August 28, 2021

Primordial features: how relevant are they?

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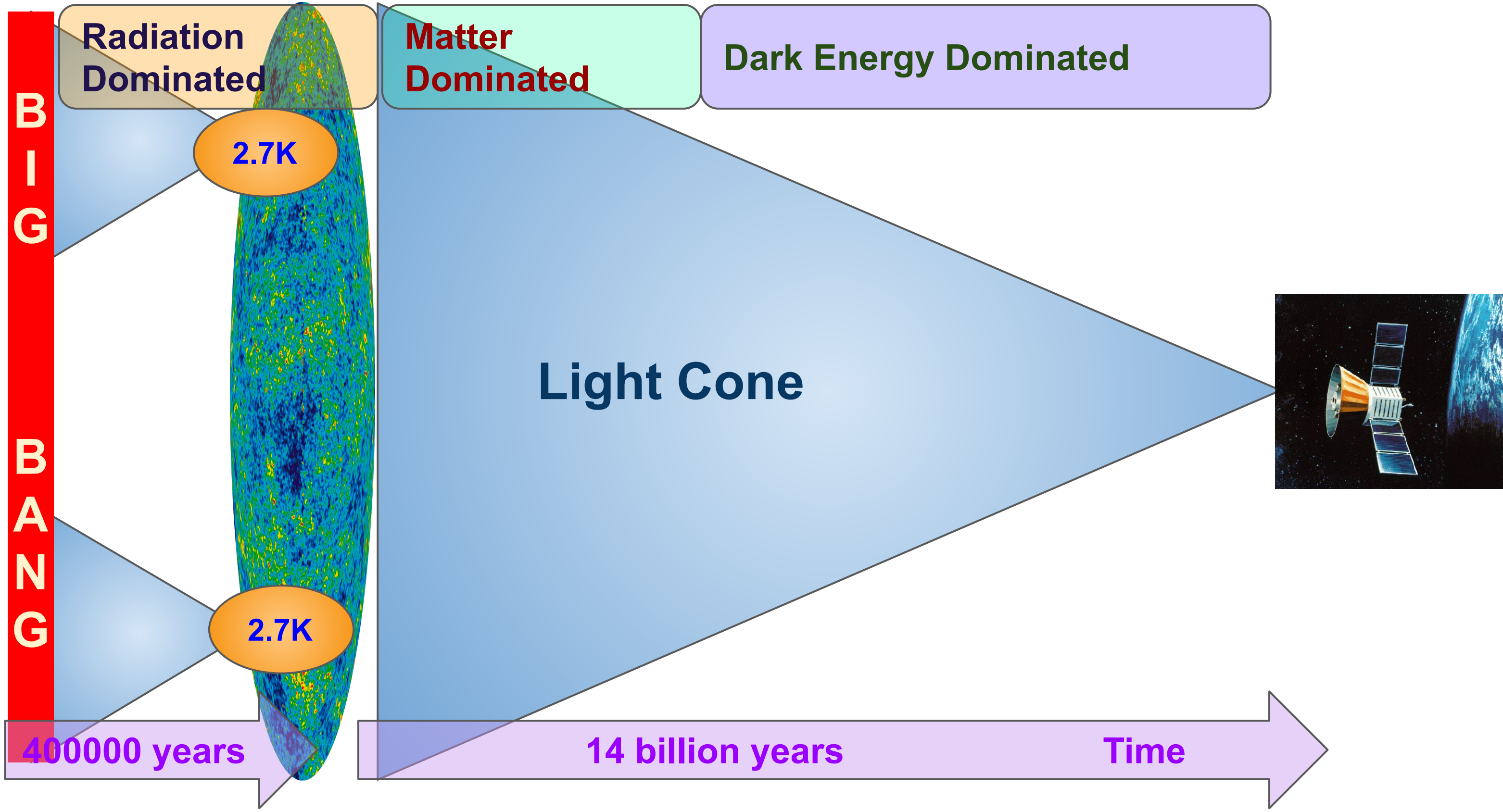
Why do we need inflation ?



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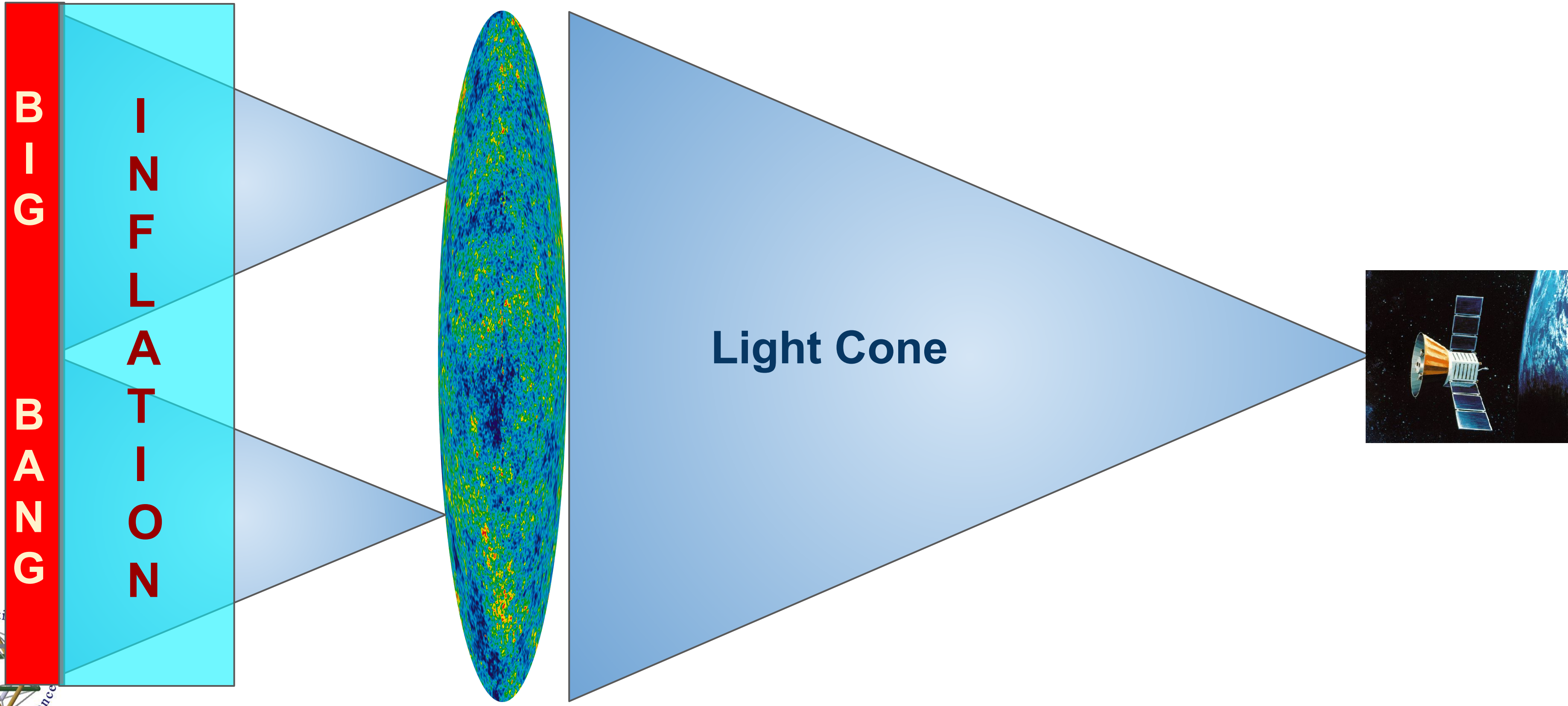
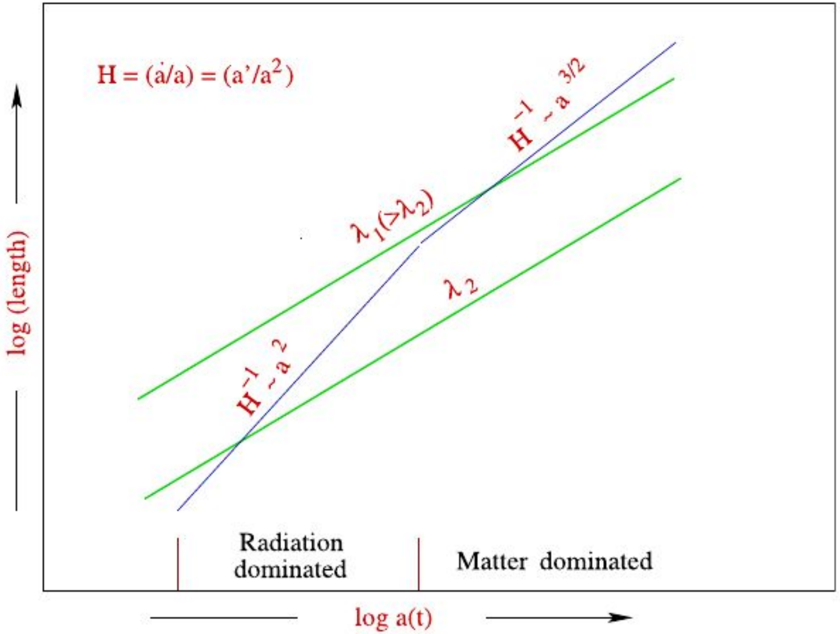
Cones do not match (Horizon problem)

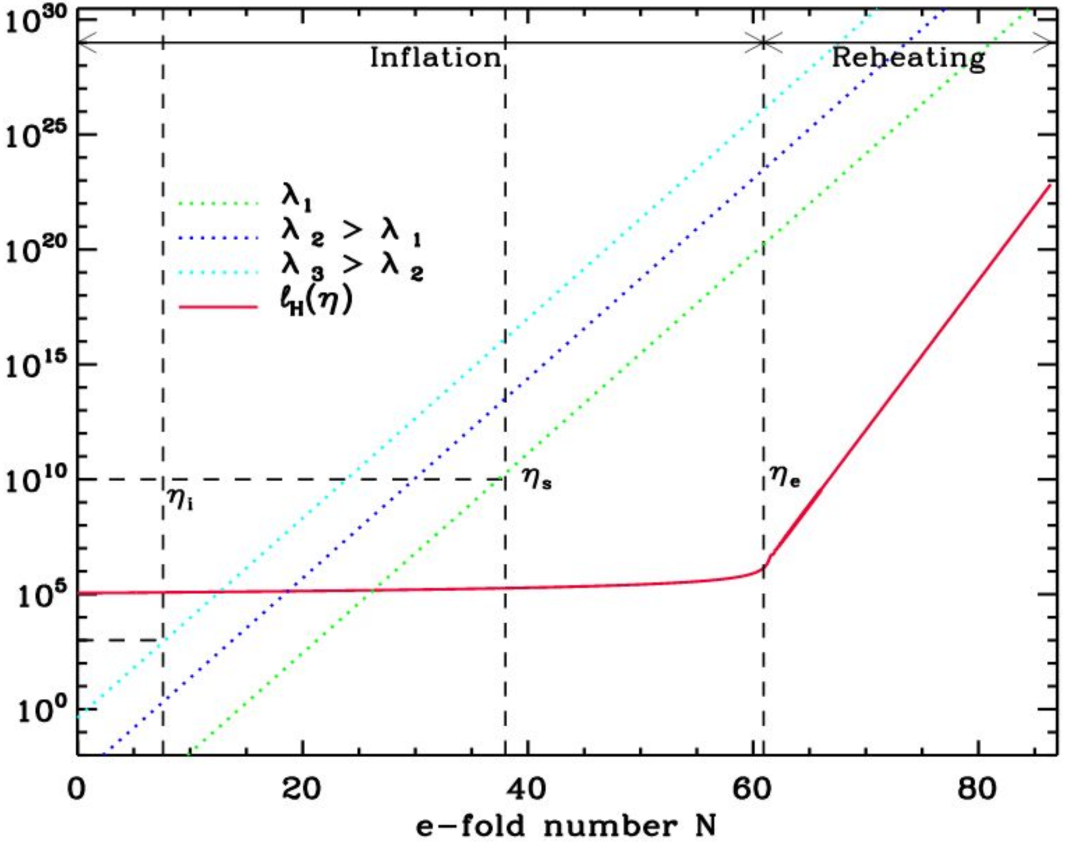


Modes

- **Modes leaving the Hubble scale**: A plot of $\log \lambda_p$ versus $\log a$ showing the modes leaving the Hubble radius $d_H = H^{-1}$ at a sufficiently early time



Solving horizon problem -- Inflation



Solution to the horizon problem

Rapid (nearly exponential) expansion of the Universe prior to the radiation dominated era -- **Inflation** -- **Starobinsky, Guth, Linde**

Matter and radiation are not able to drive such expansion

Scalar fields that roll slowly down a potential can provide the appropriate expansion

