Saturday, March 13, 2021

Prospects and Search for the stochastic gravitational-wave background induced by primordial curvature perturbations in LIGO′s second observing run

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Based on Kapadia et al (2020) arXiv:2009.05514 arXiv:2005.05693

Motivation

- Primordial Black Holes (PBHs)
  - Born from overdense regions in the early Universe
  - Very broad mass range: min mass —>10^{-5}g.
  - Predicted by several models of inflation
- Could answer many open questions.
  - Dark Matter candidates
  - Potential GRB sources
  - Formation of supermassive BHs

PBHs and LIGO

- PBH as formation channel for LIGO's BBHs
  - Speculative
  - Other less exotic channels are more popular
- Search for coalescing binary PBHs in O1 data:
  - Resolved: $\sim 10^{-1}M_\odot - 1M_\odot$ *(LIGO-Virgo 2018)*
  - Stochastic: $\sim 1\ M_\odot - 10^2 M_\odot$ (*Wang et al 2018*)

Ultralight PBHs

- Hawking Radiation (HR)
  - Masses ("**ultralight**") $\lesssim 10^{15} g$ $(10^{-18} M_{\odot})$
  - Not verified experimentally
- Non-GW searches for ultralight PBHs:
  - Extra Galactic Photon Background
  - Big Bang Nucleosynthesis
  - HR dependent

Ultralight PBHs in this work

- We focus on PBHs formed in RD era
  - Associated with mass-scale $10^{13}g - 10^{15}g$
- Would lead to formation of SB of GWs:
  - Non linear mode couplings
- **Can we detect them in LVK data ?**

Curvature Power Spectrum $P_\zeta(k)$

- Popular models (True shape unknown):
  - Monochromatic: $P_\zeta = A\delta(\log(k/k_0))$
  - Gaussian: $P_\zeta = A \exp\left(-\frac{\log(k/k_0)^2}{2\sigma^2}\right)$
  - k_0 = central wavenumber associated with a PBH mass-distribution function f(M)
  - Hyper-parameters A, k_0, σ need to be constrained from data

From Power Spectrum to Measurable Quantities

$$\Omega_{\rm GW}(f) = \frac{1}{\rho_c}\frac{d\rho_{\rm GW}}{d\log f},$$

$$\Omega_{\rm GW} \propto \left(\frac{k}{aH}\right)^2 \bar{P}_h, \quad \bar{P}_h \equiv \text{time averaged power spectrum}$$

Wavenumber k, Scale Factor a, Hubble Parameter H

From Power Spectrum to PBH Production

$$M = K M_H (\delta - \delta_c)^\gamma$$

PBH Mass

Mass within Hubble horizon

Density

Critical Density

$$\mathcal{P}_{M_H}(\delta(M)) = \frac{1}{\sqrt{2\pi\sigma_c^2(k(M_H))}} \exp\left(-\frac{\delta^2(M)}{2\sigma_c^2(k(M_H))}\right)$$

Assume density fluctuations are Gaussian distributed

Variance related to power spectrum

PBH Abundance

Distribution at Formation time

$$\tilde{\beta}_{M_H}(M) = \frac{K}{\sqrt{2\pi\gamma^2\sigma_c^2(k(M_H))}}\left(\frac{M}{KM_H}\right)^{1+1/\gamma} \times \exp\left(-\frac{1}{2\sigma_c^2(k(M_H))}\left(\delta_c + \left(\frac{M}{KM_H}\right)^{1/\gamma}\right)^2\right)$$

$$f(M) \equiv \frac{1}{\Omega_{\rm CDM}}\frac{d\Omega_{\rm PBH}}{d\log M/M_\odot}$$

PBH mass function today

Fraction of DM as PBH

$$f_{\rm PBH} \equiv \Omega_{\rm PBH}/\Omega_{\rm DM}$$

$$f_{\rm PBH} = \int_{-\infty}^{\infty} f(M)\,d\log(M/M_\odot).$$

Choice of Gaussian Width

- Monochromatic (unrealistic) —> Narrow Gaussian
- Two Gaussians:
  - Narrow —> $\sigma = 0.1$
  - Broad —> $\sigma = 0.88$

