Probing the reheating phase through primordial magnetic field and CMB

(based on arXiv:2012.10859v2 [hep-th])

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- *Motivation*
- *Inflationary Magnetogenesis*
- *Connecting Reheating and Primordial magnetic field via CMB*
- *Numerical Results*
- *Main points and outcomes*

Motivation

- *Why reheating phase is import?*

- *Indirect bound on reheating dynamics (Taking into account both the CMB and present value of the magnetic field)*

❑ *We Address :*

- In the current study we consider present day Large Scale Magnetic Field (LSMF) and CMB anisotropy to probe the reheating phase of the universe followed by the standard inflationary phase.

- Conventionally after the end of inflation, the magnetic field on the super-horizon scales redshifts with the scale factor as $B^2 \propto 1/a^4$ provided inflaton energy density transfers into plasma and the universe become good conductor instantly right after the end of inflation.

- In the reference **Kobayashi et al. Phys. Rev. D 100, no.2, 023524 (2019)** it has been shown that if the conductivity remains small redshifts of magnetic energy density becomes slower, $B^2 \propto 1/a^6 H^2$, due to electromagnetic Faraday induction. This helps one to obtain the required value of the present-day large scale magnetic field.

- Taking into account both the CMB anisotropic constraints on the inflationary power spectrum, and the present value of the the large-scale magnetic field, our analysis reveals an important connection among the reheating parameters (T_{re}, w_{re}), magnetogensis models and inflationary scalar spectral index.

❑ *Electromagnetic power spectrum :*

- FLRW metric background expressed in conformal coordinate: $ds^2 = a(\tau)^2\left(-d\tau^2 + d\mathbf{x}^2\right)$
- The gauge field action: $S = -\frac{1}{4}\int d^4x\sqrt{-g}I(\tau)^2F_{\mu\nu}F^{\mu\nu}$
- In order to quantize the field components A_μ are in terms of irreducible scalar and vector components $A_\mu = (A_0, \partial_i S + v_i)$ with $\partial_i V_i = 0$.
- One write $V_i(\tau, x)$ in terms of annihilation and creation operator:

$$V_i(\tau,x) = \sum_{p=1,2}\int \frac{d^3k}{(2\pi)^3}\epsilon_i^{(p)}(\mathbf{k})\left\{e^{i\mathbf{k}.\mathbf{x}}a_k^{(p)}u_k^{(p)}(\tau) + e^{-i\mathbf{k}.\mathbf{x}}a_k^{\dagger(p)}u_k^{*(p)}(\tau)\right\}$$

- The equation of motion of the mode function:

$$u_k^{(p)\prime\prime} + 2\frac{I'}{I} + k^2u_k^{(p)\prime} = 0$$

- Conventionally the electromagnetic power spectrum is expressed in terms of those mode functions:

$$P_E(k) = \frac{k^3}{2\pi^2a^4}\sum_{p=1,2}|u_k^{(p)\prime}|^2 \; ; \; P_B(k) = \frac{k^5}{2\pi^2a^4}\sum_{p=1,2}|u_k^{(p)}|^2$$

- In order to diagonalize the Hamiltonian, we employ the Bogoliubov transformation

$$b_k^{(p)}(\tau) = \alpha_k^{(p)}(\tau) a_k^{(p)} + \beta_k^{(p)*} a_{-k}^{(p)\dagger} \; , \; b_k^{(p)\dagger}(\tau) = \alpha_k^{(p)*}(\tau) a_k^{(p)\dagger} + \beta_k^{(p)}(\tau) a_{-k}^{(p)}$$

- $\alpha_k^{(p)}$ and $\beta_k^{(p)}$ are the Bogoliubov coefficients defined as :

$$\alpha_k^{(p)}(\tau) = I\left(\sqrt{\frac{k}{2}} u_k^{(p)} + \frac{i}{\sqrt{2k}} u_k'^{(p)}\right) \quad ; \quad \beta_k^{(p)}(\tau) = I\left(\sqrt{\frac{k}{2}} u_k^{(p)} - \frac{i}{\sqrt{2k}} u_k'^{(p)}\right)$$

- The power spectrum in terms of Bogoliubov coefficients :

$$P_E(k) = \frac{k^4}{4\pi^2 a^4 I^2} \sum_{p=1,2} |\alpha_k^{(p)} - \beta_k^{(p)}|^2 \; ; \; P_B(k) = \frac{k^4}{4\pi^2 a^4 I^2} \sum_{p=1,2} |\alpha_k^{(p)} + \beta_k^{(p)}|^2$$

