Imprints of an early primordial black hole domination in the stochastic gravitational wave background

(Based on JHEP 07, 130 (2022))

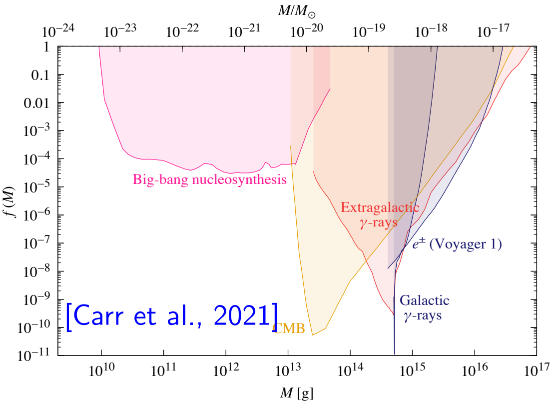
Nilanjandev Bhaumik
With A Ghoshal, M Lewicki
Weekly cosmology meeting, IIT Madras

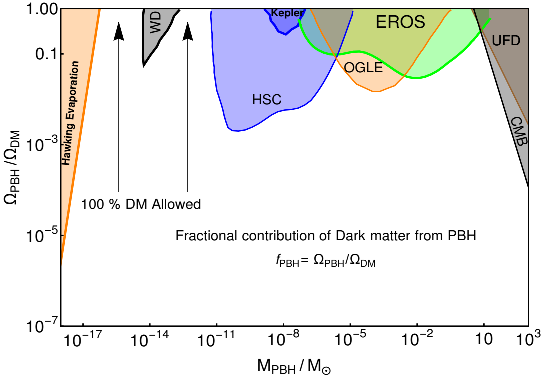
Indian Institute of Science, Bangalore

September 3, 2022

Overview

Ultra low mass PBHs

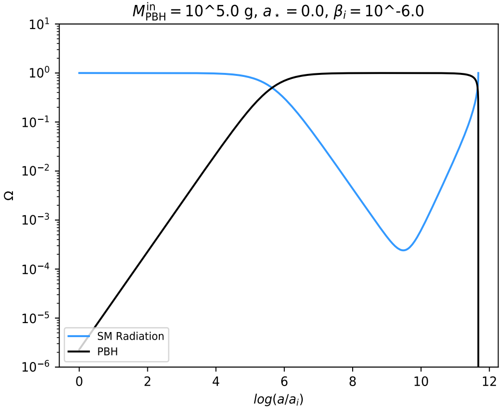




Inflation $\xrightarrow{\text{eRD}}$ PBH formation $\xrightarrow{\text{eRD}}$ PBH domination $\xrightarrow{\text{eMD}}$ PBH evaporation $\xrightarrow{\text{RD}}$ BBN

Background variables

$$a_{eRD}(\tau) = \left(\frac{a_f}{\tau_f}\right)\tau, \quad a_{eMD}(\tau) = \frac{a_f(\tau+\tau_m)^2}{4\tau_f\tau_m}, \quad a_{RD}(\tau) = \frac{a_f(\tau_r+\tau_m)(2\tau-\tau_r+\tau_m)}{4\tau_f\tau_m}$$



$$\tau_r = \sqrt{2}\left(\frac{3\Delta t_{\rm PBH}^2 \rho_{EQ}\tau_{EQ}^4}{M_{Pl}^2}\right)^{1/4}$$

$$\frac{\tau_r}{\tau_m} = 2\left(\frac{3\pi^2\gamma^2 M_{Pl}^6\beta_f^4\tau_r^4}{M_{\rm PBH}^2\rho_{EQ}\tau_{EQ}^4}\right)^{1/6}$$

$$k_r \approx 2.1\times 10^{11}\left(\frac{M_{\rm PBH}}{10^4\,{\rm g}}\right)^{-3/2} \ {\rm M}pc^{-1}$$

