Cosmological analysis of Barrow holographic dark energy model in a nonflat universe

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Based on "Barrow holographic dark energy in a nonflat universe"

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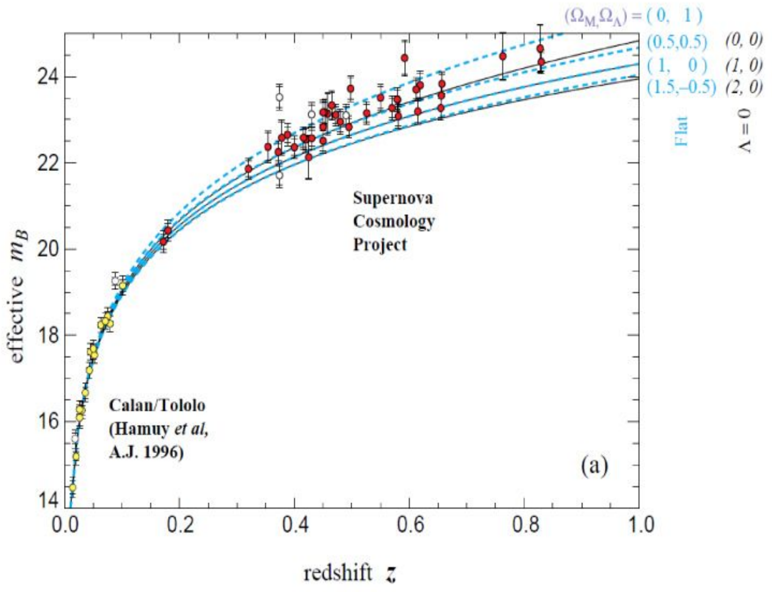
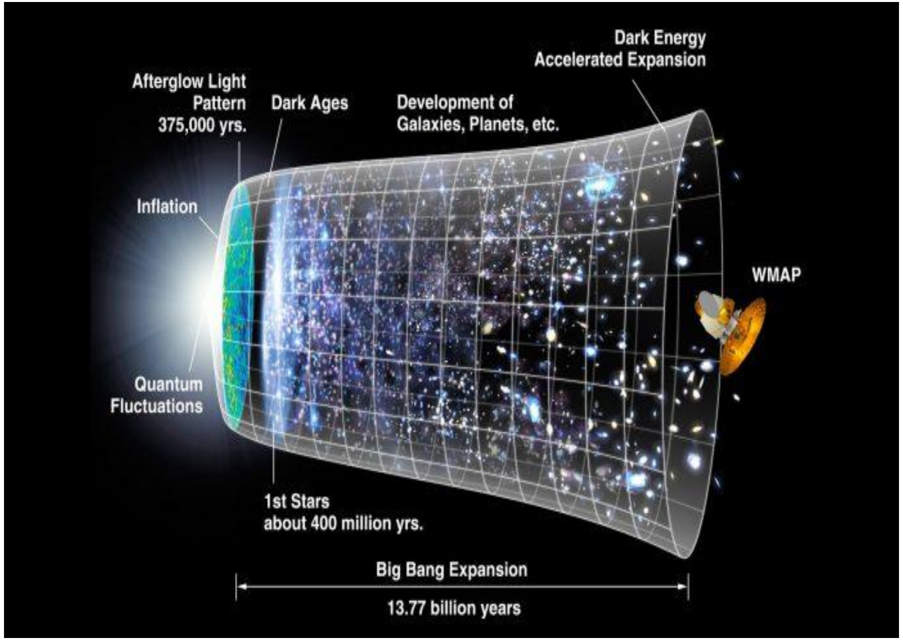
Plan of the talk

- Introduction
  - Modern cosmology
  - Holographic principle
  - Barrow holographic dark energy
- Our model and results
  - Friedmann equations
  - Barrow holographic dark energy with positive spatial curvature
  - Barrow holographic dark energy with negative spatial curvature
- Conclusions