Modelling the coupling function

- We consider the widely considered power-law form of the coupling function

$$I(\tau) = \begin{cases} \left(\frac{a_{end}}{a}\right)^n & a \leq a_{end} \\ 1 & a \geq a_{end} \end{cases}$$

- The solution of the mode function considering perfect de-sitter background Hubble parameter H_{Inf} :

$$u_k = \frac{1}{2I}\left(\frac{\pi}{aH_{inf}}\right)^{\frac{1}{2}} H^{(1)}_{-n+\frac{1}{2}}\left(\frac{k}{aH_{inf}}\right)$$

- The time dependent Bogoliubov coeffient are expressed as

$$\alpha_k = \left(\frac{\pi z}{8}\right)^{\frac{1}{2}} \left\{ H^{(1)}_{-n+\frac{1}{2}}(z) - iH^{(1)}_{-n-\frac{1}{2}}(z) \right\} \; ; \; \beta_k = \left(\frac{\pi z}{8}\right)^{\frac{1}{2}} \left\{ H^{(1)}_{-n+\frac{1}{2}}(z) + iH^{(1)}_{-n-\frac{1}{2}}(z) \right\}$$

- In the super-Horizon limit : $\frac{1}{|\beta_k^{(p)}|^2} \ll \theta_k^{(p)} \ll 1$. The simplified form of the electromagnetic power spectrum :

$$P_E(k) \simeq \frac{k^4}{4\pi^2 a^4 I^2} \sum_{p=1,2} 4|\beta_k^{(p)}|^2 \; ; \; P_B(k) \simeq \frac{k^4}{4\pi^2 a^4 I^2} \sum_{p=1,2} |\beta_k^{(p)}|^2 \left(\theta_k^{(p)}\right)^2$$

- Final expression for the spectrum:

$$P_E(k) \simeq \frac{4k^4}{2\pi^2 a^4 I^2} \left(\frac{\pi z}{8}\right) \left\{ H^{*(1)}_{-n+\frac{1}{2}}(z) H^{(1)}_{-n+\frac{1}{2}}(z) + iH^{*(1)}_{-n+\frac{1}{2}}(z) H^{(1)}_{-n-\frac{1}{2}}(z) \right\} - \frac{4k^4}{2\pi^2 a^4 I^2} \left(\frac{\pi z}{8}\right) \left\{ iH^{*(1)}_{-n-\frac{1}{2}}(z) H^{(1)}_{-n+\frac{1}{2}}(z) - H^{(1)}_{-n-\frac{1}{2}}(z) H^{*(1)}_{-n-\frac{1}{2}}(z) \right\}$$

$$P_B(k) \simeq \frac{k^4}{2\pi^2 a^4 I^2} \left(\frac{\pi z}{8}\right) \left\{ H^{*(1)}_{-n+\frac{1}{2}}(z) H^{(1)}_{-n+\frac{1}{2}}(z) + iH^{*(1)}_{-n+\frac{1}{2}}(z) H^{(1)}_{-n-\frac{1}{2}}(z) \right\} - \frac{k^4}{2\pi^2 a^4 I^2} \left(\frac{\pi z}{8}\right) \left\{ iH^{*(1)}_{-n-\frac{1}{2}}(z) H^{(1)}_{-n+\frac{1}{2}}(z) - H^{(1)}_{-n-\frac{1}{2}}(z) H^{*(1)}_{-n-\frac{1}{2}}(z) \right\}$$
$$\left\{ arg \left[\frac{\pi z}{8} \left(H^{(1)}_{-n+\frac{1}{2}}(z) - iH^{(1)}_{-n-\frac{1}{2}}(z) \right) \left(H^{*(1)}_{-n+\frac{1}{2}}(z) - iH^{*(1)}_{-n-\frac{1}{2}}(z) \right) \right] - \pi \right\}^2$$

- For n being positive integer

$$P_E(k) \simeq \frac{8\Gamma(n+\frac{1}{2})^2 H_{inf}^4}{I^2 \pi^3} \left(\frac{k}{2aH_{inf}}\right)^{-2(n-2)} \; ; \; P_B(k) \simeq \frac{8\Gamma(n-\frac{1}{2})^2 H_{inf}^4}{I^2 \pi^3} \left(\frac{k}{2aH_{inf}}\right)^{-2(n-3)}$$

- Most of the studies so far considered the fact that when I^2 becomes constant at the end of the inflation and $|\beta_k^{(p)}|^2$ is conserved.

