formula of the in-in formalism...

$$\begin{split} \langle X(\vec{k_1},\eta_I)A_i(\vec{k_2},\eta_I)A_j(\vec{k_3},\eta_I)\rangle &= -2\operatorname{Im} \int_{-\infty}^{\eta_I} d\eta_1 a(\eta_1) \langle 0|H_{XAA}(\eta_1)X(\vec{k_1},\eta_I)A_i(\vec{k_2},\eta_I)A_j(\vec{k_3},\eta_I)|0\rangle \\ \langle X(\vec{k_1},\eta_I)B_i(\vec{k_2},\eta_I)B^i(\vec{k_3},\eta_I)\rangle &= -\frac{1}{a(\eta_I)^4} \Big(\delta_{ij}\,\vec{k_2}.\,\vec{k_3} - k_{2j}k_{3i}\Big) \langle X(\vec{k_1},\eta_I)A_i(\vec{k_2},\eta_I)A_j(\vec{k_3},\eta_I)\rangle \end{split}$$

Our aim \Rightarrow Generalize to generic initial vacua for both curvature perturbation and gauge field.





Planck scale, i.e. domain of QG (breakdown of EFT)

> BD assumption ⇒ Deep inside the horizon, the initial vacuum is Minkowskian and sets initial conditions for the mode functions.

NBD counter-argument ⇒ Modes cannot be blueshifted back to infinite past due to breakdown of EFT, and the initial vacuum of inflation need not be perfectly Minkowskian.

 $X_k^{(NBD)}(\eta) \equiv \alpha_k X_k^{(BD)}(\eta) + \beta_k X_k^{(BD)*}(\eta)$

The NBD solution is a Bogolyubov rotation of the BD solution, where the Bogolyubov coefficients parametrize our ignorance of pre-inflation physics.



Only theoretical constraint (from canonical commutation):

$$|\alpha_k|^2 - |\beta_k|^2 = 1$$

We have assumed generic initial vacua for all the modes involved...

$$\begin{split} \zeta_k^{(NBD)}(\eta) &= \alpha_k \zeta_k^{(BD)}(\eta) + \beta_k \zeta_k^{(BD)*}(\eta) , \qquad |\alpha_k|^2 - |\beta_k|^2 = 1 , \\ h_k^{(NBD)}(\eta) &= \rho_k h_k^{(BD)}(\eta) + \sigma_k h_k^{(BD)*}(\eta) , \qquad |\rho_k|^2 - |\sigma_k|^2 = 1 , \\ A_k^{(NBD)}(\eta) &= \gamma_k A_k^{(BD)}(\eta) + \delta_k A_k^{(BD)*}(\eta) . \qquad |\gamma_k|^2 - |\delta_k|^2 = 1 . \end{split}$$

...and explicitly computed the three-point correlator $\langle \zeta BB \rangle$ in the kinetic coupling model.

General expressions are extremely cumbersome, but limiting cases show interesting physical features!

Three-point cross-correlations between inflationary metric perturbation and PMFs



Three-point cross-correlations between inflationary metric perturbation and PMFs



 $\mathcal{B}^{(sq)}(k_1,k_2,k_3;\eta_I)=4 imes P_\zeta(k_1)P_B(k_2)$ \Rightarrow Retains product structure!



⇒

Squeezed

Apparent divergence of bispectrum is an artifact of assumed infinite past... Can be cured by choosing finite past which then acts as regulator!

$${\cal B}^{(fl)}(k_1,k_2,k_3;\eta_I,\eta_0)pprox {\cal B}^{(fl)}_{BD}(k_1,k_2,k_3;\eta_I)+rac{4}{a(\eta_I)^4}|\zeta_{k_1^*}|^2|A_{k_2^*}|^2eta_1^*k_1^2(k_1\eta_0)^2$$

...in the limit of weak deviations from BD vacua.