Modern Cosmology



Evidence of accelerated expansion of the universe



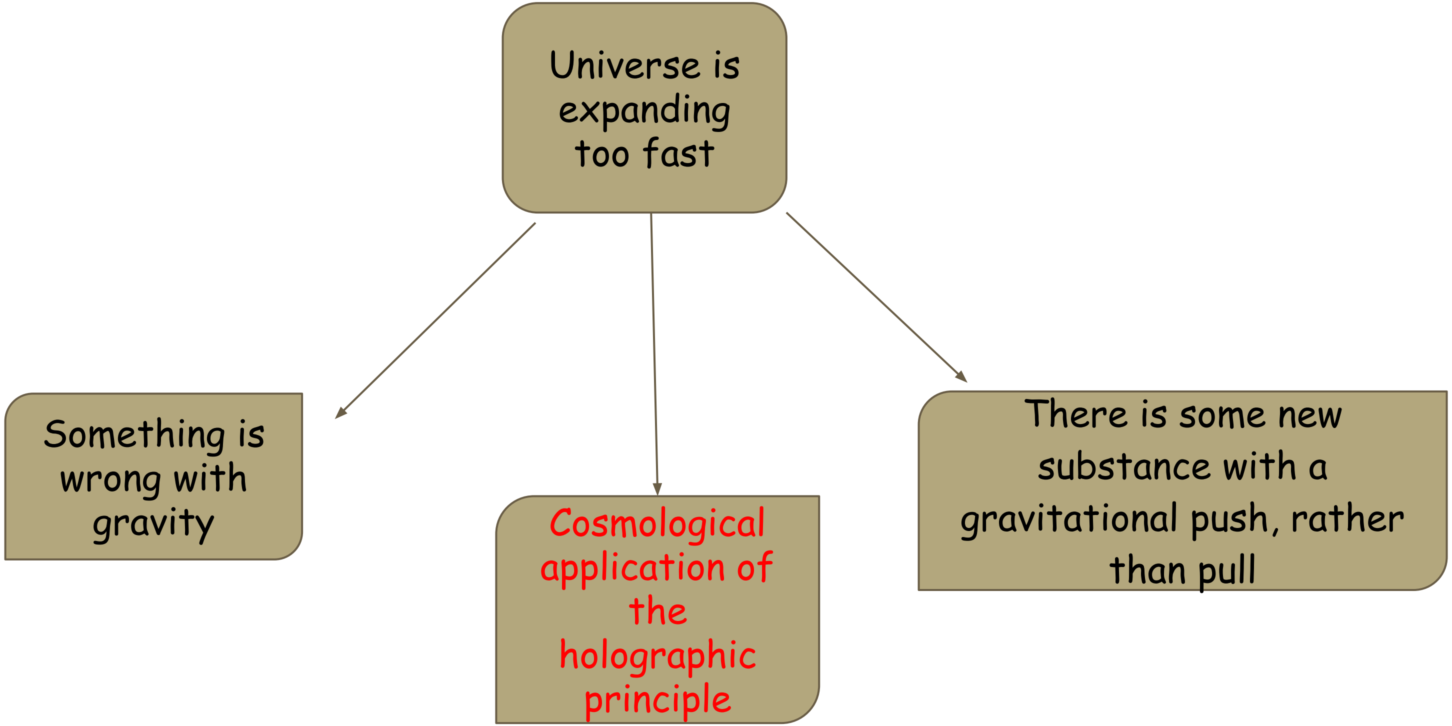
Cosmic history

Source: NASA/Supernova Cosmology Project (SCP)/WMAP Science Team

State of the Art

- Our universe is not only expanding but it is also accelerating!!
- ΛCDM model happens to be the simplest model of the universe.
- ΛCDM model has been constrained with unprecedented accuracy.
- But suffers from cosmological constant problem, coincidence problem
- We need to extent our imagination beyond standard ΛCDM.