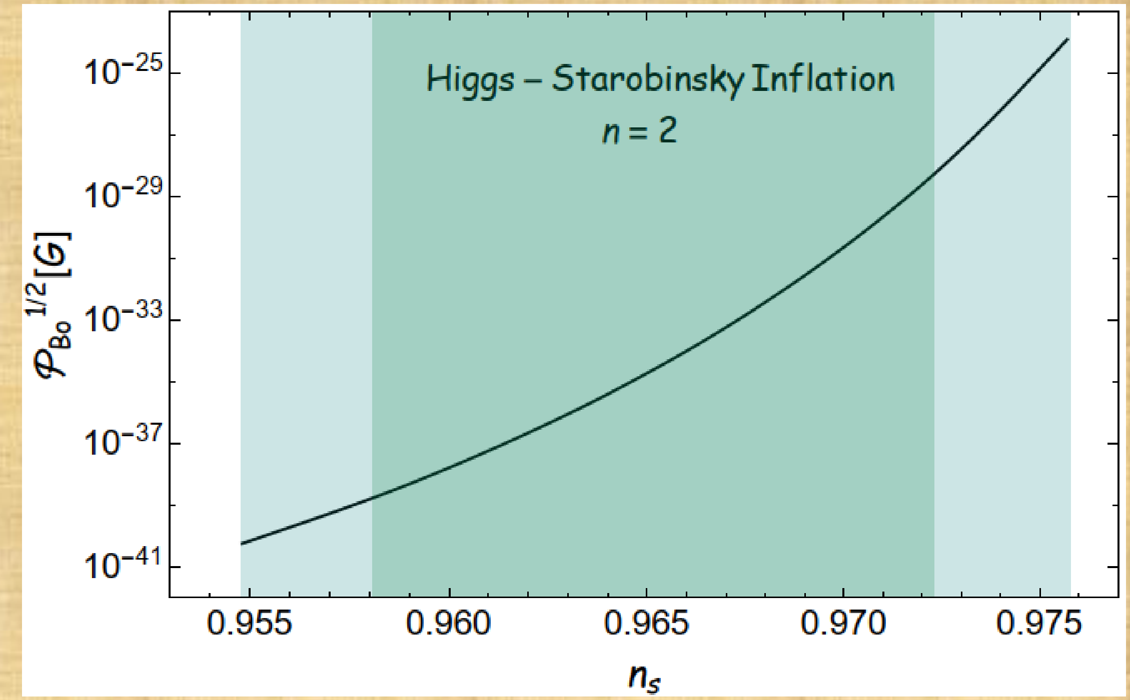
- Consequently, $\mathcal{P}_B(k) \propto a^{-4}$ until today.

- Magnetic power spectrum at the end of the inflation :

$$\mathcal{P}_B^{end}(k) \simeq \frac{k^4}{2\pi^2 a_{end}^4} \left(\theta_k^{end}\right)^2 |\beta_k^{end}|^2$$

- The magnetic power spectrum at the present universe:

$$\mathcal{P}_{B0}(k) \simeq \mathcal{P}_B^{end}(k) \left(\frac{a_{end}}{a_0}\right)^4$$



- It is clear that the required magnetic field strength $10^{-9} - 10^{-22}$ G is difficult to achieve within the conventional framework, unless one introduces slow decreasing rate of magnetic energy density in some early state of universe evolution.

After inflation dynamics: reheating

- During reheating in the Fourier space, the mode function solution of the free Maxwell equation :

$$u_k^{(p)} = \frac{1}{\sqrt{2k}}\{\alpha_k^{(p)}(z_{end})e^{-ik(\tau-\tau_{end})} + \beta_k^{(p)}(z_{end})e^{-ik(\tau-\tau_{end})}\}$$

- Now the phase factor : $\theta_k^{(p)} = \left\{Arg\left[\alpha_k^{(p)}(z_{end})\,\beta_k^{(p)*}(z_{end})\right] - \pi\right\} - 2k\,(\tau - \tau_{end})$ $\quad \tau - \tau_{end} = \int_{a_{end}}^{a} \frac{da}{a^2 H}$

- The conventional one will be associated with the redshift factor $\propto a^{-4}$ emerging from the first term in the right-hand side of the equation. Important one is associated with the redshift factor $\propto a^{-6}H^{-2}$ emerged out from the second term of the above equation.

