PBH and ISGWB from USR model of inflation with different reheating histories

(Based on JCAP01(2020)037 and arxiv: 2009.10424)

Nilanjandev Bhaumik
With Rajeev kumar Jain

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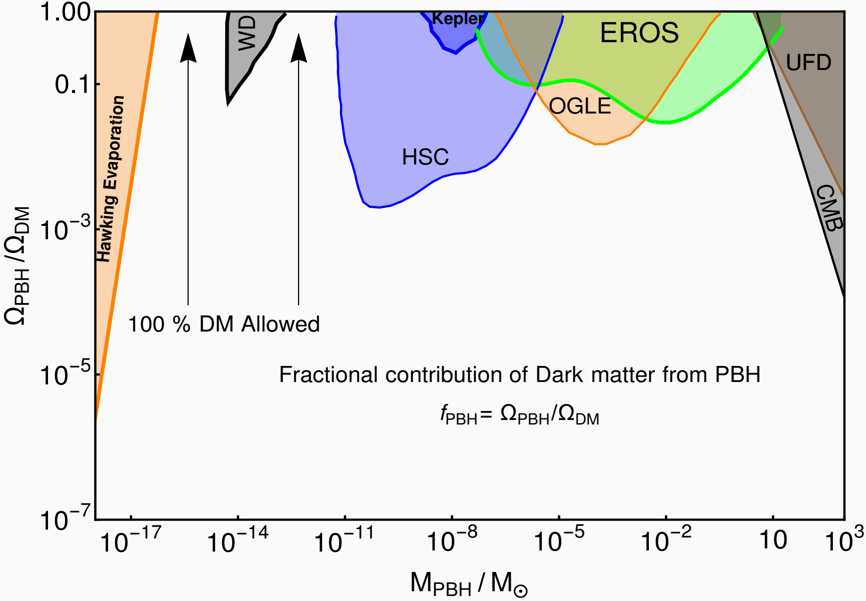
IISc Bangalore

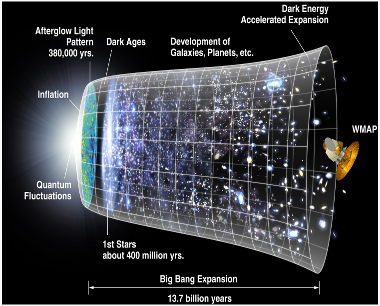
Overview

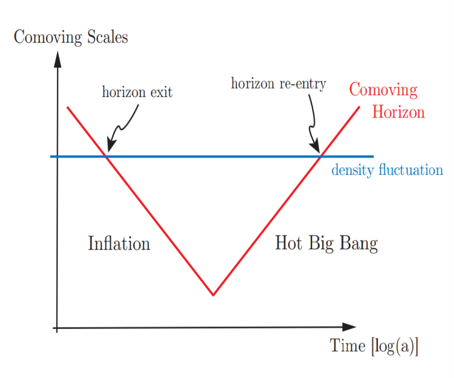
Primordial Black Holes from USR models as CDM

Primordial Black Holes as CDM and constraints



Inflation to form PBHs





Conditions on curvature perturbation

- $\mathcal{R} \sim 0.1$; $P_{\mathcal{R}} \sim 10^{-2}$ at PBH scale ($\delta_c \sim w$)
- $\mathcal{R} \sim 0.0001$; $P_{\mathcal{R}} \sim 2.25 \times 10^{-9}$ at CMB Scale

Inflection point model Potential

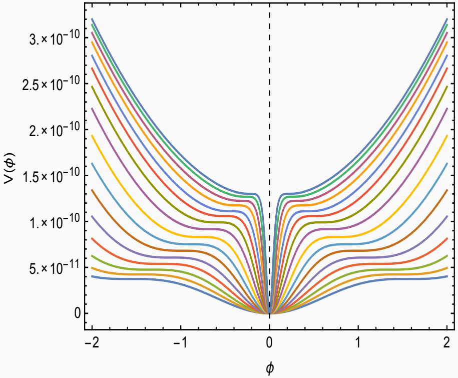
- $V'(x_0) = 0$
- $V''(x_0) = 0$
- $V'''(x_0) \neq 0$

Chosen Single field Potential of our Model

$$V(x) = V_0 \frac{ax^2 + bx^4 + cx^6}{(1 + dx^2)^2}$$

Behaviour in different regimes

- Large field limit : $V(x) \simeq \frac{V_0 c}{d^2} x^2$
- Small field limit : $V(x) \simeq V_0 a\, x^2$



Bhaumik, Jain 2019

Dynamics of inflation in our model

Friedmann equations

- $\frac{d^2\phi}{dN^2} + (3-\epsilon)\frac{d\phi}{dN} + \frac{1}{H^2}V'(\phi) = 0$
- $\frac{dH}{dN} = -\frac{H}{2M_{\rm Pl}^2}\left(\frac{d\phi}{dN}\right)^2,$ ϵ =First Hubble slow Roll parameter