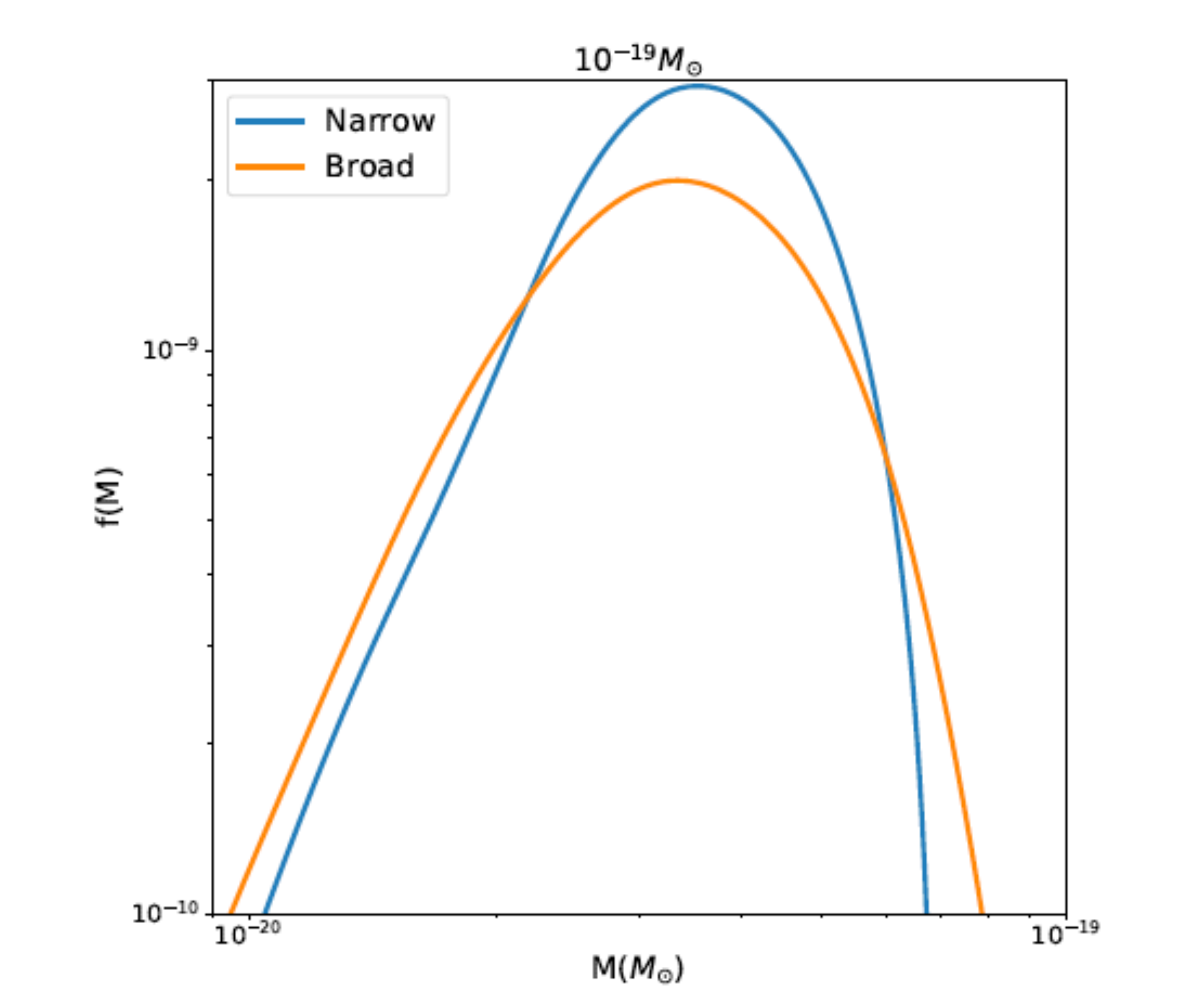
Choice of A using mass function f(M)

- f(M) is the fractional energy density of cold dark matter as PBHs per log mass bin
- Shape of f(M) set by shape of P_ζ

$$f(M) \equiv \frac{1}{\Omega_{\rm CDM}} \frac{d\Omega_{\rm PBH}}{d\log M/M_\odot}$$

- Amplitude A set by normalising f(M):
  - Upper limits from HR-based constraints
  - HR-free constraint