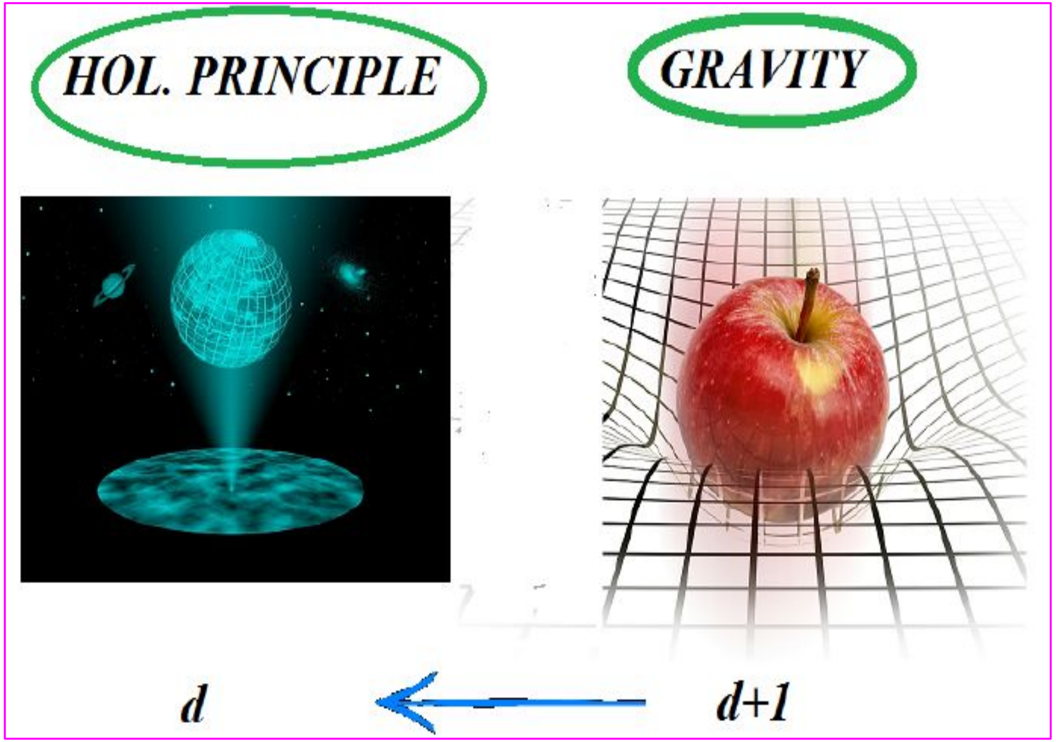
Modern Cosmology



Holographic principle

- The **holographic principle** is an axiom in string theory and a supposed property of quantum gravity which states that the description of a volume of space can be thought of as encoded on a lower-dimensional boundary to the region — such as a light-like boundary like a gravitational horizon.

- The universe filled with galaxies, stars, planets, houses, boulders, and people—is a hologram, an image of reality coded on a distant two-dimensional (2D) surface.



Holographic principle

- Einstein's equations of General Relativity contain singular solutions, named Black Holes (BH).
- Black holes are singular spacetime configurations provided by an event horizon outside which nothing can escape.
- The holographic principle was inspired by black hole thermodynamics, which conjectures that the maximum entropy in any region scales with the square of radius.

$$S_{BH} = \frac{A}{4\, l_p^2}$$

- In the case of a black hole, the insight was that the information content of all the objects that have fallen into the hole might be entirely contained in surface fluctuations of the event horizon.

Holographic principle

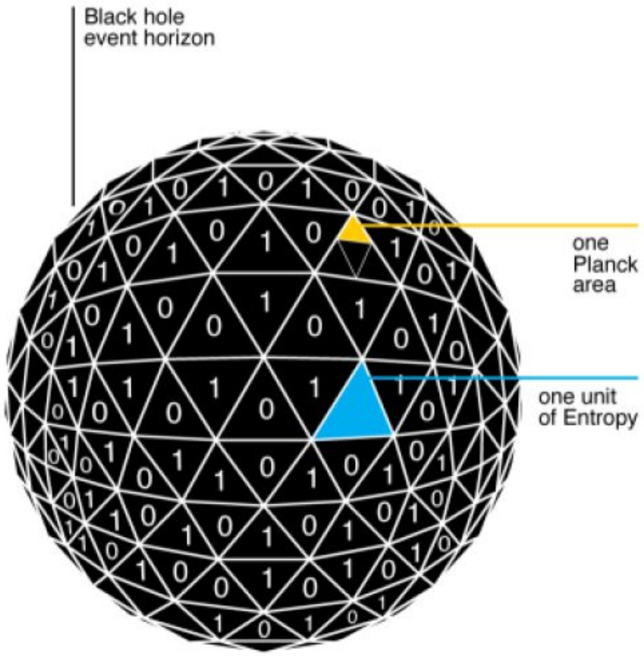
- They emit thermal radiation via production of pairs at their horizon

- For gravitational theories the Holographic principle states that:

  "the maximum entropy of a region with volume V is the entropy of the biggest BH that fits".