Slow Roll Phase

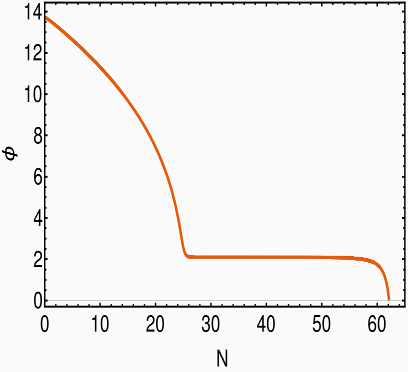
- $\frac{d\phi}{dN} + \frac{1}{3H^2}V'(\phi) \simeq 0$

$\phi(N) \simeq \phi_i - \sqrt{2\epsilon_V} M_{\rm Pl}(N - N_i)$

Ultra Slow Roll Phase

- $\frac{d^2\phi}{dN^2} + 3\frac{d\phi}{dN} \simeq 0$

$\frac{d\phi(N)}{dN} \sim \exp\left[-3(N - N_i)\right]$



Mukhanov sasaki equations

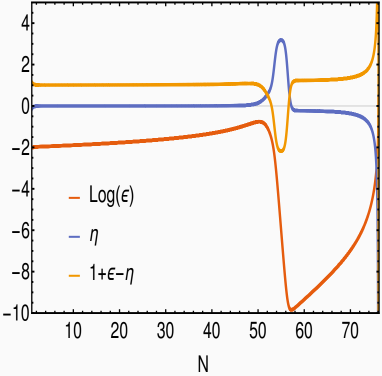
- $\mathcal{R}_k'' + 2\left(\frac{z'}{z}\right)\mathcal{R}_k' + k^2\mathcal{R}_k = 0; \qquad z = a\dot{\phi}/H = a\phi_N$
- $P_{\mathcal{R}} = \frac{k^3}{2\pi^2}|\mathcal{R}_k|^2 \qquad \mathcal{R}=$ Comoving curvature perturbation

Growing and Decaying mode

- $\mathcal{R}_k(\tau) \simeq C_1 + C_2 \int \frac{d\tau}{z^2}$

Ultra Slow Roll and need of numerical Study

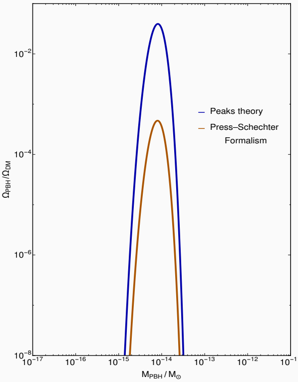
- $\frac{z'}{z} = aH(1+\epsilon-\eta)$

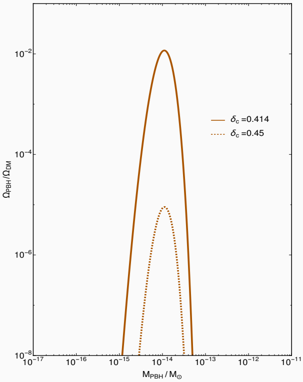


PBH mass fraction and uncertainties

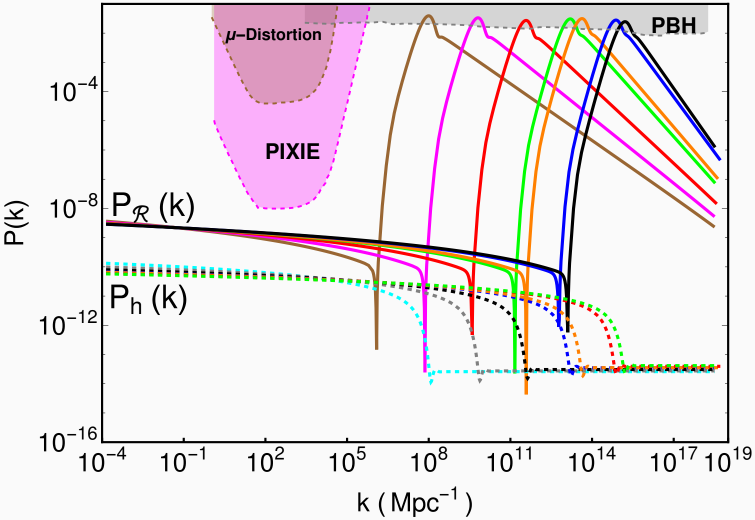
Uncertainties in the computation of mass fraction

- Peaks theory and Press Schechter : Press Schechter
- Critical density contrast $\delta_c = .414$
- The window Function $W(k, R) = exp\left(-\frac{k^2 R^2}{2}\right)$