$$k_m \approx 3.4\times 10^{17}\left(\frac{M_{\rm PBH}}{10^4\,{\rm g}}\right)^{-5/6} \beta_f^{2/3}\ {\rm M}pc^{-1}$$

$$k_f = k_m/\beta_f.$$

Adiabatic perturbations from two contributions

1. Poisson Distribution of PBHs
2. Cutoff for scales bellow PBH mean distance
3. Finite duration of PBH domination (Non-linearity bound)

$$\mathcal{P}_{\rm PBH}(k,\tau_r) = \frac{2}{3\pi}\left(\frac{k}{k_{\rm UV}}\right)^3\left(5+\frac{4}{9}\frac{k^2}{k_m^2}\right)^{-2}$$

$$k_{\rm UV} = \gamma^{-1/3}\beta^{1/3}k_f$$

$$\Phi(k,\tau_r) = \Phi_{\rm infl}(k,\tau_r) + \Phi_{\rm PBH}(k,\tau_r)$$

$$\mathcal{P}_{\rm infl}(k,\tau_r) = A_s\left(\frac{k}{k_p}\right)^{n_s-1}\theta_H(k_m-k)$$

$$\frac{\Phi(k,\tau_r)}{\Phi(k,\tau_{r0})} \approx \left(\sqrt{\frac{2}{3}}\frac{k}{k_r}\right)^{-1/3}$$

$$\mathcal{P}(k,\tau_r) = \mathcal{P}_{\rm infl}(k,\tau_r) + \mathcal{P}_{\rm PBH}(k,\tau_r)$$

Induced Stochastic Gravitational Wave Background (ISGWB)

$$h''_{\mathbf{k}}(\tau) + 2\mathcal{H}h'_{\mathbf{k}}(\tau) + k^2 h_{\mathbf{k}}(\tau) = 4S_{\mathbf{k}}(\tau)$$

$$S_k^s = \int \frac{\mathrm{d}^3 q}{(2\pi)^{3/2}} e_{ij}^s(k) q_i q_j \left[2\Phi_q \Phi_{k-q} + \frac{4}{3(1+w)} (\mathcal{H}^{-1}\Phi'_q + \Phi_q)(\mathcal{H}^{-1}\Phi'_{k-q} + \Phi_{k-q}) \right]$$

$$\Omega_{\mathrm{GW}}(\tau, k) = \frac{1}{6}\left(\frac{k}{\mathcal{H}}\right)^2 \int_0^\infty \mathrm{d}v \int_{|1-v|}^{1+v} \mathrm{d}u \left(\frac{4v^2 - (1+v^2-u^2)^2}{4uv} \right)^2$$

$$\times \overline{I^2_{\mathrm{RD}}(v, u, x)} \mathcal{P}_{\mathcal{R}}(kv) \mathcal{P}_{\mathcal{R}}(ku)$$

$$I_{RD}(u, v, x) = \quad \int_0^x d\bar{x} \frac{a(\bar{x})}{a(x)} f(u, v, \bar{x}) k G(\bar{x}, x)$$

SGWB in case of an early PBH domination

Dependence on kernel I(u,v,x)

$$\Omega_{\rm GW}(\eta, k) =$$

$$\frac{1}{6}\left(\frac{k}{\mathcal{H}}\right)^2 \int_0^\infty \mathrm{d}v \int_{|1-v|}^{1+v} \mathrm{d}u \left(\frac{4v^2-(1+v^2-u^2)^2}{4uv}\right)^2 \overline{I^2_{\rm eRD+eMD+RD}(v,u,x)} \mathcal{P}_{\mathcal{R}}(kv)\mathcal{P}_{\mathcal{R}}(ku)$$

$$I(u,v,x,x_r) \simeq I_{RD}(u,v,x,x_r) = \int_{x_r}^{x} d\bar{x} \frac{a(\bar{x})}{a(x)} f(u,v,\bar{x},x_r) kG(\bar{x},x)$$