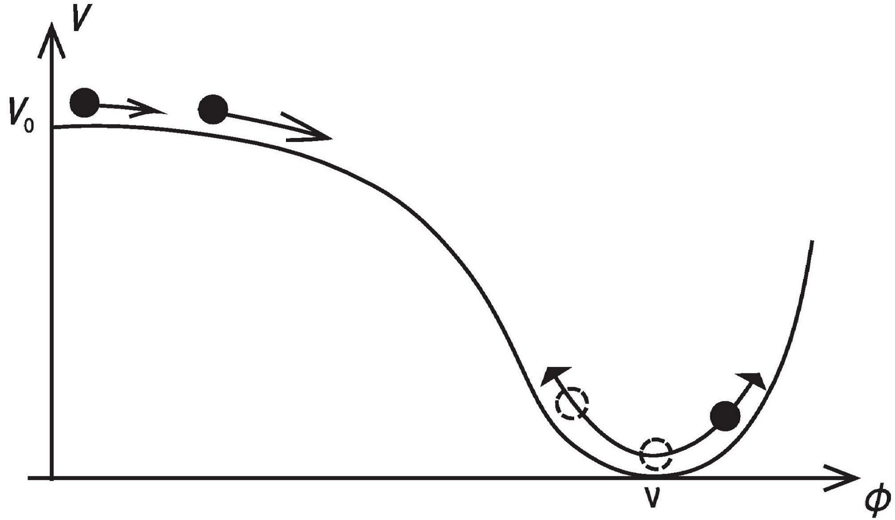
We need at least **60 *e-folds*** of inflation (the Universe expands by e^{60} times compared to its initial size)

Generates the primordial fluctuations that seeded the Large Scale Structures

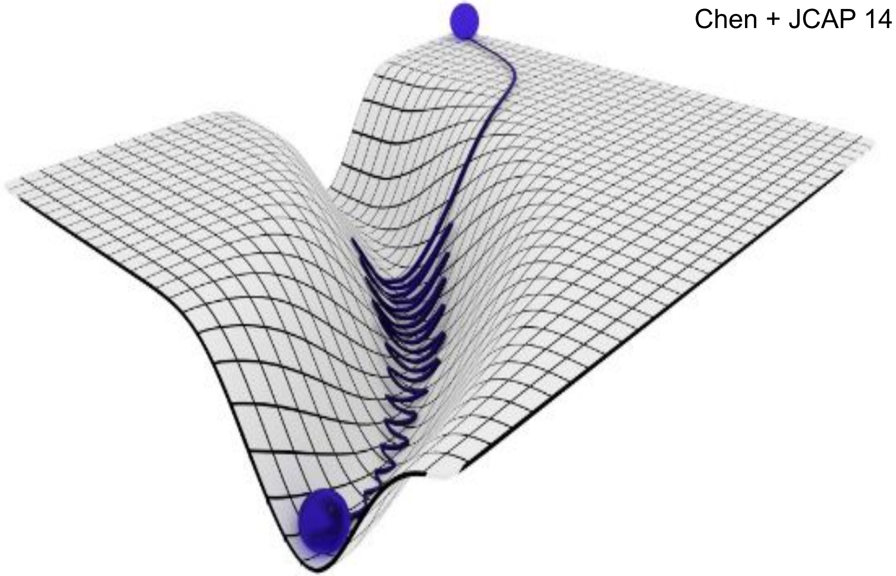
What is an inflaton ?

Scalar field that drives inflation by rolling down its `nearly flat’ potential

Inflaton can refer to a single or multiple fields



Chen + JCAP 14



Inflaton (Scalar fields -- single)

For FRW universe,the density and pressure term for the scalar field(ϕ) can be written as,

$$\rho = (\frac{\dot{\phi}^2}{2} + V)$$

$$p = (\frac{\dot{\phi}^2}{2} - V)$$

Equation of motion of the scalar field:

$$\ddot{\phi} + 3H\dot{\phi} + V_\phi = 0$$

where $V_\phi = \dfrac{dV}{d\phi}$

Inflation (necessary conditions)

To satisfy the condition for inflation we need

$$(\rho + 3p) < 0$$

For the scalar field this reduces to

$$\dot{\phi}^2 < V$$

In other words inflation can be achieved if the potential energy dominates the kinetic energy.
If we consider the field to be *slowly rolling* $i.e.$

$$\dot{\phi}^2 << V$$

then it proves to be a sufficient condition for inflation
To resolve horizon problem, we need at least ~ 60 e-folds.So the field should slow roll over a sufficiently long period of time

$$\ddot{\phi} << 3H\dot{\phi}$$

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Metric perturbation (Linear order)

$$G^{\mu}_{\nu} = 8\pi G T^{\mu}_{\nu}$$

where,

$$G^{\mu}_{\nu} = R^{\mu}_{\nu} - \frac{1}{2} R \delta^{\mu}_{\nu}$$

Introduce perturbation in FRW metric in (n+1) dimensions:

$$ds^2 = (1+2A)dt^2 - 2a(\partial_i B + S_i)dtdx^i - a^2(t)[(1-2\Psi)\delta^i_j + 2\partial_i\partial_j E + (\partial_i F_j + \partial_j F_i) + h_{ij}]dx^i dx^j$$

Degrees of freedom

Scalars=(A, B, Ψ, E)=4

Vectors=$\overrightarrow{F}$, $\overrightarrow{S}$=2(n-1)

Tensors=h_{ij}=$\frac{n(n+1)}{2}$

Metric perturbation (Linear order)

Only one scalar function in metric needed: $A = \Psi = \Phi$

$$3H(H\Phi + \dot{\Phi}) - (1/a^2)\nabla^2\Phi = -(4\pi G)\delta\rho$$

$$H\Phi + \dot{\Phi} = 4\pi G\delta q$$

$$\ddot{\Phi} + 4H\dot{\Phi} + (2\dot{\Phi} + 3H^2)\Phi = 4\pi G\delta p$$

$$\delta p = C_A^2\delta\rho + \delta p_{NA}$$ (Non adiabatic pressure pert.)

Next Step: Go to conformal time coordinate. Obtain a single scalar perturbation equation.

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Bardeen equation

$$\Phi'' + 3\mathcal{H}(1 + C_A^2)\Phi' - C_A^2\nabla^2\Phi + [2\mathcal{H}' + (1 + 3C_A^2\mathcal{H}^2)]\Phi = (4\pi G a^2)\delta p_{NA}$$

where,

$$\mathcal{H} = \frac{a'}{a} \text{The Conformal Hubble Parameter}$$

$$C_A^2 = (p'/\rho')$$

C_A^2=Adiabatic speed Of perturbation

Curvature perturbation (Conserved at SHS)

Define ,

$$\mathcal{R} = -\frac{1}{\mathcal{H}^2 - \mathcal{H}'}[\mathcal{H}\Phi' + (2\mathcal{H}^2 - \mathcal{H}')\Phi]$$

According to the Bardeen equation $\mathcal{R}$ satisfies,

$$\mathcal{R}' = -\frac{\mathcal{H}}{\mathcal{H}^2 - \mathcal{H}'}[(4\pi G a^2)\delta p_{NA} + C_A^2 \nabla^2 \Phi]$$

Neglect non-adiabatic case $\Rightarrow \delta p_{NA} = 0$, go to fourier space,

$$\mathcal{R}' = \frac{\mathcal{H} C_A^2}{\mathcal{H}^2 - \mathcal{H}'}(k^2 \Phi)$$

At super-Hubble limit,$k \to 0$ and we have $\mathcal{R}' \simeq 0$.
For adiabatic case, $\mathcal{R}_k$ is conserved at super-Hubble scales.

Curvature perturbation (Conserved at SHS)

Using the formula for the δp_{NA} , $\mathcal{R}'$ becomes

$$\mathcal{R}' = \left(\frac{\mathcal{H}}{\mathcal{H}^2 - \mathcal{H}'}\right) (\nabla^2 \Phi)$$

Combine $\mathcal{R}'$ equation with the Bardeen Equation,
$\Longrightarrow$ The equation for The curvature perturbation:

$$\boxed{\mathcal{R}'' + 2\frac{z'}{z}\mathcal{R}' - \nabla^2\mathcal{R} = 0}$$

where z is defined as,

$$z = \frac{a\phi'}{\mathcal{H}}$$

In the Fourier k-space

$$\boxed{\mathcal{R}_k'' + 2\frac{z'}{z}\mathcal{R}_k' + k^2\mathcal{R}_k = 0}$$

Mukhanov Sasaki Equations

Define v as $v = \mathcal{R}z$
after substitution we get

$$\boxed{v_k'' + \left[k^2 - \frac{z''}{z}\right] v_k = 0}$$

This is Mukhanov-Sasaki equation

$$\lim_{(k/\mathcal{H})\to\infty} v_k(\eta) \to \left(\frac{1}{\sqrt{2k}}\right) \mathrm{e}^{-ik\eta}$$

This is the initial condition of the equation at sub-Hubble scales known as Bunch-Davies initial condition

Power spectrum and its tilt

The Power Spectrum of the scalar perturbations($\mathcal{P}_S(k)$)

$$\mathcal{P}_S(k) = \left(\frac{k^3}{2\pi^2}\right)|\mathcal{R}_k|^2 = \left(\frac{k^3}{2\pi^2}\right)\left(\frac{|v_k|}{z}\right)^2$$

The $\mathcal{R}_k$ is evaluated at super-Hubble scales $i.e.$ at $(k/\mathcal{H}) << 1$
Scalar spectral index is defined as

$$n_s = 1 + \left(\frac{d\ln\mathcal{P}_s}{d\ln k}\right)$$

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Scalar amplitude and tilt and tensors

Simplest scalar power spectrum is characterized by an amplitude ($\boldsymbol{A_s}$) and a tilt ($\boldsymbol{n_s}$)

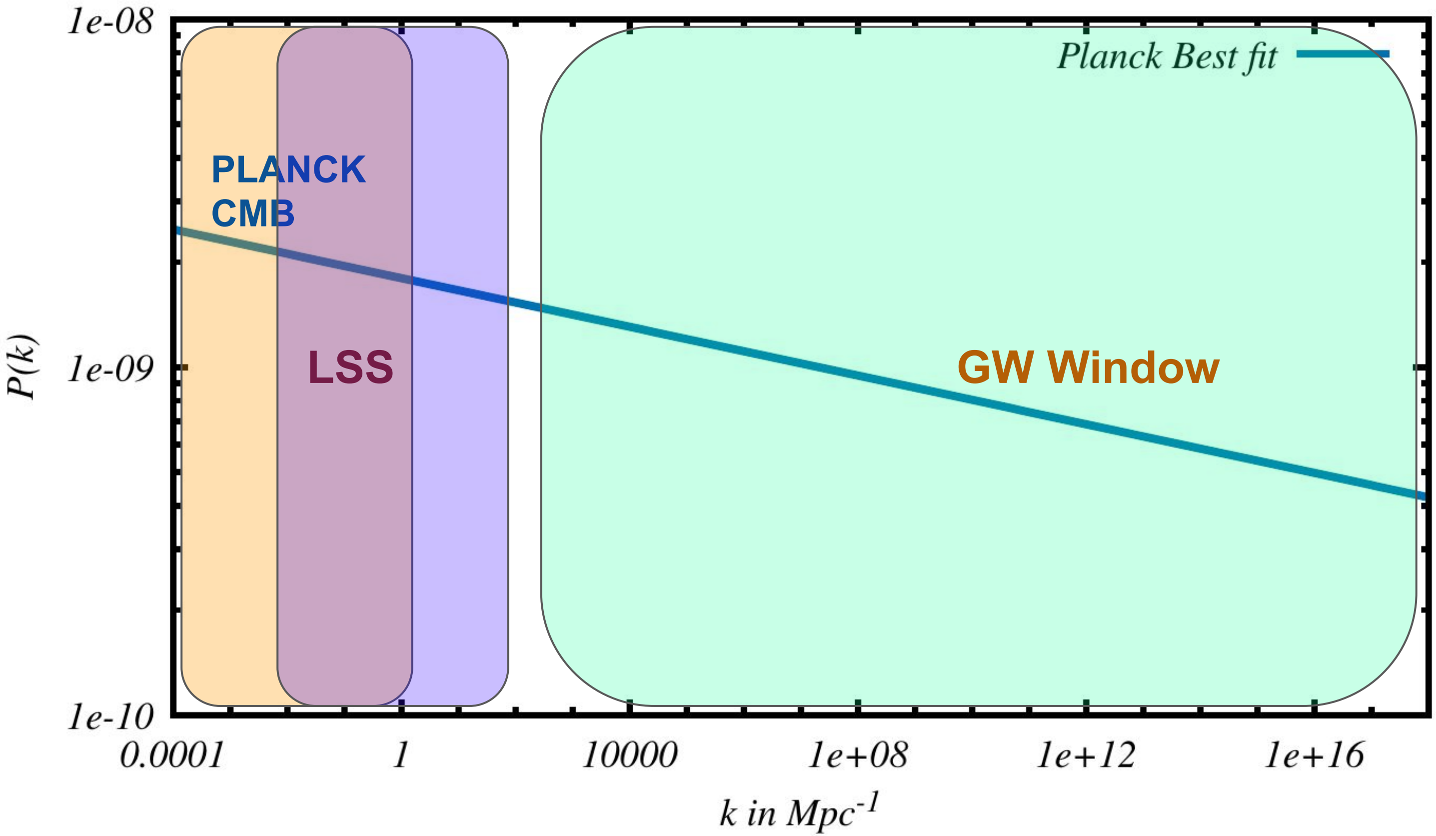
Similar to the generation of scalar perturbations inflation also produces tensor fluctuations

Ratio of tensor to scalar power spectrum amplitudes is commonly known as the tensor to scalar ratios ($\boldsymbol{r}$)