- The magnetic power spectra during reheating: $P_B(k) \simeq \frac{k^4}{2\pi^2 a_{re}^4}(\theta_k)^2|\beta_k^{end}|^2$

- Final general expression of the present-day magnetic power spectrum:

$$P_{B_0}(k) = P_{B_{re}}(k)\left(\frac{a_{re}}{a_0}\right)^4 \simeq \frac{k^4}{2\pi^2}(\theta_k^{re})^2|\beta_k^{end}|^2\frac{1}{a_0^4}$$

- For integer value of n, the expression of the present-day magnetic power spectrum:

$$P_{B0}(k) \simeq \frac{8\Gamma(n-\frac{1}{2})^2}{\pi^3}H_{inf}^4\left(\frac{k}{2a_{end}H_{inf}}\right)^{-2(n-3)}\left(\frac{a_{end}}{a_0}\right)^4\left\{1+(2n-1)\int_{a_{end}}^{a_{re}}\frac{da}{a}\frac{a_{end}H_{inf}}{aH}\right\}^2$$

Reheating dynamics: Connecting Reheating and Primordial magnetic field via CMB

Case-I:

- For this we follow the reheating dynamics proposed in **Kamionkowski et al. Phys. Rev. Lett. 113, 041302 (2014)**, where inflaton energy is assumed to converted into radiation instantaneously at the end of reheating.

- The dynamics is parametrized by an effective equation of state ω_{eff}, reheating temperture T_{re} and duration N_{re} (e-folding number during reheating era).

- The expression for T_{re} and N_{re} :

$$T_{re} = \left(\frac{43}{11 g_{s,re}}\right)^{\frac{1}{3}} \left(\frac{a_0 T_0}{k'}\right) H_k e^{-N_k} e^{-N_{re}}$$

$$N_{re} = \frac{4}{(1-3\omega_{eff})}\left[-\frac{1}{4} ln\left(\frac{45}{\pi^2 g_{re}}\right) - \frac{1}{3} ln\left(\frac{11 g_{s,re}}{43}\right) - ln\left(\frac{k'}{a_0 T_0}\right) - ln\left(\frac{V_{end}^{1/4}}{H_k}\right) - N_k\right]$$

Reheating parameters and primordial magnetic field:

- For integer value of n, the expression of the phase parameter:

$$\theta_k^{re} \simeq \frac{2}{2n-1} \frac{k}{a_{end} H_{inf}} \left\{1 + \frac{4n-2}{3\omega_{eff}+1}\left(\frac{a_{end} H_{inf}}{a_{re} H_{re}} - 1\right)\right\}$$

- The present-day magnetic field turns out as

$$P_{B_0}(k) \simeq \frac{8\Gamma(n-\frac{1}{2})^2}{\pi^3 H_{Inf}^{-4}} \left(\frac{k}{2a_{end}H_{Inf}}\right)^{-2(n-3)} \left(\frac{a_{end}}{a_0}\right)^4 \left\{1 + \left(\frac{4n-2}{3\omega_{eff}+1}\right)\left(\frac{a_{end}H_{inf}}{a_{re}H_{re}} - 1\right)\right\}^2$$

- The ratio between the scale factor a_0 and a_{re}, considering entropy conservation:

$$\frac{a_0}{a_{re}} = \left(\frac{11 g_{s,re}}{43}\right)^{\frac{1}{3}} \frac{T_{re}}{T_0}$$

- ***Case-II:***

- In this reheating model we consider perturbative reheating model where effective equation of state is time-dependent.

- In perturbative reheating the corresponding energy density satisfy the standard Boltzmann equations: $\dot{\rho}_\phi + 3H(1+w_\phi^1)\rho_\phi - \Gamma_\phi \rho_\phi (1+w_\phi^1) = 0$ $\quad \dot{\rho}_R + 4H\rho_R + \Gamma_\phi \rho_\phi (1+w_\phi^1) = 0$

- Initial conditions: $\rho_\phi(A=1) = \frac{3}{2}V_{end}(\phi) \;\; ; \;\; \rho_R(A=1) = 0$

- Reheating temperature is identified from radiation temperature at the point of $H(t_{re}) = \Gamma_\phi$, when maximum inflaton energy density transfer into radiation.

$$H(A_{re})^2 = \left(\frac{\dot{A}_{re}}{A_{re}}\right)^2 = \frac{\rho_\phi(\Gamma_\phi, A_{re}, n_s^k) + \rho_R(\Gamma_\phi, A_{re}, n_s^k))}{3M_p^2} = \Gamma_\phi^2$$