[Inomata, Kohri, Nakama, Terada; Phys. Rev. D 100, 043532 (2019)]
[Domènech, Sasaki ; Phys. Rev. D 103, 063531 (2021)]

$$f(u,v,\bar{x},x_r) = \frac{4}{9}\Big[(\bar{x}-x_r/2)\partial_{\bar{x}}\mathcal{T}(u\bar{x},ux_r)((\bar{x}-x_r/2)\partial_{\bar{x}}\mathcal{T}(v\bar{x},vx_r)$$

$$+\mathcal{T}(v\bar{x},vx_r))+\mathcal{T}(u\bar{x},ux_r)((\bar{x}-x_r/2)\partial_{\bar{x}}\mathcal{T}(v\bar{x},vx_r)+3\mathcal{T}(v\bar{x},vx_r))\Big]$$

- $x = \eta k$ $\quad x_r = \eta_r k$ $\quad \frac{a(\eta)}{a(\eta_r)} = 2\frac{\eta}{\eta_r} - 1$ $\quad \mathcal{H} = aH = \frac{1}{\eta - \eta_r/2}$

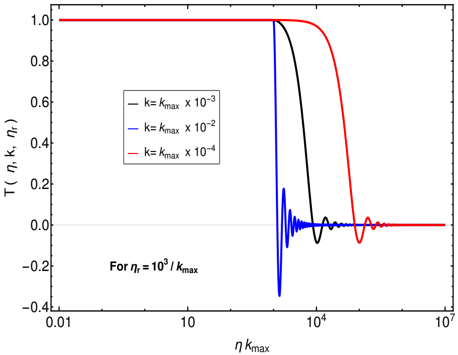
Transfer function in RD in presence of eMD

$$\mathcal{T}_k''(\eta) + 4\mathcal{H}\mathcal{T}_k'(\eta) + \frac{k^2}{3}\mathcal{T}_k(\eta) = 0$$

$$\mathcal{T}(x, x_r) = \frac{3\sqrt{3}\left[A(x_r)\, j_1\left(\frac{x - x_r/2}{\sqrt{3}}\right) + B(x_r)\, y_1\left(\frac{x - x_r/2}{\sqrt{3}}\right)\right]}{x - x_r/2}$$

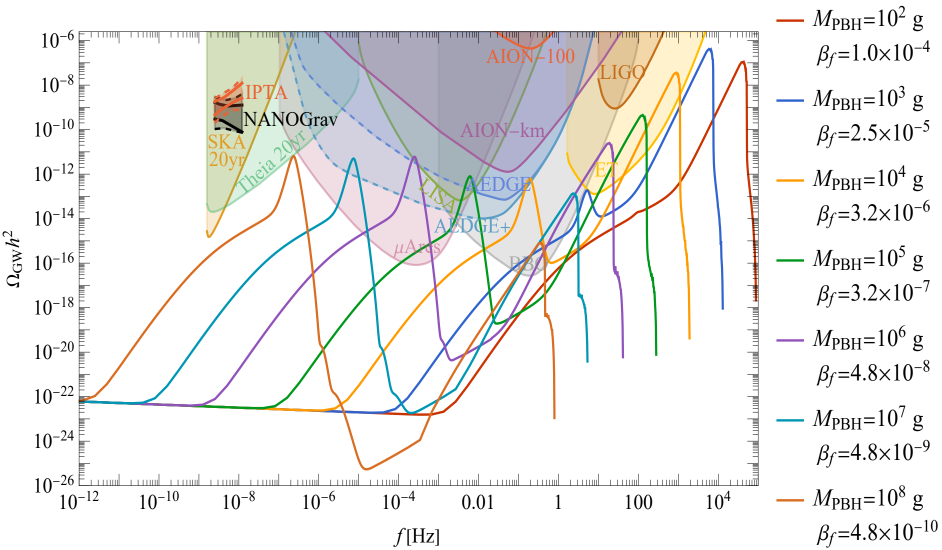
$$A(x_r) = \frac{x_r}{2\sqrt{3}} \sin\left(\frac{x_r}{2\sqrt{3}}\right) - \frac{1}{36}(x_r^2 - 36)\cos\left(\frac{x_r}{2\sqrt{3}}\right)$$