We have detected the $\boldsymbol{A_s}$ and $\boldsymbol{n_s}$ using CMB data

Detection of primordial gravitational waves can constrain $\boldsymbol{r}$

Scales of interest

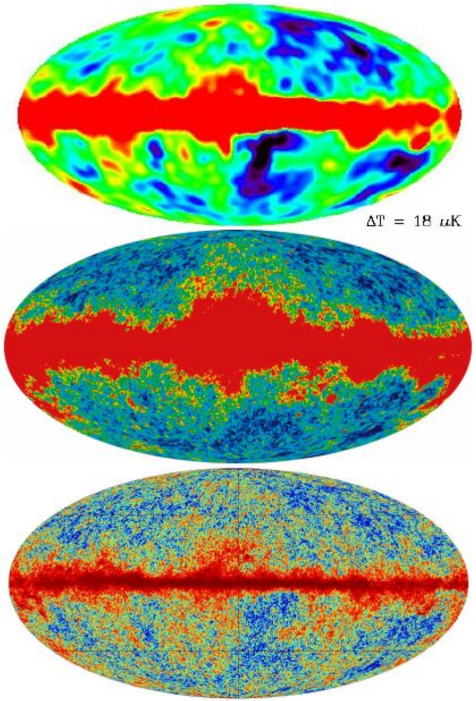


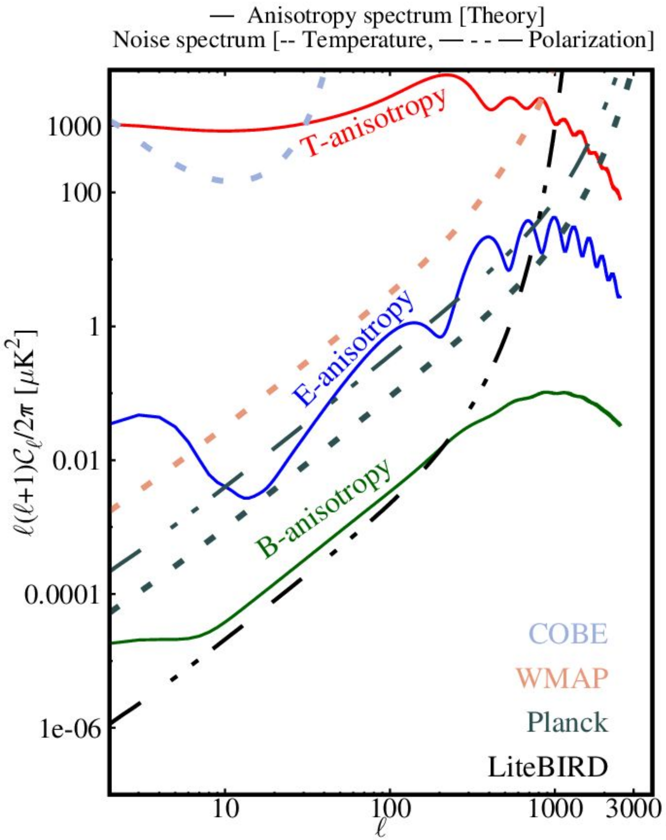
Cosmic Microwave Background

COBE

WMAP

Planck





Cosmic Microwave Background

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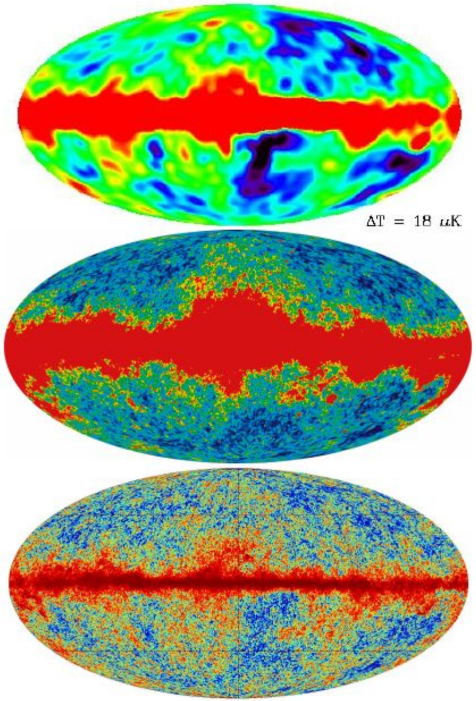

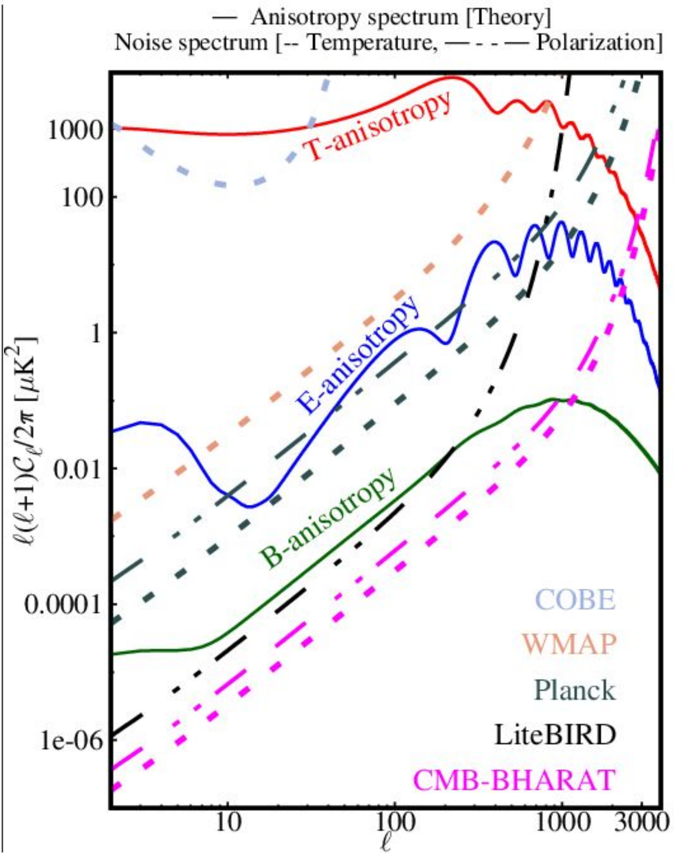

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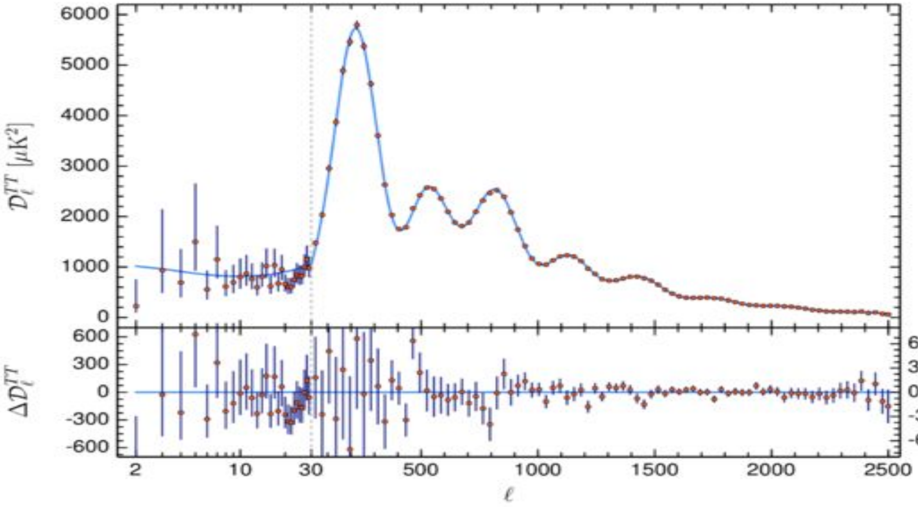
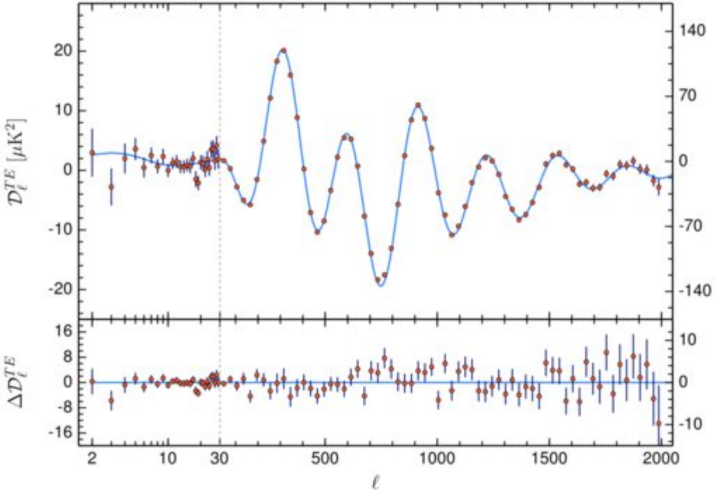
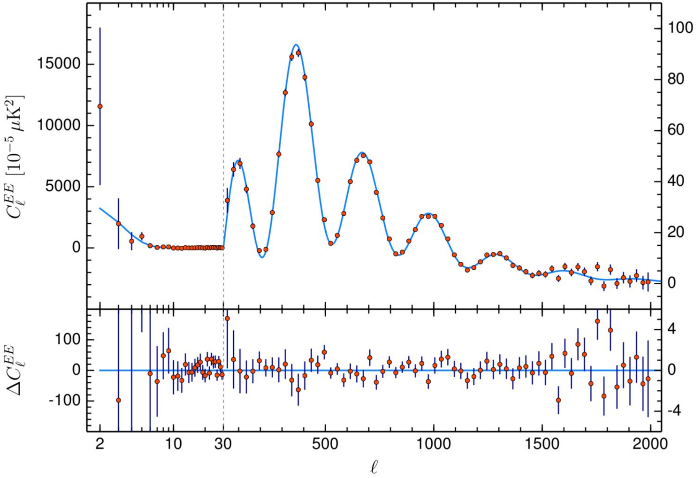
Planck power spectra







Planck 2018

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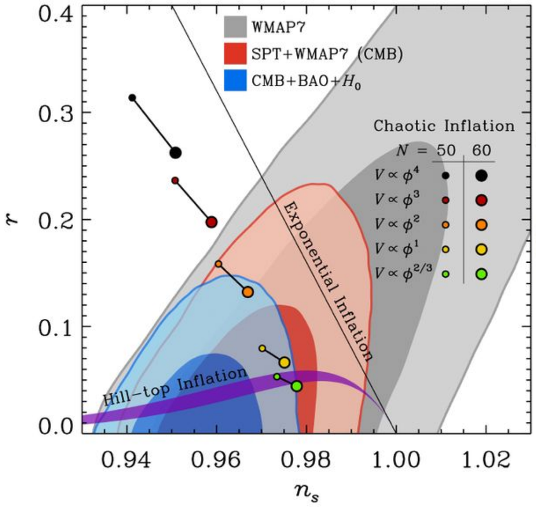
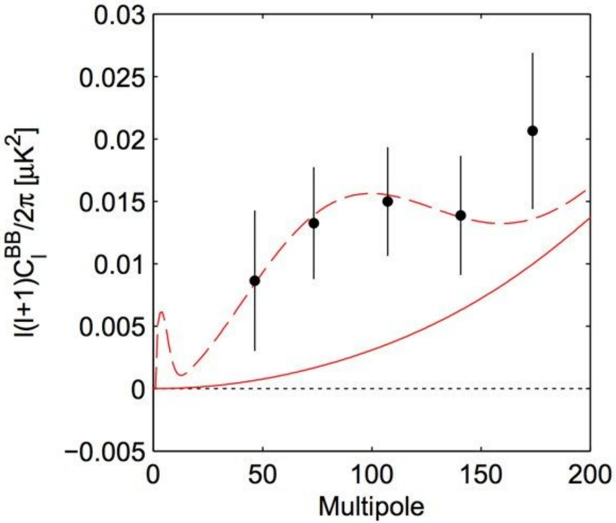
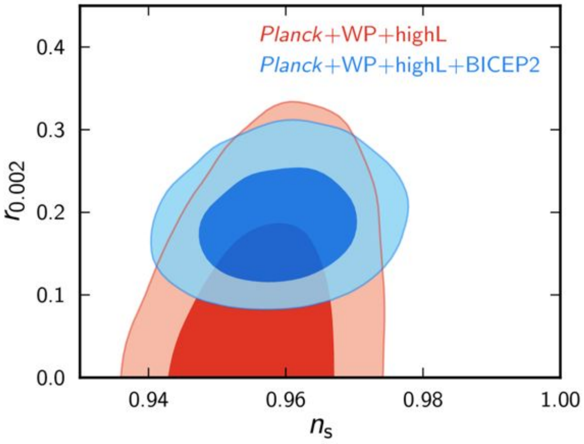
CMB constraints (WMAP)