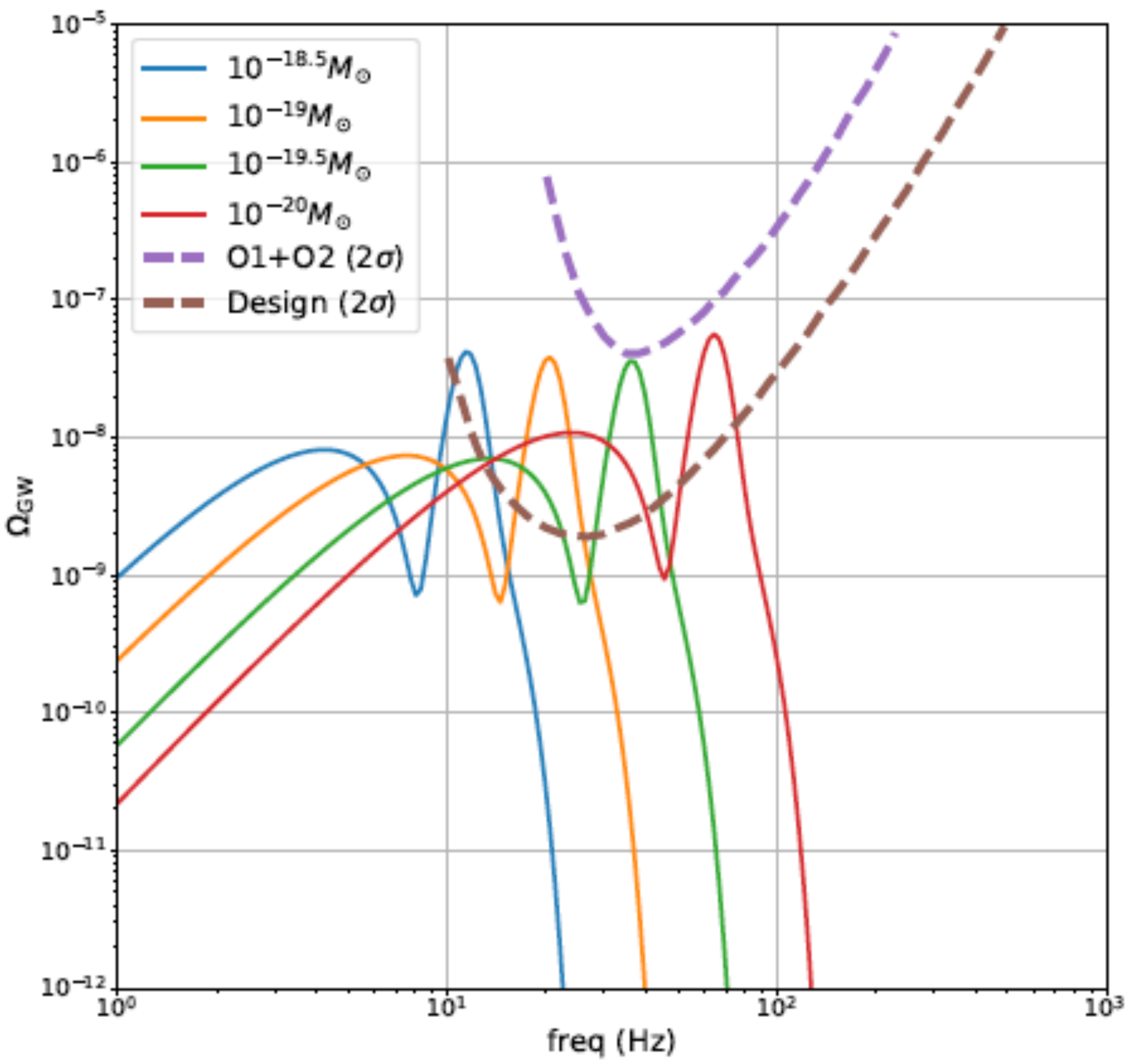
f(M) for mass-scale $10^{-19} M_{\odot}$

