Axions and helical magnetic fields

T. Kobayashi and RKJ, JCAP 03, 025 (2021)

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IITM Weekly Cosmology meeting

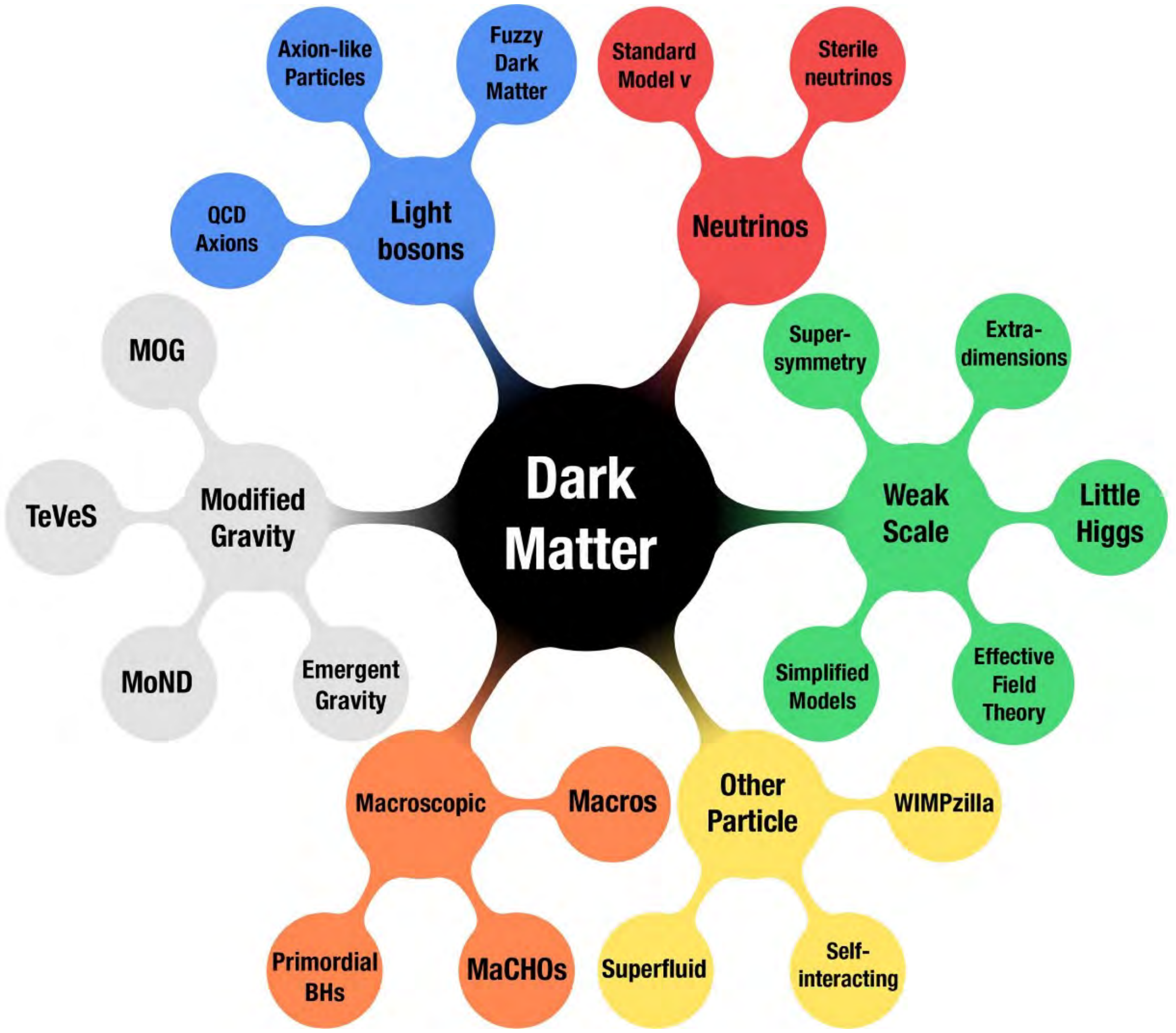
Outline of the talk

- What are Axions or Axion-like particles (ALP)?
- Axions as dark matter candidates
- Axion misalignment mechanism
    - Conventional and kinetic misalignment scenario
    - Misalignment due to helical magnetic fields
- Axion window constraints — relic abundance
    - Constant mass axion
    - QCD axion
- Conclusions and future directions

What are Axions ?

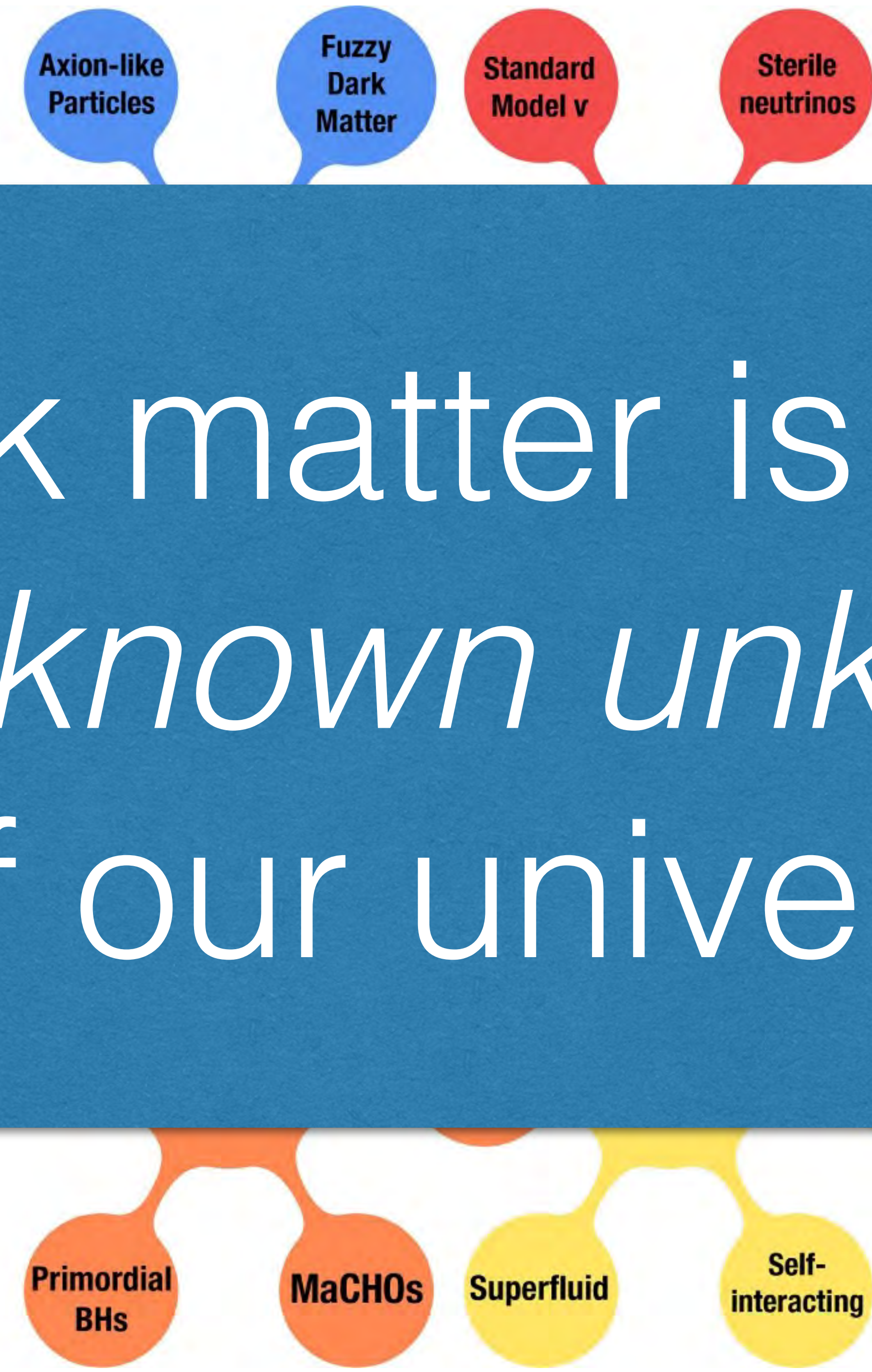
- Axions or Axion-like particles are considered promising candidates for the cold dark matter
- Introduced to solve the strong CP problem of QCD — theory predicts some degree of CP violation — but no experimental observation so far !
- Axion — origin as a goldstone boson — due to the spontaneous symmetry breaking — an additional U(1) PQ symmetry
- Similar to the Higgs mechanism — axion is not exactly massless !

