Non-Gaussian imprints on Cosmic Microwave Background

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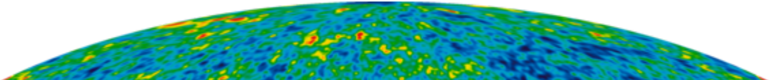
based on

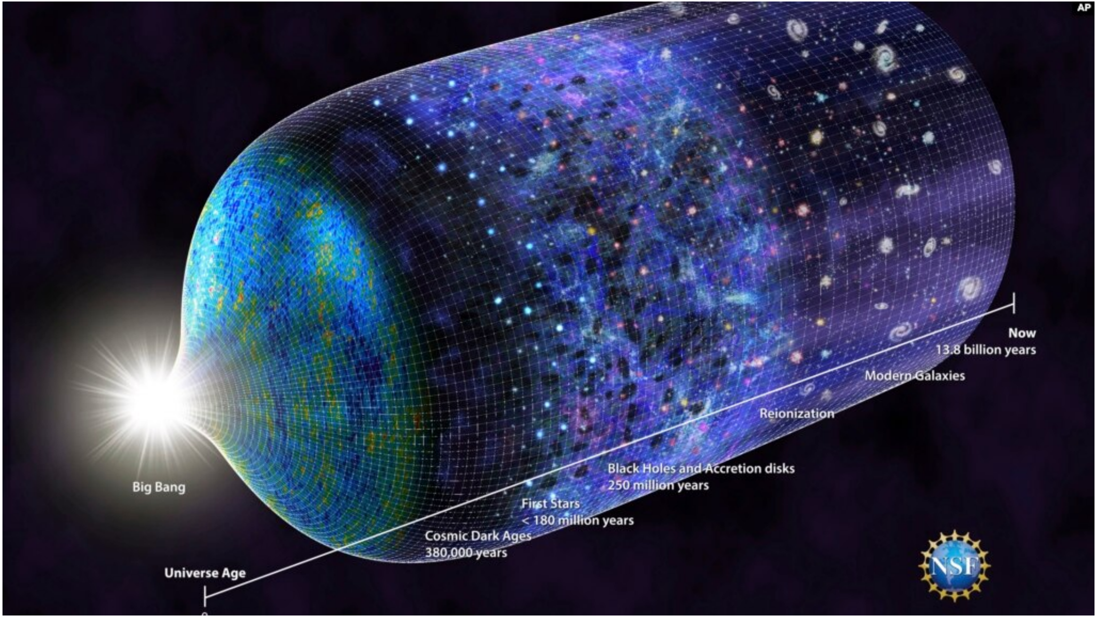
Indirect imprints of primordial non-Gaussianity on cosmic microwave background

Barnali Das and H. V. Ragavendra, arXiv:2304.05941 [astro-ph.CO]

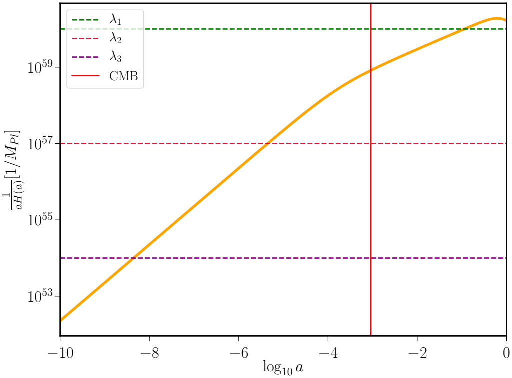


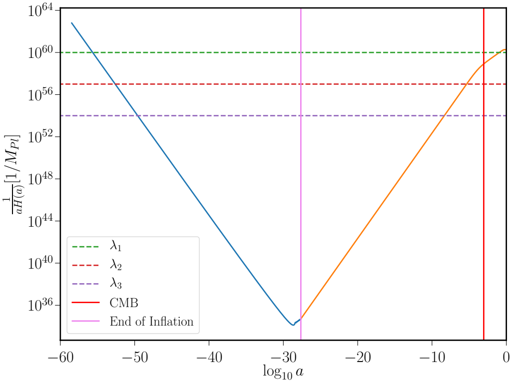
Overview

- Introduction
- Non-Gaussianity
  - Non-Gaussianity in CMB
  - Non-Gaussianity in single field inflationary models
- Methodology
- Results
  - Simple templates
  - Realistic models
- Conclusion and Outlook


AP
Now
13.8 billion years
Modern Galaxies
Reionization
Black Holes and Accretion disks
250 million years
Big Bang
First Stars
< 180 million years
Cosmic Dark Ages
380,000 years
Universe Age
NSF

Introduction





- Inflation is the exponentially expansion of the universe.
- Inflation solves horizon and flatness problem of the ΛCDM cosmology.
- It is driven by a scalar field, Inflaton (ϕ).

Perturbations

Perturbation in inflaton field:

$$\phi(t,\vec{x}) = \phi_0(t) + \delta\phi(t,\vec{x}) \tag{1}$$

Perturbation in metric:

$$\begin{aligned} ds^2 &= -(1+2\Phi)dt^2 + 2a(\partial_i B + S_i)dtdx^i \\ &\quad + a^2(t)\left[(1+2\Psi)\,\delta_{ij} + 2\partial_i\partial_j E + \left(\partial_i F_j + \partial_j F_i\right) + h_{ij}\right] dx^i dx^j \end{aligned} \tag{2}$$

where Φ, B, Ψ, E are scalar, S_i, F_i, F_j are vector and h_{ij} are tensor degrees of freedom.

