E and B modes of the CMB y-type distortions: Polarised kinetic Sunyaev-Zeldovich effect

Aritra Kumar Gon
Rishi Khatri

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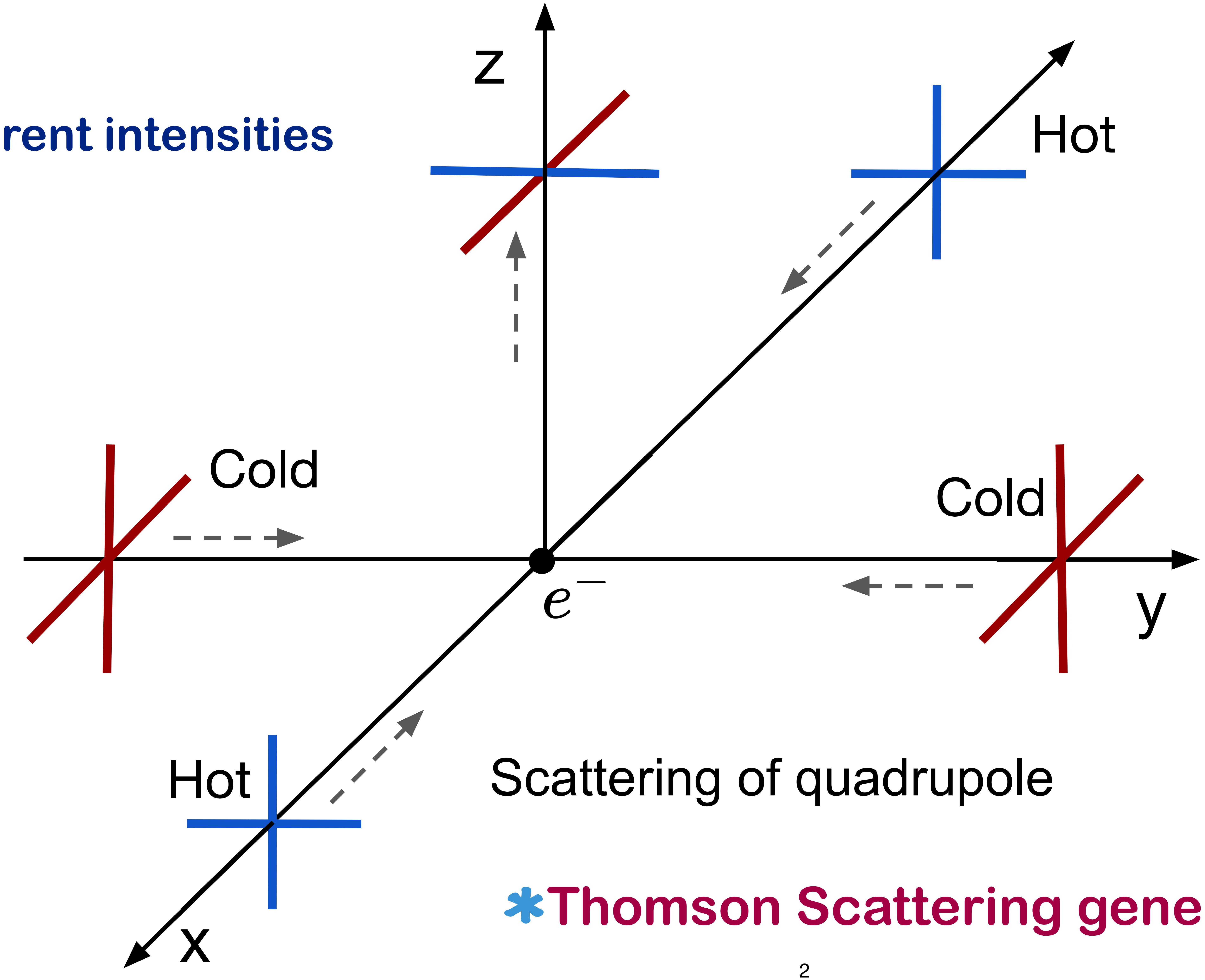
Department of Physics
IIT Madras

Tata Institute of Fundamental Research

MAX-PLANCK-GESELLSCHAFT

Electron peculiar velocities at second order generate E and B mode polarisation: The pkSZ effect

Different intensities

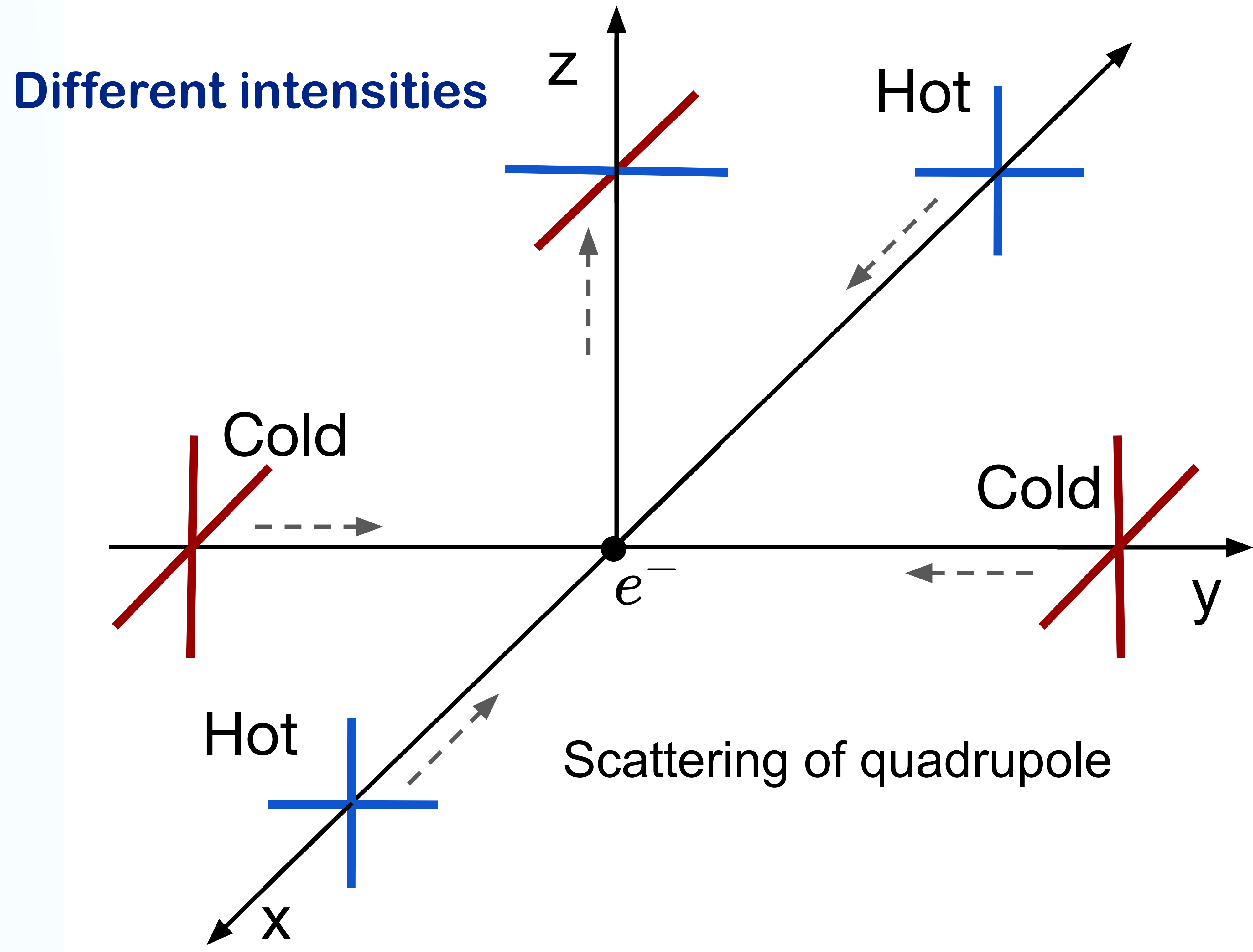


Scattering of quadrupole

✱**Thomson Scattering generates linear polarisation**.

Electron peculiar velocities at second order generate E and B mode polarisation: The pkSZ effect

- Free electrons produced during reionisation, have peculiar velocities $(\vec{\mathbf{v}})$
- In the electron rest frame, the CMB is not isotropic. Has a quadrupolar anisotropy.