- The reheating temperature in terms of radiation temperature: $T_{re} = T_{rad}^{end} = \left(\frac{30}{\pi^2 g_*(T)}\right)^{1/4} \rho_R(\Gamma_\phi, A_{re}, n_s^k)^{1/4}$

- From entropy conservation: $T_{re} = \left(\frac{43}{11 g_{re}}\right)^{1/3} \left(\frac{a_0 T_0}{k}\right) H_k e^{-N_k} e^{-N_{npre}} e^{-N_{pre}}$

- Now connecting above equations, we can establish one to one correspondence between T_{re} and $\Gamma\phi$.

- ***Connecting reheating and primordial magnetic field:***

- For pertubative decay of the inflaton field the phase parameter now explicitly depends on the evolution of the two energy components ρ_ϕ and ρ_R with time

$$\theta_k^{re} \simeq \frac{2}{2n-1} \frac{k}{a_{end} H_{Inf}} \left\{ 1 + (2n-1) \int_{a_{end}}^{a_{re}} \frac{\sqrt{3} M_p}{\sqrt{\rho_\phi(a) + \rho_R(a)}} \frac{a_{end} H_{inf}}{a} \frac{da}{a} \right\}$$

- The magnetic power spectrum in the present universe:

$$P_{B0}(k) \simeq \frac{\Gamma(n-\frac{1}{2})^2}{\pi^3} \frac{2^{2n-3}\,(2.6 \times 10^{39})}{(6.4 \times 10{-39})^{2n-6}} \left(\frac{k}{a_0} M_{pc}\right)^{-2(n-3)} \left(\frac{11 g_{s,re}}{43}\right)^{\frac{2-2n}{3}} \left(\frac{T_{re}}{T_0}\right)^{2-2n} \left(\frac{H_{inf}}{GeV} \frac{1}{A_{re}}\right)^{2(n-1)} \left\{ 1 + (2n-1) \int_1^{A_{re}} \frac{\sqrt{3} M_p H_{Inf}}{\sqrt{\rho_\phi(a) + \rho_R(a)}} \frac{dA}{A^2} \right\}^2 G^2$$

Discussion on strong coupling and backreaction problem

- As coupling function monomial function of scale factor $I \propto a^{-n}$, for negative value of n the gauge kinetic function increasing during inflation.

- The coupling function boils down to unity after inflation. The effective electromagnetic coupling, $\alpha_{eff} = \frac{e^2}{4\pi I} = \frac{\alpha}{I}$ for negative values of n becomes very large, which turns the theory non-perturbative.

- On the other hand, if one considers positive values of $n \geq 0$, throughout the inflationary period the gauge kinetic function $I(\tau)$ will always larger than unity, so there is a considerable reduction of the effective electromagnetic coupling α_{eff}. Strong coupling problem can be avoided for positive values of n but backreaction problem arise as production of electromagnetic energy can generically take over the background energy density.

- To avoid backreaction problem: $\frac{\rho_A}{\rho_{tot}} \leq \zeta$

- Total gauge field energy density at a given scale factor a_T during inflation can be calculated as:

$$\rho_A(a_T) = \rho_E(a_T)+\rho_B(a_T) = \frac{I(a_T)^2}{2}\int_{k_{IR}}^{k_T}\frac{dk}{k}\left\{\mathcal{P}_E(k,a_T)+\mathcal{P}_B(k,a_T)\right\}$$

$$\rho_A(a_T) = \frac{I(a_T)^2}{2}\int_{k_{IR}}^{k_T}\frac{dk}{4\pi k H_{end}}\left(\frac{k}{a_T}\right)^5\left\{\left|H^{(1)}_{-n-\frac{1}{2}}\left(\frac{k}{a_T H_{end}}\right)\right|^2+\left|H^{(1)}_{-n+\frac{1}{2}}\left(\frac{k}{a_T H_{end}}\right)\right|^2\right\}$$