The scalar perturbation

$$\mathcal{R} = \Psi + \frac{H}{\dot{\phi}_0}\delta\phi \tag{3}$$

where

$\delta\phi$: perturbation term from the inflaton field,

Ψ: perturbation term from metric,

$\dot{\phi}_0$: time derivative of inflaton field without perturbation,

H: Hubble parameter during inflation.

- Scalar perturbations follow the equation:

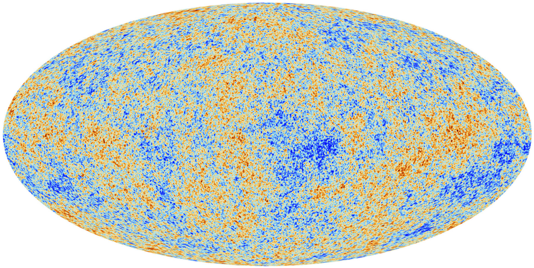
$$\mathcal{R}_{\boldsymbol{k}}'' + 2\frac{z'}{z}\mathcal{R}_{\boldsymbol{k}}' + k^2\mathcal{R}_{\boldsymbol{k}} = 0 \tag{4}$$

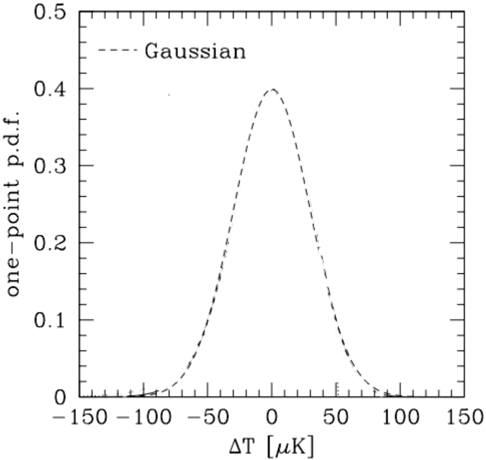
- Scalar power spectrum:

$$\mathcal{P}_{\mathrm{S}} = \frac{k^3}{2\pi^2}|\mathcal{R}_{\boldsymbol{k}}|^2\Big|_{\eta\to 0} \tag{5}$$

where $()' = \frac{d()}{d\eta}$ and η is the conformal time $\left(\eta = \int \frac{dt}{a(t)}\right)$.

Relating with observation





Mean = 0

Variance $\neq 0$

Skewness ≈ 0

[1] E. Komatsu, The Pursuit of Non-Gaussian Fluctuations in the Cosmic Microwave Background, PhD thesis, arXiv:astro-ph/0206039 (2002)

Variance $\propto C_\ell^{TT}$

$$C_\ell^{TT} = \frac{2}{\pi} \int k^2 dk \, \underbrace{\mathcal{P}_{\mathrm{S}}(k)}_{\text{inflation}} \, \underbrace{\Delta_{T\ell}(k)\Delta_{T\ell}(k)}_{\text{anisotropies}} \tag{6}$$

where the sub/superscript T is for temperature

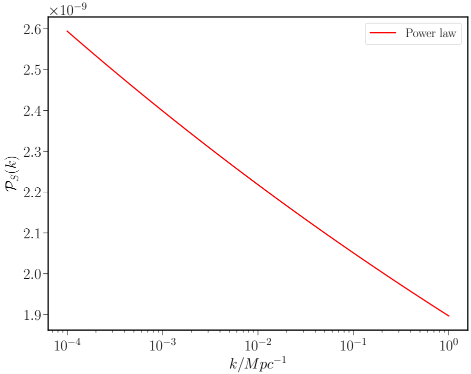
ℓ is degree of associated spherical harmonics

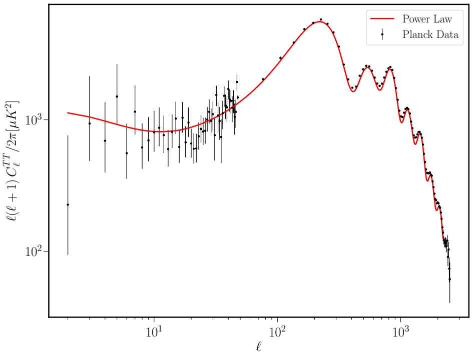
$\mathcal{P}_{\mathrm{S}}(k)$ is the scalar power spectrum

$\Delta_{T\ell}(k)$ is transfer function

CAMB

Input ($\mathcal{P}_{\mathrm{S}}(k)$) $\rightarrow$ Output (C_ℓ)



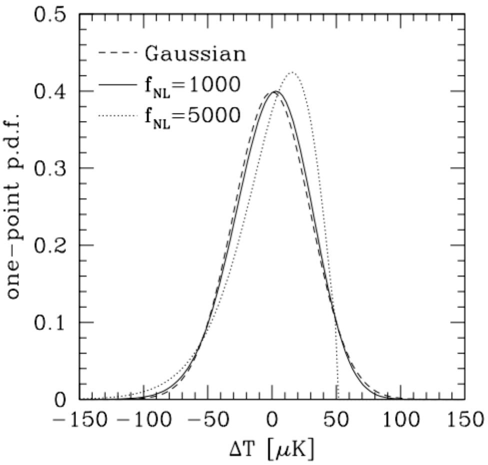


[1]A. Lewis, A. Challinor, CAMB: Code for Anisotropies in the Microwave Background, ascl (2011)

Non-Gaussianity in CMB

$$\mathcal{R}_{\rm NG} = \mathcal{R} - \frac{3}{5} f_{\rm NL} \mathcal{R}^2 \tag{7}$$

$f_{\rm NL}$ is measure of non-Gaussianity.