Dark matter candidates



Credit: G. Bertone and T. M. P. Tait

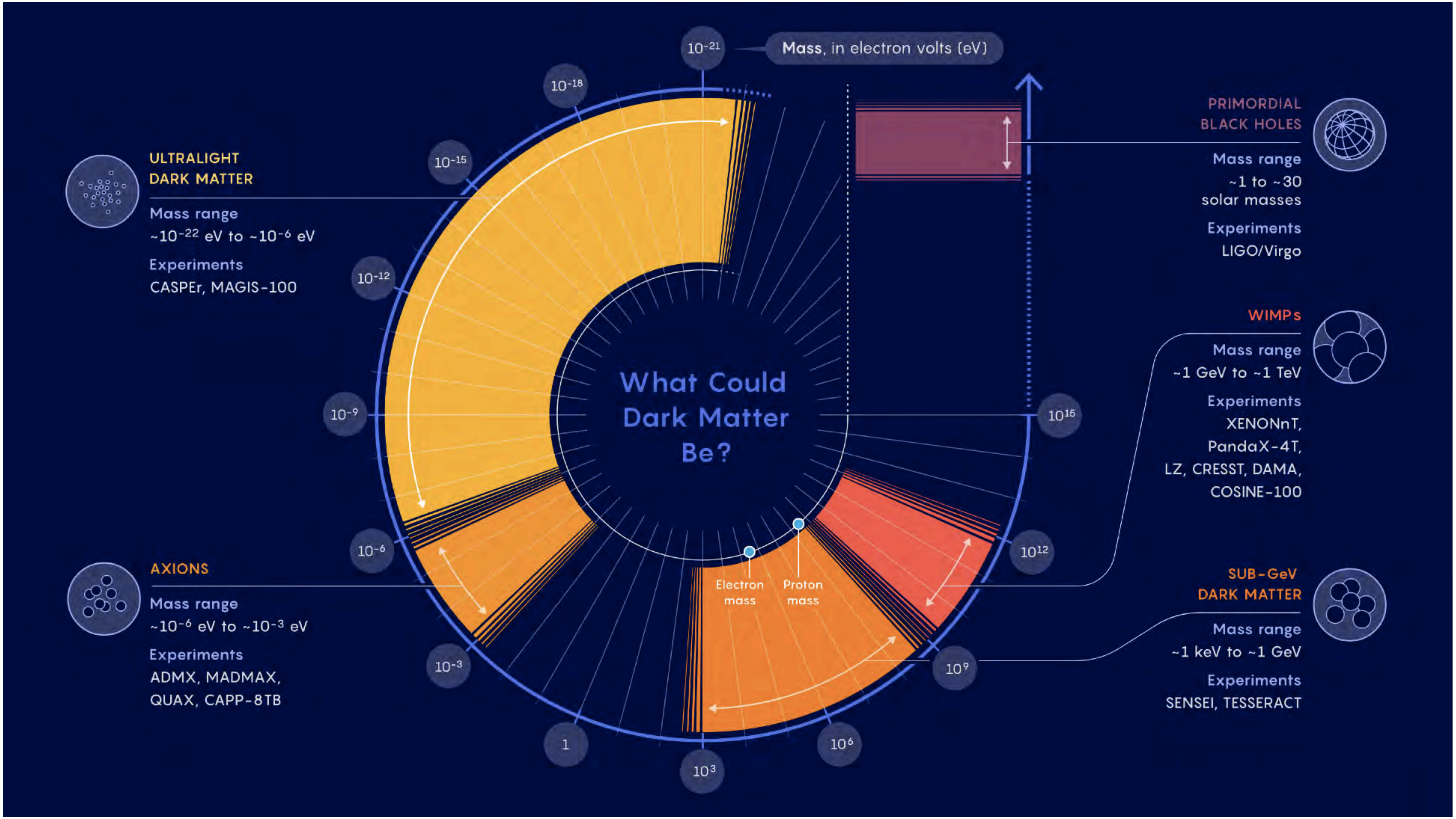
Dark matter candidates



Dark matter is one of the *known unknowns* of our universe !

Credit: G. Bertone and T. M. P. Tait

Dark matter candidates



Axion dynamics

- Consider an axion coupled to a U(1) gauge field

$$\frac{\mathcal{L}}{\sqrt{-g}} = -\frac{1}{2} f^2 g^{\mu\nu} \partial_\mu \theta \partial_\nu \theta - m^2 f^2 (1 - \cos\theta) + \frac{\alpha}{8\pi} \theta F_{\mu\nu} \tilde{F}^{\mu\nu}.$$

- Global U(1) is already broken broken by the end of inflation, and continues to be broken in the post-inflationary epoch

$$f > \frac{H_{\rm inf}}{2\pi}, T_{\rm max},$$

- Background metric and the equation of motion

$$ds^2 = -dt^2 + a(t)^2 d\boldsymbol{x}^2, \qquad 0 = \ddot{\theta} + 3H\dot{\theta} + m^2 \sin\theta - \frac{\alpha}{8\pi} \frac{F\tilde{F}}{f^2}.$$

Helical EM fields

- For visible or dark sector

$$F_{\mu\nu}\tilde{F}^{\mu\nu} = -4E_\mu B^\mu,$$

$$E^\mu = u_\nu F^{\mu\nu}, \quad B^\mu = \frac{1}{2}\eta^{\mu\nu\rho\sigma}u_\sigma F_{\nu\rho}, \quad \tilde{F}^{\mu\nu} = \frac{1}{2}\eta^{\mu\nu\rho\sigma}F_{\rho\sigma}.$$

- The energy density of the EM field is

$$\rho_A = (E_\mu E^\mu + B_\mu B^\mu)/2$$

$$(E_\mu \pm B_\mu)(E^\mu \pm B^\mu) \geq 0 \quad \rightarrow \quad |E_\mu B^\mu| \leq \rho_A$$

Helical EM fields

- If the U(1) gauge field is a hidden photon, it behaves as extra radiation and contributes to the effective extra relativistic degrees of freedom of the universe

$$\Delta N_A = \frac{8}{7}\left(\frac{11}{4}\right)^{4/3} \frac{\rho_A}{\rho_\gamma},$$