10.1088/0004-637X/779/1/86

http://www.preposterousuniverse.com/blog/2014/03/16/bicep2-updates/

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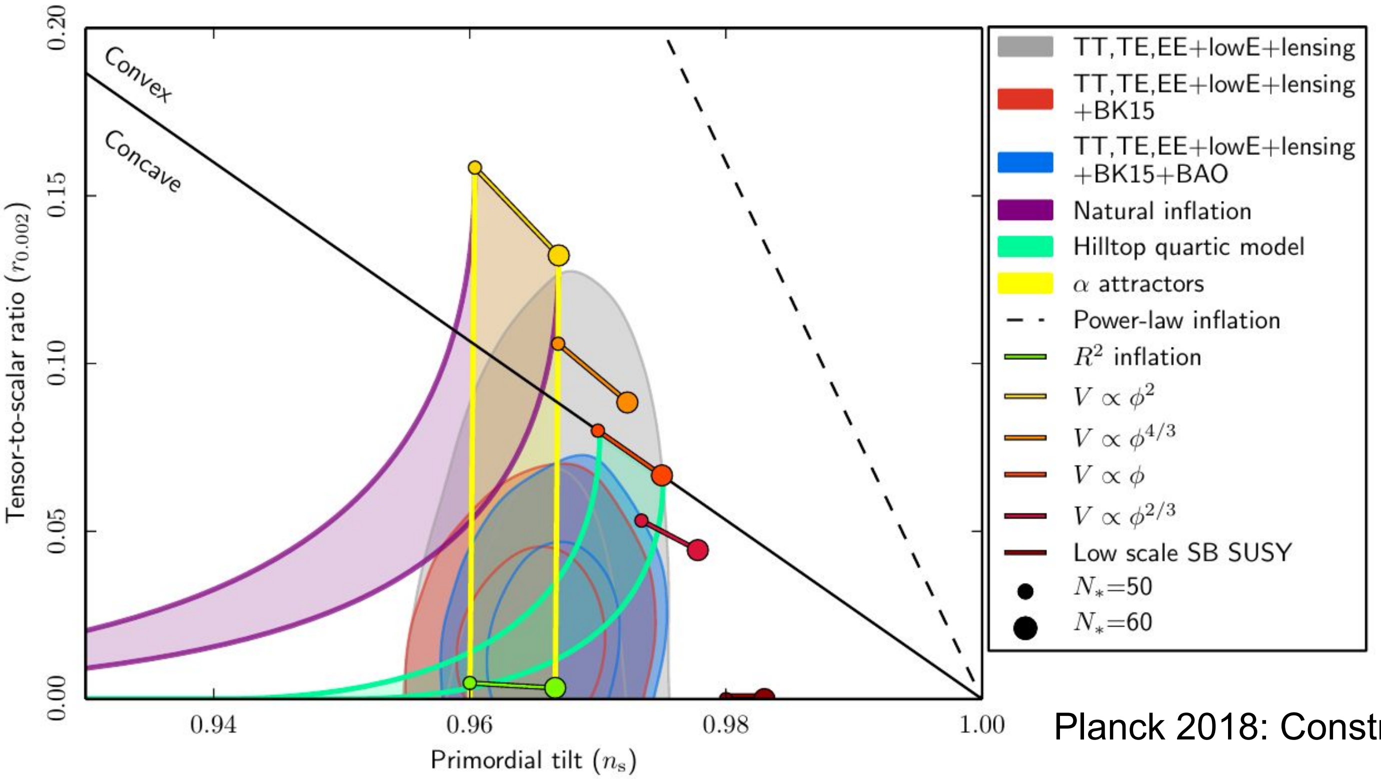
CMB constraints (Planck)



Planck 2018: Constraints on inflation

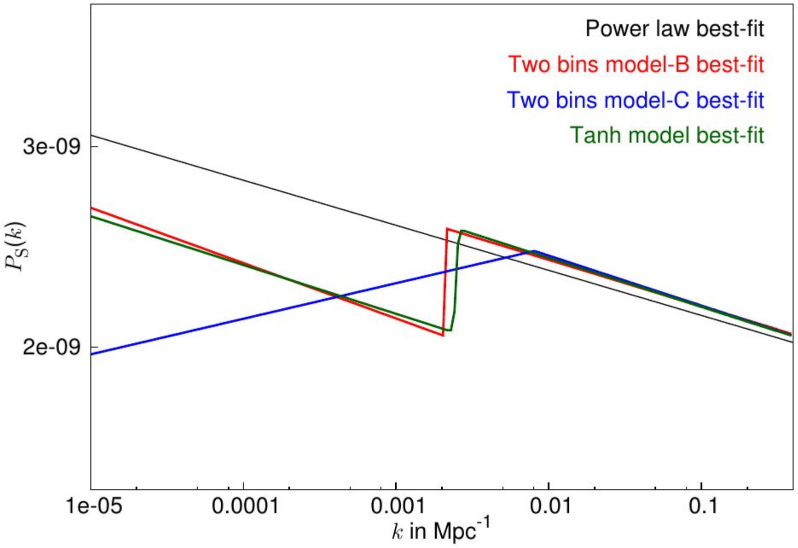
Inflation models and Planck

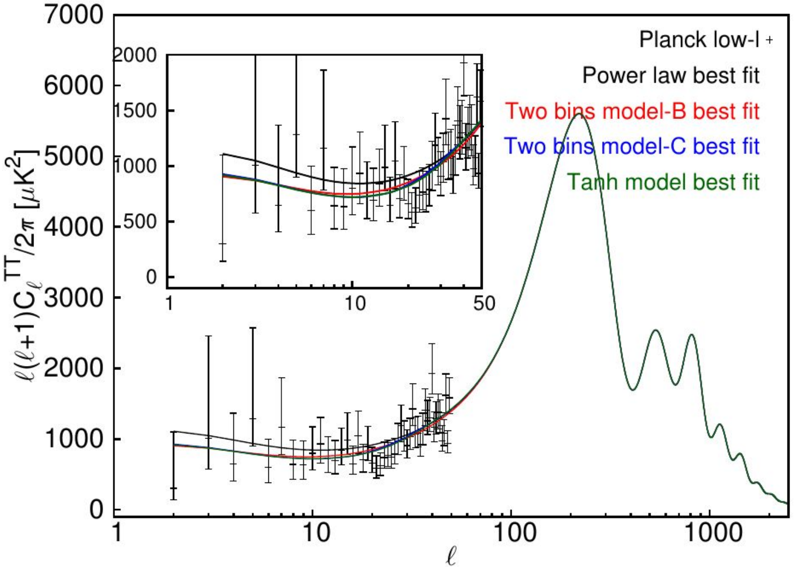
Inflationary model	Potential $V(\phi)$	Parameter range	$\Delta\chi^2$	$\ln B$
$R + R^2/(6M^2)$	$\Lambda^4\left(1 - e^{-\sqrt{2/3}\phi/M_{\rm Pl}}\right)^2$	...	...	...
Power-law potential	$\lambda M_{\rm Pl}^{10/3}\phi^{2/3}$	...	4.0	−4.6
Power-law potential	$\lambda M_{\rm Pl}^{3}\phi$	...	6.8	−3.9
Power-law potential	$\lambda M_{\rm Pl}^{8/3}\phi^{4/3}$	...	12.0	−6.4
Power-law potential	$\lambda M_{\rm Pl}^{2}\phi^{2}$	...	21.6	−11.5
Power-law potential	$\lambda M_{\rm Pl}\phi^{3}$	...	44.7	−13.2
Power-law potential	$\lambda\phi^{4}$	...	75.3	−56.0
Non-minimal coupling	$\lambda^4\phi^4 + \xi\phi^2 R/2$	$-4 < \log_{10}\xi < 4$	0.4	−2.4
Natural inflation	$\Lambda^4[1 + \cos(\phi/f)]$	$0.3 < \log_{10}(f/M_{\rm Pl}) < 2.5$	9.9	−6.6
Hilltop quadratic model	$\Lambda^4\left(1 - \phi^2/\mu_2^2 + \ldots\right)$	$0.3 < \log_{10}(\mu_2/M_{\rm Pl}) < 4.85$	1.3	−2.0
Hilltop quartic model	$\Lambda^4\left(1 - \phi^4/\mu_4^4 + \ldots\right)$	$-2 < \log_{10}(\mu_4/M_{\rm Pl}) < 2$	−0.3	−1.4
D-brane inflation ($p = 2$)	$\Lambda^4\left(1 - \mu_{D2}^2/\phi^p + \ldots\right)$	$-6 < \log_{10}(\mu_{D2}/M_{\rm Pl}) < 0.3$	−2.0	0.6
D-brane inflation ($p = 4$)	$\Lambda^4\left(1 - \mu_{D4}^4/\phi^p + \ldots\right)$	$-6 < \log_{10}(\mu_{D4}/M_{\rm Pl}) < 0.3$	−3.5	−0.4
Potential with exponential tails	$\Lambda^4[1 - \exp(-q\phi/M_{\rm Pl}) + \ldots]$	$-3 < \log_{10} q < 3$	−0.4	−1.0
Spontaneously broken SUSY	$\Lambda^4[1 + \alpha_h \log(\phi/M_{\rm Pl}) + \ldots]$	$-2.5 < \log_{10}\alpha_h < 1$	6.7	−6.8
E-model ($n = 1$)	$\Lambda^4\left\{1 - \exp\left[-\sqrt{2}\phi\left(\sqrt{3\alpha_1^{\rm E}}M_{\rm Pl}\right)^{-1}\right]\right\}^{2n}$	$-2 < \log_{10}\alpha_1^{\rm E} < 4$	0.8	−0.3
E-model ($n = 2$)	$\Lambda^4\left\{1 - \exp\left[-\sqrt{2}\phi\left(\sqrt{3\alpha_2^{\rm E}}M_{\rm Pl}\right)^{-1}\right]\right\}^{2n}$	$-2 < \log_{10}\alpha_2^{\rm E} < 4$	0.8	−1.6
T-model ($m = 1$)	$\Lambda^4 \tanh^{2m}\left[\phi\left(\sqrt{6\alpha_1^{\rm T}}M_{\rm Pl}\right)^{-1}\right]$	$-2 < \log_{10}\alpha_1^{\rm T} < 4$	−0.1	−1.2
T-model ($m = 2$)	$\Lambda^4 \tanh^{2m}\left[\phi\left(\sqrt{6\alpha_2^{\rm T}}M_{\rm Pl}\right)^{-1}\right]$	$-2 < \log_{10}\alpha_2^{\rm T} < 4$	0.8	−0.6

We were happy with these models 10 years ago

Planck 2018: Constraints on inflation

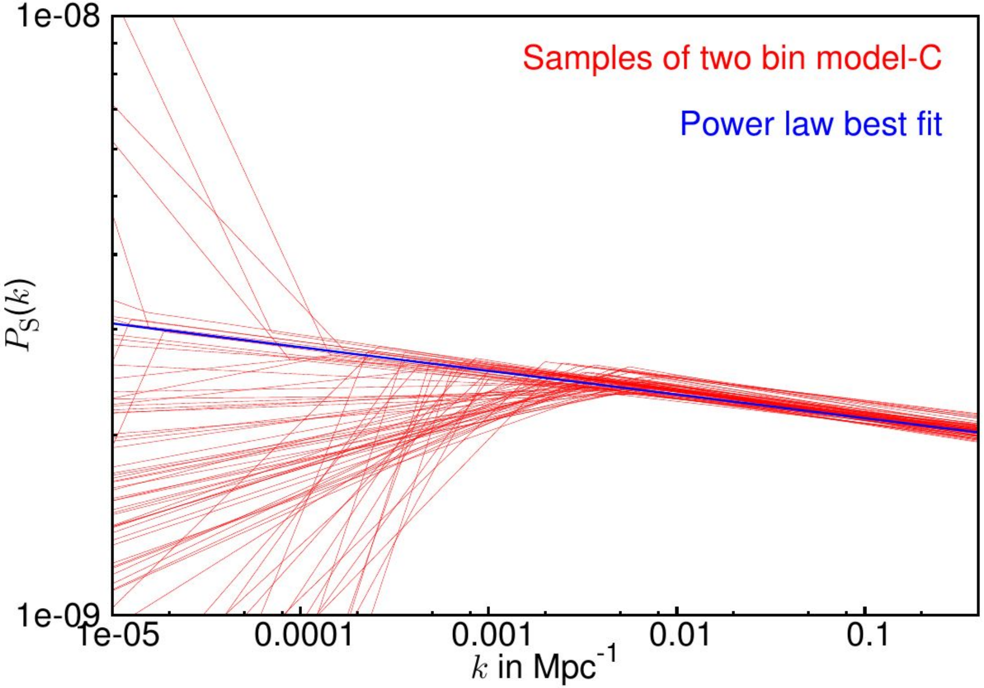
Simple test of scale dependence





Hazra, Shafieloo, Smoot JCAP 2013

Simple test of scale dependence



The spectral index is constrained to be ‘red’ after **0.01 Mpc^{-1}**

For scales larger than 0.01 Mpc^{-1} the TT data prefers a blue tilt

Hazra, Shafieloo, Smoot JCAP 2013

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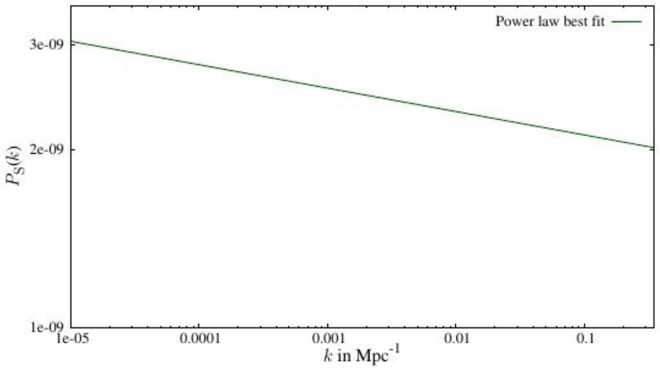
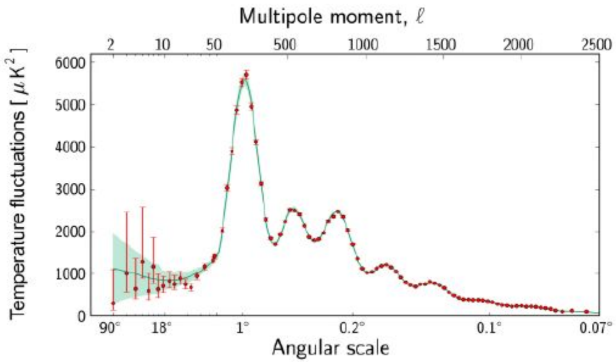
Reconstruction of localized features