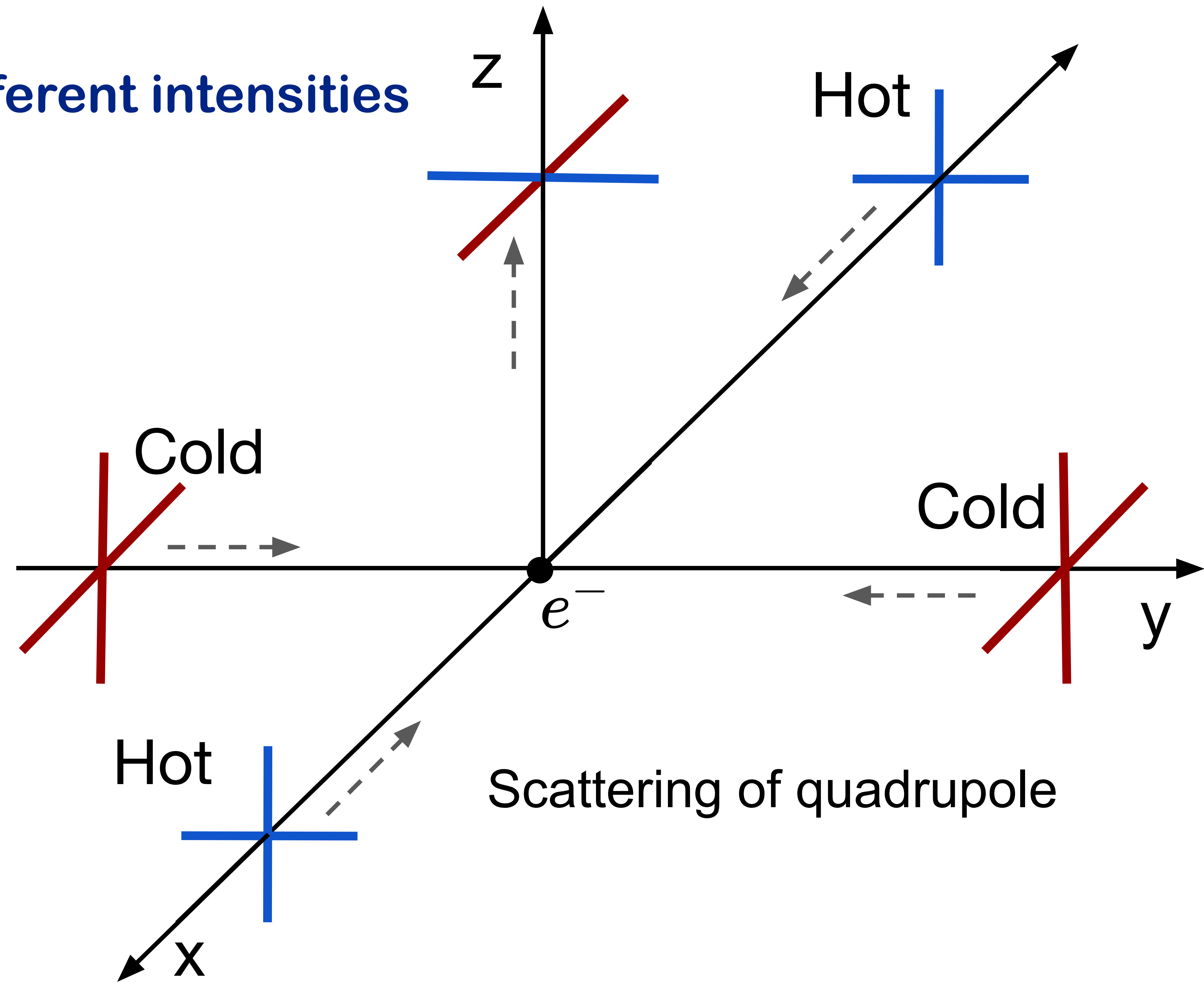
- Predicted by Sunyaev and Zeldovich in 1980. (*MNRAS,190:413-420*)

Electron peculiar velocities at second order generate E and B mode polarisation: The pkSZ effect

- Free electrons produced during reionisation, have peculiar velocities $\left(\vec{\mathbf{v}}\right)$

- In the electron rest frame, the CMB is not isotropic. Has a quadrupolar anisotropy.

  - Non-linear nature of Relativistic Doppler shift.
  - A non-linear relation between temperature and intensity in the Planck spectrum



$$T\left(\mathbf{r}, \hat{\mathbf{n}}', \eta\right) = \frac{T_0(\eta)}{\gamma\left(1 + \mathbf{v}(\mathbf{r}, \eta)\cdot \hat{\mathbf{n}}'\right)} = T_0(\eta)\left[1 + \underbrace{\frac{1}{2}v^2 - \mathbf{v}\cdot\hat{\mathbf{n}}' + \left(\mathbf{v}\cdot\hat{\mathbf{n}}'\right)^2 + \mathcal{O}\left(\left(\mathbf{v}\cdot\hat{\mathbf{n}}'\right)^3\right) + \cdots}_{\theta(\mathbf{r},\hat{\mathbf{n}}',\eta)}\right]$$

Planck Spectrum: $$n_{pl}(x) = \frac{1}{(e^x - 1)} \qquad x = \frac{h\nu}{k_B T_0}$$

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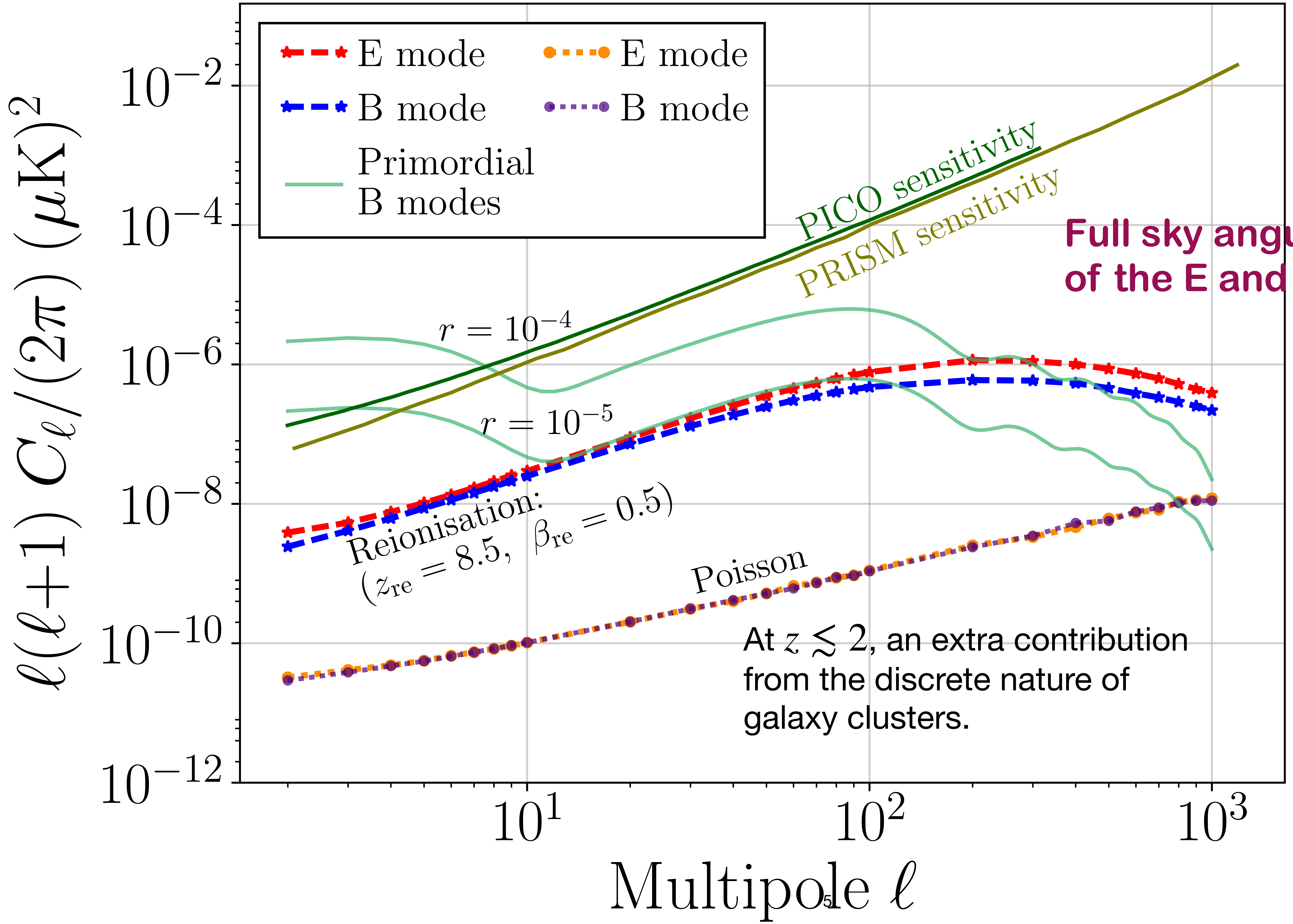
Beating the cosmic variance with pkSZ effect



Full sky angular power spectra of the E and B modes

Beating the cosmic variance with pkSZ effect

- The spectrum consists of the y-type distortions part and a blackbody part.
- Differentiates it from primary CMB signals with blackbody spectrum and other SZ-type signals, which are unpolarised.
- Free from the cosmic variance of the primary CMB polarisation signal and lensing B modes.
- Sensitive to reionisation central redshift, width, and the matter velocity power spectrum.

The scattered spectrum has a y-type distortion

- Photons from different blackbody spectra with different temperatures mix.

- Scattered spectrum not only has a differential blackbody but also a y-type distortion also.

$$\left(\frac{\delta I}{I}\right)\Bigg|_{\text{(quadrupolar)}} = 2\left(\mathbf{v}\cdot\hat{\mathbf{n}}'\right)^2 g(x) + \frac{1}{2} y(x)\left(\mathbf{v}\cdot\hat{\mathbf{n}}'\right)^2$$

$$\delta n_\nu = \frac{1}{2h\nu^3}\delta I_\nu = \left(\theta + \theta^2\right)\left(T\frac{\partial n_{pl}}{\partial T}\right)\Bigg|_{T_0} + \frac{\theta^2}{2}\left(T^4\frac{\partial}{\partial T}\left(\frac{1}{T^2}\frac{\partial n_{pl}}{\partial T}\right)\right)\Bigg|_{T_0} + \mathcal{O}(\theta^3)\ \cdots$$

$$n_{pl}(x) = \frac{1}{(e^x - 1)} \qquad x = \frac{h\nu}{k_B T_0}$$

$$g(x) = \frac{xe^x}{(e^x - 1)} \qquad y(x) = \frac{xe^x}{(e^x - 1)}\left(x\frac{e^x + 1}{e^x - 1} - 4\right)$$

The scattered spectrum has a y-type distortion

- Distinguishable from the primary polarisation signals which only have a blackbody spectrum.
- Differentiable from other y-type signals, such as the thermal SZ effect which are unpolarised.
- The blackbody part will act as a foreground for primordial B modes for $r \lesssim 3 \times 10^{-5}$ for $\ell \gtrsim 100.$

Polarisation depends on square of transverse velocity

* The polarisation field is a spin-2 field.