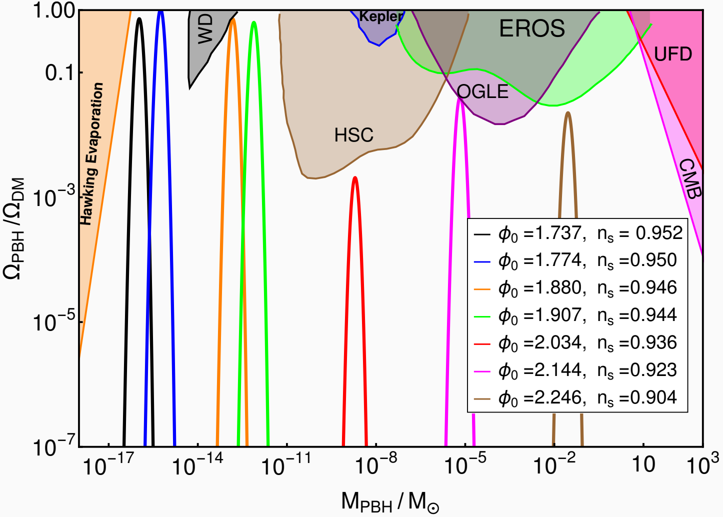




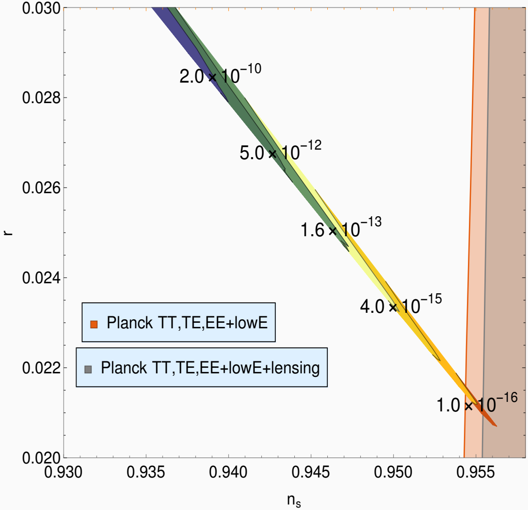
Scalar power spectra



PBH mass fraction



Scalar index and tensor to scalar ratio



Induced Stochastic GW Background (ISGWB)

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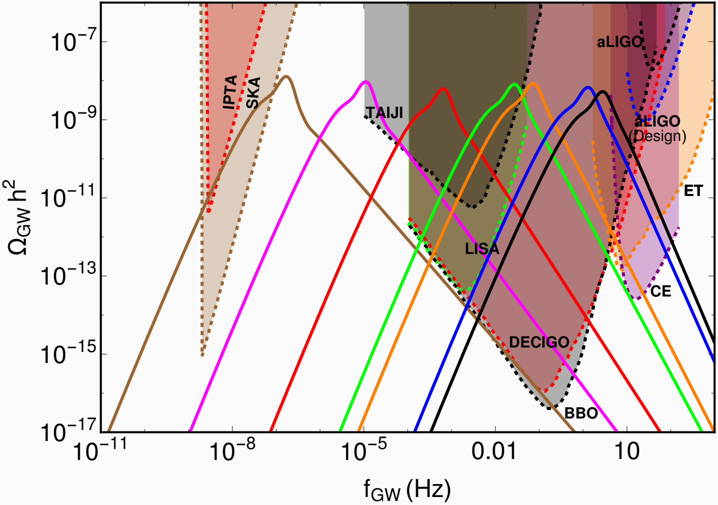
$$h''_{\mathbf{k}}(\tau) + 2\mathcal{H}h'_{\mathbf{k}}(\tau) + k^2 h_{\mathbf{k}}(\tau) = 4S_{\mathbf{k}}(\tau)$$

$$\Omega_{\rm GW}(\tau, k) = \frac{1}{6}\left(\frac{k}{\mathcal{H}}\right)^2 \int_0^\infty \mathrm{d}v \int_{|1-v|}^{1+v} \mathrm{d}u \left(\frac{4v^2 - (1+v^2-u^2)^2}{4uv}\right)^2$$

$$\times \overline{I^2_{\rm RD}(v, u, x)}\mathcal{P}_{\mathcal{R}}(kv)\mathcal{P}_{\mathcal{R}}(ku)$$

$$I_{RD}(u, v, x) = \quad \int_0^x d\bar{x} \frac{a(\bar{x})}{a(x)} f(u, v, \bar{x}) k G(\bar{x}, x)$$

ISGWB from our model

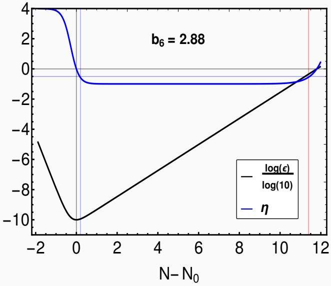


Bhaumik, Jain 2020

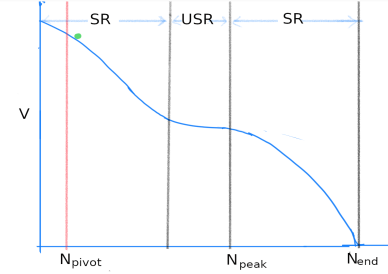
Lowest Mass PBHs for USR models

Lowest Mass PBHs for USR models (with instantaneous transition to SR)

Limits on efold number
* Assuming $|\eta| \leq 1$ in SR:
$N_{end} - N_{peak} \geq 10.4$
* $N_{end} - N_{pivot} \leq 58.92 + \frac{ln(\epsilon)}{4}$
(agrees with Liddle,Leach 2003)
* $N_{peak} - N_{pivot} \geq 47.41$