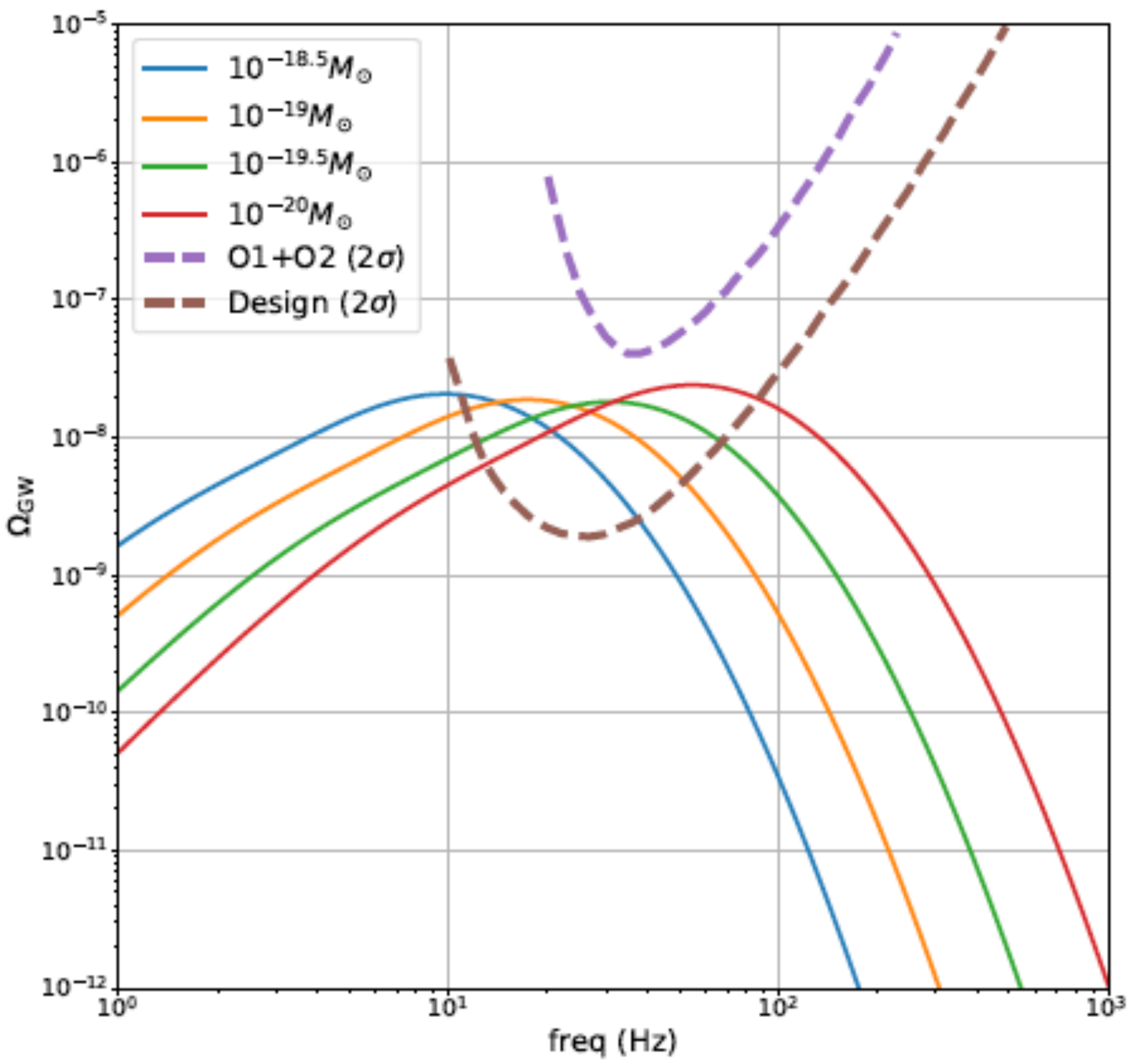


Stochastic Backgrounds

Narrow: $\sigma = 0.1$

Broad: $\sigma = 0.88$





Expected SNRs for O1, O2, O3 and Design sensitivities

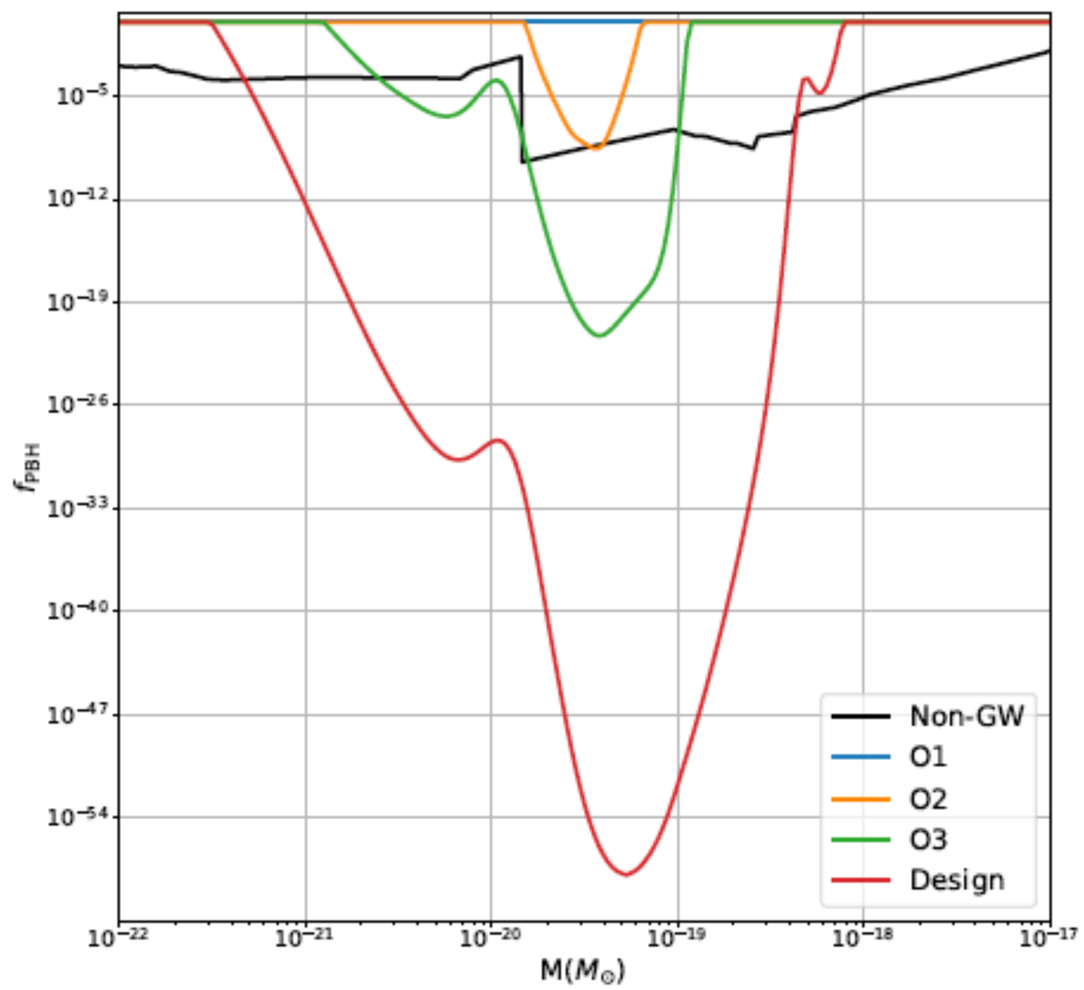
Mass ($M_\odot$)	$P_\xi(k)$	Obs. Run	$A \times 10^{-2}$	SNR
$10^{-18.5}$	Narrow	O1+O2	10.9 (16.1)	0.0 (0.0)
$10^{-18.5}$	Narrow	O3	10.9 (16.1)	0.0 (0.0)
$10^{-18.5}$	Narrow	Design	10.9 (16.1)	5.7 (12.5)
10^{-19}	Narrow	O1+O2	10.3 (15.9)	0.3 (0.6)
10^{-19}	Narrow	O3	10.3 (15.9)	2.0 (4.6)
10^{-19}	Narrow	Design	10.3 (15.9)	15.2 (35.8)
$10^{-19.5}$	Narrow	O1+O2	10.1 (15.6)	1.9 (4.5)
$10^{-19.5}$	Narrow	O3	10.1 (15.6)	4.3 (10.2)
$10^{-19.5}$	Narrow	Design	10.1 (15.6)	15.2 (36.4)