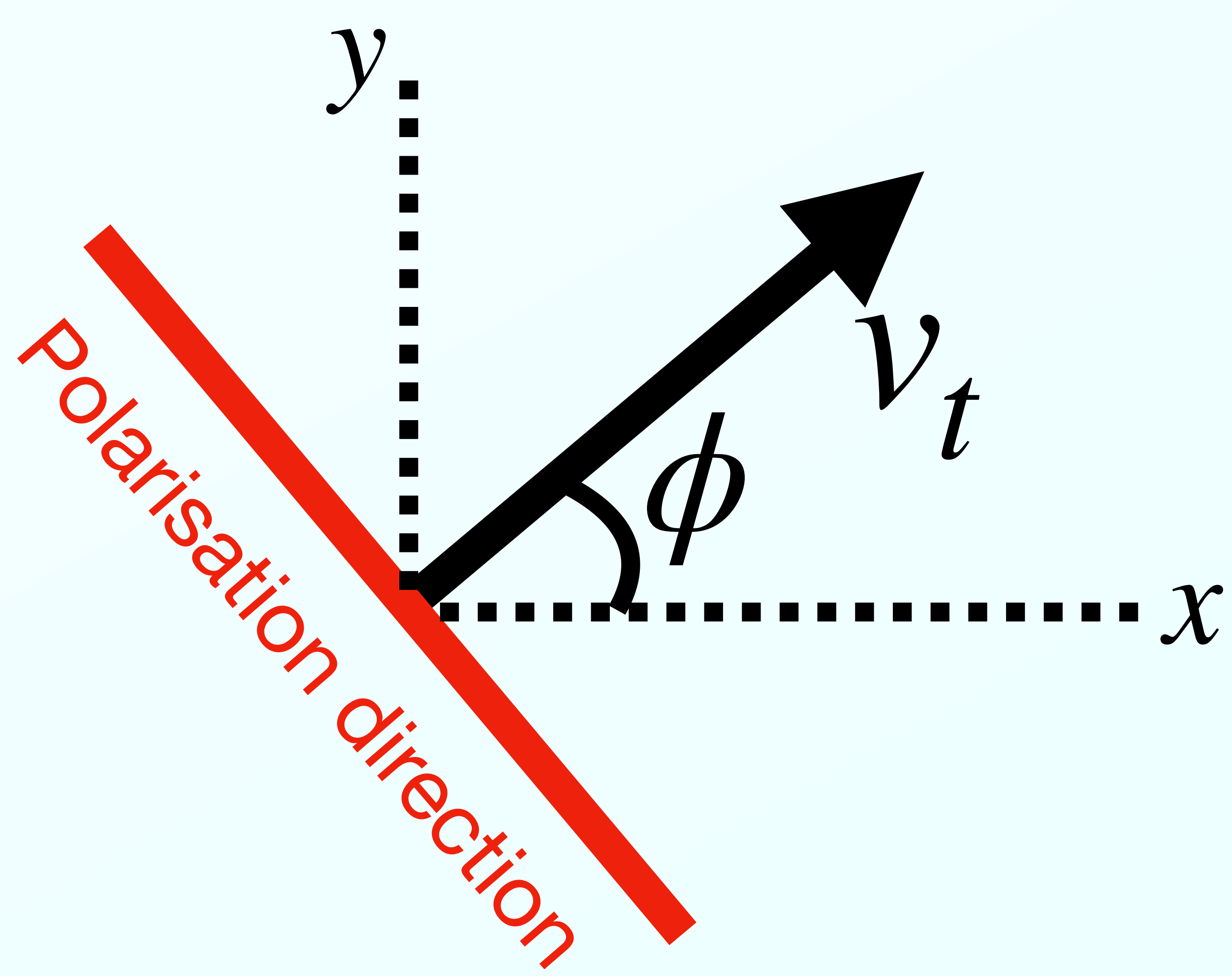
$$\left(\mathcal{Q} \pm i\mathcal{U}\right)(\hat{\mathbf{n}}) \equiv P_{\pm}(\hat{\mathbf{n}})$$

1. Electron number density - only a function of time.
2. Square of transverse velocity.

Quadrupole

$$P_{\pm}(\hat{\mathbf{n}}) = -\frac{\sqrt{6}\sigma_{\mathrm{T}}}{10}\int_0^{\chi} d\chi\, a(\chi)\, e^{-\tau(\chi)}\, \mathrm{n_e}(\chi) \sum_{\lambda=-2}^{2} {}_{\pm 2}Y_{2\lambda}(\hat{\mathbf{n}}) \int d^2\hat{\mathbf{n}}'\, Y^{*}_{2\lambda}(\hat{\mathbf{n}}') \left(\mathbf{v}(\mathbf{r},\chi)\cdot\hat{\mathbf{n}}'\right)^2.$$

$$P_{+}(\hat{\mathbf{z}}) = -\frac{\sigma_{\mathrm{T}}}{10}\int_0^{\chi_i} d\chi\, e^{-\tau(\chi)}\, n_{\mathrm{e}}(\chi)\, a(\chi)\, v_t^2\, e^{-2i\phi}$$



Polarisation field and angular power spectra

* The polarisation field is a spin-2 field.

$$\left(\mathcal{Q} \pm i\mathcal{U}\right)\left(\hat{\mathbf{n}}\right) \equiv P_{\pm}\left(\hat{\mathbf{n}}\right) \qquad a_{\ell m} = \int P_{+}\left(\hat{\mathbf{n}}\right) \, {}_{2}Y^{*}_{\ell m}\left(\hat{\mathbf{n}}\right) d^{2}\hat{\mathbf{n}}$$

$$P_{\pm}\left(\hat{\mathbf{n}}\right) = \sum_{\ell,m} \left(e_{\ell,m} + i b_{\ell,m}\right) \, {}_{\pm 2}Y_{\ell,m}$$

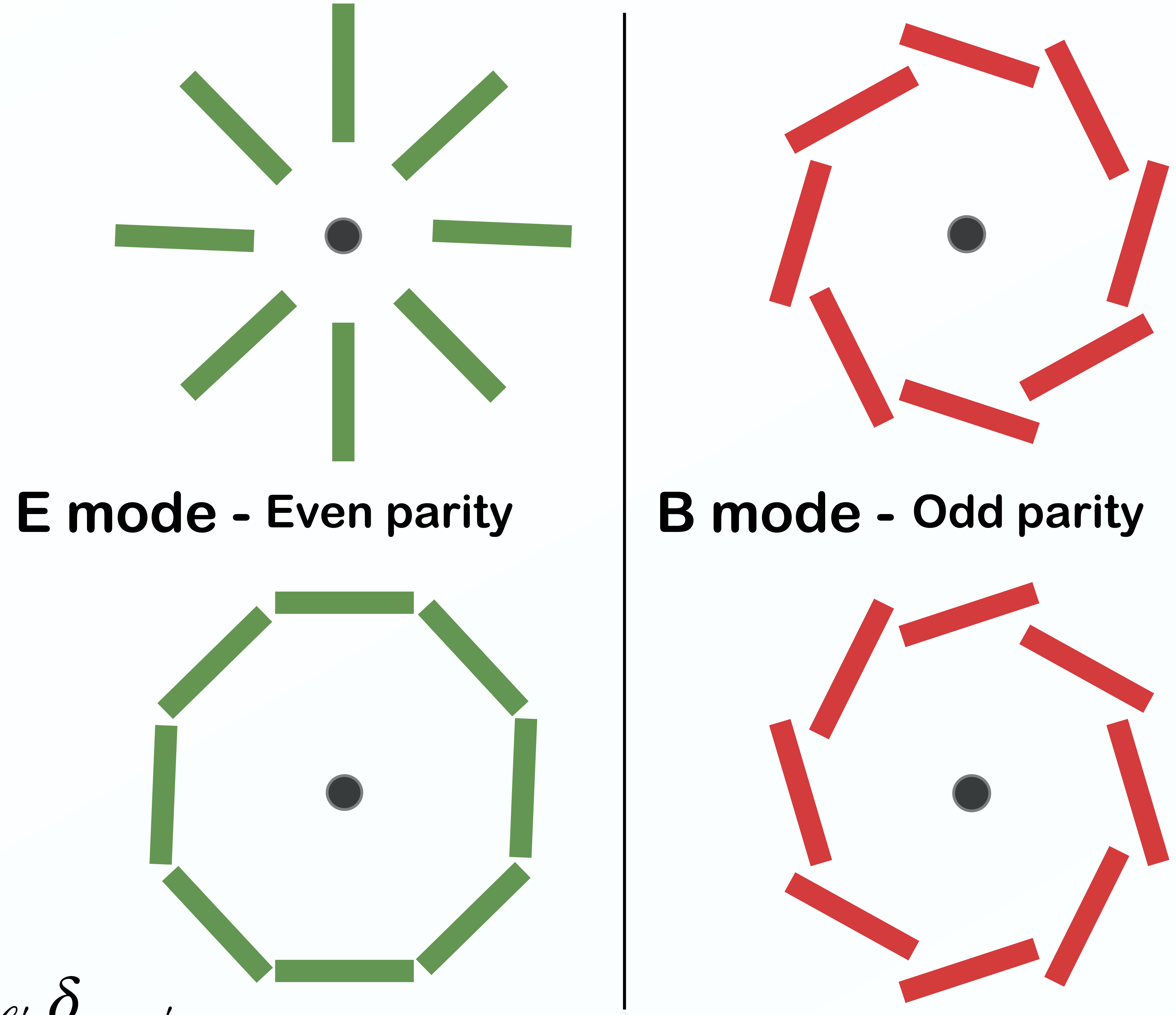
E mode   B mode

* Construct spin-0 fields related to polarisation field .