N→k→M$_{PBH}$,**f**$_{SGWB}$

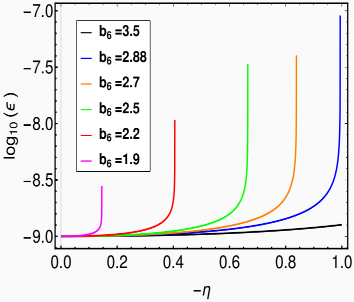


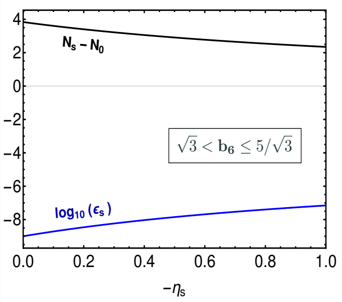
$\mathbf{M}_{PBH} \geq 6.14 \times 10^{-23} M_{\odot}$

$\mathbf{f}_{SGWB} \leq 2.91 \times 10^{2} Hz$

[Bhaumik, Jain 2020]

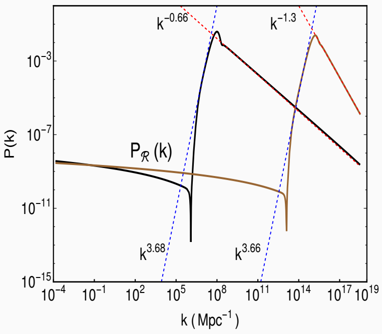
Smooth transition to SR:
Potential: $V(x) = b_0 + b_1(\phi - \phi_0) + b_2(\phi - \phi_0)^2$
2nd Slow Roll parameter: $\eta = \epsilon - \phi_{NN}/\phi_N = \eta(N, b_6)$

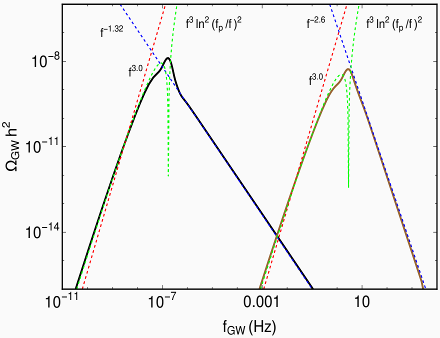




Minimum efold interval:
$N_{end} - N_{peak} = \Delta N_{transition} + \Delta N_{SR} \geq 10.58$

Effects on the slope of SGWB peak





Slope in red tilted regimes

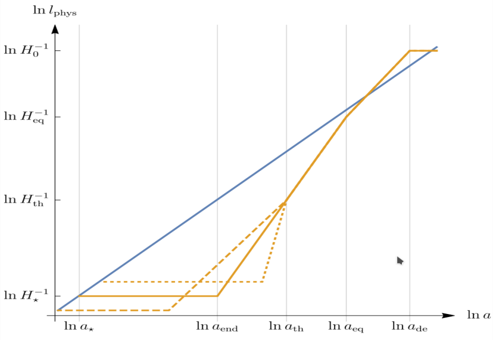
- $P_\zeta \sim k^{-\gamma} \rightarrow \Omega_{SGWB} \sim f^{-2\gamma}$ $(\gamma \leq 4.0)$ [Xu et al, Phys.Rev.D 2020]
- $N_{end} - N_{peak} \geq 10.4 \rightarrow \gamma \leq 2.0$
- Maximum slope of SGWB ≤ 4.0 .

Effects of reheating

We assume reheating phase with a constant equation of state w_{reh}, and duration N_{reh}.

Remapping of scales :

$$k_e = k_{no-reheating} \times e^{-\frac{1}{4} N_{reh}(1-3w_{reh})}$$



$$N_e - N_p = ln\left(\frac{k_e}{k_p}\right) = \mathbf{58.92} + \frac{\mathbf{1}}{\mathbf{4}}\mathbf{ln}(\epsilon) - \frac{\mathbf{1}}{\mathbf{4}}\mathbf{N_{reh}(1-3w_{reh})}$$

Effects of reheating on lowest mass limit

Non-instantaneous reheating history

- $N_{peak} - N_{pivot} \simeq 47.41 - \frac{1}{4} N_{reh}(1 - 3w_{reh})$
- $M_{PBH} \geq 6.14 \times 10^{-23} e^{\frac{1}{2} N_{reh}(1-3w_{reh})} M_{\odot}$
- $f_{SGWB} \leq 2.91 \times 10^{2} e^{-\frac{1}{4} N_{reh}(1-3w_{reh})} Hz.$

Matter dominated reheating
(w=0, $\mathbf{N}_{reh} = \mathbf{10}$) :