$$B(x_r) = -\frac{1}{36}(x_r^2 - 36)\sin\left(\frac{x_r}{2\sqrt{3}}\right) - \frac{x_r}{2\sqrt{3}}\cos\left(\frac{x_r}{2\sqrt{3}}\right)$$

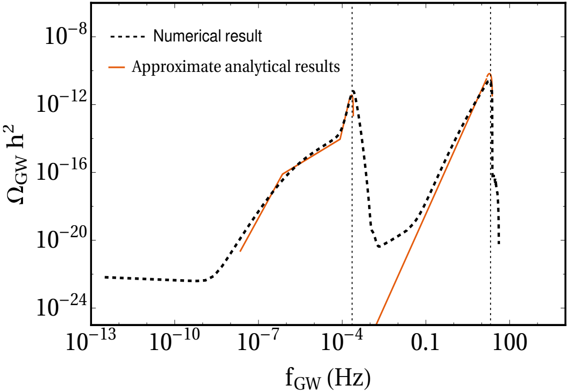


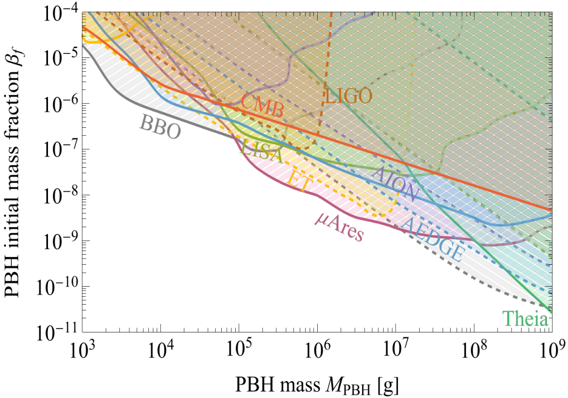
$$x = \eta k \qquad x_r = \eta_r k \qquad \frac{a(\eta)}{a(\eta_r)} = 2\frac{\eta}{\eta_r} - 1 \qquad \mathcal{H} = aH = \frac{1}{\eta - \eta_r/2} \qquad \mathcal{T}_k(\eta_r) = 1 \qquad \mathcal{T}_k'(\eta_r) = 0$$

Two Peaks of ISGWB



Detection Probability





$$\mathrm{SNR} \equiv \sqrt{\mathcal{T} \int \mathrm{d}f \left[\frac{\Omega_{\rm GW}(f)}{\Omega_{\rm noise}(f)}\right]^2}$$

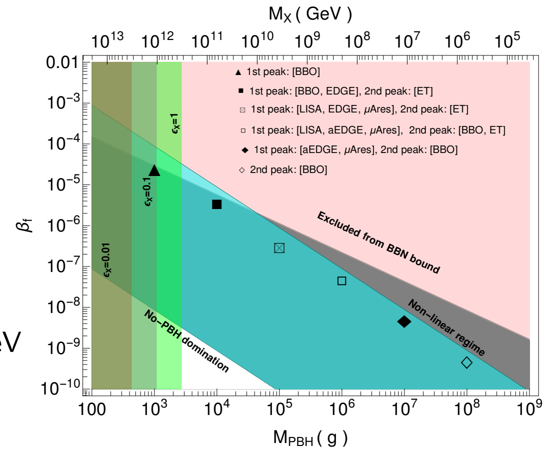
Testing the PBH induced Baryogenesis

$$Y_B = \frac{n_{\mathrm{PBH}}}{s}\epsilon_X N_X$$

$$Y_B \approx 8.8 \times 10^{-11}$$

$$M_X \approx 6.34 \times 10^{15}\sqrt{\epsilon_X \left(\frac{1\ \mathrm{g}}{M_{\mathrm{PBH}}}\right)^{5/2}}\ \mathrm{GeV}$$