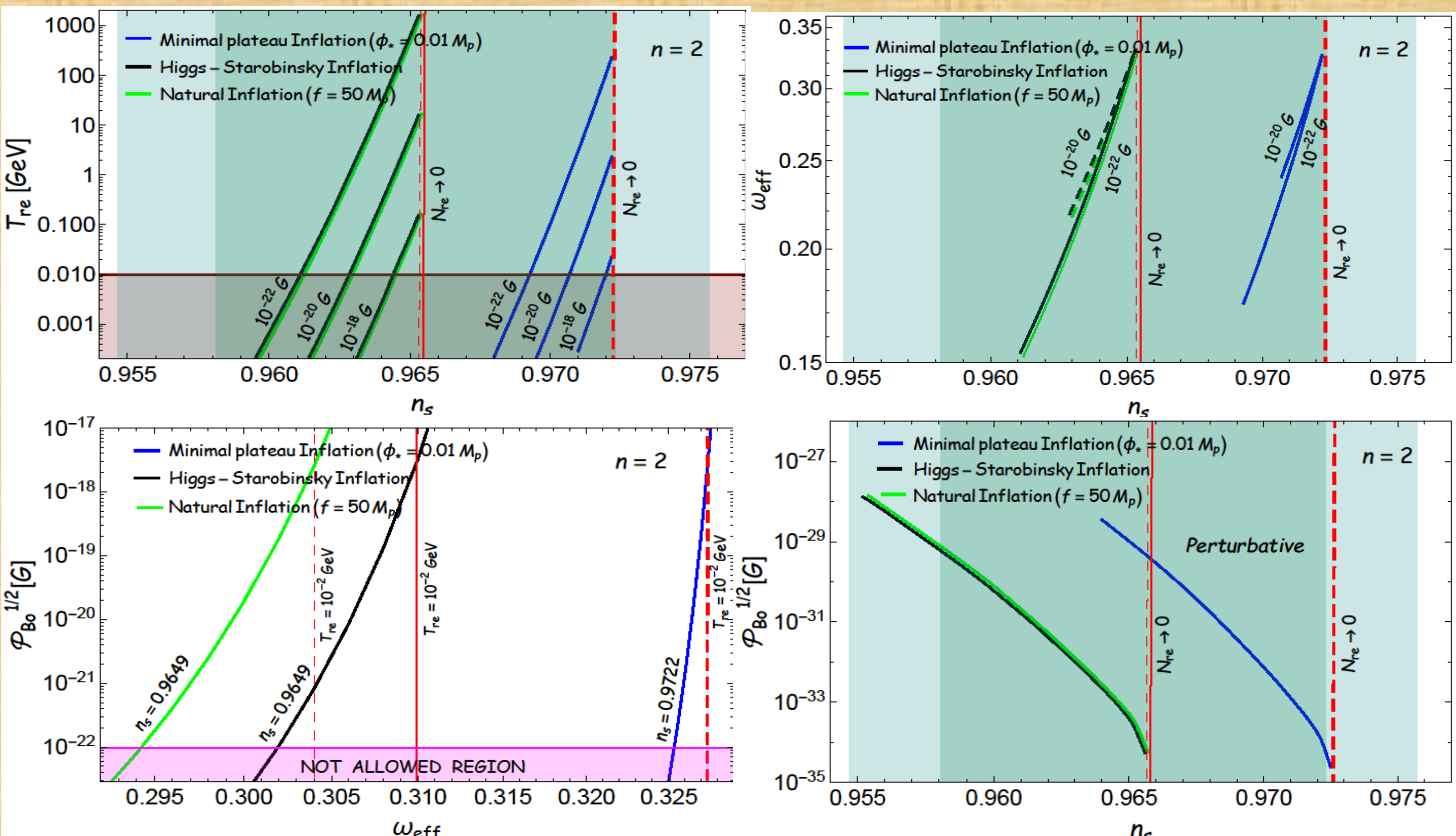
- For example, the scale-invariant electric power spectrum which corresponds to n=2, the total gauge field energy density at the end of the inflation

$$\rho_A \sim \frac{9}{4}\frac{H_{end}^4}{\pi^2}N_k + \frac{H_{end}^4}{8\pi^2}\left(1-e^{-2N_k}\right) \sim 10^{-10}\rho_{tot}$$

- However, scale-invariant magnetic power spectrum (n=3), immediate back-reaction problem as the electric field power spectrum, $\mathcal{P}_E(k) \propto \left(\frac{k}{aH_{end}}\right)^{-2} \rightarrow \infty$ for large scale limit $\frac{k}{aH_{end}} \rightarrow 0$.