- $M_{PBH} \geq 9.09 \times 10^{-21} M_{\odot}$
- $f_{SGWB} \leq 23.94 Hz.$

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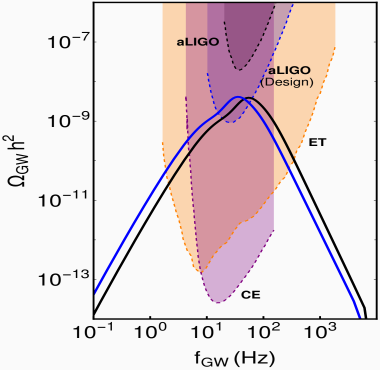
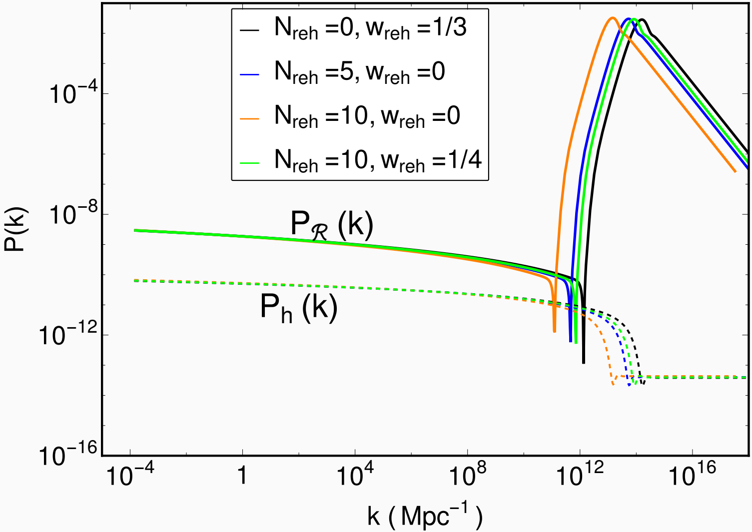
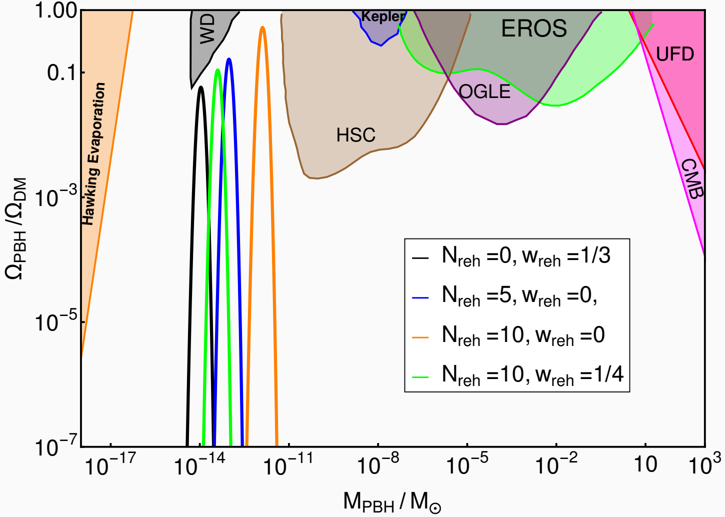


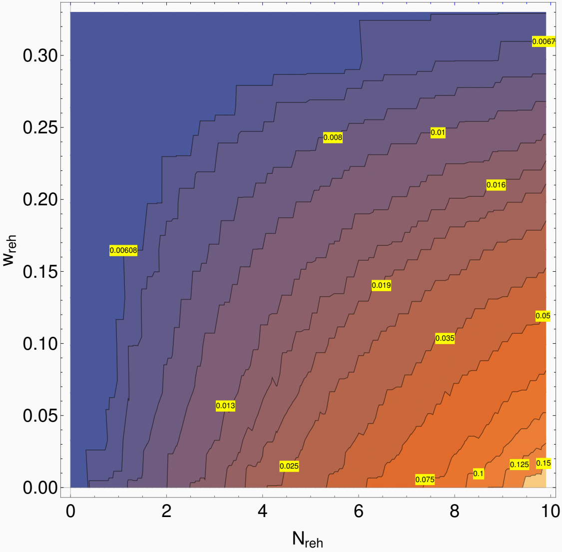
Power Spectra



Mass fraction



Contour plot for mass fraction



Effects of reheating on ISGWB

Dependence on kernel I(u,v,x)

$$\Omega_{\rm GW}(\tau,k)=\frac{1}{6}\left(\frac{k}{\mathcal{H}}\right)^2\int_0^\infty \mathrm{d}v\int_{|1-v|}^{1+v}\mathrm{d}u\left(\frac{4v^2-(1+v^2-u^2)^2}{4uv}\right)^2$$

$$\times\overline{I^2_{\rm MD+RD}(v,u,x)}\mathcal{P}_{\mathcal{R}}(kv)\mathcal{P}_{\mathcal{R}}(ku)$$

$$I(u,v,x,x_r)\simeq I_{RD}(u,v,x,x_r)=\int_{x_r}^{x}d\bar{x}\frac{a(\bar{x})}{a(x)}f(u,v,\bar{x},x_r)kG(\bar{x},x)$$

(Inomata, Phys. Rev. D (2019))

$$f(u,v,\bar{x},x_r)=\frac{4}{9}\Big[(\bar{x}-x_r/2)\partial_{\bar{x}}\mathcal{T}(u\bar{x},ux_r)((\bar{x}-x_r/2)\partial_{\bar{x}}\mathcal{T}(v\bar{x},vx_r)$$
$$+\mathcal{T}(v\bar{x},vx_r))+\mathcal{T}(u\bar{x},ux_r)((\bar{x}-x_r/2)\partial_{\bar{x}}\mathcal{T}(v\bar{x},vx_r)+3\mathcal{T}(v\bar{x},vx_r))\Big]$$