[Hooper, Krnjaic, Phys.Rev.D(2021)]



Summary of Results

- Ultra-low mass PBHs can dominate the universe before BBN and contribute to isocurvature perturbations.
- Such an early PBH dominated universe leads to an uniquely shaped doubly peaked ISGWB spectrum; one peak from the inflationary adiabatic and another from isocurvature induced adiabatic scalar modes.
- The amplitude of the ISGWB peaks enables us to constrain the initial abundance of PBHs.
- The uniquely shaped spectrum of ISGWB also works as a potential probe for all such scenarios where the Hawkig radiation of ultra-low mass PBHs plays a significant role.

Thank You

Scalar transfer function $\mathcal{T}_k(\eta)$ and Kernel I(u,v,x,x_r)

Oscillating terms

$\mathcal{I} = I(u, v, x, x_r) \times (x - x_r/2)$
$\mathcal{I} = \mathcal{I}_s \sin(x) + \mathcal{I}_c \cos(x) +$ 4 other terms

Oscillation avarage

$$\overline{\mathcal{I}^2} = \frac{1}{2}\left(\mathcal{I}_s^2 + \mathcal{I}_c^2\right)$$

Different k regiemes

$\mathcal{I}_s \simeq \mathcal{I}_{ss} + \boxed{\mathcal{I}_{sl} x_r^4} +$ other terms

$\mathcal{I}_c \simeq \mathcal{I}_{cs} + \boxed{\mathcal{I}_{cl} x_r^4} +$ other terms

$I = I_{eRD} + I_{eMD} + I_{RD}$
$\Phi(k, \tau_r) = \Phi_{\mathrm{infl}}(k, \tau_r) + \Phi_{\mathrm{PBH}}(k, \tau_r)$

Isocurvature Perturbation

$$d_f \equiv \left(\frac{3M_{\mathrm{PBH},f}}{4\pi\rho_{\mathrm{PBH},f}}\right)^{1/3} = \gamma^{1/3}\beta^{-1/3}{H_f}^{-1}$$

$$\langle \delta\rho_{\mathrm{PBH}}(k)\delta\rho_{\mathrm{PBH}}(k')\rangle = \frac{4\pi}{3}\left(\frac{d}{a}\right)^3 \rho_{\mathrm{PBH}}^2\,\delta(k+k')$$

$$S = \frac{\delta\rho_{\mathrm{PBH}}}{\rho_{\mathrm{PBH}}} - \frac{3}{4}\frac{\delta\rho_r}{\rho_r} = \frac{\delta\rho_{\mathrm{PBH}}}{\rho_{\mathrm{PBH}}} + \frac{3}{4}\frac{\delta\rho_{\mathrm{PBH}}}{\rho_r} \approx \frac{\delta\rho_{\mathrm{PBH}}}{\rho_{\mathrm{PBH}}} \quad \text{for } \rho_r \gg \rho_{\mathrm{PBH}}$$

$$\mathcal{P}_S(k) = \frac{2}{3\pi}\left(\frac{k}{k_{UV}}\right)^3$$

$$\Phi_{\mathrm{eMD}}(k; a \gg a_{\mathrm{eq}}) = S\begin{cases} \dfrac{1}{5} & k \ll k_{\mathrm{eq}} \\ \dfrac{3}{4}\left(\dfrac{k_{\mathrm{eq}}}{k}\right)^2 & k \gg k_{\mathrm{eq}} \end{cases}$$