[1] E. Komatsu, The Pursuit of Non-Gaussian Fluctuations in the Cosmic Microwave Background, PhD thesis, arXiv:astro-ph/0206039 (2002)

Non-Gaussianity in single field inflationary models and our methodology

Action for Inflaton

$$S = S^{(0)}(\phi) + S^{(2)}(\mathcal{R}^2) + S^{(3)}(\mathcal{R}^3) + \dots \quad (8)$$

informs about	background dynamics during inflation	perturbations during inflation	non-Gaussianity of perturbations
		↓	↓
		Power spectrum $\mathcal{P}_{\mathrm{S}}(k)$	Bispectrum $\mathcal{B}(k_1, k_2, k_3)$

Scalar power spectrum and bispectrum

Scalar power spectrum:

$$\langle \mathcal{R}^{\mathrm{G}}_{\boldsymbol{k}_1} \mathcal{R}^{\mathrm{G}}_{\boldsymbol{k}_2} \rangle = \frac{2\pi^2}{k_1^3} \mathcal{P}_{\mathrm{S}}(k_1) \delta^{(3)}(\boldsymbol{k}_1 + \boldsymbol{k}_2) \,. \tag{9}$$

Scalar bispectrum:

$$\langle \mathcal{R}_{\boldsymbol{k}_1} \mathcal{R}_{\boldsymbol{k}_2} \mathcal{R}_{\boldsymbol{k}_3} \rangle = (2\pi)^3 \, \mathcal{B}(k_1, k_2, k_3) \delta^{(3)}(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3) \,. \tag{10}$$

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[1] J. Maldacena, Non-gaussian features of primordial fluctuations in single field inflationary models, JHEP (2003)

Correction to the power spectrum

Scalar perturbation with a Gaussian and non-Gaussian component:

$$\mathcal{R}_{\boldsymbol{k}}(\eta) \quad = \quad \mathcal{R}^{\mathrm{G}}_{\boldsymbol{k}}(\eta) - \frac{3}{5}\int \frac{\mathrm{d}^3\boldsymbol{k}_1}{(2\,\pi)^{3/2}}\mathcal{R}^{\mathrm{G}}_{\boldsymbol{k_1}}(\eta)\mathcal{R}^{\mathrm{G}}_{\boldsymbol{k}-\boldsymbol{k_1}}(\eta)\, f_{\mathrm{NL}}[\boldsymbol{k}, (\boldsymbol{k}_1 - \boldsymbol{k}), -\boldsymbol{k}_1] \tag{11}$$

Power spectrum with correction:

$$\mathcal{P}^{\mathrm{Total}}_{\mathrm{S}}(k) \quad = \quad \underbrace{\mathcal{P}^{\mathrm{G}}_{\mathrm{S}}(k)}_{\mathcal{P}_{\mathrm{S}}(k)} + \underbrace{\frac{9}{50\pi}k^3 \int \mathrm{d}^3\boldsymbol{k}_1\, \frac{\mathcal{P}^{\mathrm{G}}_{\mathrm{S}}(k_1)}{k_1^3}\frac{\mathcal{P}^{\mathrm{G}}_{\mathrm{S}}(|\boldsymbol{k} - \boldsymbol{k}_1|)}{|\boldsymbol{k} - \boldsymbol{k}_1|^3}\, f^2_{\mathrm{NL}}[k, |\boldsymbol{k}_1 - \boldsymbol{k}|, k_1]}_{\mathcal{P}_{\mathrm{C}}(k)} \tag{12}$$

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[1] H. V. Ragavendra, Accounting for scalar non-Gaussianity in secondary gravitational waves, PRD (2022)

[2] I. Agullo, D. Kranas, V. Sreenath, Anomalies in the Cosmic Microwave Background and their Non-Gaussian Origin in Loop Quantum Cosmology, FrASS (2021)

[3] F. Schmidt, M. Kamionkowski, Halo clustering with nonlocal non-Gaussianity, PRD (2010)

Definition of f_{NL}

Non-Gaussianity parameter f_{NL}:

$$f_{\mathrm{NL}}(k_1,k_2,k_3) = -\frac{10\sqrt{2\pi}}{3}\frac{(k_1k_2k_3)^3\mathcal{B}(k_1,k_2,k_3)}{\left[k_1^3\mathcal{P}_{\mathrm{S}}(k_2)\mathcal{P}_{\mathrm{S}}(k_3)+k_2^3\mathcal{P}_{\mathrm{S}}(k_1)\mathcal{P}_{\mathrm{S}}(k_3)+k_3^3\mathcal{P}_{\mathrm{S}}(k_1)\mathcal{P}_{\mathrm{S}}(k_2)\right]}. \quad (13)$$

Given $\mathcal{P}_{\mathrm{S}}(k)$ and $\mathcal{B}(k_1,k_2,k_3)$, for a model, we know $f_{\mathrm{NL}}(k_1,k_2,k_3)$ and then $\mathcal{P}_{\mathrm{C}}(k)$.