The scalar transfer function in RD

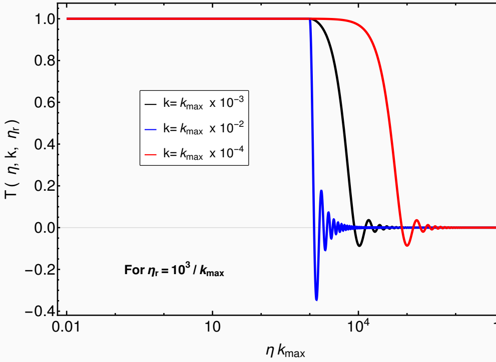
$$\mathcal{T}_k''(\eta) + 4\mathcal{H}\mathcal{T}_k'(\eta) + \frac{k^2}{3}\mathcal{T}_k(\eta) = 0 \qquad (1$$

Background with eMD

$$\frac{a(\eta)}{a(\eta_r)} = 2\frac{\eta}{\eta_r} - 1$$
$$\mathcal{H} = \frac{1}{\eta - \eta_r/2}$$

Initial condition with eMD $(x = \eta k)$

- $\mathcal{T}(x, x_r)|_{x=x_r} = 1$
- $\partial_x \mathcal{T}(x, x_r)|_{x=x_r} = 0$



Different oscillating terms

$$\mathcal{I}(u,v,x,x_r) = I(u,v,x,x_r) \times (x - x_r/2)$$

$$\mathcal{I}(u,v,x,x_r) = \mathcal{I}_s(u,v,x,x_r)\sin(x) + \mathcal{I}_c(u,v,x,x_r)\cos(x) + \text{4 other terms}$$

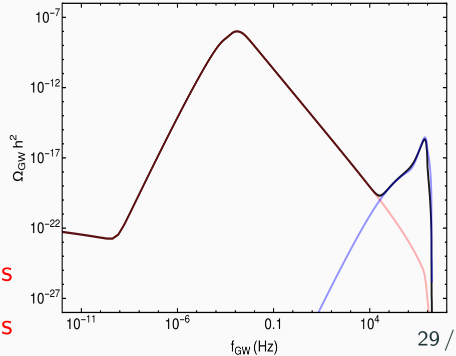
Oscillation avarage

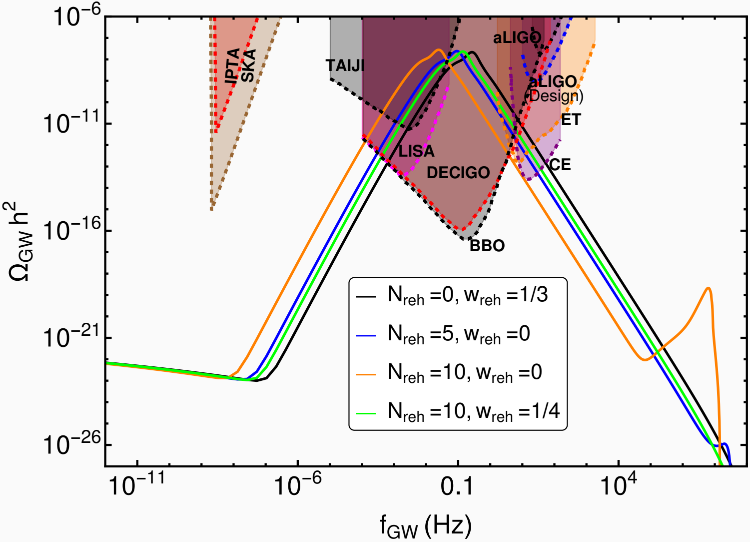
$$\overline{\mathcal{I}^2} = \frac{1}{2}\left(\mathcal{I}_s^2 + \mathcal{I}_c^2\right)$$

Different k regiemes

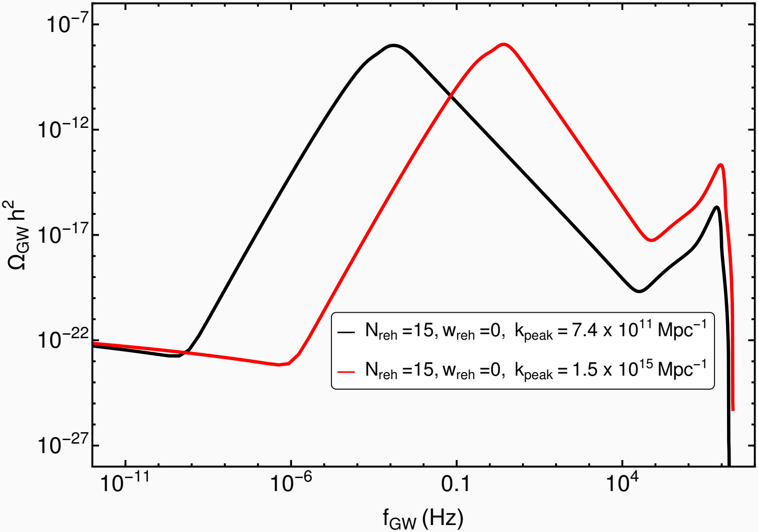
$$\mathcal{I}_s \simeq \mathcal{I}_{ss} + \boxed{\mathcal{I}_{sl}} + \text{other terms}$$

$$\mathcal{I}_c \simeq \mathcal{I}_{cs} + \boxed{\mathcal{I}_{cl}} + \text{other terms}$$