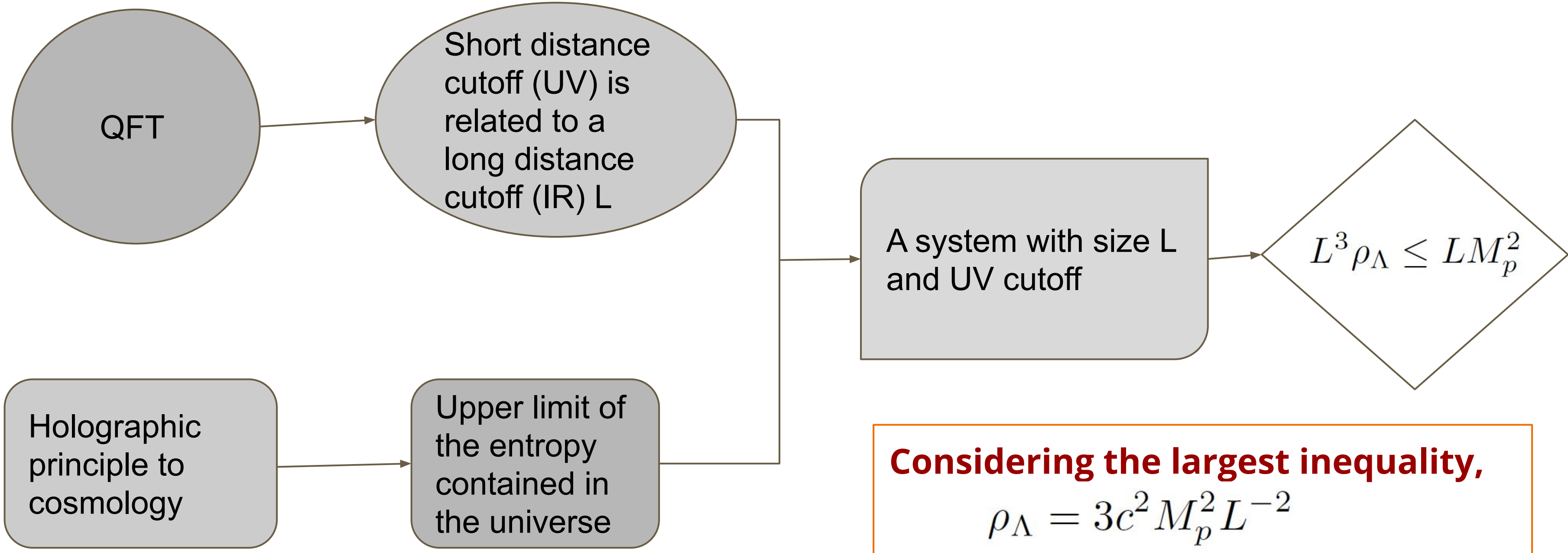
  This means

$$S_{max} = S_{BH} = \frac{1}{4\pi G_N} \times \text{horizon area}$$



Schematic representation of the BH entropy and Bekenstein-Hawking idea

Holographic dark energy



Considering the largest inequality,

$$\rho_\Lambda = 3c^2 M_p^2 L^{-2}$$

A. G. Cohen et al., “Effective Field Theory, Black Holes, and the Cosmological Constant”, PRL 82, 4971 (1999); arXiv: hep-th/9803132
E. N. Saridakis, “Barrow holographic dark energy”, Phys. Rev. D 102, no.12, 123525 (2020); [arXiv:2005.04115]