(Further reduction of one order when taking angular average for CMB)



$$\begin{split} \mathcal{B}^{(eq)}(k_{1},k_{2},k_{3};\eta_{I}) &= \frac{|\zeta_{k^{*}}|^{2}|A_{k^{*}}|^{2}}{9a(\eta_{I})^{4}}k^{2}\operatorname{Re}\left[\left(\gamma+\delta\right)^{2}\left[4(\alpha+\beta)\left\{\alpha^{*}\left(17\gamma^{*2}+54\gamma^{*}\delta^{*}-27\delta^{*2}\right)\right.\right.\right.\\ &+\beta^{*}\left(27\gamma^{*2}-54\gamma^{*}\delta^{*}-17\delta^{*2}\right)\right\}+5\left\{27(\alpha+\beta)\left(\ln(-3k\eta_{I})\left(\beta^{*}\delta^{*2}-\alpha^{*}\gamma^{*2}\right)\right.\\ &+\ln(-k\eta_{I})(\beta^{*}\gamma^{*}(\gamma^{*}+2\delta^{*})-\alpha^{*}\delta^{*}(2\gamma^{*}+\delta^{*}))\right)-27\gamma_{E}(\alpha+\beta)(\alpha^{*}-\beta^{*})(\gamma^{*}+\delta^{*})^{2}\\ &+2\gamma^{*}\left(\gamma^{*}(47\alpha\alpha^{*}+56\alpha^{*}\beta+9\beta\beta^{*})+9\delta^{*}(4\alpha\alpha^{*}+5\alpha^{*}\beta-5\alpha\beta^{*}-4\beta\beta^{*})\right)\\ &+2\delta^{*}\left(9\gamma^{*}(4\alpha\alpha^{*}+5\alpha^{*}\beta-5\alpha\beta^{*}-4\beta\beta^{*})-\delta^{*}(9\alpha\alpha^{*}+56\alpha\beta^{*}+47\beta\beta^{*})\right)\right\}\right]\right]\\ &-\frac{15\pi}{2}\frac{|\zeta_{k^{*}}|^{2}|A_{k^{*}}|^{2}}{a(\eta_{I})^{4}}k^{2}\operatorname{Im}\left[(\alpha+\beta)(\gamma+\delta)^{2}\left\{\alpha^{*}(\gamma^{*2}-\delta^{*2}+2\gamma^{*}\delta^{*})+\beta^{*}\left(-\gamma^{*2}+2\gamma^{*}\delta^{*}+\delta^{*2}\right)\right\}\right],\end{split}$$