Primordial power spectrum



Angular power spectrum (Planck)

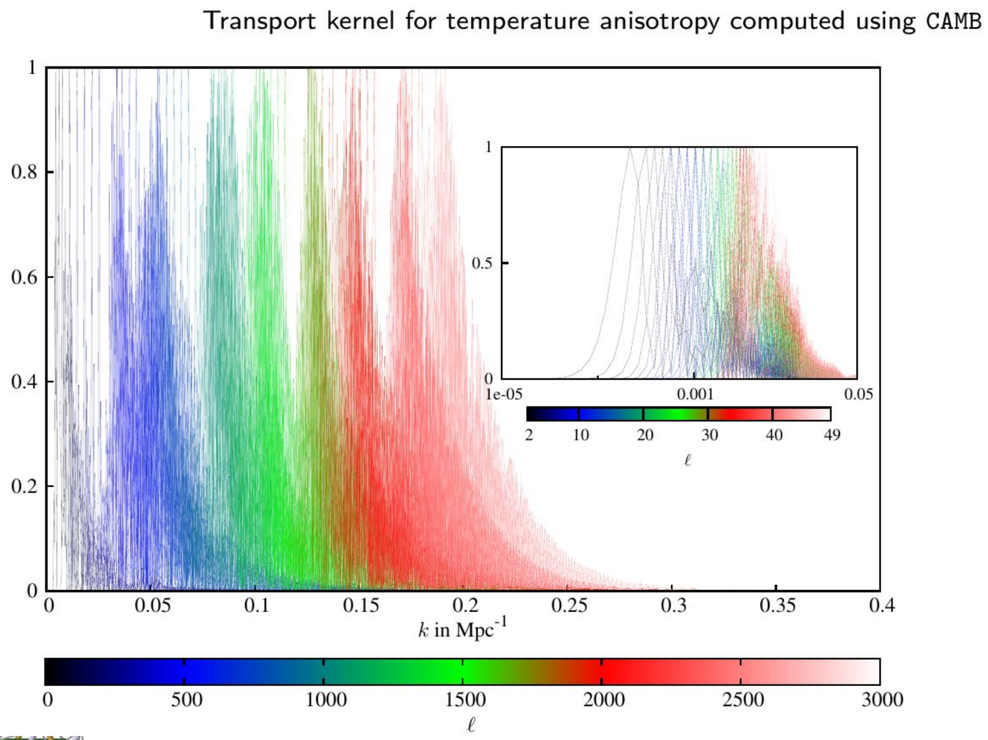


$$C_\ell^{\mathrm{T}} = \sum_i G_{\ell k_i} P_{k_i}$$

$G_{\ell k}$ is the radiative transport kernel

Hazra, Shafieloo, Souradeep JCAP 2014

Reconstruction of localized features



The transport kernel depends on background cosmology

Using a baseline cosmology we attempt to reconstruct the primordial power spectrum from the CMB angular power spectrum data

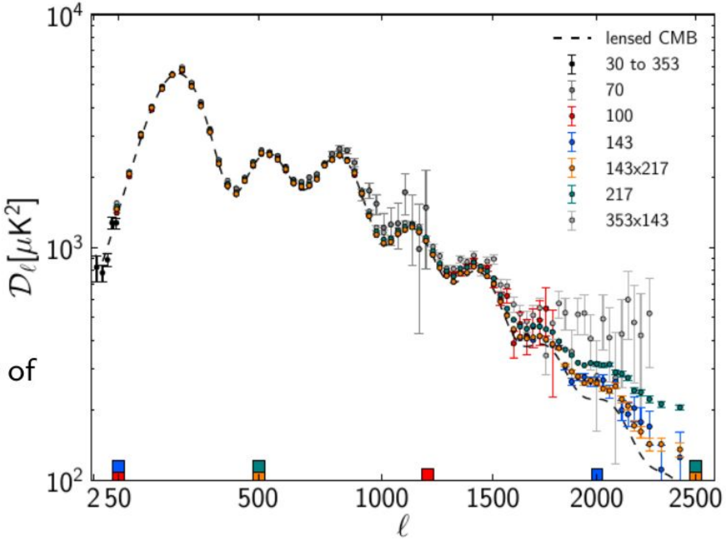
Hazra, Shafieloo, Souradeep JCAP 2014

Reconstruction (Richardson-Lucy algorithm)

Richardson (1972) and Lucy (1974)

$$P_k^{(i+1)} - P_k^{(i)} = P_k^{(i)} \times \left[\sum_\ell \widetilde{G}_{\ell k} \left(\frac{C_\ell^{\mathrm{D}} - C_\ell^{\mathrm{T}(i)}}{C_\ell^{\mathrm{T}(i)}} \right) \right]$$

Angular power spectra (in different Planck frequencies)



Planck 2013

- 5 different spectra for parameter estimation, calculated from combinations of maps in different frequency channels
- Foreground and calibration effects
- Substantial lensing

Hazra, Shafieloo, Souradeep JCAP 2014

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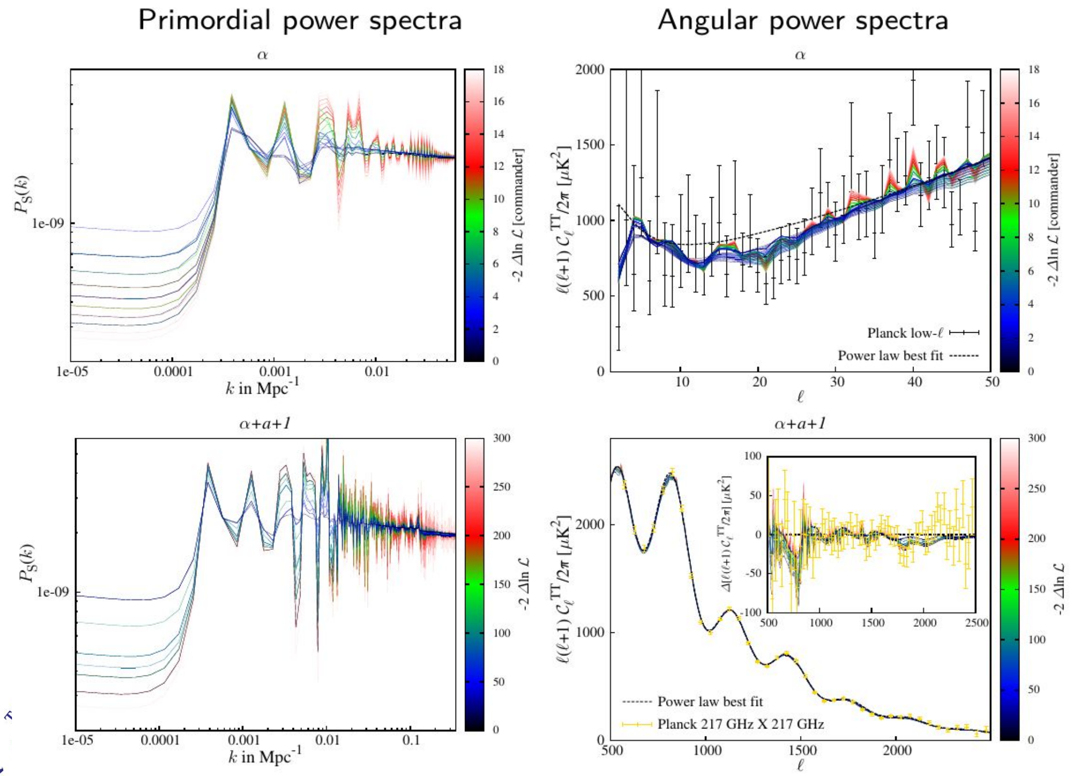
Reconstruction (Modified Richardson-Lucy)



MRL reconstructs the free-form primordial power spectrum from different combinations of frequency channels

Helps to identify features present in all frequencies

Also helps to check consistencies between frequencies

łazra, Shafieloo, Souradeep JCAP 2014

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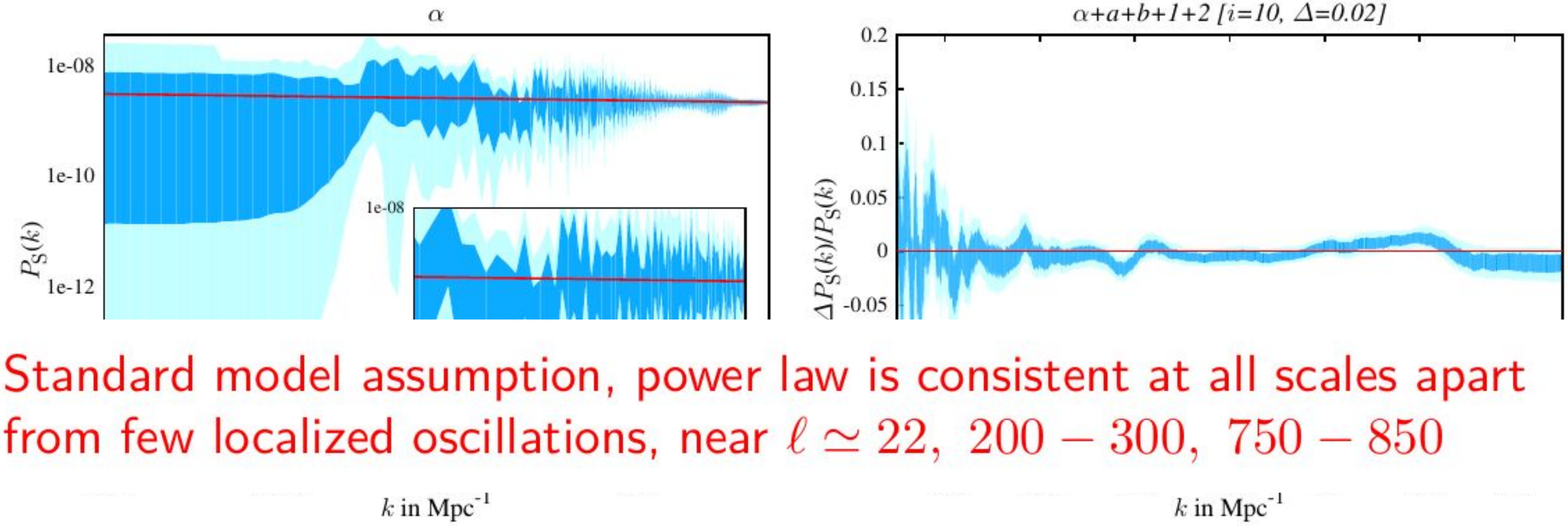
Features that seem '*important*'



Standard model assumption, power law is consistent at all scales apart from few localized oscillations, near $\ell \simeq 22,\ 200-300,\ 750-850$

Hazra, Shafieloo, Souradeep JCAP 2014

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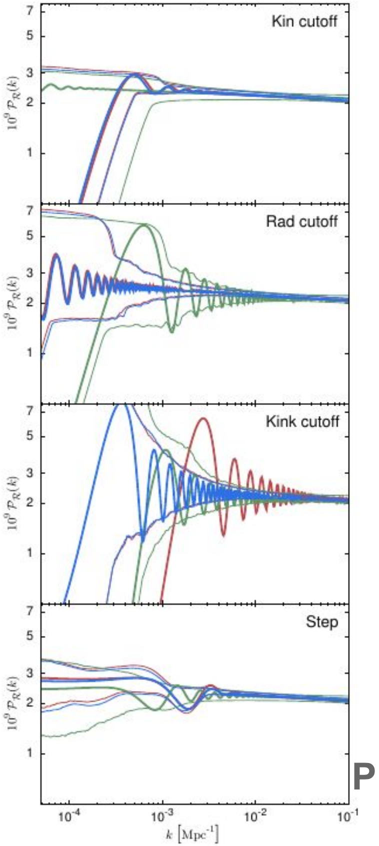
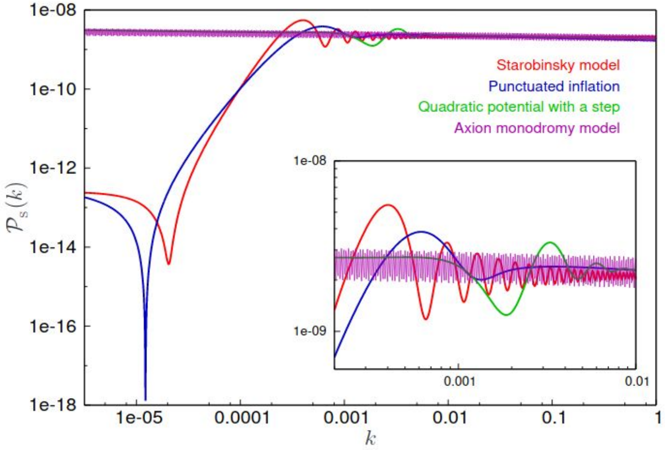
Possible *'features'*