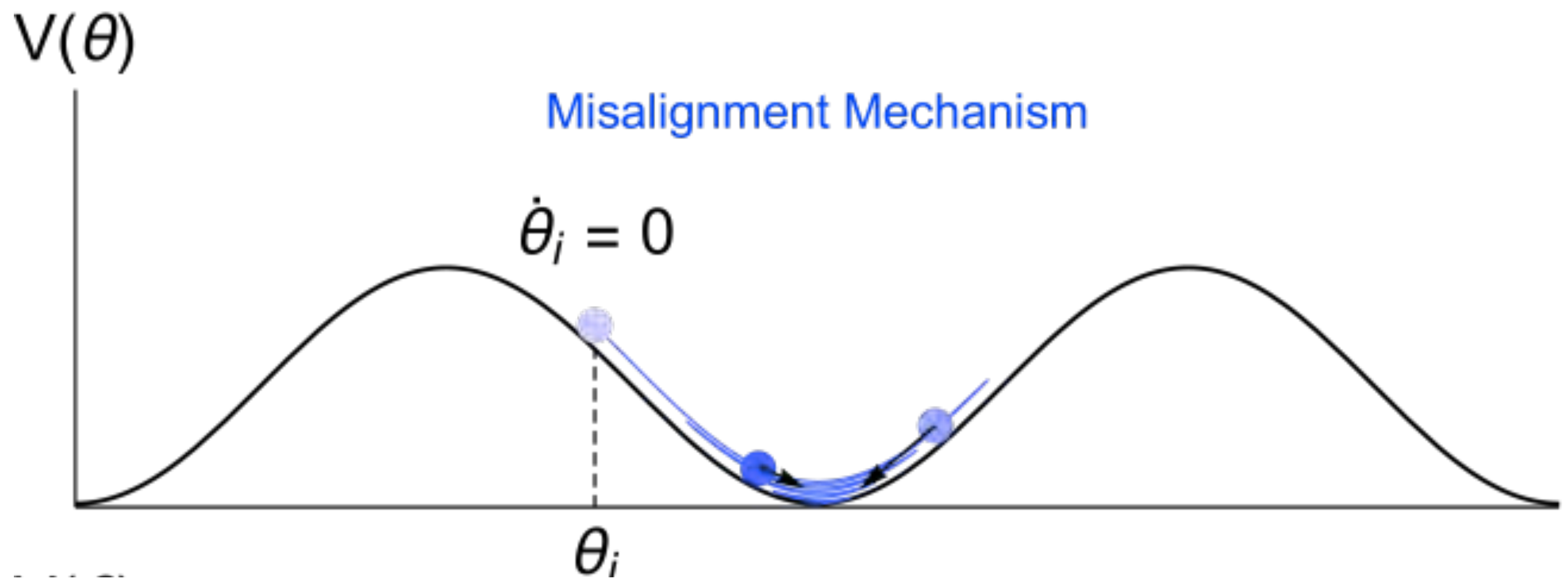
- This allows us to parameterize

$$\Delta N_{E\cdot B} \equiv \frac{8}{7}\left(\frac{11}{4}\right)^{4/3} \frac{|E_\mu B^\mu|}{\rho_\gamma} \leq \Delta N_A \lesssim 10^{-1},$$

If $|E_\mu B^\mu| \propto \rho_\gamma \propto a^{-4}$ then $\Delta N_{E\cdot B}$ is time-independent !

Conventional (vacuum) axion misalignment

- Conventional Axion misalignment — coherent initial displacement from its minimum
- Axion produced via misalignment behave as cold dark matter once the field starts oscillating around the minimum



Conventional axion dynamics

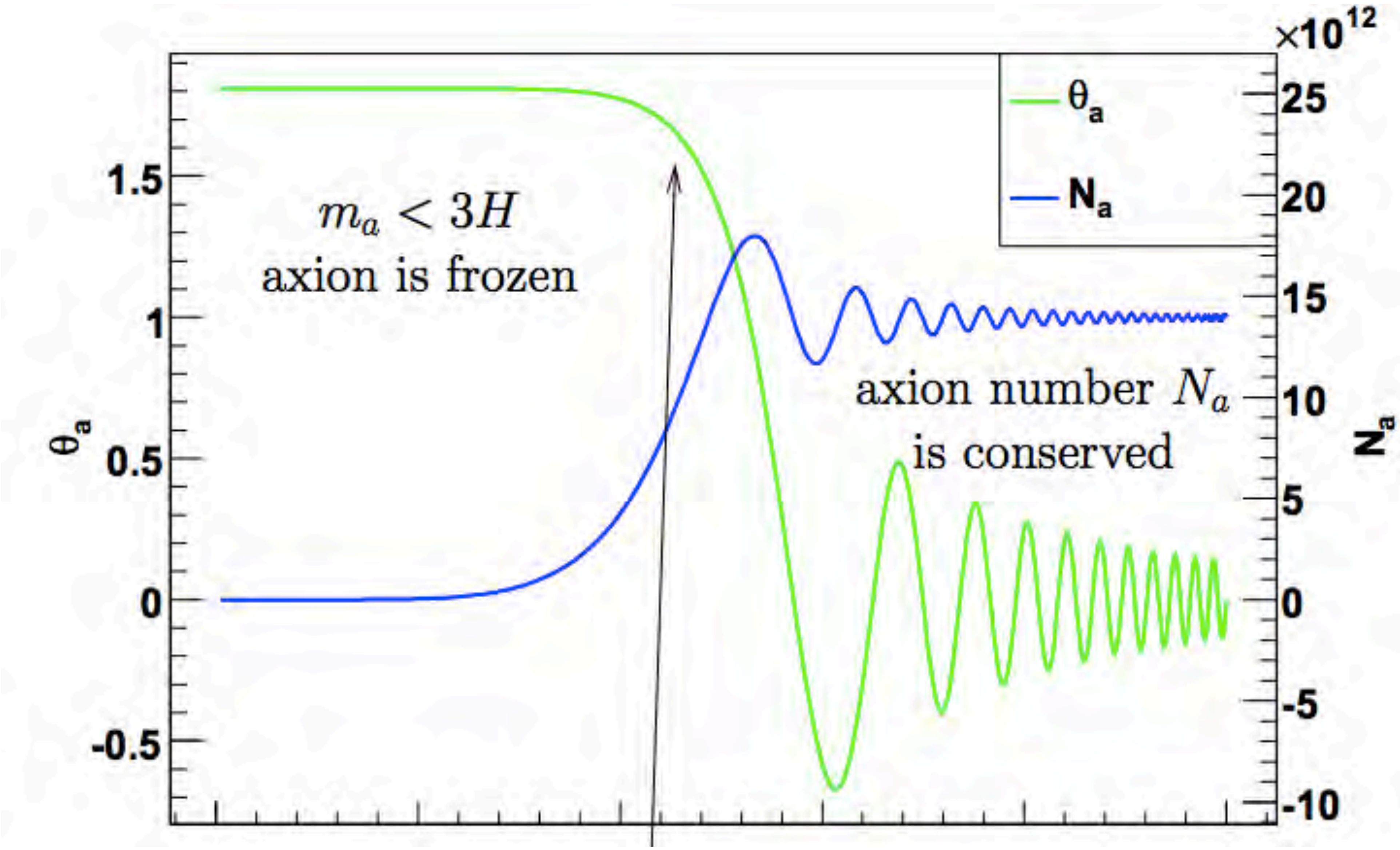
- The axion field stays frozen at some initial value while H > m, then begins to oscillate about the potential minimum when H ~ m.
- The energy density of the axion at the onset of the oscillation is

$$\rho_{\theta m} = b\,(m^2 f^2 \theta^2)_m,$$

- Since this epoch, the axion oscillates and its particle number is conserved. The relic abundance today is

$$\rho_{\theta 0} = m_0 n_{\theta 0} = b\, m_0 m_m f^2 \theta_m^2 \left(\frac{a_m}{a_0}\right)^3,$$