Numerical Results

❑ *k-independent electric power spectrum: Case-I*



❖ For numerical results, here we consider a class of inflationary models :

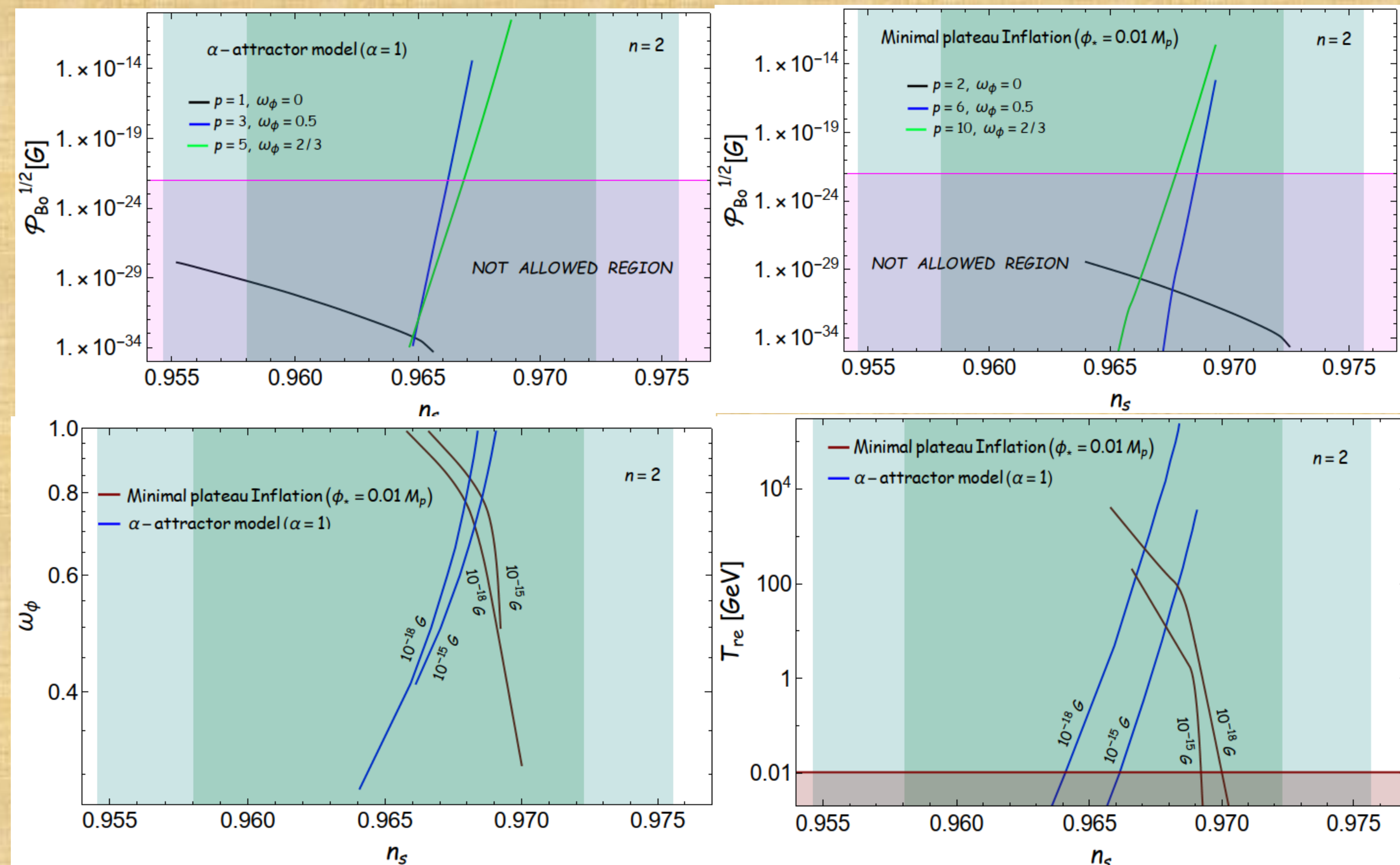
1. Minimal plateau model $V_{min} = \Lambda \dfrac{m^{4-p}\phi^p}{1+\left(\frac{\phi}{\phi_*}\right)^p}$