Divergence in $\mathcal{P}_{\rm C}(k)$

Let $x = k_1/k$ and $y = u/k$, then

$$\mathcal{P}_{\rm C}(k) \quad = \quad \frac{9}{25}\int_0^\infty {\rm d}x \int_{|1-x|}^{|1+x|} {\rm d}y \frac{\mathcal{P}_{\rm S}(kx)}{x^2}\,\frac{\mathcal{P}_{\rm S}(ky)}{y^2}\, f_{\rm NL}^2[k, kx, ky]\,. \tag{14}$$

Problem:

Divergence at $(x, y) = (1, 0)$ and $(x, y) = (0, 1)$

Solution:

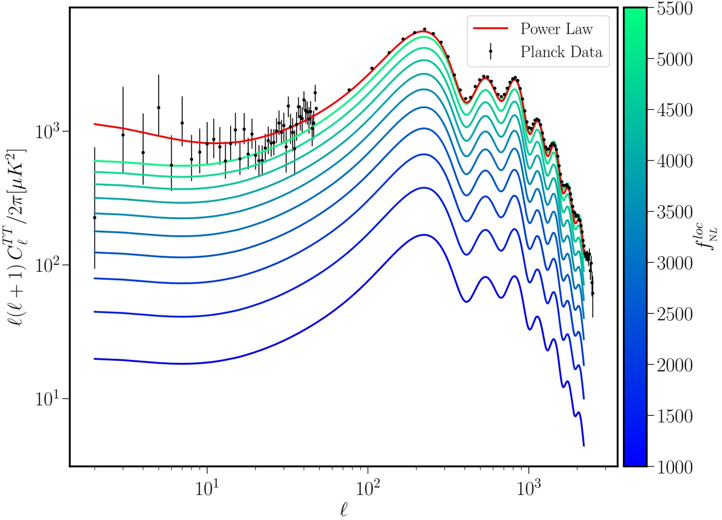
1. Form of $\mathcal{P}_{\rm S}$ or $f_{\rm NL}$ which cancels the denominator,
2. Introduction of k_{min}.

Results

Local type

$$
\begin{aligned}
\mathcal{P}_{\mathrm{S}}^{\mathrm{loc}}(k) &= A_s(k)\left(\frac{k}{k_*}\right)^{n_{\mathrm{S}}-1} \text{.(Power Law)} \\
\mathcal{B}^{\mathrm{loc}}(k_1,k_2,k_3) &= -\frac{3}{10\sqrt{2\pi}} A_S{}^2 f_{\mathrm{NL}}^{loc}\left[\frac{1}{k_1^3 k_2^3} + 2\ \text{perm}\right] \\
\mathcal{P}_{\mathrm{C}}^{\mathrm{loc}}(k) &= -\frac{18}{25}\mathcal{P}_{\mathrm{S}}^2(k)\,(f_{\mathrm{NL}}^{\mathrm{loc}})^2\left(1 + 2\lim_{k_{\mathrm{min}}\to 0}\ln\frac{k_{\mathrm{min}}}{k}\right).
\end{aligned}
$$

Free parameter: $f_{\mathrm{NL}}^{\mathrm{loc}}$



$f_{\rm NL}^{loc} = -0.9 \pm 5.1$

[1] Planck Collaboration, Planck 2018 results. IX. Constraints on primordial non-Gaussianity, A&A (2020)

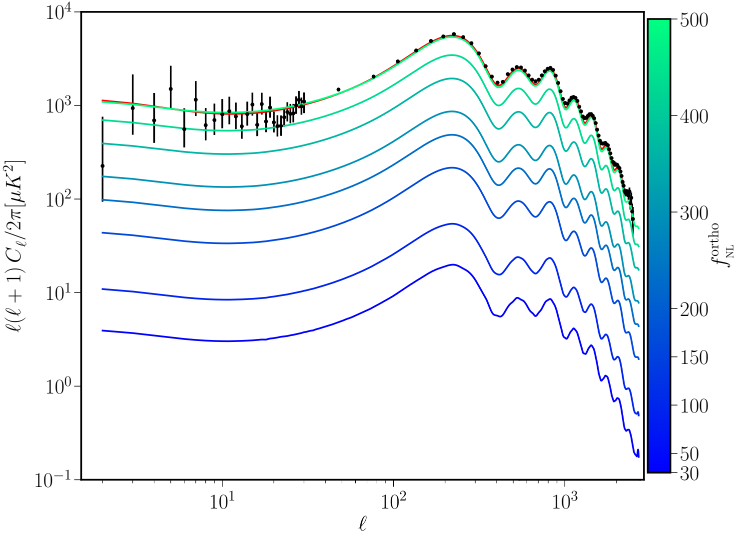
Orthogonal type

$$\mathcal{P}_{\mathrm{S}}^{\mathrm{eq}}(k) = A_s(k)\left(\frac{k}{k_*}\right)^{n_{\mathrm{S}}-1}.$$