Conventional axion dynamics



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Kinetic axion misalignment

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Axion Kinetic Misalignment Mechanism

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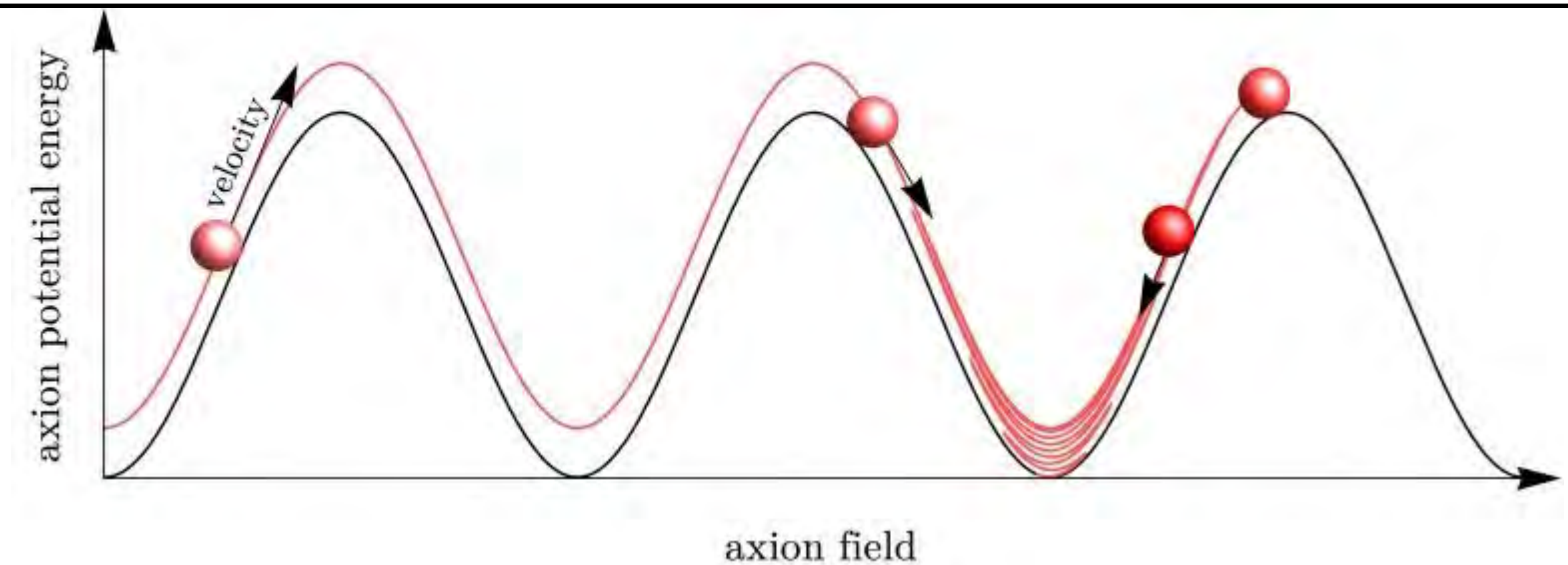
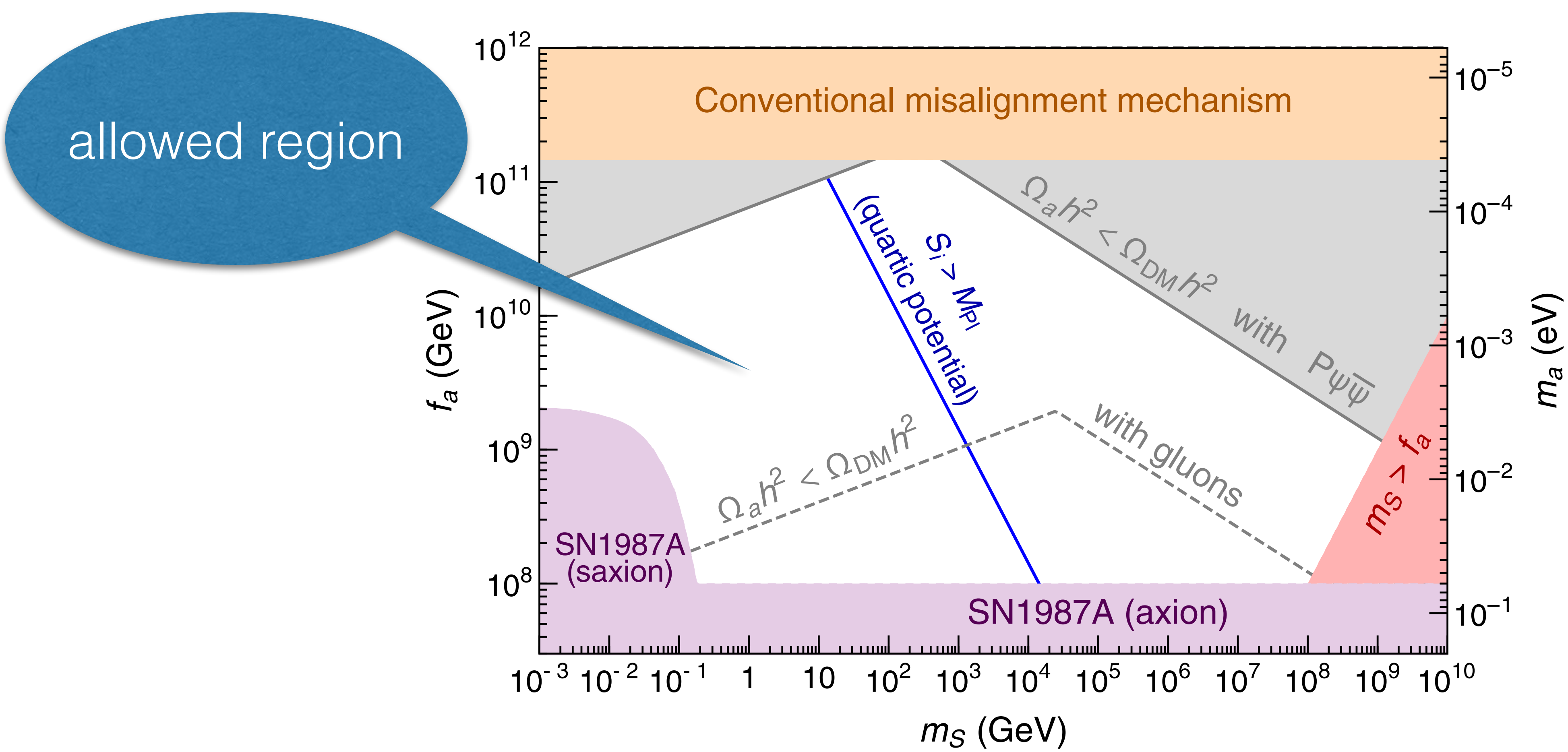
- A new set of initial conditions — both the field and it velocity are non-zero !
- Larger axion DM abundance for larger m
- A sufficient velocity may arise from the explicit breaking of the axion shift symmetry in the early universe