$$\begin{split} \mathcal{B}^{(or)}(k_{1},k_{2},k_{3};\eta_{I}) &= -\frac{6\sqrt{2}\pi}{a(\eta_{I})^{4}} |\zeta_{k_{1}^{*}}|^{2} |A_{k_{2}^{*}}|^{2} k_{1}^{2} \mathrm{Im} \Big[\left(\alpha_{1}+\beta_{1}\right) (\gamma_{2}+\delta_{2})^{2} \\ &\times \Big\{ \alpha_{1}^{*} \Big(\gamma_{2}^{*}(\gamma_{2}^{*}+\delta_{2}^{*})+\delta_{2}^{*}(\gamma_{2}^{*}-\delta_{2}^{*}) \Big) + \beta_{1}^{*} \Big(\gamma_{2}^{*}(\delta_{2}^{*}-\gamma_{2}^{*})+\delta_{2}^{*}(\gamma_{2}^{*}+\delta_{2}^{*}) \Big) \Big\} \Big] \\ &- \frac{2}{a(\eta_{I})^{4}} |\zeta_{k_{1}^{*}}|^{2} |A_{k_{2}^{*}}|^{2} k_{1}^{2} \mathrm{Re} \Bigg[\left(\gamma_{2}+\delta_{2}\right) (\gamma_{3}+\delta_{3}\right) \Big[\alpha_{1} \Big\{ \alpha_{1}^{*} \Big(\gamma_{2}^{*} \big((6\sqrt{2}\gamma_{E}-10\sqrt{2}-7) \\ &+ 6\sqrt{2} \ln(\sqrt{2}+1) \big) \gamma_{2}^{*} + 2\sqrt{2} \left(3\gamma_{E}-5 \right) \delta_{2}^{*} \Big) + \delta_{2}^{*} \left(2\sqrt{2} (3\gamma_{E}-5) \gamma_{2}^{*} + \delta_{2}^{*} (6\sqrt{2}\gamma_{E}-10\sqrt{2} \\ &+ 7+6\sqrt{2} \ln(\sqrt{2}-1) \big) \Big) + \beta_{1}^{*} \Big(\delta_{2}^{*} \big(2\sqrt{2} (7-3\gamma_{E}) \gamma_{2}^{*} + (-6\sqrt{2}\gamma_{E}+14\sqrt{2}+7 \\ &+ 3\sqrt{2} \ln(\sqrt{2}-1) - 3\sqrt{2} \ln(\sqrt{2}+1) \big) \delta_{2}^{*} \Big) - \gamma_{2}^{*} \big((6\sqrt{2}\gamma_{E}-14\sqrt{2}+7+6\sqrt{2} \ln(\sqrt{2}-1)) \gamma_{2}^{*} \\ &+ 2\sqrt{2} (3\gamma_{E}-7) \delta_{2}^{*} \Big) \Big\} + \beta_{1} \Big\{ \alpha_{1}^{*} \Big(\gamma_{2}^{*} \big((6\sqrt{2}\gamma_{E}-14\sqrt{2}+7+6\sqrt{2} \ln(\sqrt{2}-1)) \right) \gamma_{2}^{*} \\ &+ 2\sqrt{2} (3\gamma_{E}-7) \delta_{2}^{*} \Big) + \delta_{2}^{*} \big(2\sqrt{2} (3\gamma_{E}-7) \gamma_{2}^{*} + (6\sqrt{2}\gamma_{E}-14\sqrt{2}+7+6\sqrt{2} \ln(\sqrt{2}-1)) \delta_{2}^{*} \big) \Big) \\ &+ \beta_{1}^{*} \Big(\delta_{2}^{*} \big(2\sqrt{2} (5-3\gamma_{E}) \gamma_{2}^{*} + (-6\sqrt{2}\gamma_{E}+10\sqrt{2}+7+3\sqrt{2} \ln(\sqrt{2}-1) - 3\sqrt{2} \ln(\sqrt{2}+1)) \delta_{2}^{*} \\ &- \gamma_{2}^{*} \big((6\sqrt{2}\gamma_{E}-10\sqrt{2}+7+6\sqrt{2} \ln(\sqrt{2}-1)) \gamma_{2}^{*} + 2\sqrt{2} (3\gamma_{E}-5) \delta_{2}^{*} \big) \Big) \Big\} \\ &+ 6\sqrt{2} (\alpha_{1}+\beta_{1}) (\alpha_{1}^{*}-\beta_{1}^{*}) (\gamma_{2}^{*}+\delta_{2}^{*}) (\gamma_{2}^{*}+\delta_{2}^{*}) \ln(-k_{1}\eta_{I}) \Big] \Bigg]. \end{split}$$

Presence of additional logarithmic terms even at tree-level, which may dominate at CMB scales for suitable choices of NBD parameters!



Can be caused by damping of acoustic waves, magnetic fields, etc. ⇒ Pajer & Zaldarriaga (2012), Ganc & Sloth (2014), etc.



Bose-Einstein distribution (non-zero chemical potential)

$$\mu = 1.4 \times \left(\frac{\Delta E}{E}\right)$$



⇒ Pajer & Zaldarriaga (2012), Ganc & Sloth (2014), etc.