$$e_{\ell m} = \frac{1}{2}\left(a_{\ell m} + (-1)^{m} a^{*}_{\ell -m}\right) \qquad b_{\ell m} = \frac{-i}{2}\left(a_{\ell m} - (-1)^{m} a^{*}_{\ell -m}\right)$$

* The E and B mode power spectra :

$$\langle e_{\ell m} e^{*}_{\ell' m'} \rangle = C^{EE}_{\ell} \, \delta_{\ell,\ell'} \, \delta_{m,m'} \qquad \langle b_{\ell m} b^{*}_{\ell' m'} \rangle = C^{BB}_{\ell} \, \delta_{\ell,\ell'} \, \delta_{m,m'}$$



The power spectra at second order is a high dimensional integral.

$$C_\ell^{BB} = \frac{T_{CMB}^2}{2}\left[(4\pi)\left(\frac{4\pi}{3}\right)^2\sqrt{\frac{3}{2\pi}}\frac{\sqrt{6}\sigma_T}{10}\right]^2 \sum_{\lambda,\lambda'=-2}^{2}(-1)^{(\lambda+\lambda')}\int_0^{\chi} d\chi\, e^{-\tau(\chi)}\, a(\chi)\int_0^{\chi} d\chi'\, e^{-\tau(\chi')}\times$$

$$a(\chi')\mathrm{n_e}(\chi)\mathrm{n_e}(\chi')\sum_{\substack{L,M\\ L',M'}}\sum_{\substack{p_1,p_2\\ p_1',p_2'}} i^{(L-L')}\begin{pmatrix}1 & 1 & 2\\ p_1 & p_2 & -\lambda\end{pmatrix}\begin{pmatrix}1 & 1 & 2\\ p_1' & p_2' & -\lambda'\end{pmatrix}\iint\frac{k_1^2dk_1\, k_2^2dk_2}{(2\pi)^6}\times$$

$$P_{uu}(k_1)P_{uu}(k_2)\, j_L(k\chi)\, j_{L'}(k'\chi')\int d\Omega_{\mathbf{k_1}}\int d\Omega_{\mathbf{k_2}}\; Y_{LM}^*(\hat{\mathbf{k}})Y_{L'M'}(\hat{\mathbf{k}})Y_{1p_1}^*(\hat{\mathbf{k}}_1)Y_{1p_2}^*(\hat{\mathbf{k}}_2)\times$$

$$Y_{1p_1'}(\hat{\mathbf{k}}_1')Y_{1p_2'}(\hat{\mathbf{k}}_2')\, A_{\ell m}^{\lambda LM}\, A_{\ell m}^{\lambda' L' M'}\left(1-(-1)^{(L+\ell)}\right)\left(1-(-1)^{(L'+\ell)}\right)$$

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Matter velocity power spectrum

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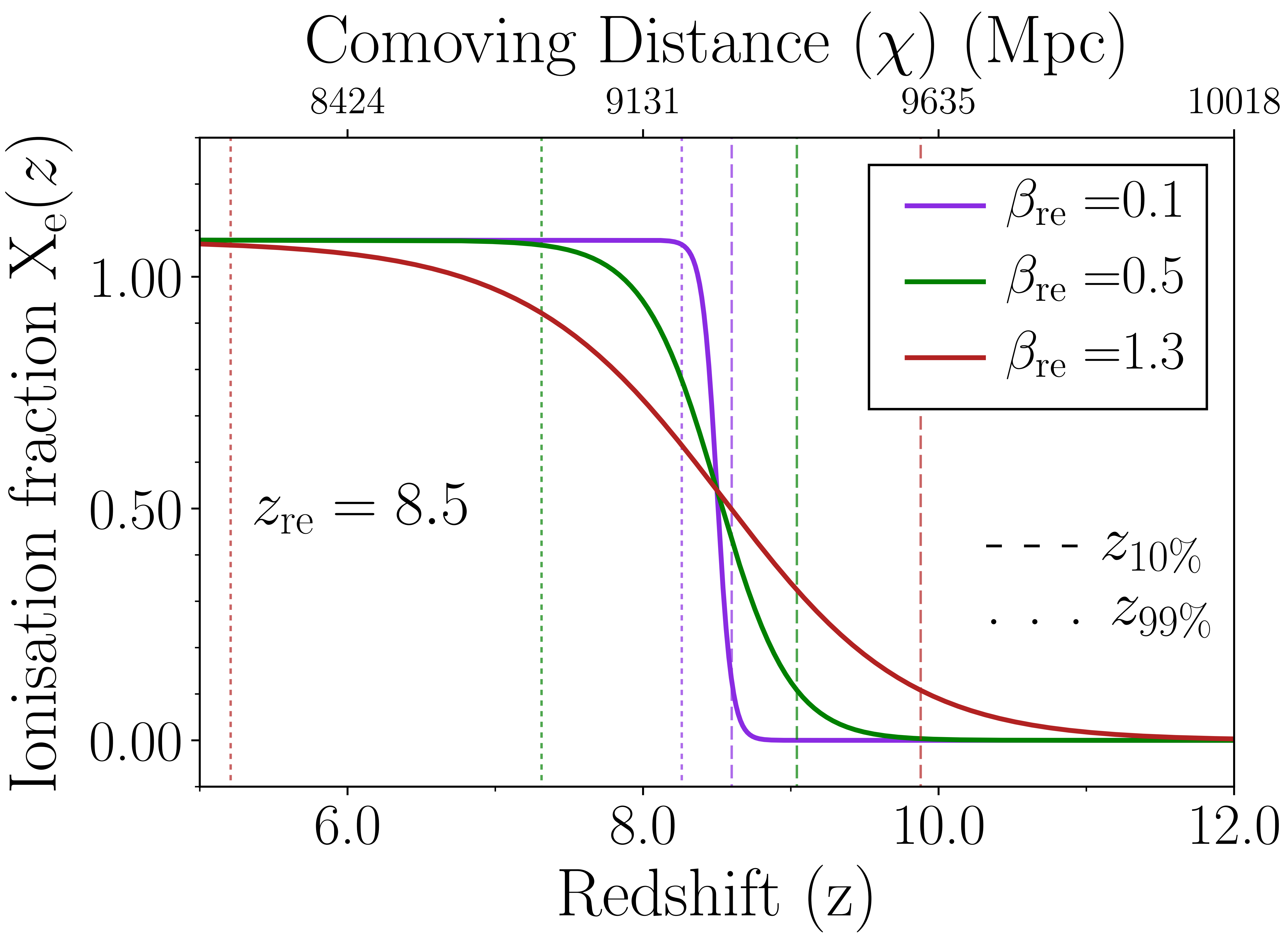
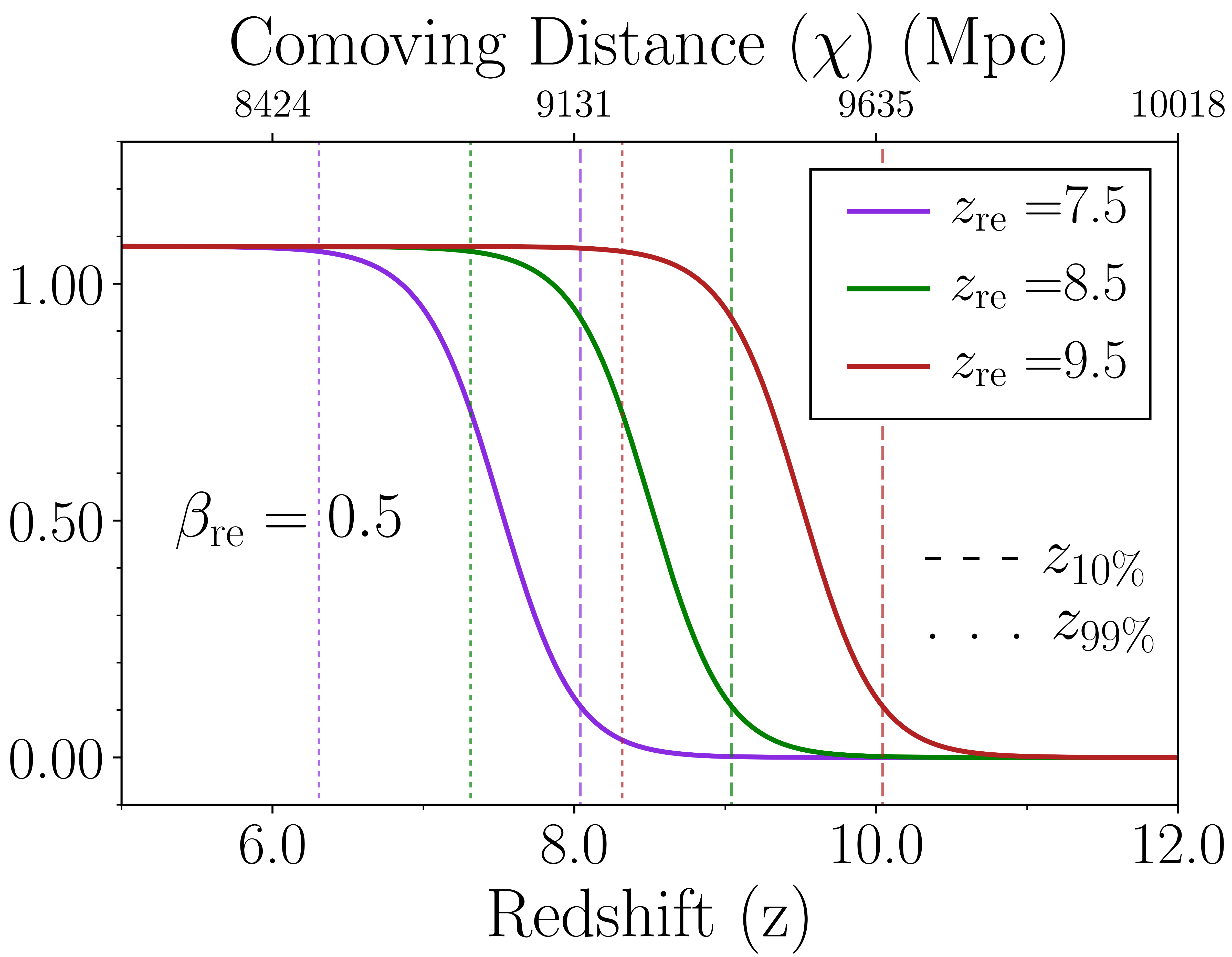
Matter velocity power spectrum