10^−6
10^−11
10^−16
10^−21
10^−26
10^−11
10^−6
0.1
10^4
f_GW (Hz)
Ω_GW h^2
IPTA
SKA
TAIJI
aLIGO
aLIGO
(Design)
ET
LISA
DECIGO
CE
BBO
N_reh =0, w_reh =1/3
N_reh =5, w_reh =0
N_reh =10, w_reh =0
N_reh =10, w_reh =1/4

SGWB for different power spectras, same reheating histories



Summary

Summary of Results

- It is possible to produce PBHs in different mass ranges, and ISGWB in different frequncy bands in these class of models.
- An upper mass limit comes from n_s problem.
- Also there is lower mass limit (and an upper limit on f_{SGWB}) for USR models which has no phase of $\eta \leq -1.0$.
- For non-instantaneous reheating history, the lower mass bound gets even stronger.
- Even for monochromatic case, a matter dominated reheating phase results in higher PBH mass range with more abundant PBH formation, lower frequency range for primary ISGWB peak, along with a second ISGWB peak in very high frequency.

(Based on JCAP01(2020)037 and arXiv:2009.10424)

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Thank you

Generation of Primordial Black Holes

- Different Classes of generation mechanism
- PBH from scalar perturbation of inflation
  - Radiation dominated epoch
  - Matter dominated epoch
  - Reheating epoch

We shall deal with PBH formed in radiation dominated epoch, and study the effects of non-instantaneous reheating phase.

Choice of initial scale factor

For Non-instantaneous reheating :

$$\frac{a_i}{a_0} = \left(\frac{g_{*,0}}{g_{*,r}}\right)^{1/3} \frac{T_\gamma}{T_r}\, e^{-N-N_{reh}},$$

$$T_r = \left(\frac{90}{\pi^2 g_{*,r}}\right)^{1/4} \left(H_e^2 M_{\rm Pl}^2\, e^{-3N_{reh}(1+w_{reh})}\right)^{1/4}$$

Steps :

- Choose a_{0i}
- Choose inflection point
- Scale the field for optimized efold Number
- Normalize $P_{\mathcal{R}}$ at k_p
- Calculate a_{0c} from the model
- Rerun loop with new a_{0i}

Need to scale the field

Requirement while scanning the parameter space:

1. Finite Number of Efolds
2. The Fixed PBH Mass Range

$$\phi = x * \nu$$

Scaling of the field

1. Does not harm the inflection point condition
3. It effects duration of USR phase most .
4. Changing only ν we get desired efold requirement

Computation of numerical power spectra

- Background evolution
- Sub horizon condition :
  - Bunch-Davies vacuum conditions
    $\mathcal{R}_k = \frac{1}{\sqrt{2k}} \frac{e^{-ik\tau}}{z}, \qquad h_k = \frac{1}{\sqrt{2k}} \frac{e^{-ik\tau}}{a}.$
- Evolve the perturbation along with back ground evolution
- Obtain Power spectra after:
  - Super horizon condition satisfies: $(k/aH \sim 10^{-3})$
  - Decaying mode dies

Computation of Mass function

Press Schechter Formalism

(Radiation dominated Epoch $\quad w = 1/3$
$R = 1/k = (aH)^{-1}$)

$$\sigma_\delta^2(R) = \frac{16}{81}\int \frac{dk}{k}(kR)^4 P_{\mathcal{R}}(k) W^2(k,R) \tag{2}$$

$$\beta_f(M) = \frac{1}{2}\mathrm{erfc}\left(\frac{\delta_c}{\sqrt{2}\,\sigma_\delta(M(R))}\right) \tag{3}$$

$$M(R_f) = 4\pi\gamma M_{\mathrm{Pl}}^2\left(\frac{a_{eq}}{R_{eq}}\right)R_f^2$$

$$\beta_{eq}(M) = \beta_f(M)\left(\frac{a_{eq}}{a_f}\right) = \beta_f(M)\left(\frac{R_{eq}}{R_f}\right)$$

$$f_{PBH} = \frac{\beta_{eq}(M)}{\Omega_{DM}(M)}$$

Total efolds

In radiation dominated universe, $a \propto t^{1/2}$, $H \propto t^{-1}$ and $k = aH \propto t^{-1/2} \propto H^{1/2}$ .As $\rho = 3H^2 M_{pl}^2$, $k = aH \propto \rho^{1/4}$.