Holographic dark energy

A. Cohen and collaborators suggested sometime ago [3], that in quantum field theory a short distance cut-off is related to a long distance cut-off due to the limit set by formation of a black hole, namely, if ρ_Λ is the quantum zero-point energy density caused by a short distance cut-off, the total energy in a region of size L should not exceed the mass of a black hole of the same size, thus $L^3\rho_\Lambda \leq LM_p^2$. The largest L allowed is the one saturating this inequality, thus

$$\rho_\Lambda = 3c^2M_p^2L^{-2}. \quad (1)$$

principle to cosmology

contained in the universe

considering the largest inequality,

$$\rho_\Lambda = 3c^2M_p^2L^{-2}$$

A. G. Cohen et al., “Effective Field Theory, Black Holes, and the Cosmological Constant”, PRL 82, 4971 (1999); arXiv: hep-th/9803132
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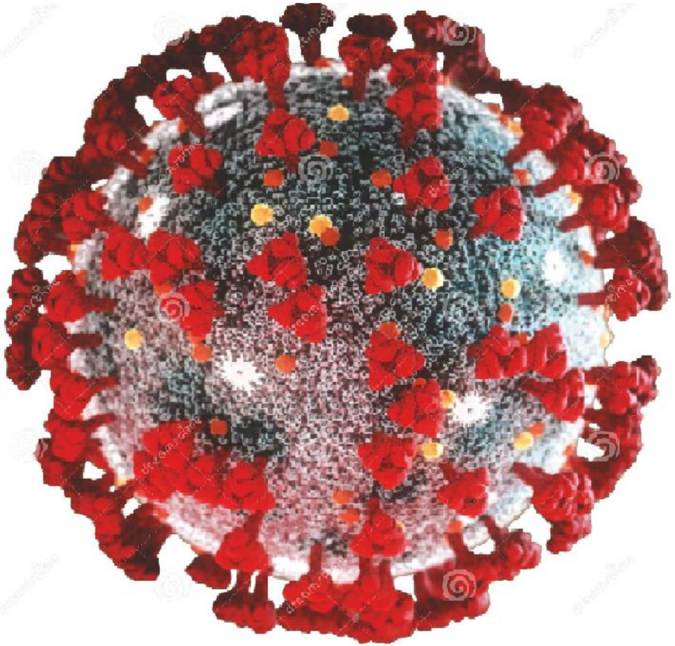
Barrow entropy



- Recently John D. Barrow, inspired by representations of the Covid-19 virus, demonstrated that incorporation of quantum gravitational effects may introduce intricate fractal features into the surface of a black hole.

- The entropy relation gets modified as

$$S_B = \left(\frac{A}{A_0}\right)^{1+\frac{\Delta}{2}}$$

where A the standard area of the horizon, A_0 is the Planck area and the Barrow exponent Δ lies in the range $0 \leqslant \Delta \leqslant 1$.

- $\Delta = 0$ corresponds to the standard smooth structure (standard Bekenstein-Hawking ones), and with $\Delta = 1$ corresponds to the most intricate structure.

J. D. Barrow, “The area of a rough black hole”, Phys. Lett. B **808**, 135643 (2020) [arXiv:2004.09444]

Barrow Holographic dark energy

- By applying this extended entropy relation in the holographic framework, one obtains a holographic dark energy density of the form

$$\rho_{DE} = CL^{\Delta-2}$$

where *L* the holographic horizon length and $C = 3c^2 {M_p}^2$ has dimension of $[L]^{-2-\Delta}$.

- The resulting energy density is considered to be the present day dark energy.

E. N. Saridakis, "Barrow holographic dark energy", Phys. Rev. D 102, no.12, 123525 (2020); [arXiv:2005.04115]

Our model

- Recently there is a reheated debate on whether the spatial curvature of the universe is zero or not. In particular, there are arguments that if one considers the combined analysis of Cosmic Microwave Background (CMB) anisotropy power spectra of the Planck Collaboration with the luminosity distance data, then a non-flat universe is favored at 99% confidence level.[#]

- Additionally the enhanced lensing amplitude in the CMB power spectrum seems to suggest that the curvature index k may be positive.[*]

- These make us interested in constructing and investigating Barrow Holographic dark energy in a non-flat universe.

- We carry out the investigation for both positive and negative spatial curvature index and will compare the results.