Kinetic axion misalignment





Induced axion velocity from helical EM field

- Background evolution

$$H \propto a^{-\frac{3(w+1)}{2}}, \quad F\tilde{F} \propto a^{-n}.$$

- Solution in the presence of helical EM field

$$\dot{\theta} = \left\{ -n + \frac{3(w+3)}{2} \right\}^{-1} \frac{\alpha}{8\pi} \frac{F\tilde{F}}{f^2 H} + K a^{-3},$$

- Assume that the axion is totally kicked by the helical fields i.e. ignore the axion potential

$$\left| \frac{\alpha}{8\pi} \frac{F\tilde{F}}{f^2} \right| > m^2.$$

Induced axion velocity from helical EM field

- The axion potential can also be neglected if the induced kinetic energy of the axion is larger than the height of the periodic axion potential

$$\left|\frac{\alpha}{8\pi}\frac{F\tilde{F}}{f^2}\right| > mH,$$

- A coherent helical field background sources an axion velocity of

$$|\dot{\theta}| \sim \left|\frac{\alpha}{8\pi}\frac{F\tilde{F}}{f^2 H}\right|,$$

- Finally, the condition for ignoring the axion potential becomes

$$\left|\frac{\alpha}{8\pi}\frac{F\tilde{F}}{f^2}\right| > \min.\{m^2, mH\}.$$

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Backreaction from the axion

- The axion can backreact to the gauge field, as a rolling axion itself induces excitation of the coupled gauge field — induced kinetic energy of the axion should remain smaller

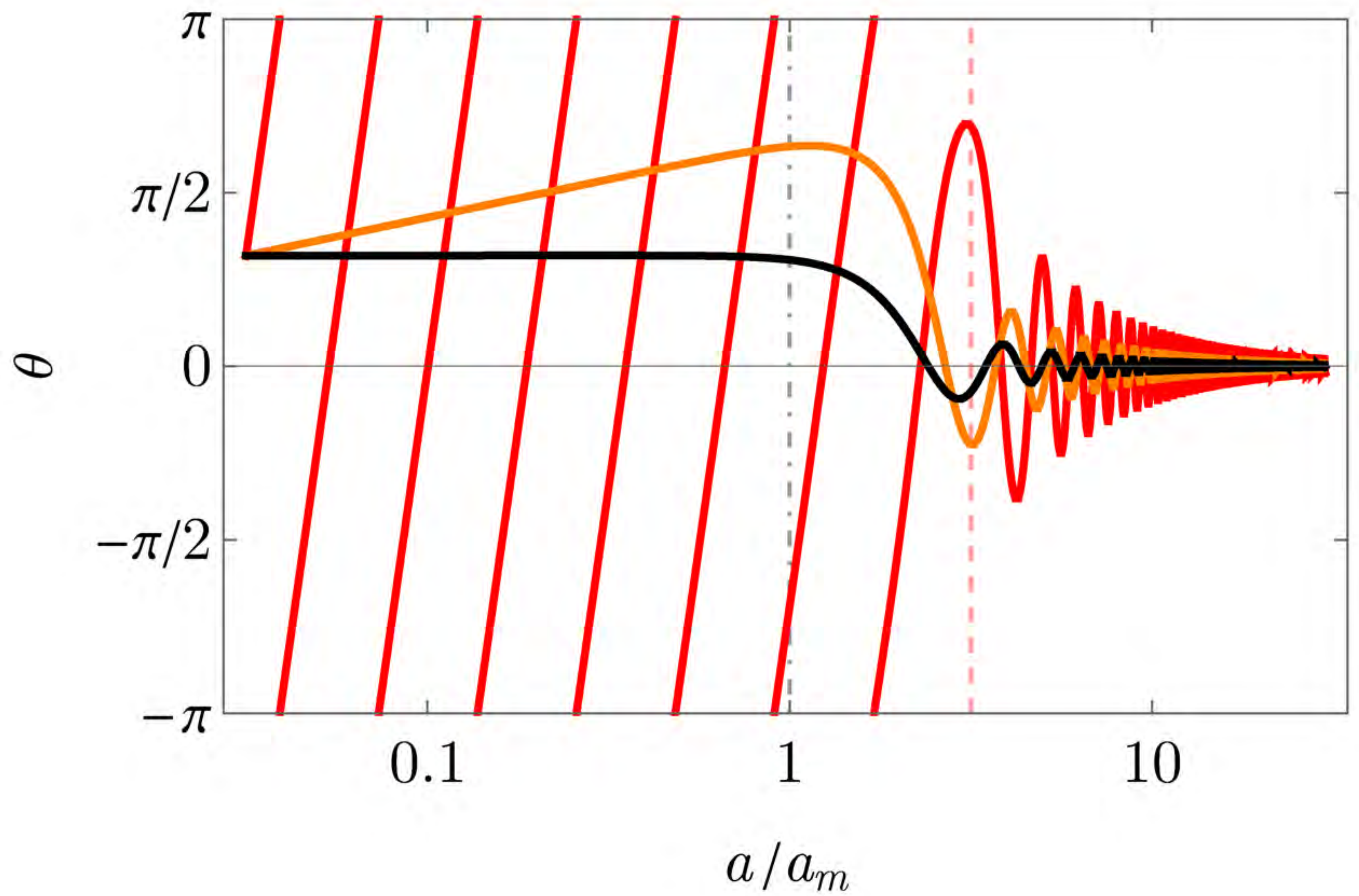
$$\left|\frac{\alpha}{8\pi}\frac{\dot{\theta}}{H}\right| \sim \left(\frac{\alpha}{8\pi}\right)^2 \frac{|F\tilde{F}|}{f^2 H^2} > 1.$$

- A coherent $F\tilde{F}$ moves the axion rapidly in one direction — expected to produce gauge bosons with momenta typically of order the Hubble scale at that time.
- In our scenario, the gauge fields are *not* produced by the helical coupling.

Axion dynamics — numerical

$$R_m = \left| \frac{\alpha}{8\pi} \frac{(F\tilde{F})_m}{m^2 f^2} \right| ,$$

Dimensionless strength of helical EM fields



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Comoving axion number density