10^{-20}	Narrow	O1+O2	12.5 (15.4)	0.7 (1.0)
10^{-20}	Narrow	O3	12.5 (15.4)	2.1 (3.1)
10^{-20}	Narrow	Design	12.5 (15.4)	9.8 (14.8)
$10^{-18.5}$	Broad	O1+O2	3.07 (4.52)	0.4 (0.8)
$10^{-18.5}$	Broad	O3	3.07 (4.52)	1.3 (2.9)
$10^{-18.5}$	Broad	Design	3.07 (4.52)	10.4 (22.5)
10^{-19}	Broad	O1+O2	2.92 (4.45)	1.0 (2.4)
10^{-19}	Broad	O3	2.92 (4.45)	3.2 (7.3)
10^{-19}	Broad	Design	2.92 (4.45)	16.8 (38.9)
$10^{-19.5}$	Broad	O1+O2	2.87 (4.38)	1.6 (3.7)
$10^{-19.5}$	Broad	O3	2.87 (4.38)	4.3 (9.9)
$10^{-19.5}$	Broad	Design	2.87 (4.38)	18.2 (42.4)
10^{-20}	Broad	O1+O2	3.30 (4.32)	1.9 (3.2)
10^{-20}	Broad	O3	3.30 (4.32)	4.6 (7.9)
10^{-20}	Broad	Design	3.30 (4.32)	17.6 (30.1)