Electron number density—(Reionisation history)

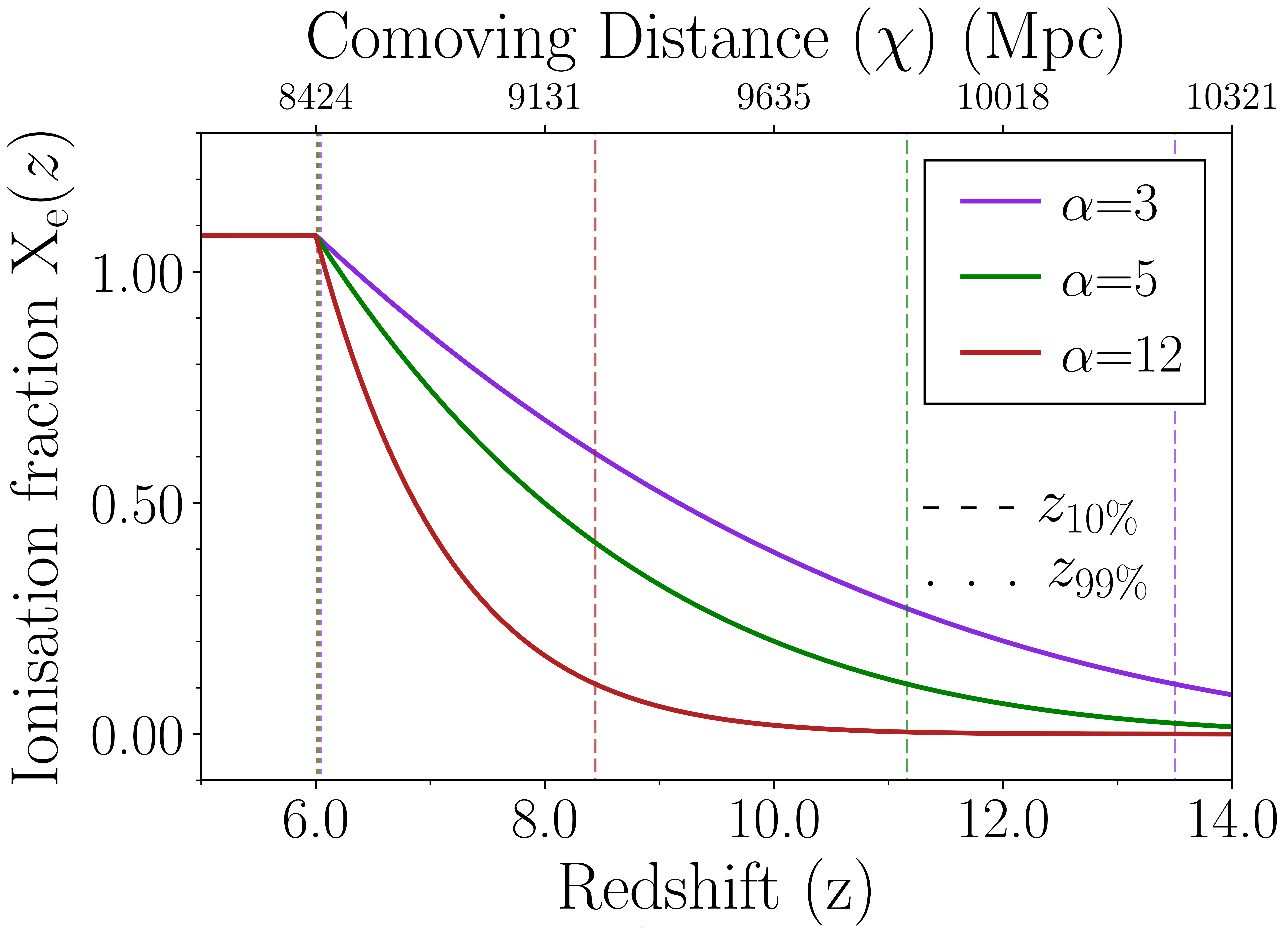
$$A^{\lambda LM}_{\ell m} = \sqrt{\frac{5(2L+1)(2\ell+1)}{4\pi}}(-1)^{(m)}\begin{pmatrix}L & 2 & \ell\\ 0 & -2 & 2\end{pmatrix}\begin{pmatrix}L & 2 & \ell\\ M & \lambda & -m\end{pmatrix}$$