- Comoving axion number density

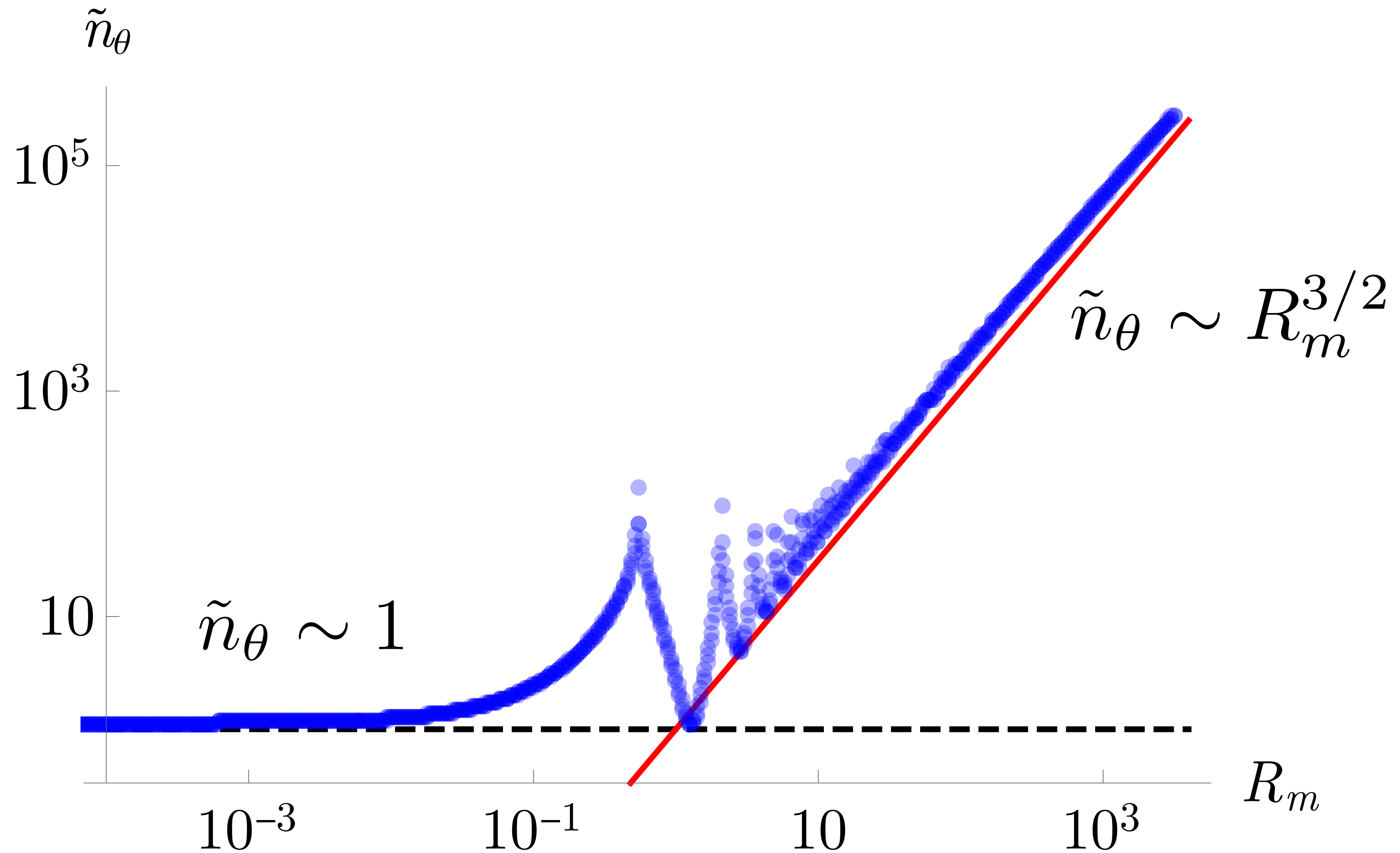
$$\tilde{n}_\theta = \frac{n_\theta}{mf^2}\left(\frac{a}{a_m}\right)^3$$

- Our analytical estimates indicate the behaviour as

$$\tilde{n}_\theta = \begin{cases} 1 & \text{for } R_m < 1, \\ R_m^{\frac{6}{2n-3(w+1)}} = R_m^{3/2} & \text{for } R_m \geq 1. \end{cases}$$

- It captures the behaviour in the asymptotic regimes.

Comoving axion number density



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Axion relic abundance — Constant mass axion

- The condition for the helical fields to delay the onset of the axion oscillation is translated into a lower bound on the field values today as

$$\left|\frac{\alpha}{8\pi}(F\tilde{F})_0\right| > \left(\frac{45}{128\pi^2}\right)^{1/3} \frac{g_*(T_m)}{g_{*s}(T_m)^{4/3}} \frac{f^2 s_0^{4/3}}{M_{\rm Pl}^2}.$$

$$\left|\frac{\alpha}{8\pi}\frac{(F\tilde{F})_m}{f^2}\right| > m_m^2.$$

- The axion abundance today is

$$\rho_{\theta 0} = c\left(\frac{128\pi^2}{45}\right)^{1/4} \frac{g_{*s}(T_{\rm tr})}{g_*(T_{\rm tr})^{3/4}} \left|\frac{\alpha}{8\pi}(F\tilde{F})_0\right|^{3/2} \frac{m^{1/2} M_{\rm Pl}^{3/2}}{f s_0},$$

- These two conditions together lead to

$$\Omega_\theta h^2 \sim 10^{-1}\left(\frac{|\alpha|\Delta N_{E.B}}{10^{-2}}\right)^{3/2}\left(\frac{f}{10^{17}\,{\rm GeV}}\right)^{-1}\left(\frac{m}{10^{-22}\,{\rm eV}}\right)^{1/2}$$

$$\text{for}\ \ |\alpha|\Delta N_{E.B} \gtrsim 10^{-2}\left(\frac{f}{10^{17}\,{\rm GeV}}\right)^2.$$

Axion relic abundance — Constant mass axion

- What if this condition is not satisfied ?

$$\left|\frac{\alpha}{8\pi}\frac{(F\tilde{F})_m}{f^2}\right| > m_m^2.$$

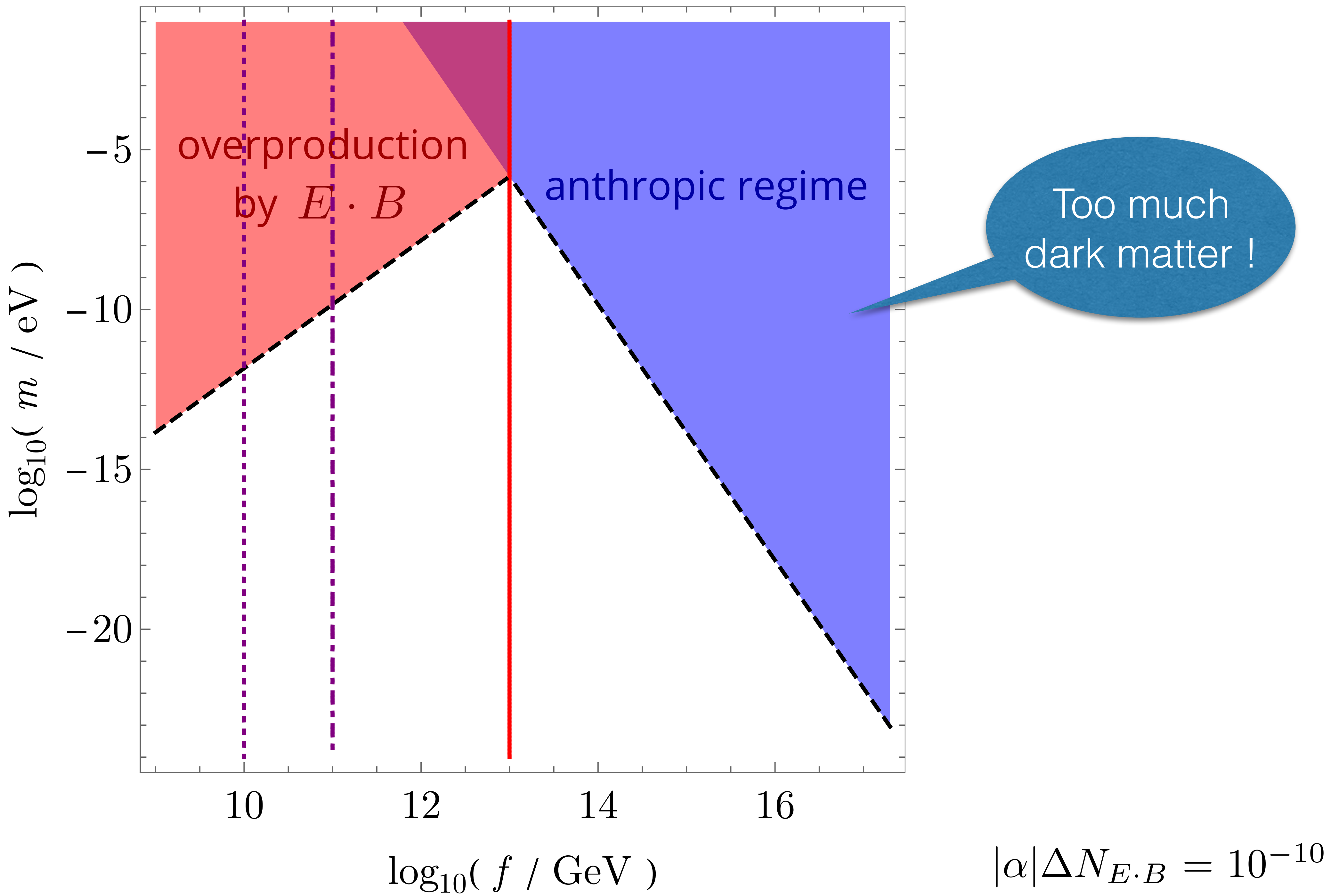
- One then recovers the conventional vacuum misalignment scenario and the axion abundance is