$$S_h(f) = \frac{3H_0^2}{2\pi^2}\frac{\Omega_{\rm GW}(f)}{f^3}$$

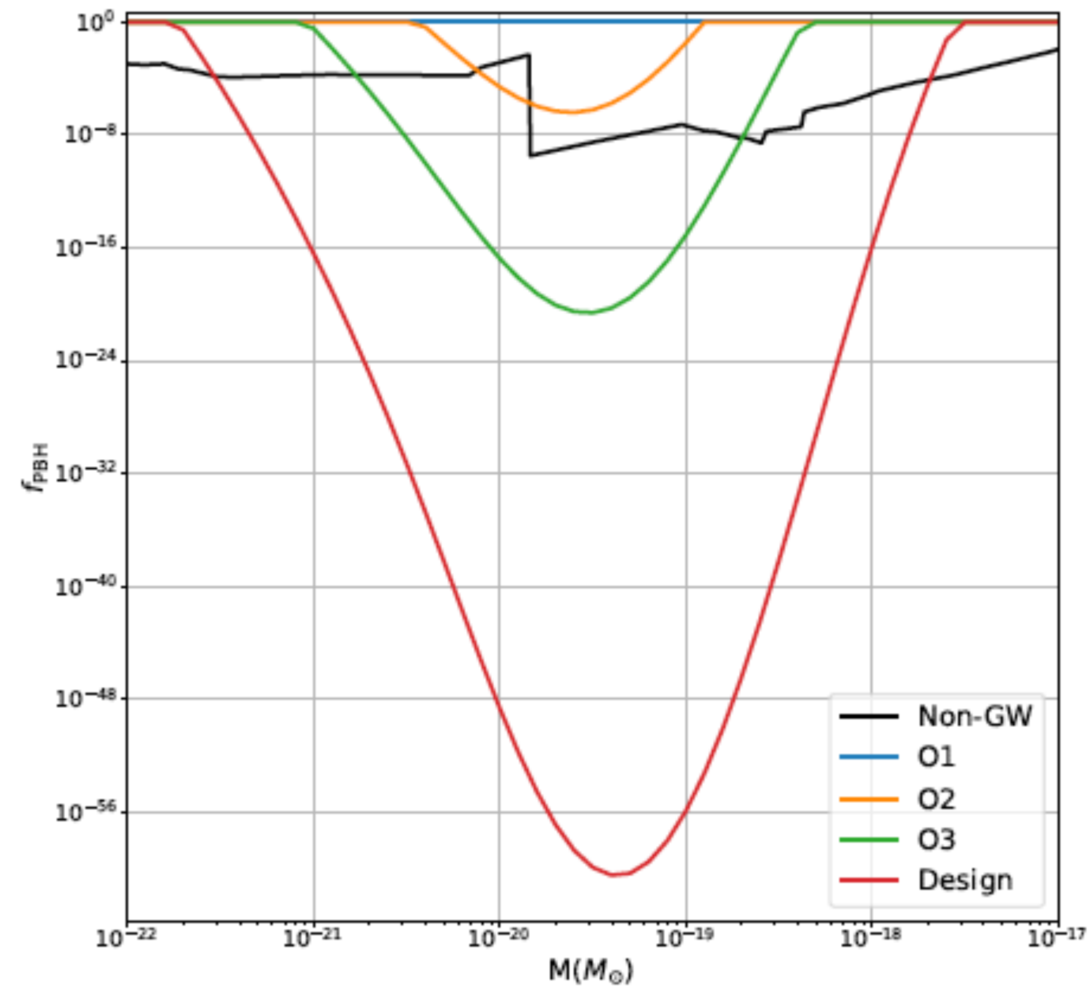
$$\rho = \sqrt{2T\int_{f_{low}}^{f_{high}} df \left[\sum_{I=1}^{M}\sum_{J>I}^{M}\frac{\Gamma_{IJ}^2 S_h^2(f)}{S_{0I}(f)S_{0J}(f)}\right]}$$

HR-Independent $f_{\rm DM}$ upper limits

Narrow: $\sigma = 0.1$



Broad: $\sigma = 0.88$



$$f_{\rm PBH} = \int_{-\infty}^{\infty} f(M) d\log(M/M_\odot).$$

Implications

- Based on our prospective SNRs, SB detectable (SNR=2) in:
    - O1 and O2:
        - Narrow and Broad, if HR not considered
    - O3 and Design:
        - Detectable with/without HR

Motivation and Goals for a Search

- Probe the curvature power spectrum
- Shape unknown:
  - Amplitude
  - Width
- Use GW (O2) data
  - Would be the first GW data-driven constraints
  - Complementary to EM-constraints
  - Hawking radiation independent

Slight Redefining of the Curvature Power Spectrum $P_\zeta(k)$

- Gaussian: $P_\zeta = \frac{A}{\sqrt{2\pi\sigma}} \exp\left(-\frac{\log(k/k_0)^2}{2\sigma^2}\right)$
- k_0 = central wavenumber associated with a PBH mass-distribution function f(M)
- Hyper-parameters A, k_0, σ need to be constrained from data

Cross-Correlation Search & PE

LIGO-T1900058/public

$$\hat{\Omega}_{\rm ref} = \frac{\sum_j w(f_j)^{-1}\hat{C}(f_j)\sigma_C^{-2}(f_j)}{\sum_j w(f_j)^{-2}\sigma_C^{-2}(f_j)},$$