E. Di Valentino, A. Melchiorri and J. Silk, "*Investigating Cosmic Discordance*", Astrophys. J. Lett. 908, L9 (2021); [arXiv:2003.04935]
* N. Aghanim et al., "*Planck 2018 results. I. Overview and the cosmological legacy of Planck*", Astron. Astrophys. 641, A1(2020); [arXiv:1807.06205]

Friedmann equations

$$ds^2 = -\,dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right]$$

$$3H^2 + 3\frac{k}{a^2} = \rho_m + \rho_{DE}$$
$$2\dot{H} + 3H^2 + \frac{k}{a^2} = -p_{DE},$$

$$\dot{\rho}_m + 3H\rho_m = 0$$
$$\dot{\rho}_{DE} + 3H\left(1 + w_{DE}\right)\rho_{DE} = 0$$

$$\Omega_m \equiv \frac{\rho_m}{3M_p^2H^2},\ \Omega_{DE} \equiv \frac{\rho_{DE}}{3M_p^2H^2}\ \text{and}\ \Omega_k \equiv \frac{k}{a^2H^2}$$

- ❏ We need to suitably define the holographic horizon that enters in the definition of holographic dark energy.
- ❏ Although there are many possible choices, the most common one is to use the future event horizon

$$R_h \equiv a\int_t^\infty \frac{dt}{a} = a\int_a^\infty \frac{da}{Ha^2}$$

Can't we set *L* itself as the horizon ??

Stephen D. H. Hsu, "Entropy Bounds and Dark Energy", Phys. Lett. B 594, 13 (2004); [arXiv:hep-th/0403052]

Barrow holographic dark energy with positive spatial curvature

- The horizon length is given by $L = ar(t)$ where *r(t)* is determined through

$$\int_0^{r(t)} \frac{dr'}{\sqrt{1-kr'^2}} = \frac{R_h}{a}$$

- This gives $r(t) = \frac{1}{\sqrt{k}} \sin y$, where $y = \sqrt{k}\frac{R_h}{a} = \sqrt{k}\int_x^{\infty} \frac{dx}{aH}$ $x = \ln a$

- Finally putting this expression of L in $\rho_{DE} = CL^{\Delta-2}$, one gets

$$\rho_{DE} = Ca^{\Delta-2}\left(\frac{1}{\sqrt{k}}\sin y\right)^{\Delta-2}$$

Barrow holographic dark energy with positive spatial curvature

- Using this expression for ρ_{DE} in the field equations, one gets

$$\frac{1}{aH} = \frac{1}{\sqrt{\Omega_{m0}}H_0} \left(\frac{1-\Omega_{DE}}{a^{-1}-\gamma}\right)^{\frac{1}{2}}$$

which leads to

$$L = \frac{a}{\sqrt{k}} \sin\left[\sqrt{k}\int_x^{\infty} \frac{dx}{H_0\sqrt{\Omega_{m0}}} \left(\frac{1-\Omega_{DE}}{a^{-1}-\gamma}\right)^{\frac{1}{2}}\right]$$

$$\gamma \equiv \frac{\Omega_{k0}}{\Omega_{m0}}$$

- Again, $$L = \left[\frac{(1-\Omega_{DE})}{\Omega_{DE}} \frac{C}{3M_p^2H_0^2\Omega_{m0}} \frac{a^2}{(a^{-1}-\gamma)}\right]^{\frac{1}{2-\Delta}}$$

- Thus, we get

$$\frac{a}{\sqrt{k}} \sin\left[\sqrt{k}\int_x^{\infty} \frac{dx}{H_0\sqrt{\Omega_{m0}}} \left(\frac{1-\Omega_{DE}}{a^{-1}-\gamma}\right)^{\frac{1}{2}}\right] = \left[\frac{(1-\Omega_{DE})}{\Omega_{DE}} \frac{C}{3M_p^2H_0^2\Omega_{m0}} \frac{a^2}{(a^{-1}-\gamma)}\right]^{\frac{1}{2-\Delta}}$$

Barrow holographic dark energy with positive spatial curvature

- Evolution equation for Barrow holographic dark energy for closed universe

$$\frac{\Omega'_{DE}}{\Omega_{DE}(1-\Omega_{DE})} = \Delta + 1 + \gamma e^{x}(1-\gamma e^{x})^{-1} + \left[Q\cos y\ (\Omega_{DE})^{\frac{1}{2-\Delta}}(1-\Omega_{DE})^{\frac{\Delta}{2(\Delta-2)}} e^{\frac{3\Delta x}{2(\Delta-2)}}(1-\gamma e^{x})^{\frac{\Delta}{2(2-\Delta)}}\right]$$