2. Higgs-Starobinsky inflation $V(\phi) = \Lambda^4 \left[1 - e^{-\sqrt{\frac{2}{3\alpha}}\frac{\phi}{M_p}}\right]^{2p}$

3. Natural inflation $V(\phi) = \Lambda^4 \left[1 - \cos\left(\dfrac{\phi}{f}\right)\right]$

Numerical Results

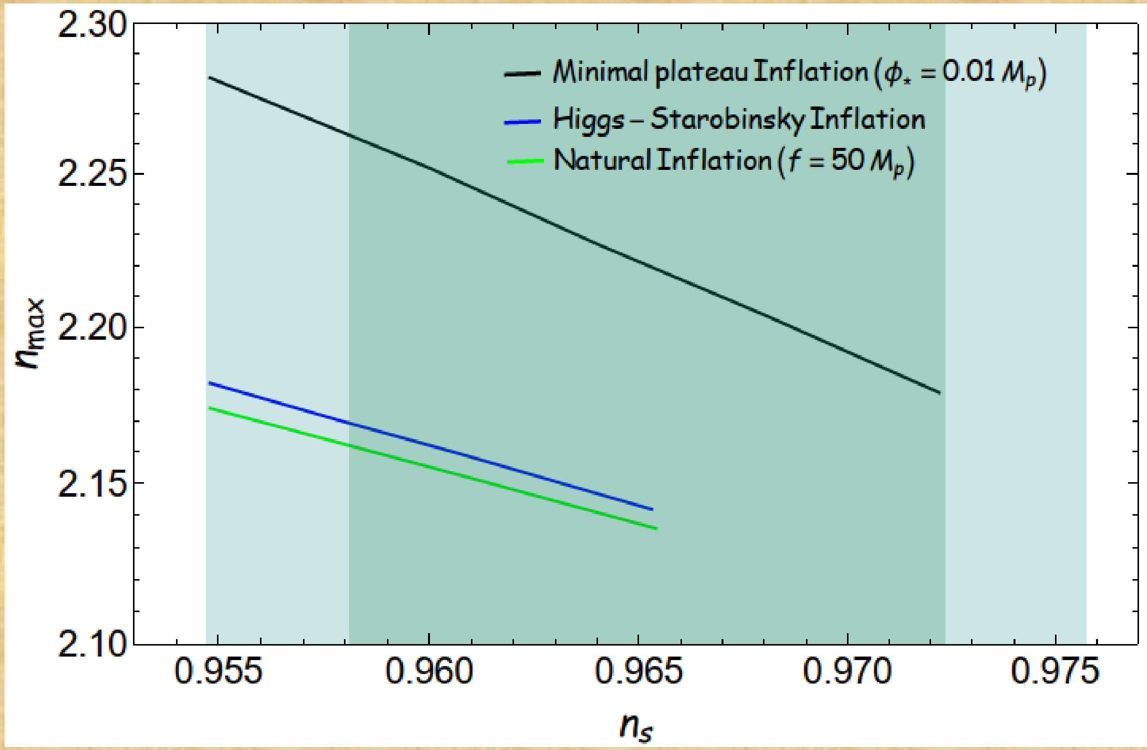
❑ *k-independent electric power spectrum: Case-II (Perturbative reheating)*



❖ Here, we assume that the average value of the inflaton equation of state $\omega_\phi = (p-2)/(p+2)$

Numerical Results

❑ *Constraining magnetogensis model: maximum possible value of n*



- The direct constraint on the value of n will come from the strong coupling and the back-reaction problem leading to the fact that n should lie within $0 \leq n \leq n_{max}$.

- Irrespective of the inflationary models under consideration, reheating temperature associated with n_{max} can be observed as high as $\sim 10^7$ GeV.

Main points and outcomes

- we can clearly predict a unique value of ω_{eff} associated with a specific choice of the present magnetic field. Additionally, for a given ω_{eff} the reheating temperature is also determined uniquely.
- Taking into account CMB constraints the reheating phase can be uniquely probed by the evolution of the primordial magnetic field

- ***Scale invariant electric power spectra:***
- For case-I reheating scenario, irrespective of models under consideration large scale magnetic field constrains the effective equation of state within $0.150 < \omega_{eff} < 0.325$ and consequently predicts the low value of reheating temperature $(10^{-2}, 10^{3})$ GeV.

- For the perturbative reheating scenario the range of inflaton equation of state $0 < \omega_{\phi} \lesssim 0.28$ is found to be observationally not viable for $\mathcal{P}_{B0}^{\frac{1}{2}} \gtrsim 10^{-18}$ G.

- This observation provides us a strong constraint on the possible form of the inflaton potential near its minimum with $p \gtrsim 3.6$ considering the aforesaid observable limit of the present-day magnetic field strength.

- ***Constraining magnetogensis model:***
- Considering both backreaction and strong coupling problems into account, the maximum allowed value of n would be $n_{max} \sim 2.3$.

THANK YOU