$$\mathcal{B}^{\mathrm{ortho}}(k_1,k_2,k_3) = \frac{6A_s^2 f_{\mathrm{NL}}^{\mathrm{ortho}}}{k_*^{2(n_{\mathrm{S}}-1)}}\left\{-\frac{8}{(k_1k_2k_3)^{\frac{2}{3}(4-n_{\mathrm{S}})}} - \frac{3}{(k_1k_2)^{4-n_{\mathrm{S}}}} - \frac{3}{(k_2k_3)^{4-n_{\mathrm{S}}}} - \frac{3}{(k_3k_1)^{4-n_{\mathrm{S}}}} + \left[\frac{3}{k_1^{(4-n_{\mathrm{S}})/3}k_2^{2(4-n_{\mathrm{S}})/3}k_3^{4-n_{\mathrm{S}}}} + 5\text{ perms}\right]\right\}.$$

$$\mathcal{P}_{\mathrm{C}}^{\mathrm{ortho}}(k) \simeq 1.71\times 10^3\,\mathcal{P}_{\mathrm{S}}^2(k)\,(f_{\mathrm{NL}}^{\mathrm{ortho}})^2.$$

Free parameter: $f_{\mathrm{NL}}^{\mathrm{ortho}}$



$f_{\rm NL}^{ortho} = -38 \pm 24$

[1] Planck Collaboration, Planck 2018 results. IX. Constraints on primordial non-Gaussianity, A&A (2020)

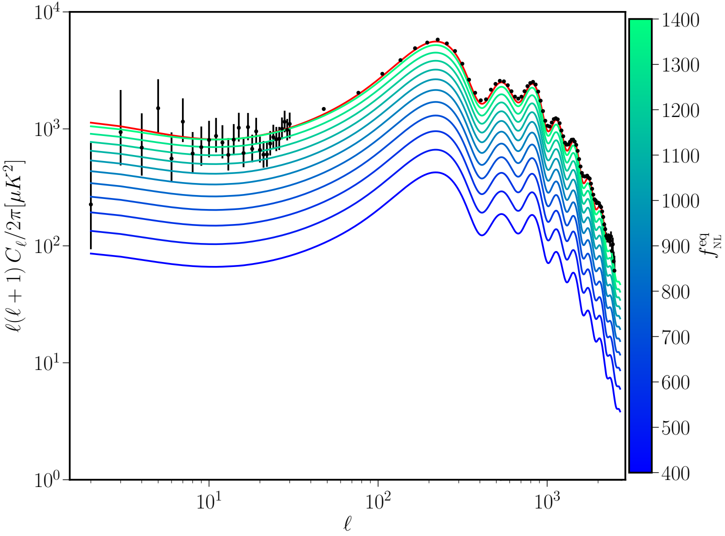
Equilateral type

$$\mathcal{P}_{\rm S}^{\rm eq}(k) = A_s(k)\left(\frac{k}{k_*}\right)^{n_{\rm S}-1}.$$

$$\mathcal{B}^{\rm eq}(k_1,k_2,k_3) = \frac{6A_s^2 f_{\rm NL}^{\rm eq}}{k_*^{2(n_{\rm S}-1)}}\left\{-\frac{2}{(k_1k_2k_3)^{\frac{2}{3}(4-n_{\rm S})}} - \frac{1}{(k_1k_2)^{4-n_{\rm S}}} - \frac{1}{(k_2k_3)^{4-n_{\rm S}}} - \frac{1}{(k_3k_1)^{4-n_{\rm S}}} + \left[\frac{1}{k_1^{(4-n_{\rm S})/3}k_2^{2(4-n_{\rm S})/3}k_3^{4-n_{\rm S}}} + 5\text{ perms}\right]\right\}.$$

$$\mathcal{P}_{\rm C}^{\rm eq}(k) \simeq 2.135\times 10^2\,\mathcal{P}_{\rm S}^2(k)\,(f_{\rm NL}^{\rm eq})^2.$$

Free parameter: $f_{\rm NL}^{\rm eq}$



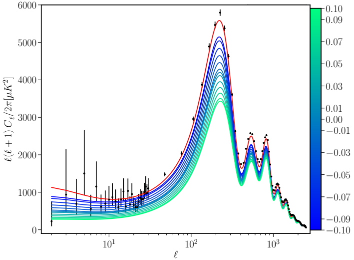
$f^{eq}_{\rm NL} = -26 \pm 47$

[1] Planck Collaboration, Planck 2018 results. IX. Constraints on primordial non-Gaussianity, A&A (2020)

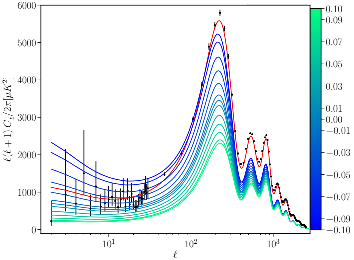
Running in $f_{\rm NL}$

$$f_{\rm NL}^{\rm type-run}(k_1,k_2,k_3) = f_{\rm NL}^{\rm type}(k_1,k_2,k_3)\left(\frac{k_1+k_2+k_3}{3\,k_*}\right)^{n_{\rm NG}},$$