- Expression for the dark-energy equation-of-state parameter

$$w_{DE} = -\left(\frac{1+\Delta}{3}\right) - \frac{Q}{3}(\Omega_{DE})^{\frac{1}{2-\Delta}}\cos y\ \left(\frac{1-\Omega_{DE}}{1-\gamma e^{x}}\right)^{\frac{\Delta}{2(\Delta-2)}} e^{\frac{3\Delta x}{2(\Delta-2)}}$$

where

$$Q \equiv (2-\Delta)\left(\frac{C}{3M_p^2}\right)^{\frac{1}{\Delta-2}}\left(H_0\sqrt{\Omega_{m0}}\right)^{\frac{\Delta}{2-\Delta}}$$

Barrow holographic dark energy with negative spatial curvature

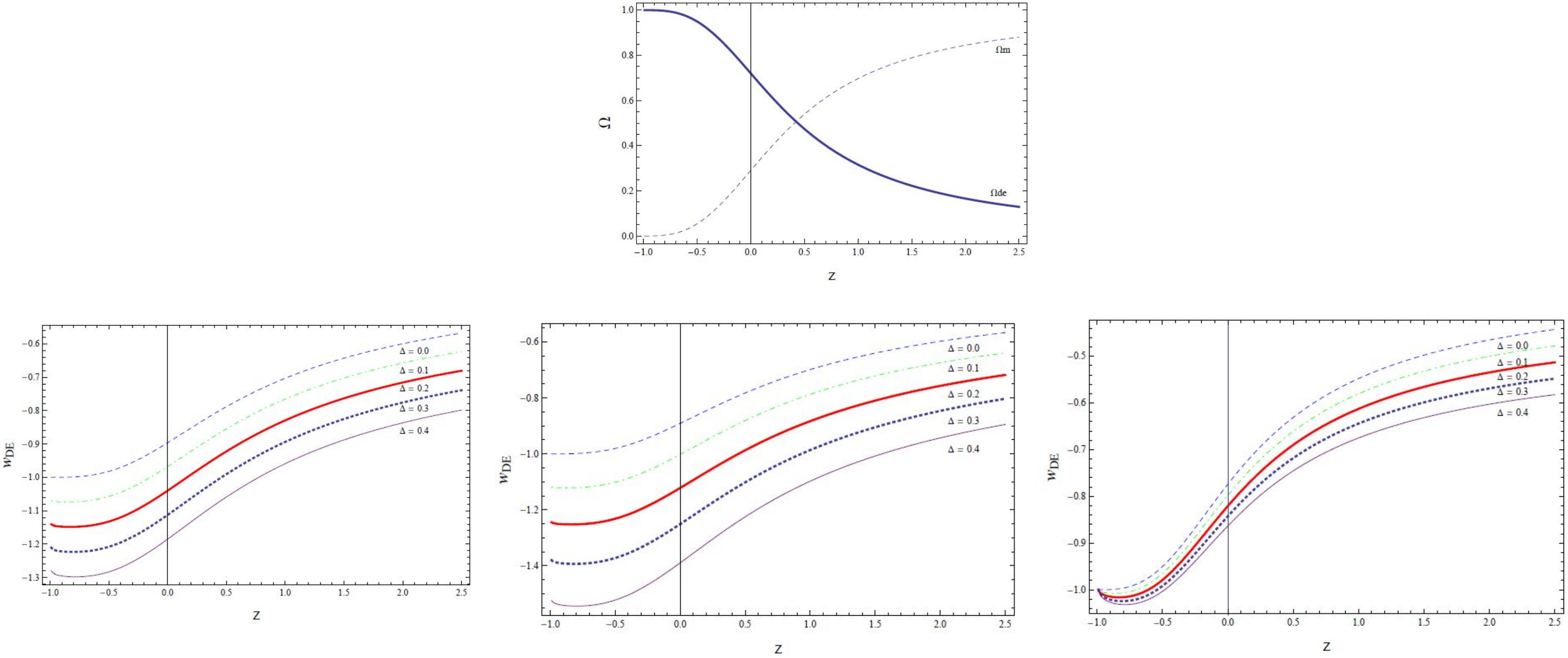
- Following the same method, *r(t)* can be determined for *k=-1* as