Symmetric Reionisation



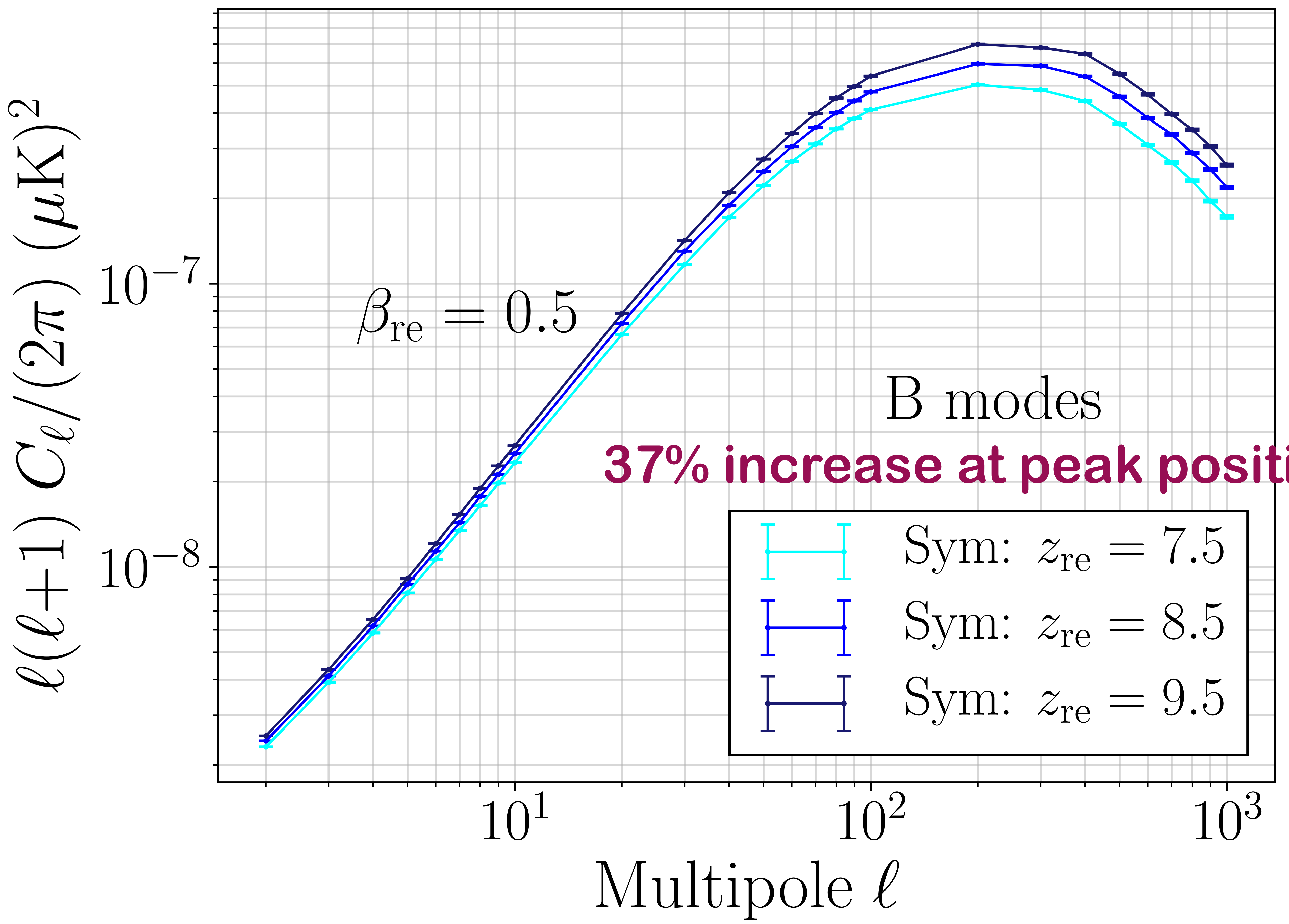


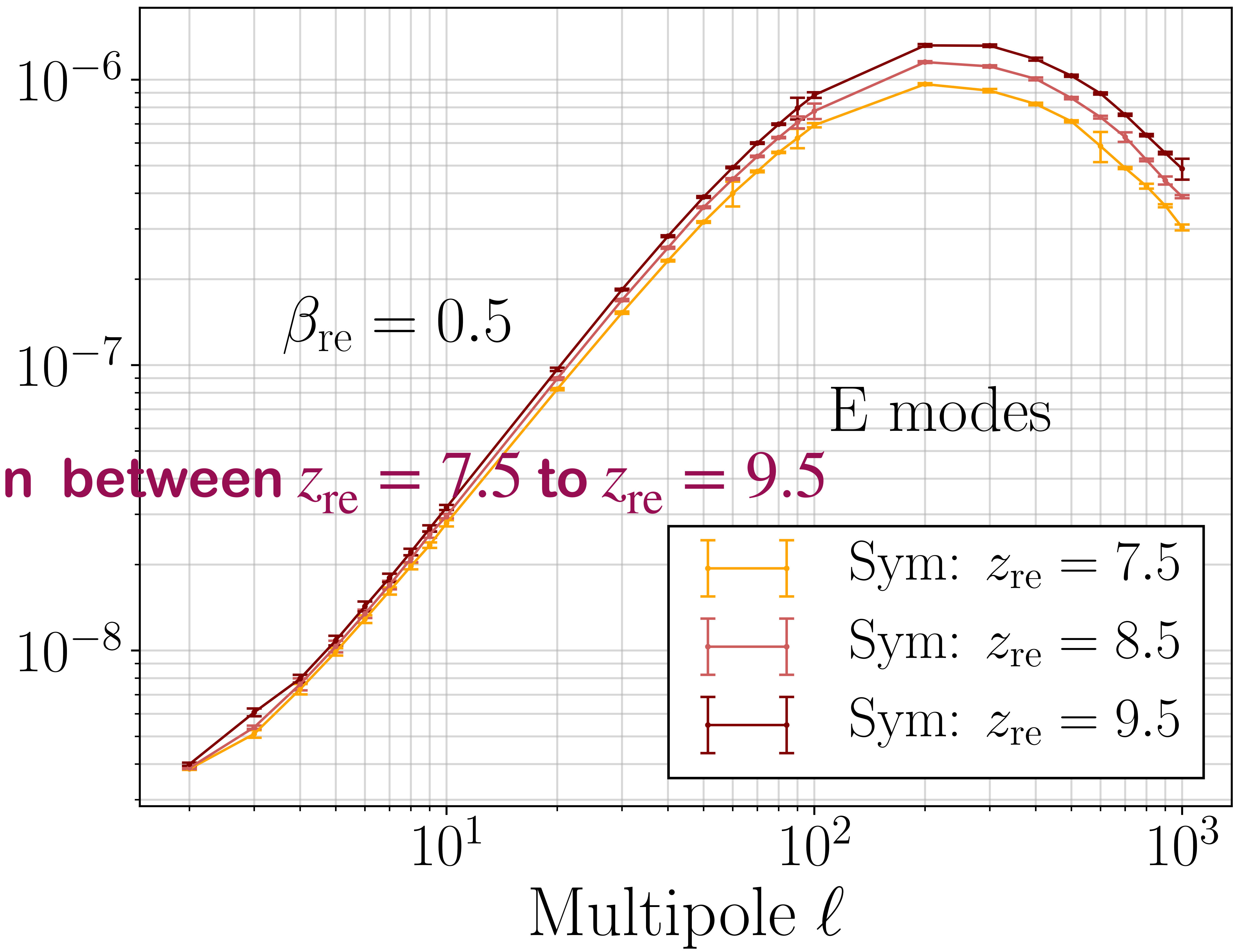
Asymmetric Reionisation



pkSZ effect is sensitive to the redshift of central reionization

- The power spectra increase with the increase in the central redshift of reionisation.
- Increasing the central redshift increases the total Thomson optical depth.