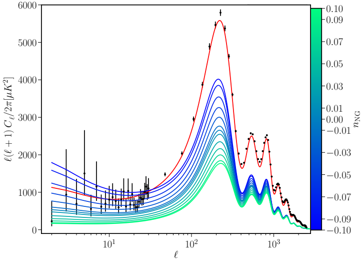
$$\mathcal{P}_{\rm C}^{\rm type-run}(k) = \mathcal{P}_{\rm C}^{\rm type}(k)\left(\frac{k}{3\,k_*}\right)^{2n_{\rm NG}}.$$



local



orthogonal



equilateral

Oscillatory type

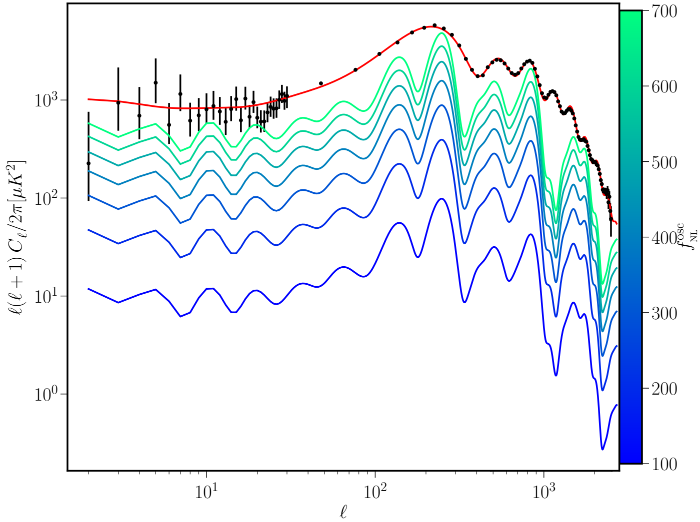
$$\mathcal{P}_{\rm S}^{\rm osc}(k) = A_{\rm S}\left(\frac{k}{k_*}\right)^{n_{\rm S}-1}\left\{1 + b\,\sin\left[\omega\ln\left(\frac{k}{k_o}\right)\right]\right\},$$

$$\mathcal{B}^{\rm osc}(k_1,k_2,k_3) = \frac{6A_{\rm S}^2 f_{\rm NL}^{\rm osc}}{(k_1k_2k_3)^2}\,\sin\left[\omega\ln\left(\frac{k_1+k_2+k_3}{k_o}\right)\right].$$

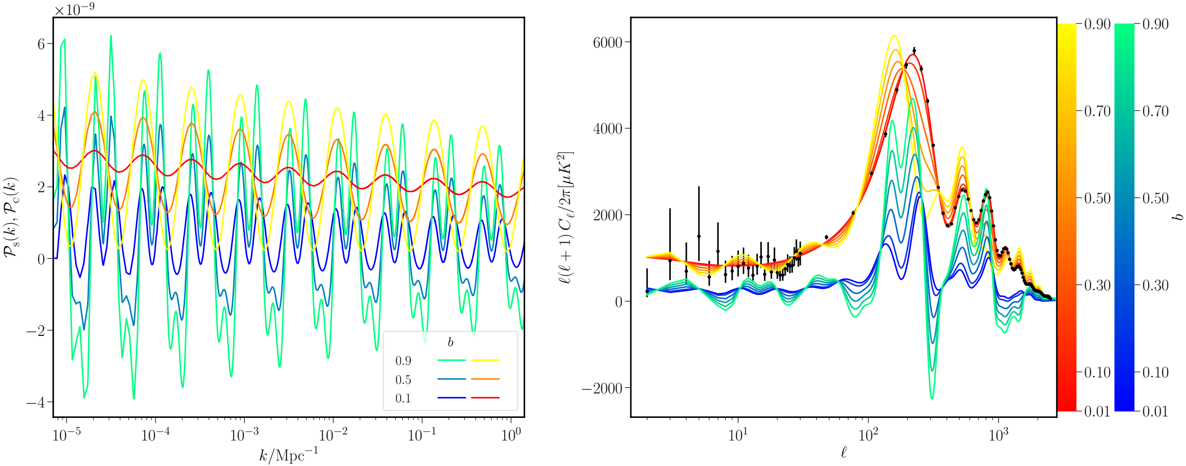
$$\mathcal{P}_{\rm C}^{\rm osc}(k) = \frac{288\pi\left[A_{\rm S}\left(\frac{k}{k_*}\right)^{n_{\rm S}-1} f_{\rm NL}^{\rm osc}\right]^2 \int_0^\infty {\rm d}x \int_{|1-x|}^{1+x} {\rm d}y\, x\, y\, F(x)\, F(y)\, \sin^2\left[\omega\ln\left(\frac{k}{k_o}(1+x+y)\right)\right]}{F(x)\,F(y) + x^3\,F(1)\,F(y) + y^3\,F(1)\,F(x)}.$$

where $F(z) = \left[1 + b\sin\left(\omega\ln\frac{kz}{k_o}\right)\right], z = 1, x, y$

Free parameter: $f_{\rm NL}^{\rm osc}$, b & ω



[1] B. Das, H. V. Ragavendra, Indirect imprints of primordial non-Gaussianity on cosmic microwave background, arXiv:2304.05941 (2023)