Bose-Einstein distribution (non-zero chemical potential)

$$\mu = 1.4 \times \left(\frac{\Delta E}{E}\right)$$

Two powers of magnetic field $\Rightarrow \mu$ -distortion multipoles

Scalar curvature perturbation \Rightarrow T-anisotropy multipoles

In the squeezed limit which is relevant to the μ T correlation:

$$C_l^{\mu T} = 1.4 imes rac{12}{5(2\pi)^3} rac{b_{NL}^{(loc)}}{\mu_0
ho_{\gamma 0}} \int\limits_0^{10k_s} dk k^2 P_{\zeta}(k) W\left(rac{k}{k_s}
ight) j_l(kr_L) g_{Tl}(k) \int\limits_{ ilde{k}_D}^{ ilde{k}_D^i} dk_1 k_1^2 P_B(k_1) = 0$$

Scalar-magnetic bispectrum $\Rightarrow \mu T$ correlation $\langle a_{lm}^{\mu} a_{l'm'}^{T} \rangle \equiv C_{l}^{\mu T} \delta_{ll'} \delta_{mm'}$

Window function W(x)=3x⁻³(sinx-xcosx)
 Scalar radiation transfer function g_{TI}(k)
 Damping limits 2.1x10⁴ Mpc⁻¹ to 83 Mpc⁻¹
 Smoothing scale k_a ~ 0.084 Mpc⁻¹

Apart from overall normalization, spectral shape governed by $\Rightarrow S_l = \int_{0}^{10k_s} dk_1 k_1^{n_s-3} j_1\left(\frac{k_1}{k_s}\right) j_l(kr_L) g_{Tl}(k)$



Analytical approximation for $l \le 1200 \Rightarrow$

$$\Rightarrow \left(\begin{array}{c} S_{l} \approx \frac{\bar{A}}{l^{3}} + \frac{\bar{B}}{l^{2}} + \bar{C} + \bar{D}l^{3/2} + \bar{E}l^{4} + \bar{F}\ln l + \bar{G}J_{1}\left(\frac{l}{1200}\right), \\ \bar{A} = 0.287492, \ \bar{B} = -0.458365, \ \bar{C} = 0.0006464, \\ \bar{D} = -6.96227 \times 10^{-9}, \ \bar{E} = 2.13935 \times 10^{-17}, \\ \bar{F} = -0.000130573, \ \bar{G} = 0.00119243. \end{array} \right)$$



$$C_l^{\mu T}pprox (1.34 imes 10^{-7}) imes heta_B imes S_l \ heta_B \equiv \left[|\gamma_2 + \delta_2| (H/M_{Pl})
ight]^2$$

Where did the scalar NBD coefficients go??



Variation of H vs a_1 (for positive β_1), error bars scaled x10.



Dependence of μ T correlation strength on choice of initial vacua (with positive real values of all the NBD parameters). Cannot be enhanced much further while being consistent with the constraint $B_0 \leq 27$ nG on μ -distortion scales.

Signals potentially detectable by several next-generation CMB missions...

Fisher signal-to-noise ratio for μT spectrum at upcoming missions

$$\left(F_{ij} = \sum_{l} \sum_{PP', QQ'} \frac{\partial C_l^{PP'}}{\partial \theta_i} \left(\operatorname{Cov}_l^{-1} \right)_{PP', QQ'} \frac{\partial C_l^{QQ'}}{\partial \theta_j} \right)$$

$$\left((\text{Cov}_l)_{PP',QQ'} = \frac{1}{(2l+1)f_{sky}} \left(C_l^{PQ} C_l^{P'Q'} + C_l^{PQ'} C_l^{P'Q} \right) \right)$$