Analytical form of ISGWB for inflationary adiabatic perturbation

$$\frac{\Omega_{GW}(\tau_0,k)}{A_s^2 c_g\,\Omega_{r,0}} \simeq \begin{cases} 3\times 10^{-7} x_r^3 x_{\rm max}^5 & 150 x_{\rm max}^{-5/3} \lesssim x_r \ll 1 \\ 6.6\times 10^{-7} x_r x_{\rm max}^5 & 1 \ll x_r \lesssim x_{\rm max}^{5/6} \\ 3\times 10^{-7} x_r^7 & x_{\rm max}^{5/6} \lesssim x_r \lesssim \frac{2}{1+\sqrt{3}} x_{\rm max} \\ \mathcal{C}(k) & \frac{2}{1+\sqrt{3}} \le \frac{x_r}{x_{\rm max}} \le \frac{2}{\sqrt{3}} \end{cases}, \tag{2}$$

where

$$\mathcal{C}(k) = 0.00638 \times 2^{-2n_s-13}\, 3^{n_s}\, x_r^7\, s_0 \left(\frac{x_r}{x_{max}}\right)^{2n_s-2} \times \\ \left(-s_0^2\, {}_2F_1(\frac{3}{2},-n_s;\frac{5}{2};\frac{s_0^2}{3}) + 4\, {}_2F_1(\frac{1}{2},1-n_s;\frac{3}{2};\frac{s_0^2}{3}) - 3\, {}_2F_1(\frac{1}{2},-n_s;\frac{3}{2};\frac{s_0^2}{3})\right). \tag{3}$$

Analytical form for isocurvature induced adiabatic contribution to ISGWB

$$\Omega_{GW}(\tau_0, k) = c_g\, \Omega_{r,0}\; \mathcal{J} \int_{-s_0}^{s_0} \frac{27\sqrt[3]{3}\left(s^2-1\right)^2}{\left(9-3s^2\right)^{5/3}} \mathrm{d}s \tag{4}$$

$$= c_g\, \Omega_{r,0}\; \mathcal{J} \frac{2}{5} s_0 \left(\frac{3\left(14-3s_0^2\right)}{\left(1-\frac{s_0^2}{3}\right)^{2/3}} - 37\, {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{3}{2}; \frac{s_0^2}{3}\right)\right) , \tag{5}$$

where,

$$\mathcal{J} = \frac{k^3 k_m^8 \left(\frac{k}{k_r}\right)^{2/3}}{1327104\sqrt[3]{2}\sqrt{3}\pi\, k_r^5 k_{\mathrm{UV}}^6} .$$

and

$$s_0 = \begin{cases} 1 & \frac{k}{k_{\mathrm{UV}}} \leq \frac{2}{1+\sqrt{3}} \\ 2\frac{k_{\mathrm{UV}}}{k} - \sqrt{3} & \frac{2}{1+\sqrt{3}} \leq \frac{k}{k_{\mathrm{UV}}} \leq \frac{2}{\sqrt{3}} \end{cases} . \tag{6}$$

Inflationary adiabatic and isocurvature-induced adiabatic perturbation

$$\Phi^{\mathrm{eISO}}_{\mathrm{eMD}}(k; a \gg a_{\mathrm{eq}}) \approx \begin{cases} \dfrac{1}{5} & k \ll k_{\mathrm{eq}} \\ \dfrac{3}{4}\left(\dfrac{k_{\mathrm{eq}}}{k}\right)^2 & k \gg k_{\mathrm{eq}} \end{cases},$$

$$\Phi^{e\mathrm{CVT}}_{e\mathrm{MD}}(k \gg k_{\mathrm{eq}}; a \gg a_{\mathrm{eq}}) \approx \frac{135}{16}\left(\frac{k_{\mathrm{eq}}}{k}\right)^4 \left(\ln 4 - \frac{7}{2} + \gamma_E + \ln\left(\sqrt{\frac{2}{3}}\frac{k}{k_{\mathrm{eq}}}\right)\right)$$

[KODAMA, SASAKI (1987). International Journal of Modern Physics A, 02(02), 491–560.]