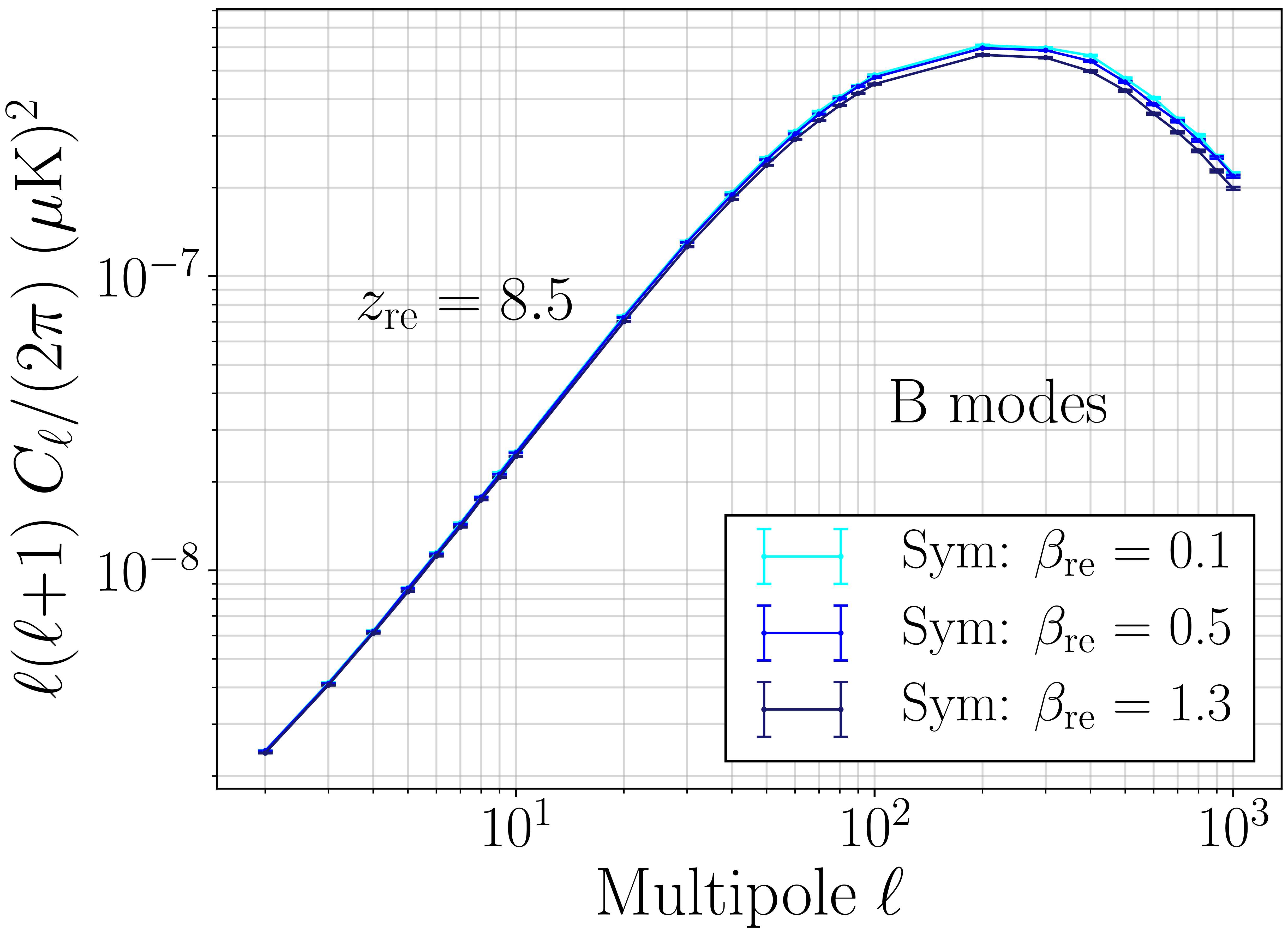


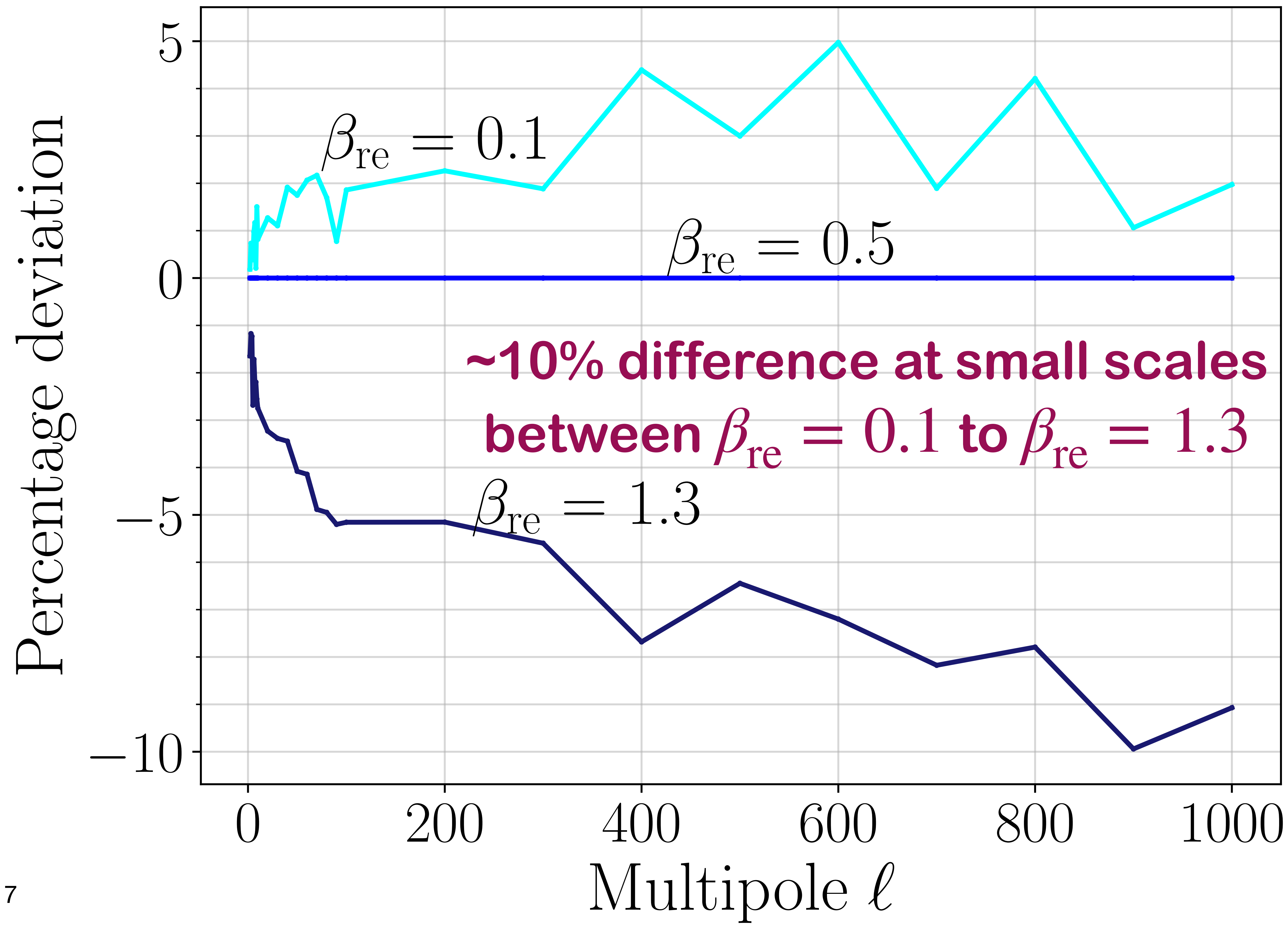
37% increase at peak position between $z_{re} = 7.5$ to $z_{re} = 9.5$

pkSZ effect is sensitive to the reionisation width

- **Changing the width at a fixed central redshift has a negligible effect on the optical depth**
- **The power spectra still decrease with the increase in the duration of reionisation.**

$$\text{Width} = z_{99\%} - z_{10\%}$$





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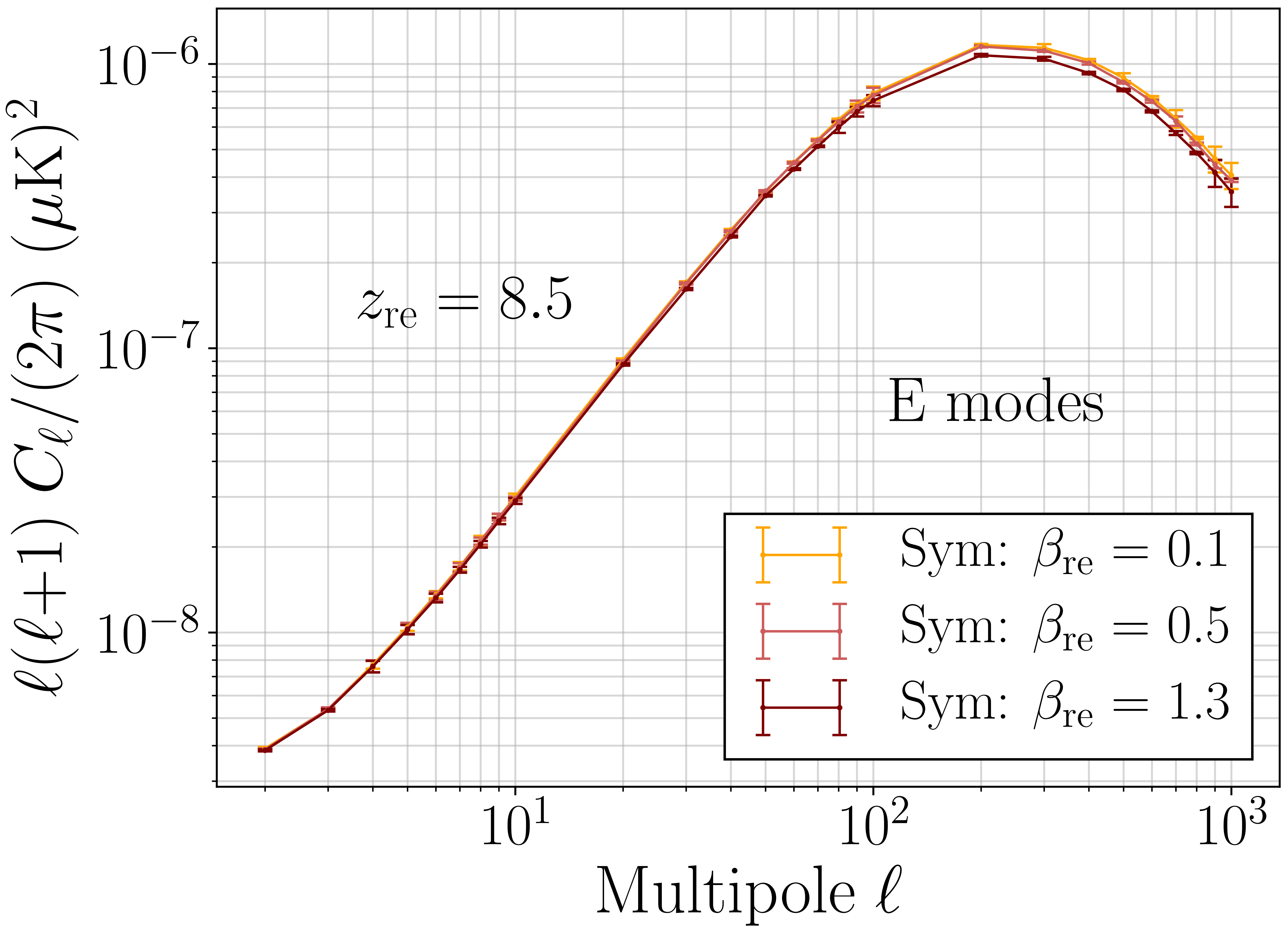
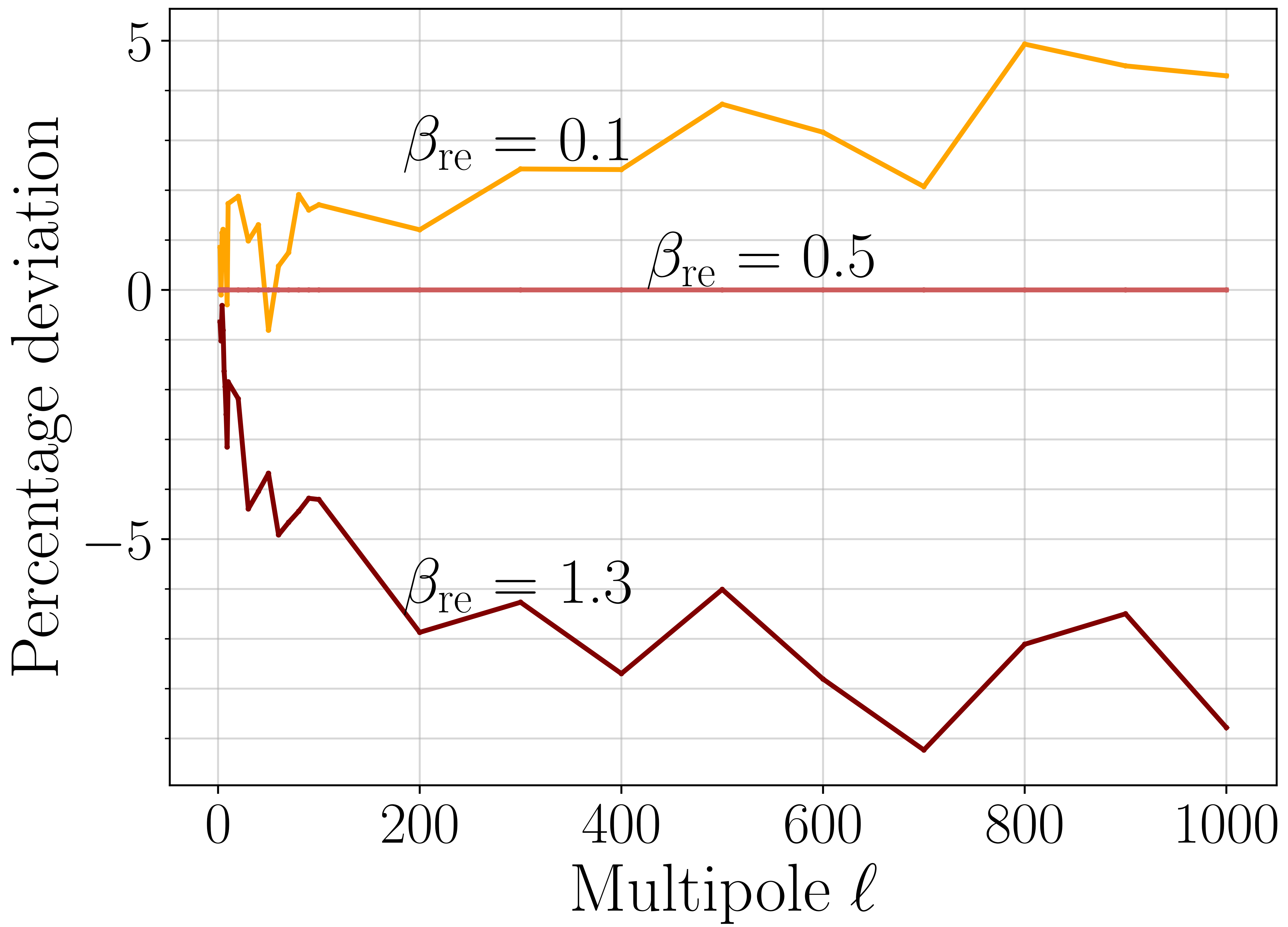