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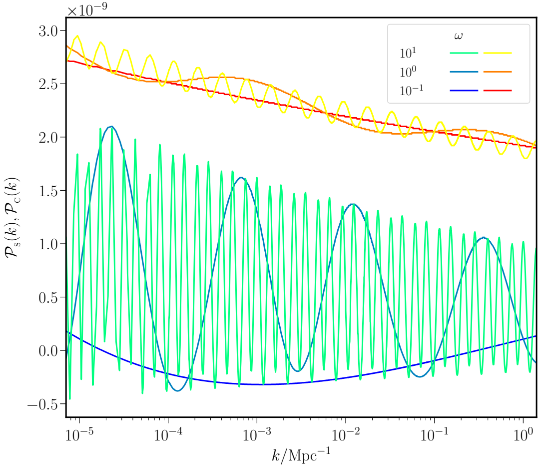

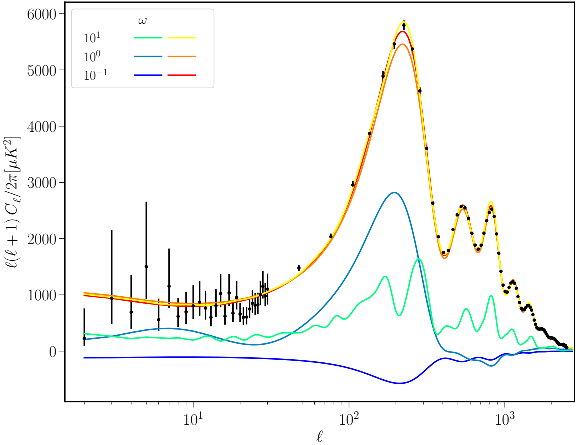

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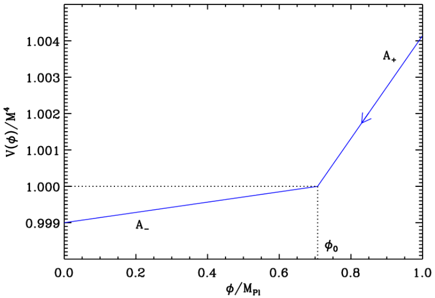
Starobinsky model

$$V(\phi) = \begin{cases} V_0 + A_+(\phi - \phi_0), & \text{for } \phi > \phi_0\,, \\ V_0 + A_-(\phi - \phi_0), & \text{for } \phi < \phi_0, \end{cases}$$



$$\mathcal{P}_{\rm S}(k) = \mathcal{P}_{\rm S}^0|\alpha_k - \beta_k|^2 = \mathcal{P}_{\rm S}^0\left\{1 - \frac{3\Delta A}{A_+}\frac{k_0}{k}\left[\left(1 - \frac{k_0^2}{k^2}\right)\sin\left(\frac{2k}{k_0}\right) + \frac{2k_0}{k}\cos\left(\frac{2k}{k_0}\right)\right]\right.$$
$$\left. + \frac{9\Delta A^2}{2A_+^2}\frac{k_0^2}{k^2}\left(1 + \frac{k_0^2}{k^2}\right)\left[1 + \frac{k_0^2}{k^2} - \frac{2k_0}{k}\sin\left(\frac{2k}{k_0}\right) + \left(1 - \frac{k_0^2}{k^2}\right)\cos\left(\frac{2k}{k_0}\right)\right]\right\}.$$

$$\mathcal{P}_{\mathrm{C}}(k) \simeq \frac{9}{16}\frac{k_0^2}{k^2}\left(\frac{A_-}{A_+}\right)^2\left(1-\frac{A_-}{A_+}\right)^2\left(\mathcal{P}_{\mathrm{S}}^0\right)^2\int_0^\infty \mathrm{d}x\int_{|1-x|}^{1+x}\mathrm{d}y\,\frac{|\alpha_{kx}-\beta_{kx}|^2}{x^2}\,\frac{|\alpha_{ky}-\beta_{ky}|^2}{y^2}$$

$$\times\left(\frac{Z(k,x,y)}{|\alpha_{kx}-\beta_{kx}|^2|\alpha_{ky}-\beta_{ky}|^2+y^3|\alpha_k-\beta_k|^2|\alpha_{kx}-\beta_{kx}|^2+x^3|\alpha_k-\beta_k|^2|\alpha_{ky}-\beta_{ky}|^2}\right)^2.$$

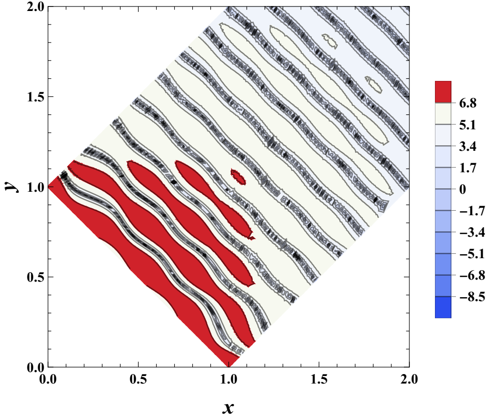
where $\mathcal{P}_{\mathrm{S}}^0 \simeq \frac{1}{12\pi^2}\frac{V_0}{M_{\mathrm{Pl}}^4}\left(\frac{V_0}{A_- M_{\mathrm{Pl}}}\right)^2$,

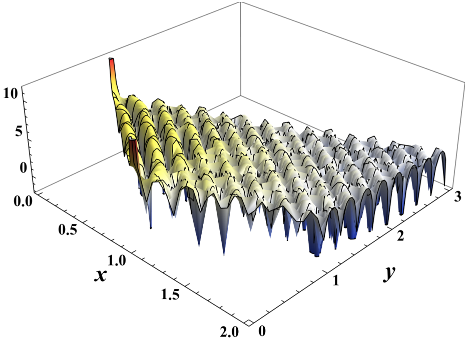
$\alpha_k = 1+\frac{3i\Delta A}{2A_+}\frac{k_0}{k}\left(1+\frac{k_0^2}{k^2}\right)$ and $\beta_k = -\frac{3i\Delta A}{2A_+}\frac{k_0}{k}\left(1+\frac{ik_0}{k}\right)^2 e^{2ik/k_0}$

$Z(k,x,y)$ function captures shape of the bispectrum.