$$r(t) = \frac{1}{\sqrt{|k|}} \sinh y$$

- Expression for the dark-energy equation-of-state parameter

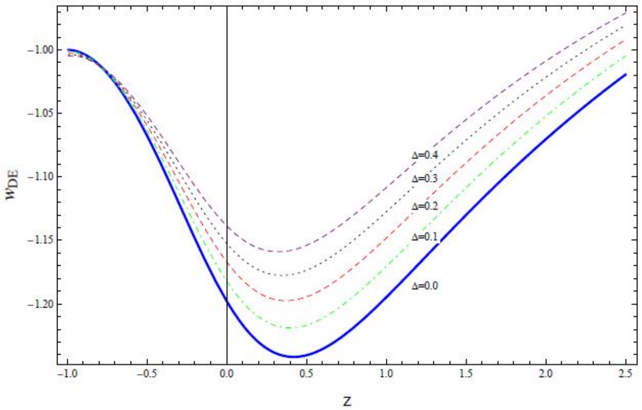
$$w_{DE} = -\left(\frac{1+\Delta}{3}\right) - \frac{Q}{3}(\Omega_{DE})^{\frac{1}{2-\Delta}} \cosh y \left(\frac{1-\Omega_{DE}}{1-\gamma e^{x}}\right)^{\frac{\Delta}{2(\Delta-2)}} e^{\frac{3\Delta x}{2(\Delta-2)}}$$

Cosmological Behavior

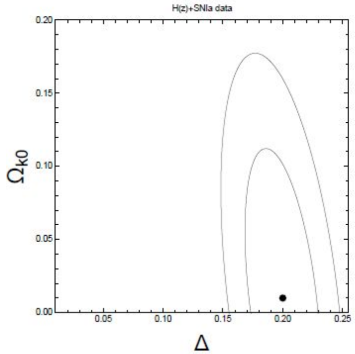


The evolution of the density parameters for matter and Barrow holographic dark energy in the case of a closed universe (k = +1). We have set Δ = 0.1 and C = 3 for the upper panel. The lower panel shows the evolution of the dark-energy equation-of-state parameter $w_{DE}(z)$ for various values Ω_{k0}=0.01, 0.001 and 0.1 respectively.

Cosmological Behavior



The evolution of the dark-energy equation-of-state parameter $w_{DE}(z)$ for various values Ω_{k0}=-0.01.



The 1 and 2 iso-likelihood contours for the Barrow holographic dark energy in non-flat universe, for the combined H(z)+SNIa analysis,

OBSERVATIONAL CONSTRAINTS

Dataset	k	Δ	Ω_{k0}	χ_{min}/dof
$H(z)$	Positive	0.19	0.032	8.42
	Negative	0.1	−0.001	9.24
SNIa	Positive	0.18	0.01	9.74
	Negative	0.21	−0.15	8.90
$H(z)$ + SNIa	Positive	0.2	0.01	9.14
	Negative	0.06	−0.09	16.23

For negative values of k the Barrow exponent is constrained to smaller values.

Concerning Ω_{k0}, although the best-fit values are small, comparatively larger values of Ω_{k0} are allowed.

Conclusions

- The scenario at hand can describe the evolution history of the universe, with the sequence of matter and dark energy epochs.

- We examined the effect of the Barrow exponent , as well as of the curvature density parameter at present, on the dark-energy equation-of-state parameter.

- For $\Delta = 0$ the dark-energy equation-of-state parameter lies completely in the quintessence regime.

- For $\Delta > 0.03$ the phantom-divide crossing has been realized in the past, namely Barrow holographic dark favors the phantom regime.

Conclusions

- In summary, the incorporation of slightly non-flat spatial geometry leads the universe into the phantom regime for significantly smaller values.

- In the case of negative curvature we found a reversed behavior, namely for increased Δ, we obtained algebraically higher w_{DE} values.

- For all cases the universe is currently in the phantom regime.

- Hence, comparing to the flat and closed universe, negative curvature favors the phantom regime more intensively.

Thank you