Fisher signal-to-noise ratio for μT spectrum at upcoming missions

α_1	$\left(\frac{H}{M_{\rm Pl}}\right) imes 10^5$	γ_2	\widetilde{B}_{μ} (in nG)	(S/N) _{PIXIE}	$(S/N)_{Super-PIXIE}$	$(S/N)_{CMBPol}$	$(S/N)_{LiteBIRD}$
1.0	5.37	60.0	26.262	4.591	46.693	92.950	83.021
1.0	5.37	40.0	17.506	2.040	20.749	41.303	36.892
1.0	5.37	20.0	8.749	0.510	5.182	10.316	9.214
1.0	5.37	10.0	4.366	0.127	1.291	2.569	2.295
1.0	5.37	5.0	2.167	0.031	0.318	0.633	0.565
1.0	5.37	2.0	0.817	0.004	0.045	0.090	0.080
1.0	5.37	1.0	0.219	3.188×10^{-4}	0.003	0.006	0.006
1.5	2.07	1.0	0.084	4.738×10^{-5}	4.819×10^{-4}	9.593×10^{-4}	8.568×10^{-4}
2.0	1.44	1.0	0.059	2.293×10^{-5}	2.332×10^{-4}	4.642×10^{-4}	4.146×10^{-4}
2.5	1.12	1.0	0.046	1.387×10^{-5}	1.411×10^{-4}	2.802×10^{-4}	2.508×10^{-4}
3.0	0.92	1.0	0.037	9.359×10^{-6}	9.519×10^{-5}	1.895×10^{-4}	1.692×10^{-4}

Signal-to-noise ratio (SNR) at four upcoming CMB missions as a function of the initial vacuum conditions.

<u>Part 1:</u>

- Assuming generic initial vacua for all the perturbative sectors, we have computed the three-point correlation function of the scalar curvature perturbation with PMFs generated in the gauge-inflaton coupling scheme.
- In the squeezed limit, the correlator reduces to a product between the scalar and the magnetic power spectra. In the flattened limit, NBD enhancement of quadratic order in the finite past appears. In the equilateral & orthogonal limits, NBD logarithmic terms appear that might possibly dominate on CMB scales.

<u>Part 2:</u>

- The squeezed limit scalar-magnetic bispectrum can source CMB µT cross-correlation. The presence of NBD initial conditions can significantly enhance as well as suppress the strength of the cross-power spectrum.
- > In cases where it is enhanced, the μ T signal should be detectable by several upcoming CMB experiments. This, in turn, could be instrumental in putting constraints on PMFs as well as on non-standard inflationary vacua.



Appendix:

Reserve Slides



What about alternative sources?

$$C_{l,\ f_{NL}}^{\mu T} \approx 1.4 \times \frac{576}{25(2\pi)^3} \times f_{NL} \times \int_{0}^{10k_s} dkk^2 P_{\zeta}(k) W\left(\frac{k}{k_s}\right) j_l(kr_L) g_{Tl}(k) \int_{\tilde{k}_{D,\rho}^{f}}^{\tilde{k}_{D,\rho}^i} dk_1 k_1^2 P_{\zeta}(k_1)$$

$$SNR \ll 1 \Rightarrow Not \ significant \ as \ competing \ signals$$

$$C_{l,\ \zeta_{ev}}^{\mu T} \approx -1.4 \times \frac{16\xi}{75(\mu_0 \rho_{\gamma 0})} \times \int_{0}^{10k_s} dkk^4 W\left(\frac{k}{k_s}\right) j_l(kr_L) g_{Tl}(k) \int_{\tilde{k}_D^f}^{\tilde{k}_D^i} dk_1 P_B(k_1)^2$$



Hints of a magnetic Universe...



Effects of nearly scale-invariant, non-helical PMFs of amplitude B_a=4.5 nG on CMB temperature and polarization power spectra.

<u>Source</u>: Zucca et al (PRD, 2017), 1611.00757



Effects of nearly scale-invariant, non-helical PMFs on matter power spectrum (nonlinear Jeans effect and diffusion damping included).

Source: Shaw & Lewis (PRD, 2012), 1006.4242