$$w(f) := \Omega_{\rm GW}(f_{\rm ref})/\Omega_{\rm GW}(f)$$

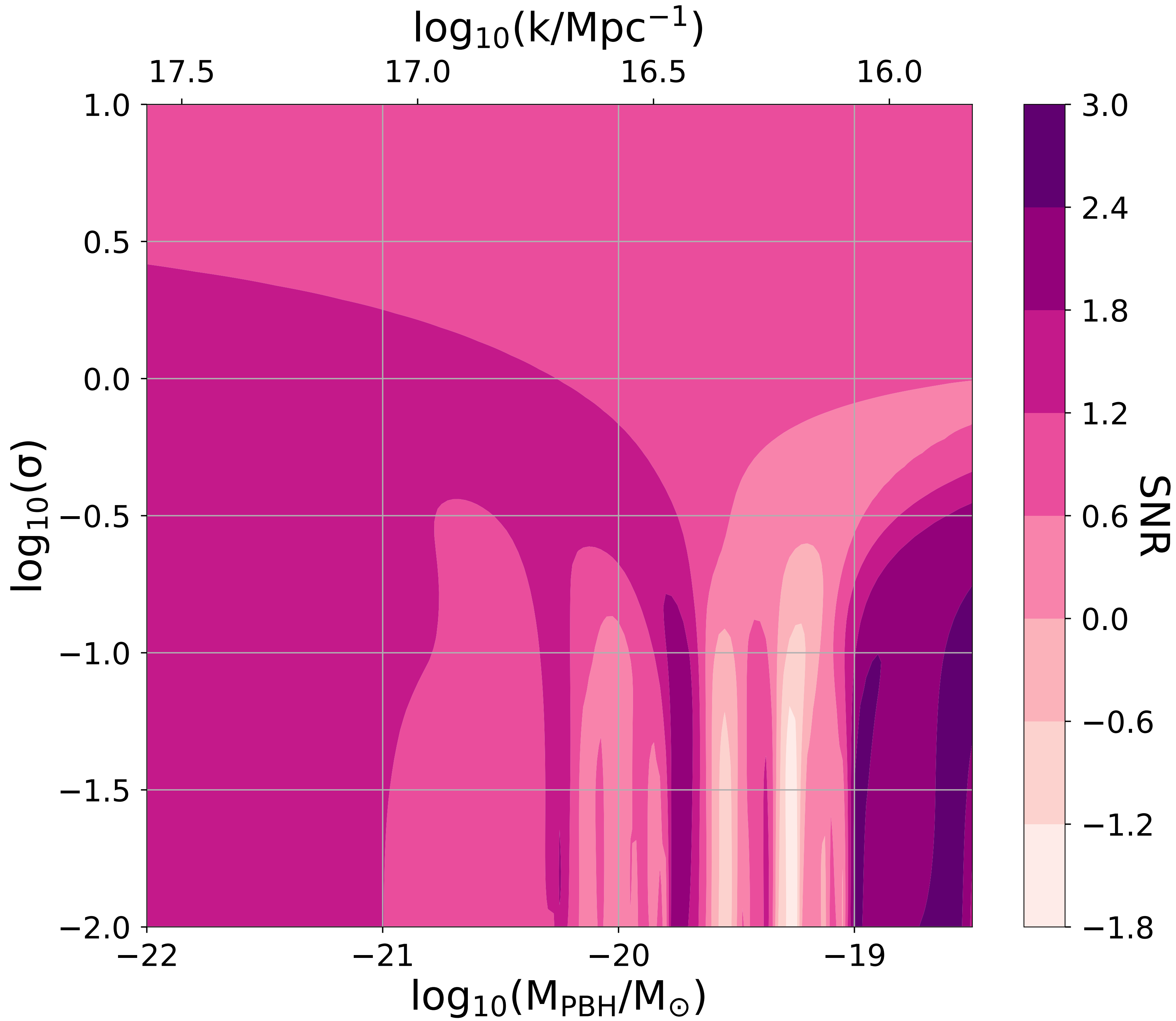
$$\sigma_\Omega^{-2} = \sum_j w(f_j)^{-2}\sigma_C^{-2}(f_j),$$