PI: Jain+.2009

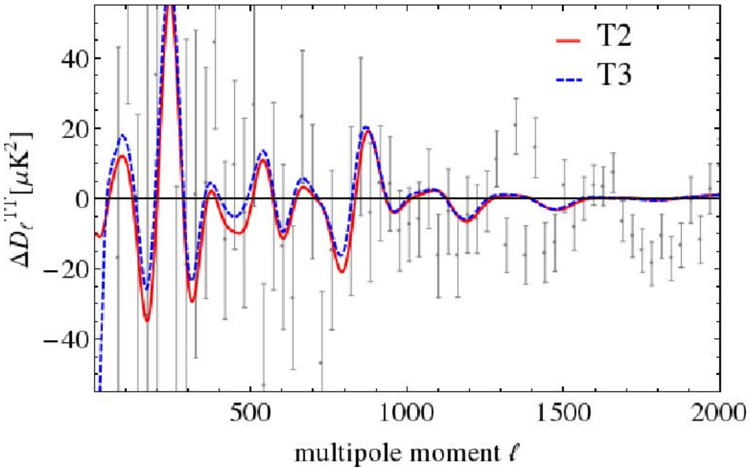
Hints: large scale suppression, few intermediate and smaller scale oscillations



Planck 2018: Constraints on Inflation



Hazra, Sriramkumar, Martin 2013



Chen + 2015

Two point and three point statistics

We can use both power spectrum and bispectra information from the models to compare with the data

Since departure from slow roll can not be solved accurately with analytical approximations, we need efficient numerical tools

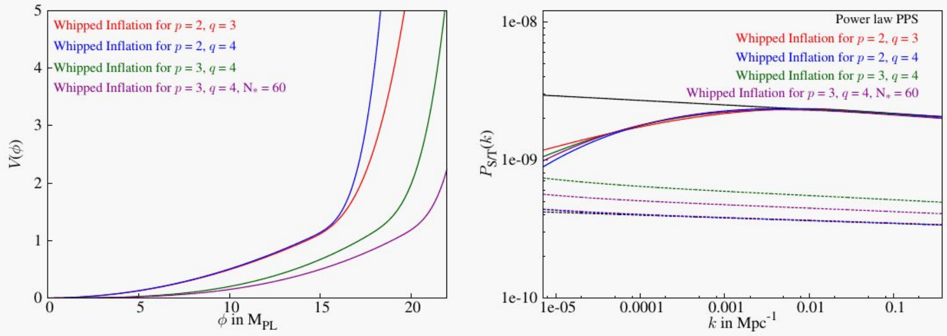
BINGO -- **BI**-spectra and **N**on-**G**aussianity **O**perator (Hazra, Sriramkumar, Martin JCAP 2013) solves for power spectra and bispectra in arbitrary triangular configurations. BINGO is the first public code to compute the bispectra and probably the fastest to compute the power spectra

What about the potential ?

Whipped Inflation potential

$$V(\phi) = V_S(\phi) + \gamma V_R(\phi)$$

Moderate fast-roll $\Longrightarrow$ strict slow-roll



A plan to construct a framework of inflation potential that can generate features with the hints from reconstruction.

Hazra, Shafieloo, Smoot, Starobinsky PRL 2014

What about the potential ?

To have features in the PPS, generate low amplitude tensor perturbations we propose : **WWI**

$$V(\phi) = V_i \left(1 - \left(\frac{\phi}{\mu}\right)^p\right) + \Theta(\phi_{\mathrm{T}} - \phi) V_i \left(\gamma(\phi_{\mathrm{T}} - \phi)^q + \phi_{01}^q\right),$$

and : **WWI′**

$$V(\phi) = \Theta(\phi_{\mathrm{T}} - \phi) V_i \left(1 - \exp\left[-\alpha\kappa\phi\right]\right) + \Theta(\phi - \phi_{\mathrm{T}}) V_{ii} \left(1 - \exp\left[-\alpha\kappa(\phi - \phi_{01})\right]\right)$$

Inflaton transits from a **moderate fast roll** to a **strict slow roll** through a **discontinuity**

A plan to construct a framework of inflation potential that can generate features with the hints from reconstruction.

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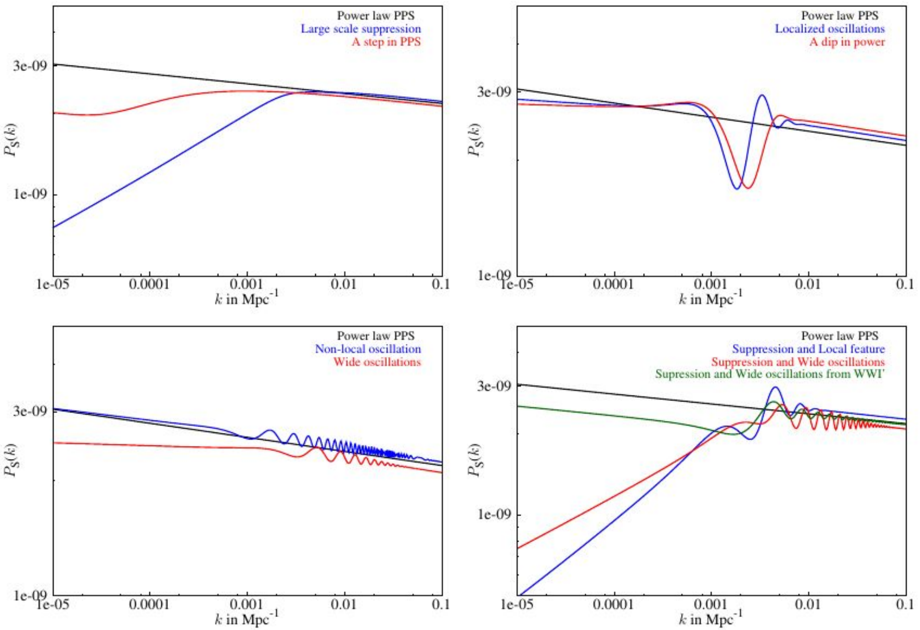
Hazra, Shafieloo, Smoot, Starobinsky JCAP 2016

Wiggly Whipped Inflation



Several classes of features can be generated with WWI, such as large scale suppression, localized and non-local oscillations

Hazra, Paoletti, Debono, Shafieloo, Smoot, Starobinsky, arXiv 2021

Analysis with post-Planck 2018 (WWI)

Likelihood type	Key	Multipole range (ℓ)	Likelihood name in official release
Commander	lowT	2-29	commander_dx12_v3_2_29
Simall	lowE	2-29	simall_100x143_offlike5_ EE_Aplanck_B
PlikHM binned	Plik-TT Plik-EE Plik-TTTEEE	TT: 30-2508 EE: 30-1996 TE: 30-1996	plik_rd12_HM_v22_TT plik_rd12_HM_v22_EE plik_rd12_HM_v22b_TTTEEE
Clean CamSpec (EG20)	TT EE TTTEEE	TT: 30-2500 EE: 30-2000 TE: 30-2000	v12.5HMclnTT v12.5HMclnEE v12.5HMclnTE v12.5HMclnTTTEEE
BICEP-KECK 2015	BK15	50-300	BK15_bandpowers_20180920

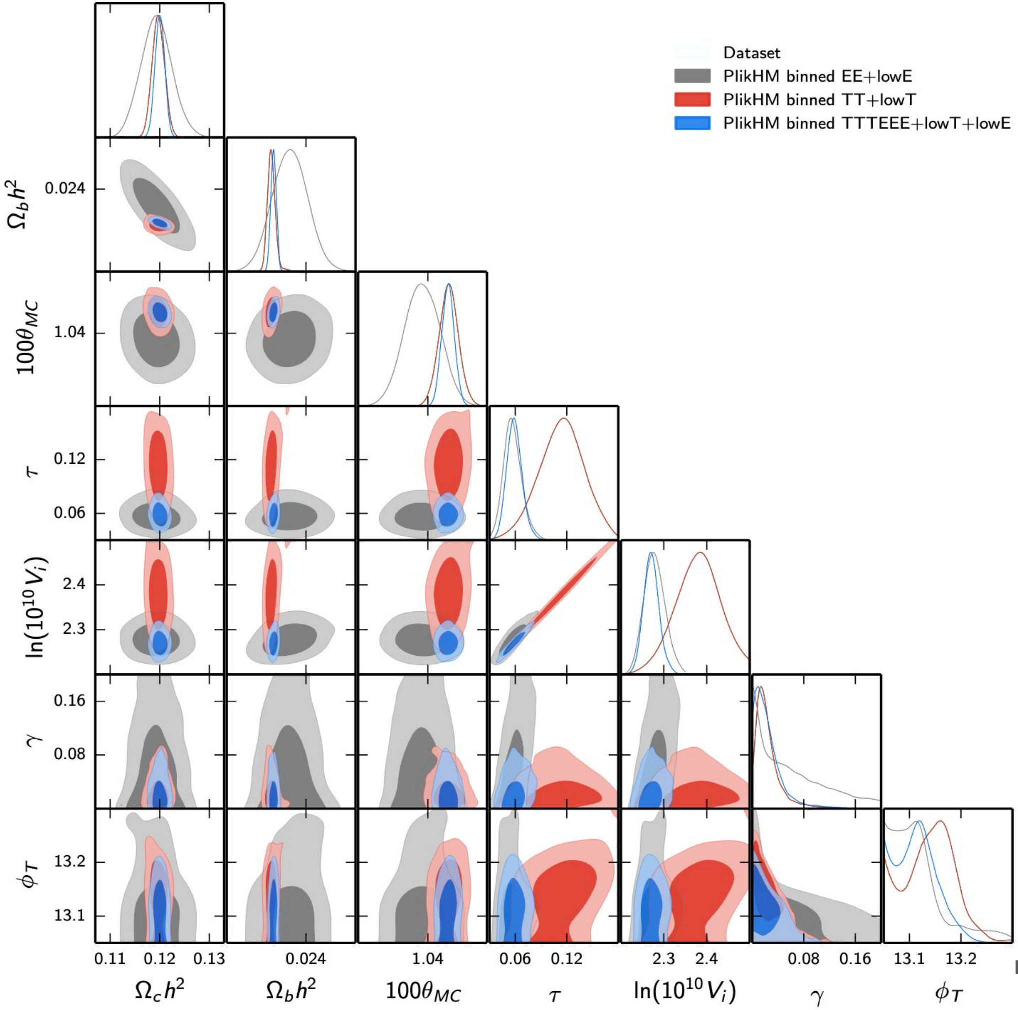
Hazra, Paoletti, Debono, Shafieloo, Smoot, Starobinsky, arXiv 2021

Analysis with post-Planck 2018 (WWI)

	Model		
Data set	WI	WWI	WWIP
TT+lowT	0.12	NA	NA
TT+lowT+lowE	NA	-2.3	-2.5
TE+lowE	NA	-2.3	-2.8
EE+lowE	-1.8	-2.4	-2.1
TTTEEE+lowT+lowE	-1.4	-2.5	-1.61