$$\Omega_\theta h^2 \sim 10^{-1}\,\theta_m^2 \left(\frac{f}{10^{17}\,\mathrm{GeV}}\right)^2 \left(\frac{m}{10^{-22}\,\mathrm{eV}}\right)^{1/2} \quad \text{for } \; |\alpha|\Delta N_{E\cdot B} \lesssim 10^{-2}\left(\frac{f}{10^{17}\,\mathrm{GeV}}\right)^2$$

- These two conditions collectively lead to

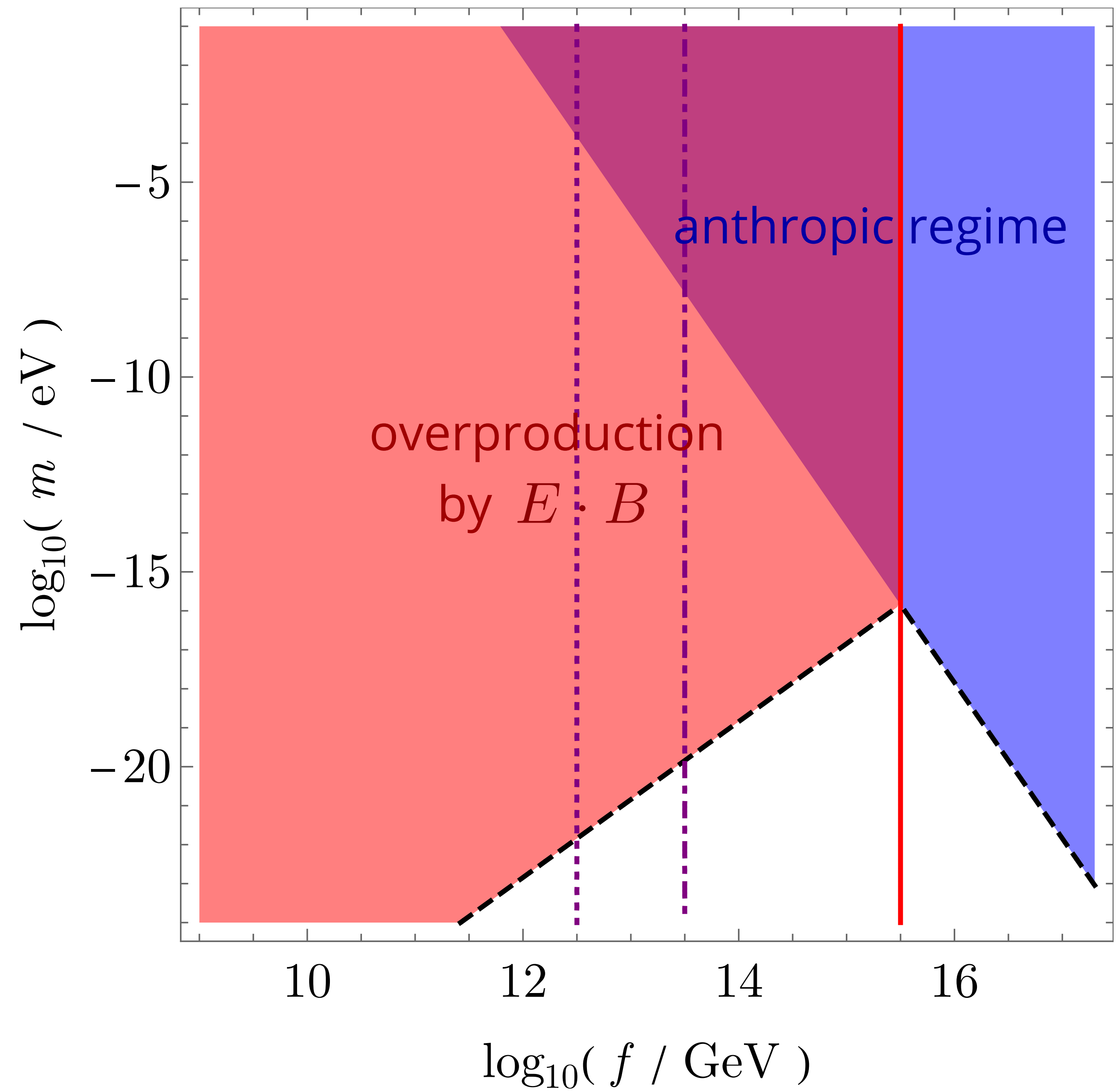
$$\Omega_\theta h^2 \sim 10^{-1}\left(\frac{m}{10^{-22}\,\mathrm{eV}}\right)^{1/2}\left(\frac{f}{10^{17}\,\mathrm{GeV}}\right)^{-1}\left[\max.\left\{\left(\frac{|\alpha|\Delta N_{E\cdot B}}{10^{-2}}\right), \left(\frac{f}{10^{17}\,\mathrm{GeV}}\right)^2\right\}\right]^{3/2}$$

Allowed window of constant mass axion



Constant mass axion window in the presence of helical EM fields

Allowed window of constant mass axion



$|\alpha|\Delta N_{E\cdot B} = 10^{-5}$

Constant mass axion window in the presence of helical EM fields

QCD axion

- QCD axion mass depends on the temperature

$$m(T) \simeq \begin{cases} \lambda\, m_0 \left(\frac{\Lambda_{\rm QCD}}{T}\right)^p & \text{for } T \gg \Lambda_{\rm QCD}, \\ m_0 & \text{for } T \ll \Lambda_{\rm QCD}. \end{cases}$$

- For $\Lambda_{\rm QCD} \approx 200\,{\rm MeV},\ \lambda \approx 0.1,\ p \approx 4,$

$$m_0 \approx 6 \times 10^{-6}\,{\rm eV} \left(\frac{10^{12}\,{\rm GeV}}{f}\right).$$

- For simplicity, we assume that $F\tilde{F} \propto a^{-4}$ and the universe become radiation dominated by H=m.