$$\frac{k}{k_{eq}} = \left(\frac{\rho}{\rho_{r,eq}}\right)^{1/4}$$

$$\rho_{r,eq} = \Omega_r (z_{eq} + 1)^4 \rho_{critical} = \Omega_r (z_{eq} + 1)^4 (3H_{present}^2 M_{pl}^2)$$

$$H_e = H_{ri} = \left(P_\zeta(k) 8\pi^2 M_{Pl}^2 \epsilon\right)^{1/2}$$

$$\rho_e = 3H_e^2 M_{pl}^2 = 8\pi^2 \left(P_\zeta(k) M_{Pl}^2 \epsilon\right)$$

$$k_{ri} = \frac{k_{eq}}{(z_{eq}+1)} \left(\frac{P_\zeta(k) 8\pi^2 \epsilon}{\Omega_r H_{present}^2}\right)^{1/4}$$

$$k_{ri} = \mathbf{1.94 \times 10^{24}} (\epsilon)^{\mathbf{1/4}} \mathbf{Mpc^{-1}}$$

$$N_e - N_p = ln\left(\frac{k_{ri}}{k_p}\right) = \mathbf{58.92} + \frac{\mathbf{1}}{\mathbf{4}} \mathbf{ln}(\epsilon)$$

Transfer function in RD in presence of an eMD

$$\mathcal{T}(x, x_r) = \frac{3\sqrt{3}\left[A(x_r)\, j_1\left(\frac{x - x_r/2}{\sqrt{3}}\right) + B(x_r)\, y_1\left(\frac{x - x_r/2}{\sqrt{3}}\right)\right]}{x - x_r/2}$$

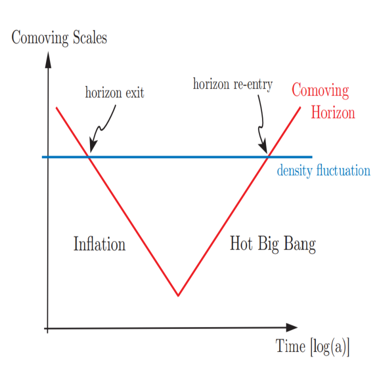
$$\begin{aligned} A(x_r) &= \tfrac{x_r}{2\sqrt{3}} \sin\left(\tfrac{x_r}{2\sqrt{3}}\right) - \tfrac{1}{36}(x_r^2 - 36)\cos\left(\tfrac{x_r}{2\sqrt{3}}\right) \\ B(x_r) &= -\tfrac{1}{36}(x_r^2 - 36)\sin\left(\tfrac{x_r}{2\sqrt{3}}\right) - \tfrac{x_r}{2\sqrt{3}} \cos\left(\tfrac{x_r}{2\sqrt{3}}\right) \end{aligned}$$

Assumptions of Instantaneous collapse

- Spherical Collapse
- $M_{PBH}(R) \simeq M_H$

Perturbation to density contrast

$\delta(k,t) \simeq \frac{2(w+1)}{(3w+5)} \left(\frac{k}{aH}\right)^2 \mathcal{R}_k$



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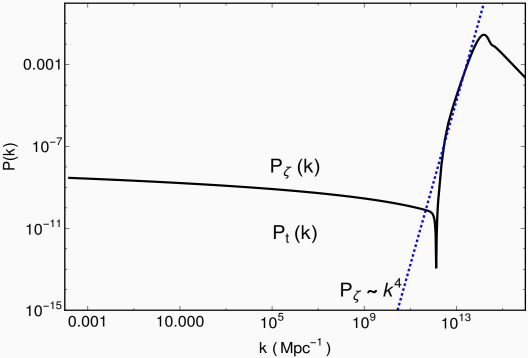
- $\mathcal{R}\sim .0001$; $P_{\mathcal{R}} \sim 2.25 \times 10^{-9}$ at CMB Scale

Steepest Possible Growth



This steep growth is consistent with Byrnes, Cole and Patil (arXiv:1811.11158).

Effects of reheating

We assume reheating phase with a constant equation of state w_{reh}, and duration N_{reh}.

Remapping of scales :

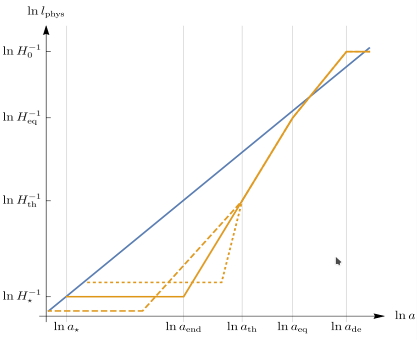
$$H_{ri} = H_e e^{-\frac{3}{2} N_{reh}(1+w_{reh})}$$

$$k_{ri} = \frac{k_{eq}}{\sqrt{H_{eq}}} H_e^{1/2} e^{-\frac{3}{4} N_{reh}(1+w_{reh})}$$

$$\frac{k_e}{k_{ri}} = \frac{a_e H_e}{a_{ri} H_{ri}} = e^{-N_{reh}+\frac{3}{2} N_{reh}(1+w_{reh})}$$

$$k_e = \frac{k_{eq}}{\sqrt{H_{eq}}} H_e^{1/2} e^{-N_{reh}(1-3w_{reh})/4}$$

$$= k_{no-reheating} \times e^{-\frac{1}{4} N_{reh}(1-3w_{reh})}$$



$$N_e - N_p = ln\left(\frac{k_e}{k_p}\right) = \mathbf{58.92} + \frac{\mathbf{1}}{\mathbf{4}}\mathbf{ln}(\epsilon) - \frac{\mathbf{1}}{\mathbf{4}}\mathbf{N_{reh}(1 - 3w_{reh})}$$