Hazra, Paoletti, Debono, Shafieloo, Smoot, Starobinsky, arXiv 2021

August 28, 2021



Hazra, Paoletti, Debono, Shafieloo, Smoot, Starobinsky, arXiv 2021

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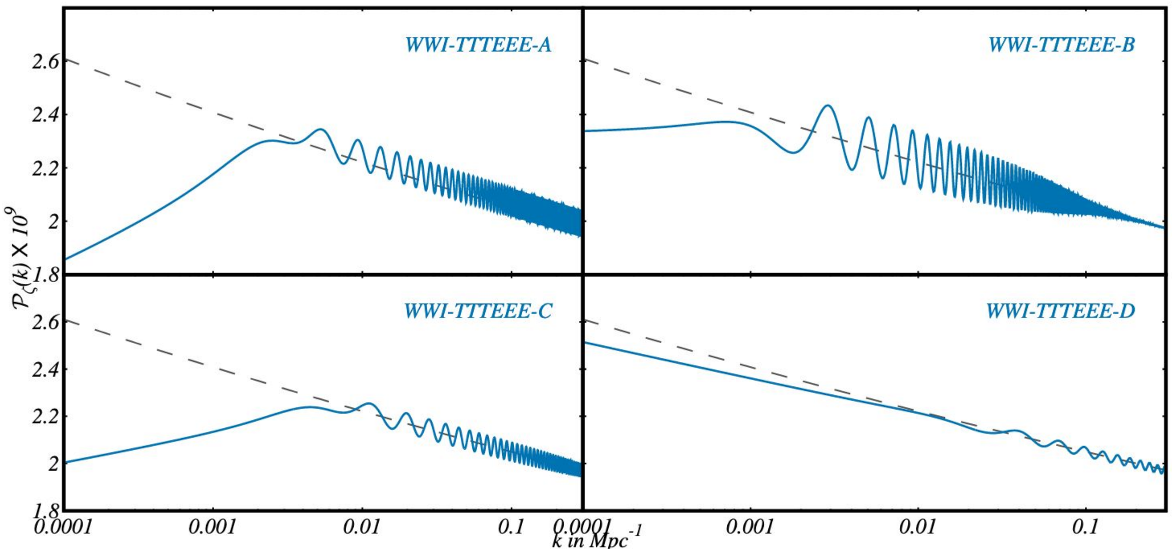
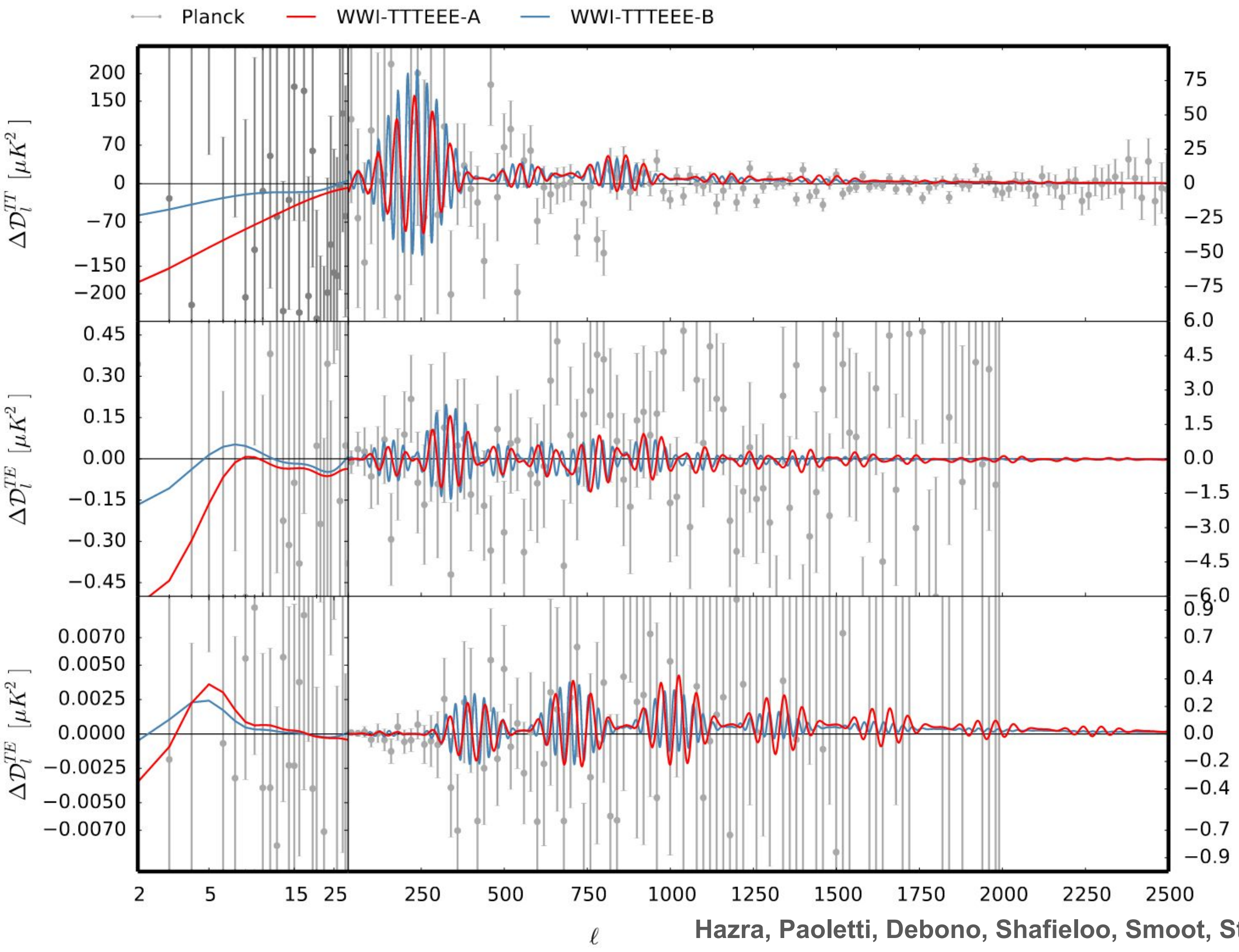
Analysis with post-Planck 2018 (WWI)



Hazra, Paoletti, Debono, Shafieloo, Smoot, Starobinsky, arXiv 2021

August 28, 2021



Hazra, Paoletti, Debono, Shafieloo, Smoot, Starobinsky, arXiv 2021

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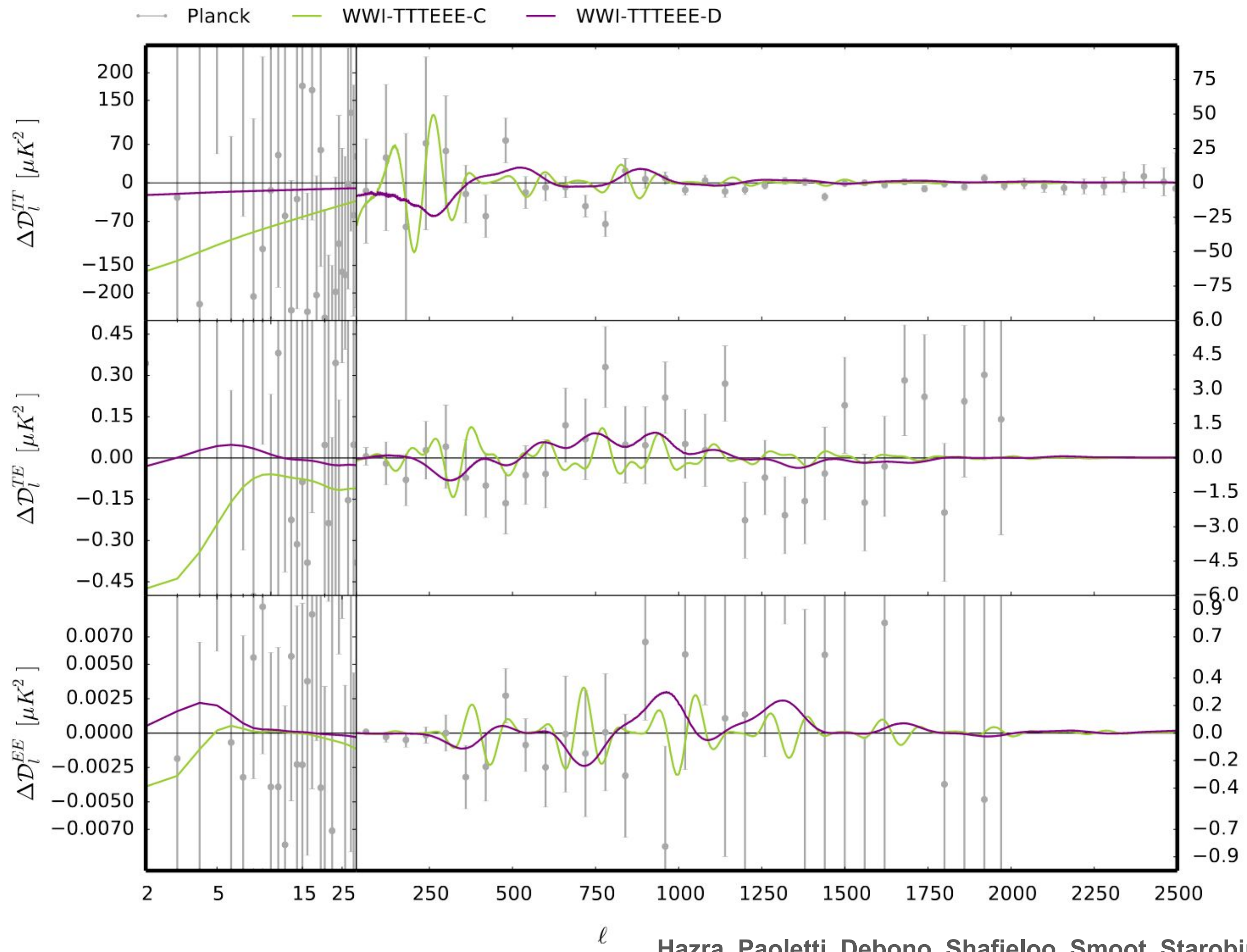

Hazra, Paoletti, Debono, Shafieloo, Smoot, Starobinsky, arXiv 2021

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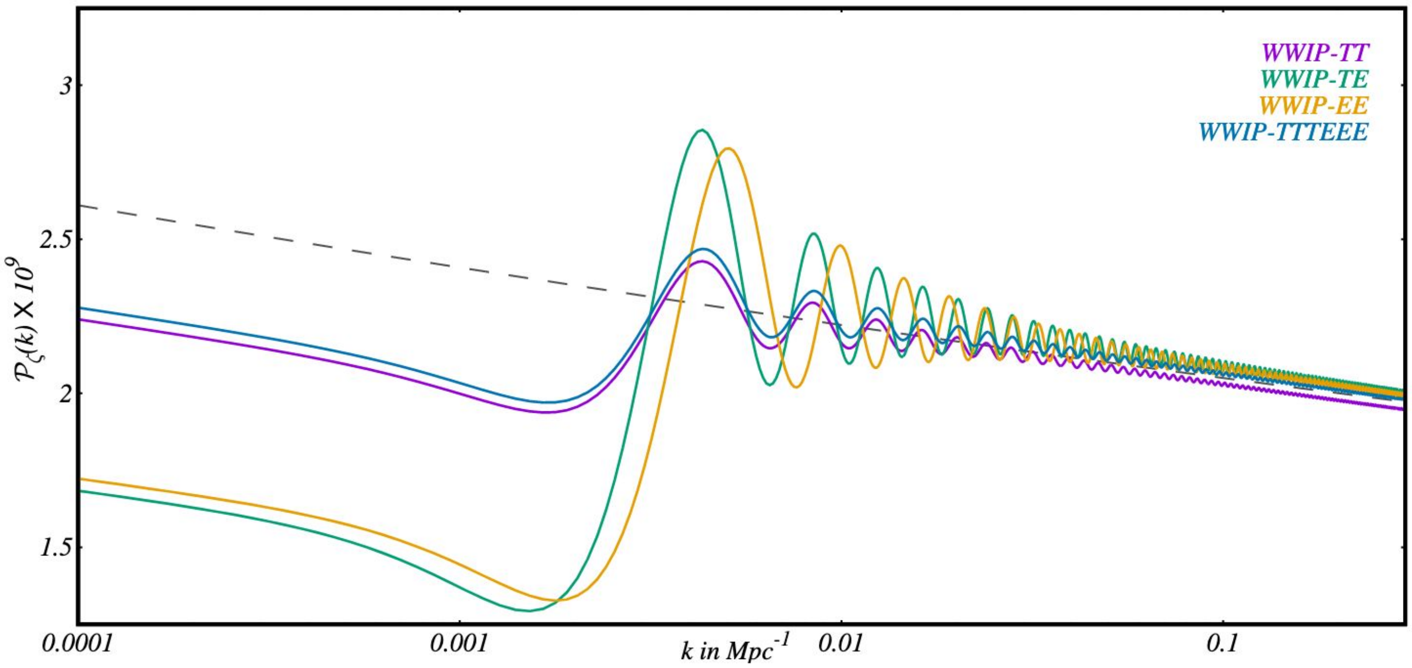
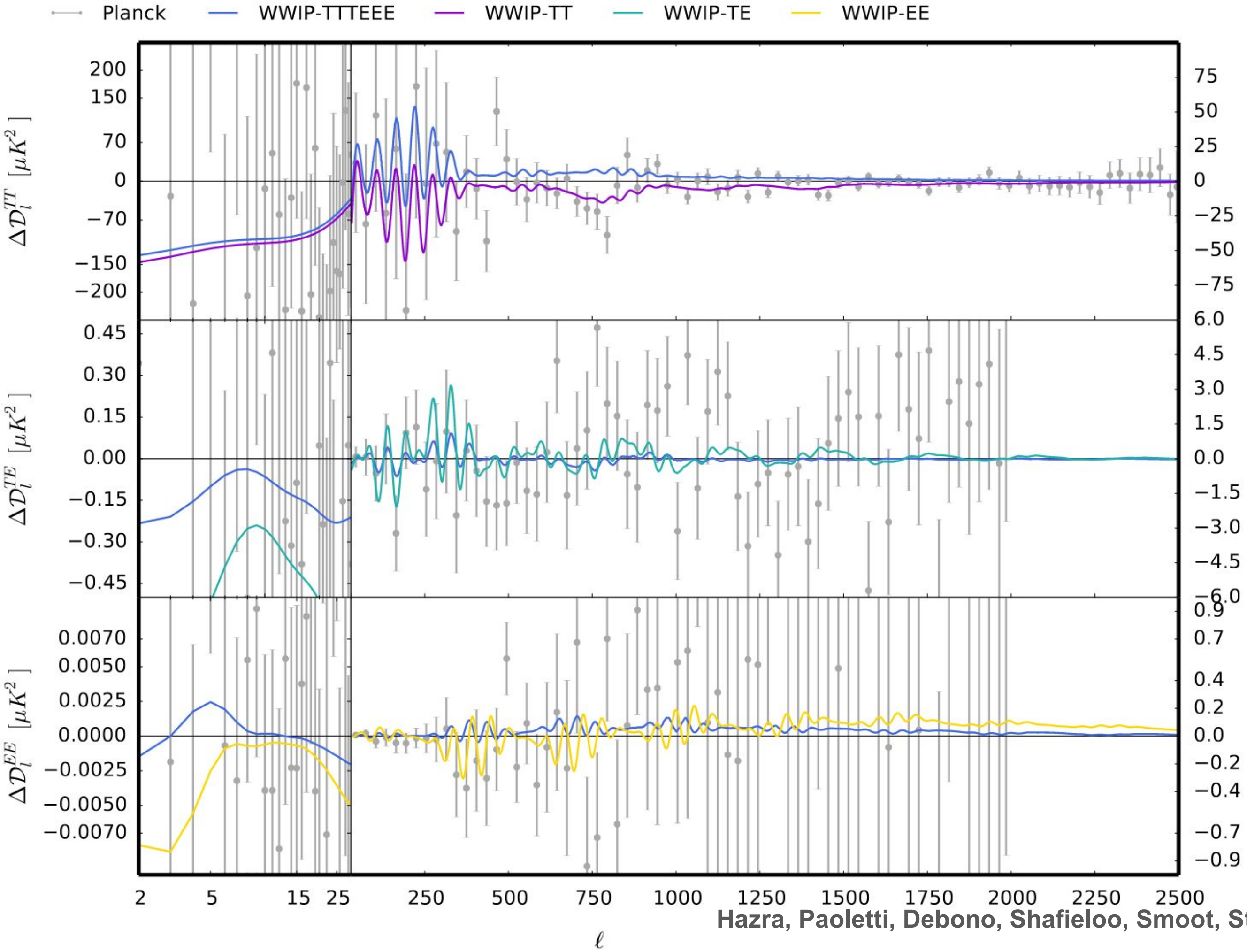