QCD axion

- For $T_{\rm tr} \lesssim \Lambda_{\rm QCD}$, a similar condition discussed earlier for the constant mass axion

$$\left|\frac{\alpha}{8\pi}(F\tilde{F})_0\right| > \frac{1}{2}\left(\frac{45}{2\pi^2}\right)^{5/6} \frac{g_*(T_{\rm tr})^{1/2}}{g_{*s}(T_{\rm tr})^{4/3}} \frac{m_0 f^2 s_0^{4/3}}{\lambda^{2/p}\Lambda_{\rm QCD}^2 M_{\rm Pl}}.$$

- The axion abundance today is

$$\Omega_\theta h^2 \sim 10^{-1}\left(\frac{|\alpha|\Delta N_{E\cdot B}}{10^{-11}}\right)^{3/2}\left(\frac{f}{10^{12}\,{\rm GeV}}\right)^{-3/2} \quad {\rm for}\ \ |\alpha|\Delta N_{E\cdot B} \gtrsim 10^{-6}\left(\frac{f}{10^{12}\,{\rm GeV}}\right)$$

- Even of the helical fields are not so large, it still affects the axion abundance if $\Lambda_{\rm QCD} \lesssim T_{\rm tr} < T_m$.

$$\Omega_\theta h^2 \sim 10^{-1}\left(\frac{|\alpha|\Delta N_{E\cdot B}}{10^{-12}}\right)^{7/6}\left(\frac{f}{10^{12}\,{\rm GeV}}\right)^{-7/6}$$

$${\rm for}\ \ 10^{-12}\left(\frac{f}{10^{12}\,{\rm GeV}}\right)^2 \lesssim |\alpha|\Delta N_{E\cdot B} \lesssim 10^{-6}\left(\frac{f}{10^{12}\,{\rm GeV}}\right)$$

QCD axion

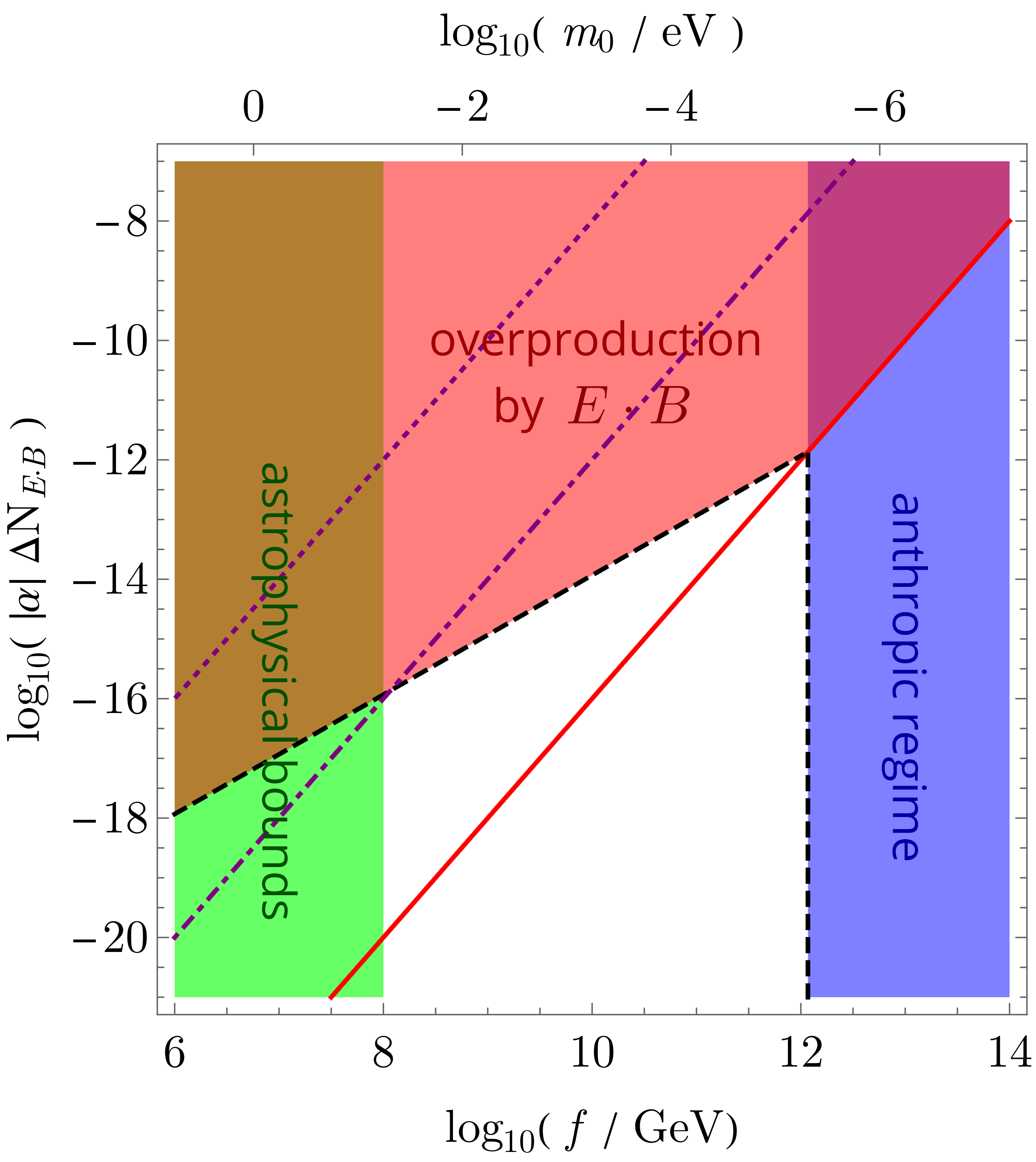
- If the helical fields are even smaller, one recovers the axion abundance of the conventional mechanism as

$$\Omega_\theta h^2 \sim 10^{-1}\,\theta_m^2 \left(\frac{f}{10^{12}\,\mathrm{GeV}}\right)^{7/6} \quad \text{for} \;\; |\alpha|\Delta N_{E\cdot B} \lesssim 10^{-12}\left(\frac{f}{10^{12}\,\mathrm{GeV}}\right)^2$$

- The relic abundance of the QCD axion crucially depends on the amplitude of the helical fields background.

- The backreaction constraint remains the same even if for a QCD axion.

QCD axion window



QCD axion window in the presence of helical EM fields

Conclusions

- Axions are (beyond BSM) well motivated particles, required for solving the strong CP problem
- A possible CDM candidate but must be very light — ALP — or must be misaligned by some mechanism
- Interesting implications of helical magnetic fields on axions
    - Effects on the Axion abundance
    - Effects on the Axion parameters window — even for tiny magnetic fields
    - Axion to photon conversion — used for DM search — both in observatories and in labs
    - Cosmic birefringence — rotation of plane of polarization of photons — strong constraints from observations

Future directions

- We only considered hidden U(1) gauge fields — it will be interesting to apply these results to non-Abelian gauge fields
- Inhomogeneous magnetic fields — small wavelength component will force the axion to move differently in different Hubble patches — axion iso-curvature perturbations — constraints from observations
- For non-trivial redshifting of EM fields — different than radiation — axion abundance would be modified
- Spontaneous symmetry breaking before/after inflation
- Axion-induced UV cascade of helical EM fields
- Parity violating signatures — constraints from cosmological observations

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