Free parameter: V_0, A_+, A_- & k_0

Integrand of $\mathcal{P}_{\rm C}$ for Starobinsky





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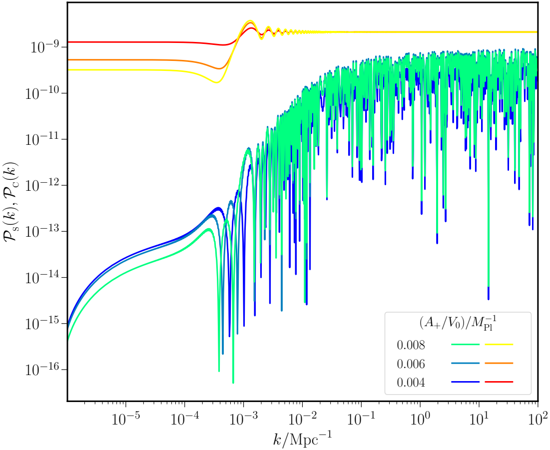

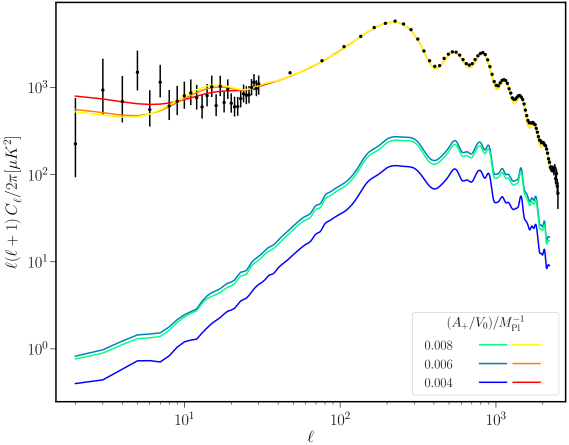

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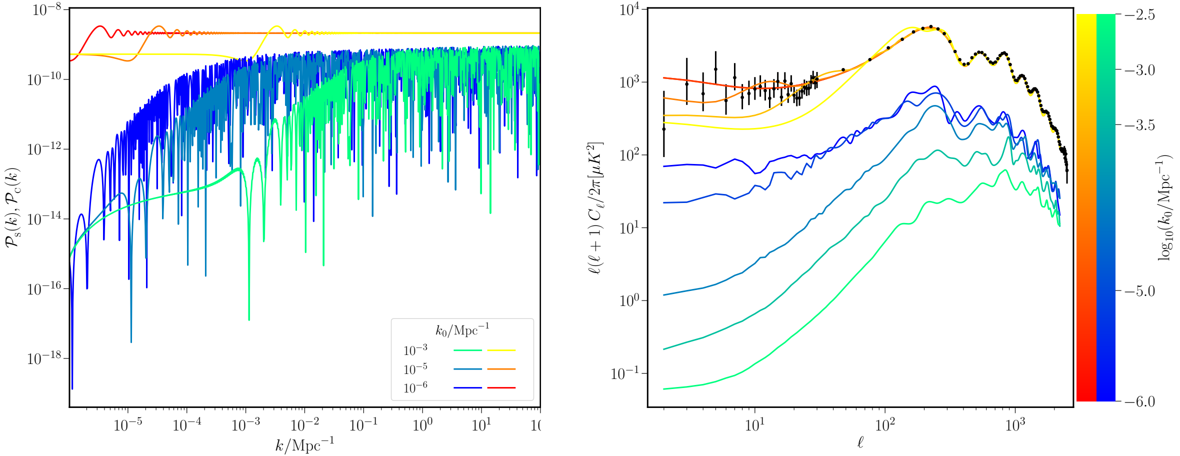

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Conclusion and Outlook

Conclusion

- There has been ways to constrain f_{NL} directly. But, it involves computing three point function using CMB maps which are numerically time taking for non-trivial models.
- In our paper, we present a new method of constraining f_{NL} indirectly. It introduces a correction term, $\mathcal{P}_{\mathrm{C}}(k)$, to the Gaussian power spectrum $\mathcal{P}_{\mathrm{S}}(k)$.
- We computed the corrected power spectrum for popular templates as well as a few realistic models.
- We found significant non-Gaussian contribution for oscillatory template and Starobinsky model.

Outlook

- Researchers have tried to compute non-Gausssian corrections using loop-diagrams, especially over small scales. Our work uses a method equivalent to it but it is applied over large scales.
- We are performing Monte-Carlo analysis of these models against Planck 2018 dataset to constrain f_{NL} and related parameters.

References

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Thank you!

The talk was based on:

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Barnali Das and H. V. Ragavendra, arXiv:2304.05941 [astro-ph.CO]