Hazra, Paoletti, Debono, Shafieloo, Smoot, Starobinsky, arXiv 2021

August 28, 2021



Hazra, Paoletti, Debono, Shafieloo, Smoot, Starobinsky, arXiv 2021

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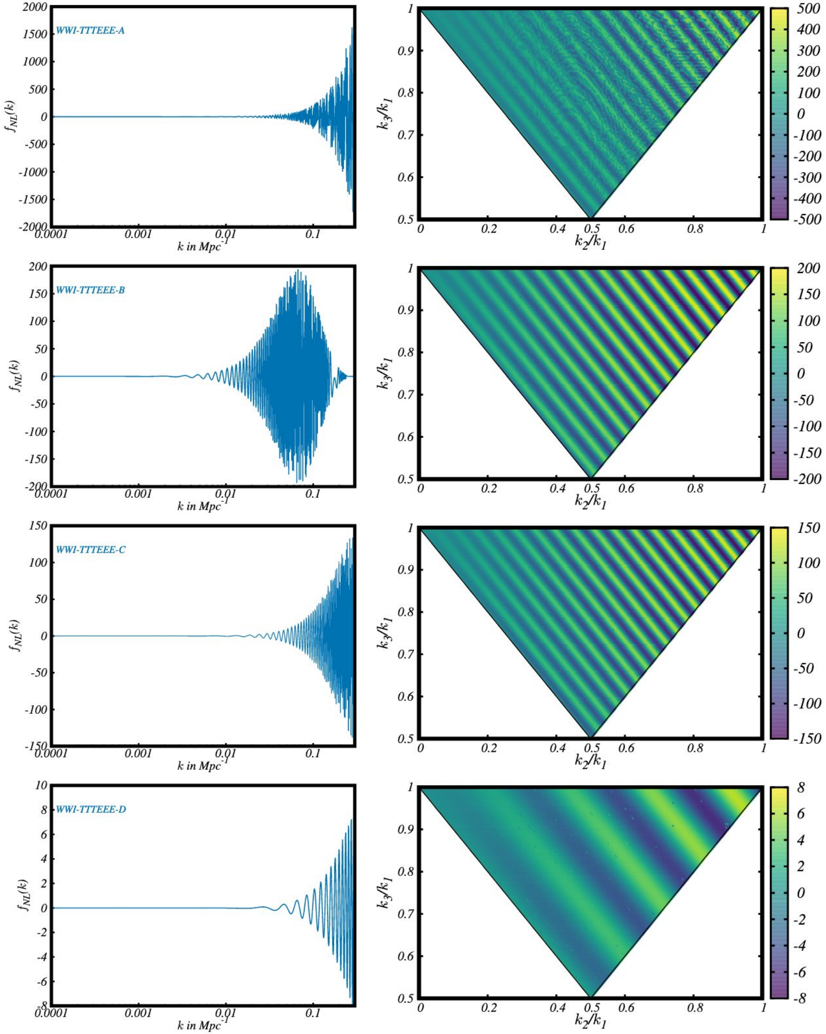
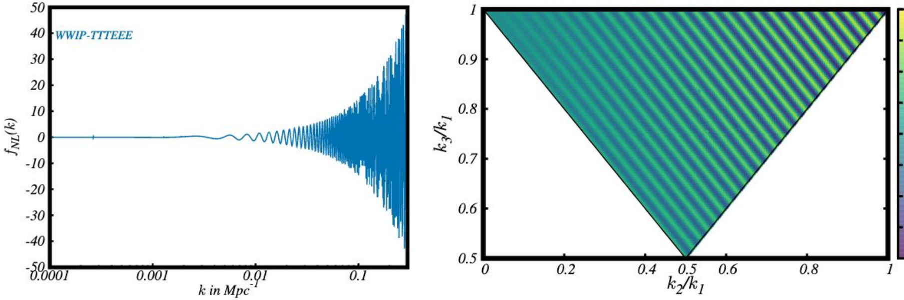
Three point correlation

WWI

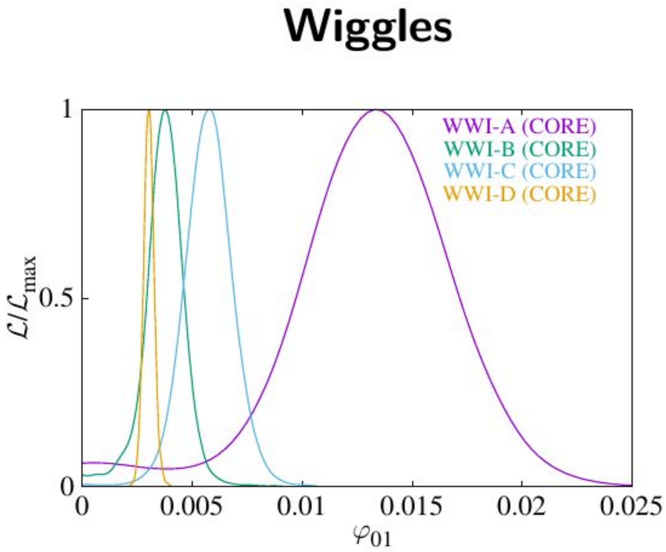


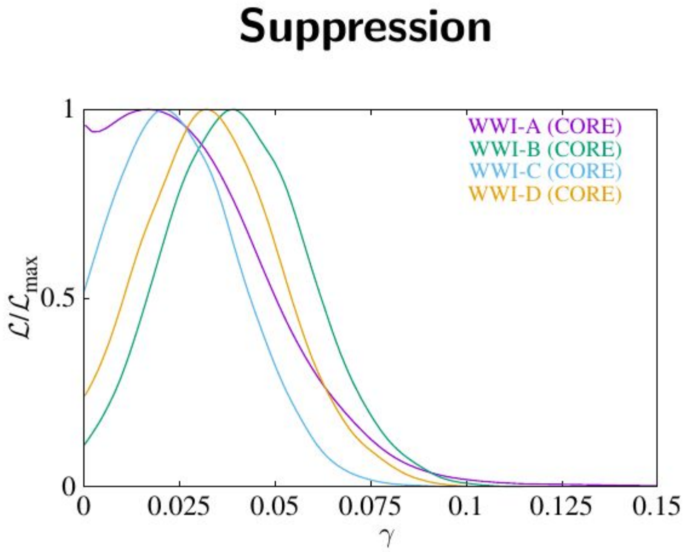
WWI'



Hazra, Paoletti, Debono, Shafieloo, Smoot, Starobinsky, arXiv 2021

Features in the future(CORE/CMBBHARAT forecast)





Even with Cosmic Variance limited surveys, it will be difficult to detect large scale suppression.

Intermediate and small scale oscillations, if present, can be detected with high significance

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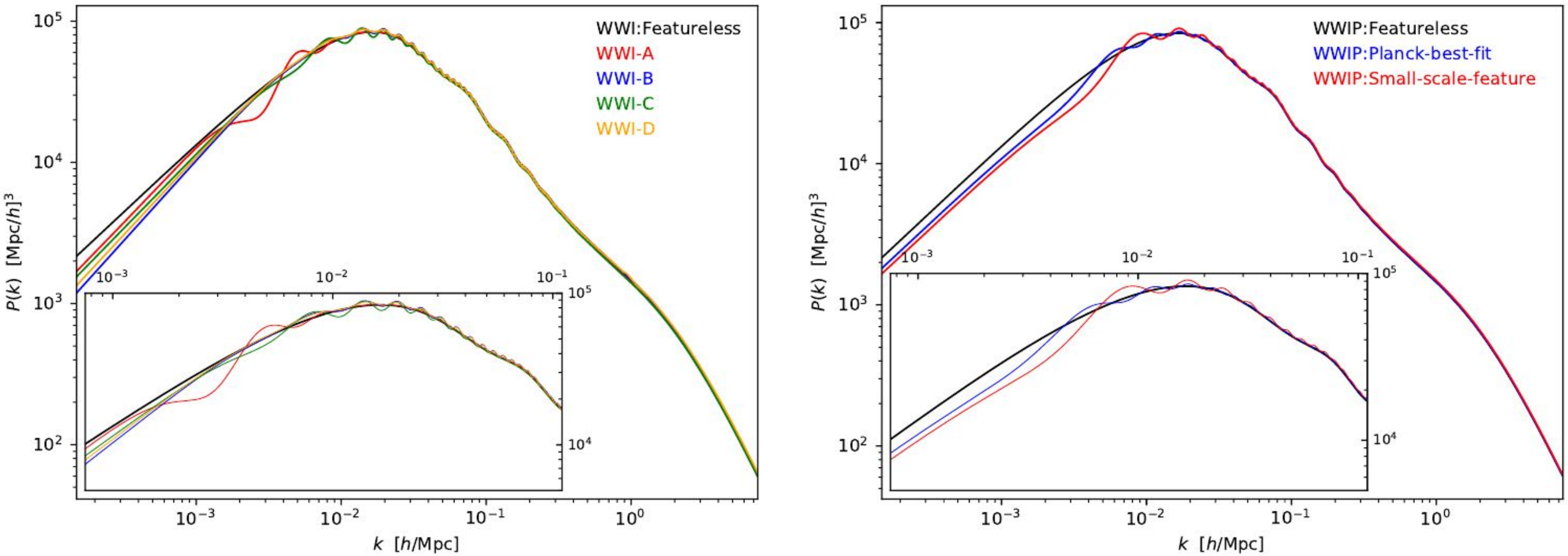
Hazra, Paoletti, Ballardini, Finelli, Shafieloo, Smoot, Starobinsky JCAP 2017

Large Scale Structure



Debono, Hazra, Shafieloo, Smoot, Starobinsky MNRAS 2020

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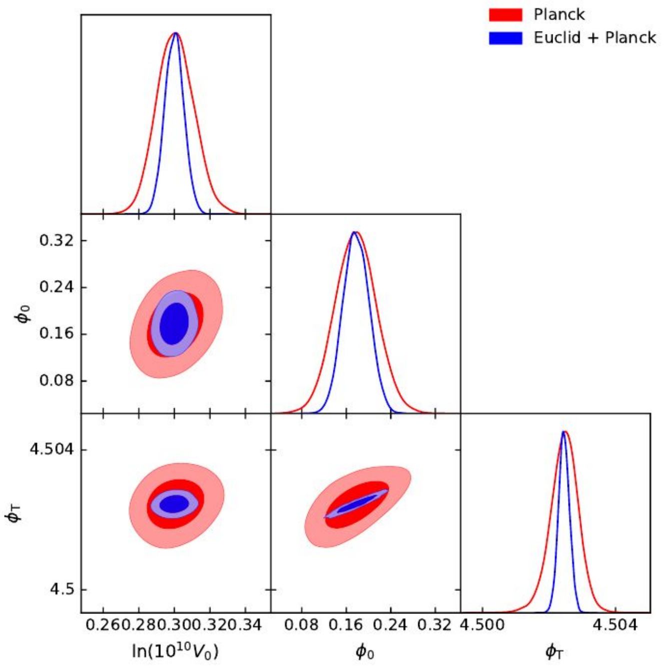
Features in the future (**LSS**: Euclid-like forecast)



Large Scale Structure survey such as Euclid can help in detecting features that are present within 0.02-0.2 Mpc^{-1}

Larger scale features will be hard to detect with Euclid as well

Debono, Hazra, Shafieloo, Smoot, Starobinsky MNRAS 2020

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Features that are relevant

- A suppression at the large scales is not favored by polarization data
- Importance of linear oscillations have decreased
- Dip at large scales remain marginally significant
- Resonant features are more favored as they are ***supported by both T & E***
- ***A combination of dip-feature and resonant oscillation*** provides the best fit. Even with 6 extra parameters, the model is as good as the standard model
- Future observations (LSS/CMB) will be able to identify these features, if present

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Thank you

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