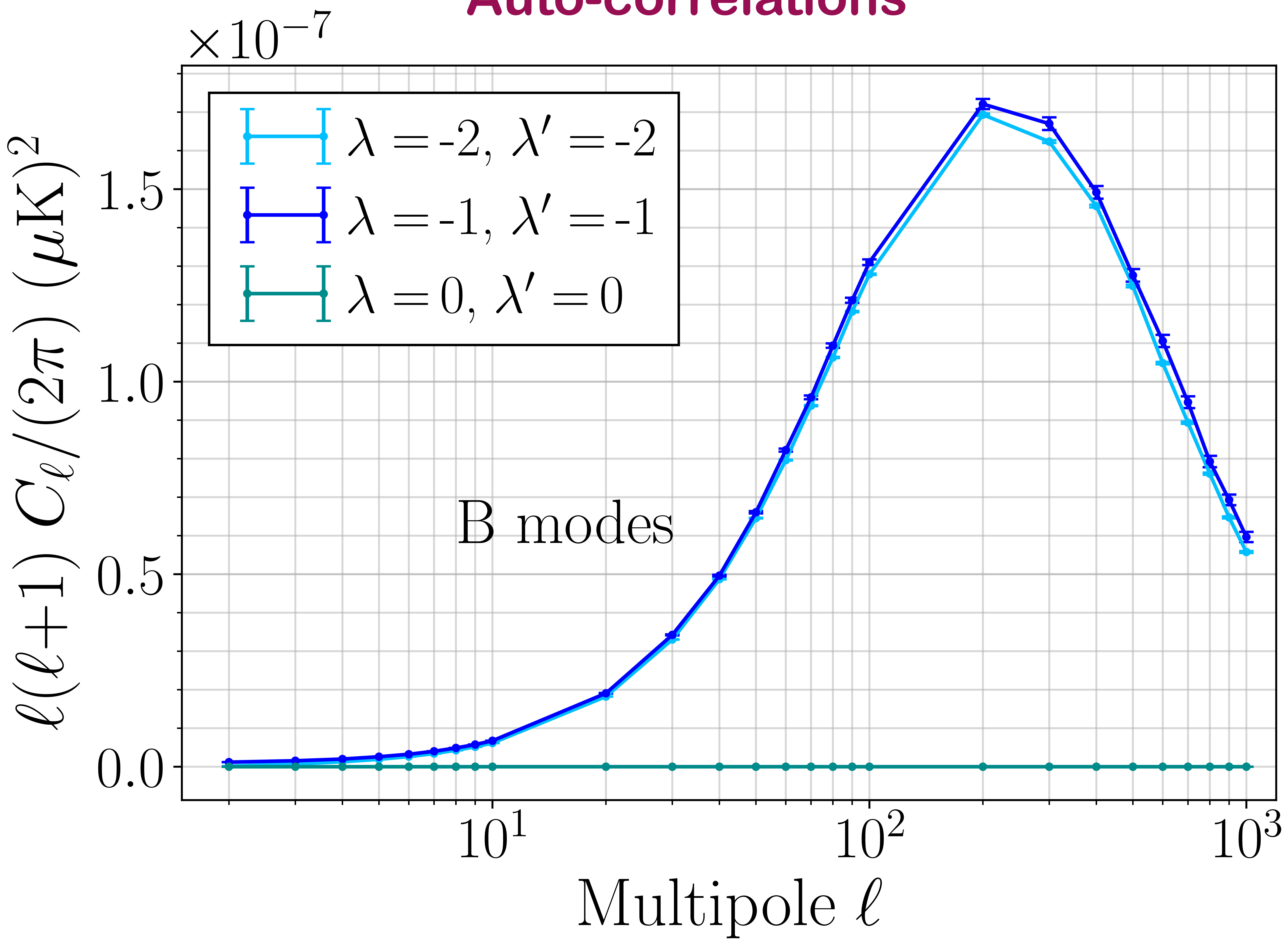
pkSZ effect is sensitive to the rapidity of reionisation

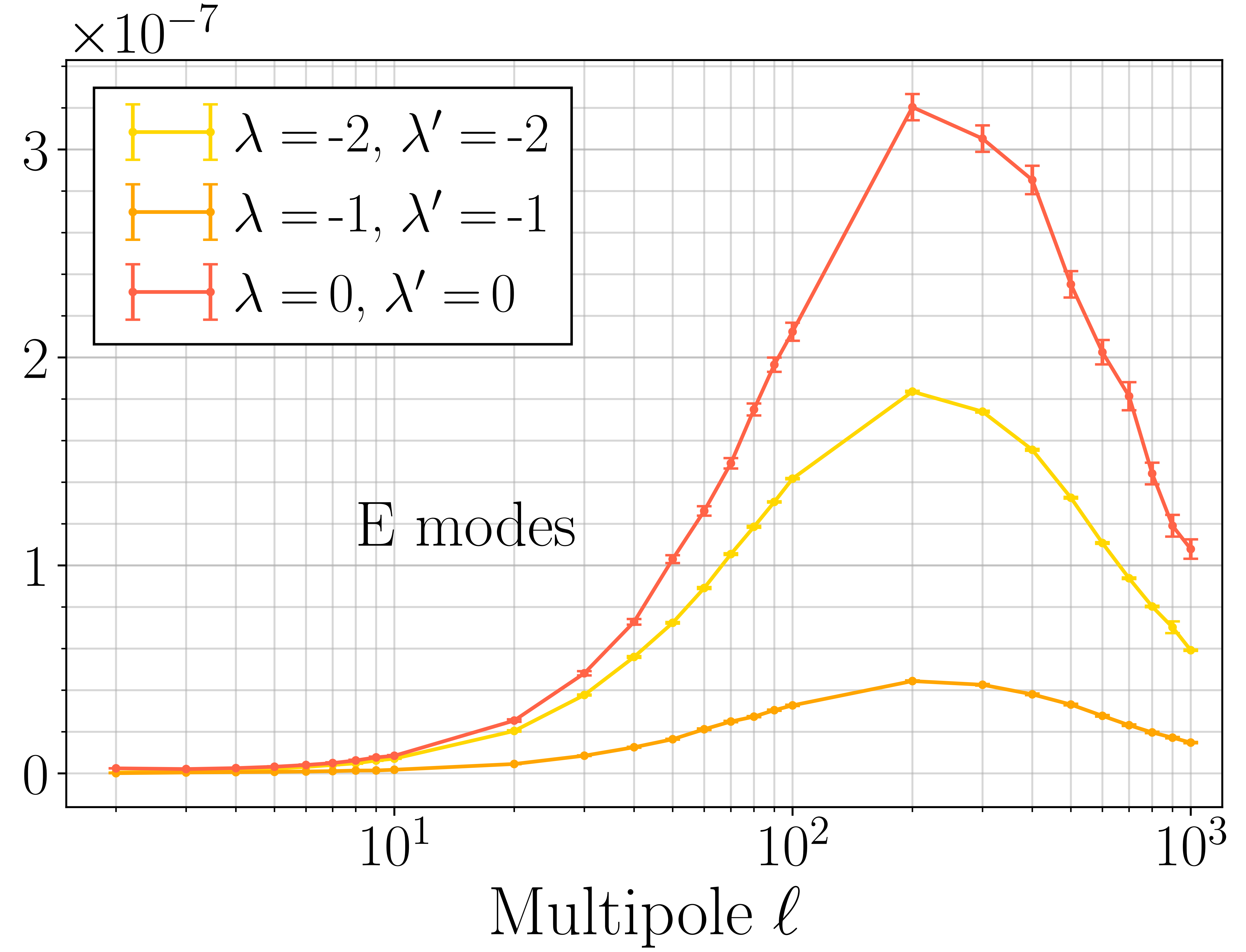
* In the case of **asymmetric reionisation**, the power spectra are **sensitive to how quickly** reionisation occurs.

E modes are greater than the B modes

Scalar $(\lambda = 0)$, Vector $(\lambda = 1)$ and Tensor $(\lambda = 2)$ Decomposition

Auto-correlations





Previous Works

* Renaux-Petel et al. arXiv: 1312.4448 - They claimed to observe only a 2% effect on the width of the Reionisation but we observed a much larger effect.

* Hotinli et al. arXiv: 2204.12503 - They concentrated on a non-Gaussianity and birefringence and not on reionisation

Concluding Remarks

* Sensitive to reionisation central redshift, width, rapidity and the matter velocity power spectrum.

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Thank You !!