$$\mathrm{SNR} = \frac{\hat{\Omega}_{\rm ref}}{\sigma_\Omega}$$

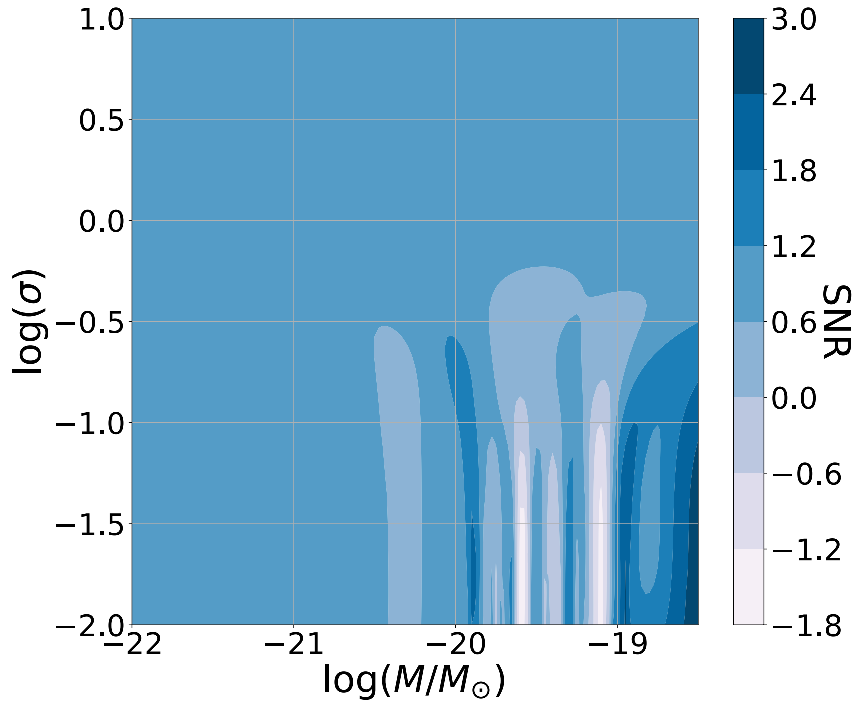
Parameter Estimation Likelihood

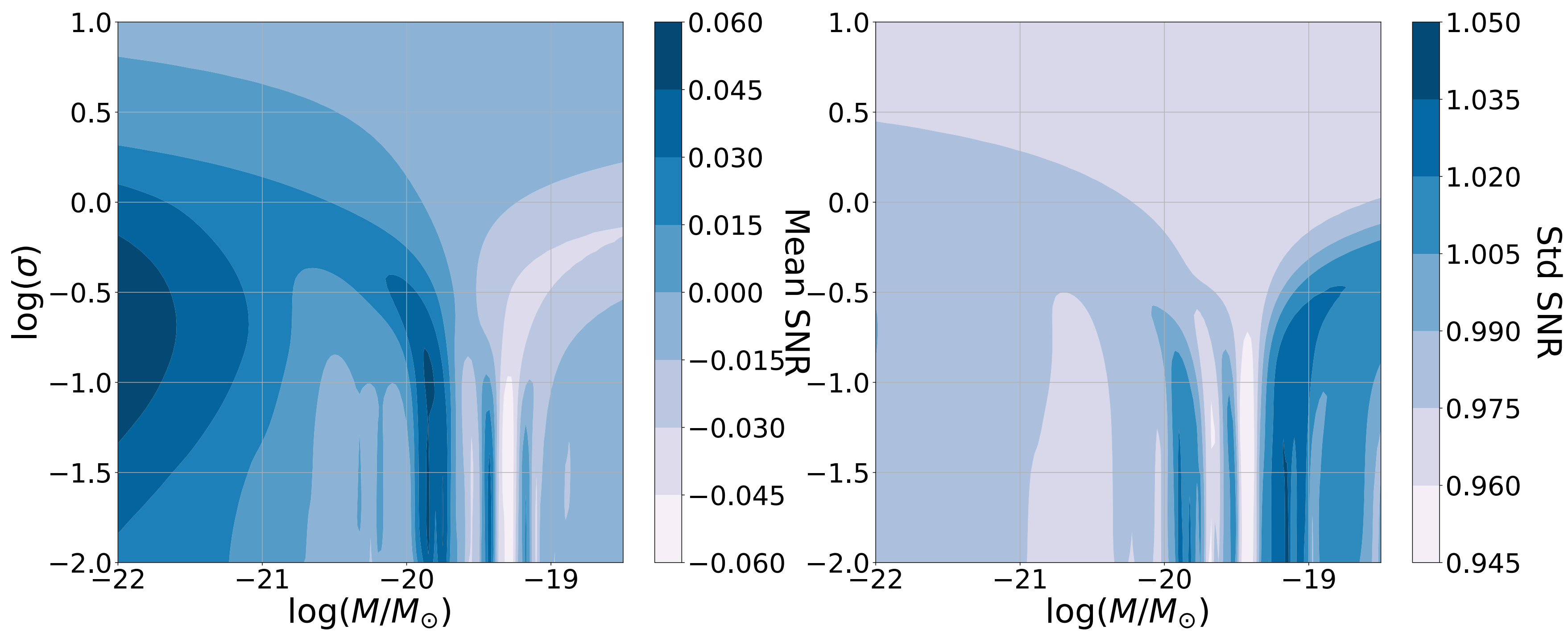
$$p(\hat{C}\,|\,\Theta) \propto \prod_j \exp\left(\frac{-[\hat{C}(f_j) - \Omega_{\rm GW}(f_j;\Theta)]^2}{2\sigma_C^2(f_j)}\right).$$

Search SNRs

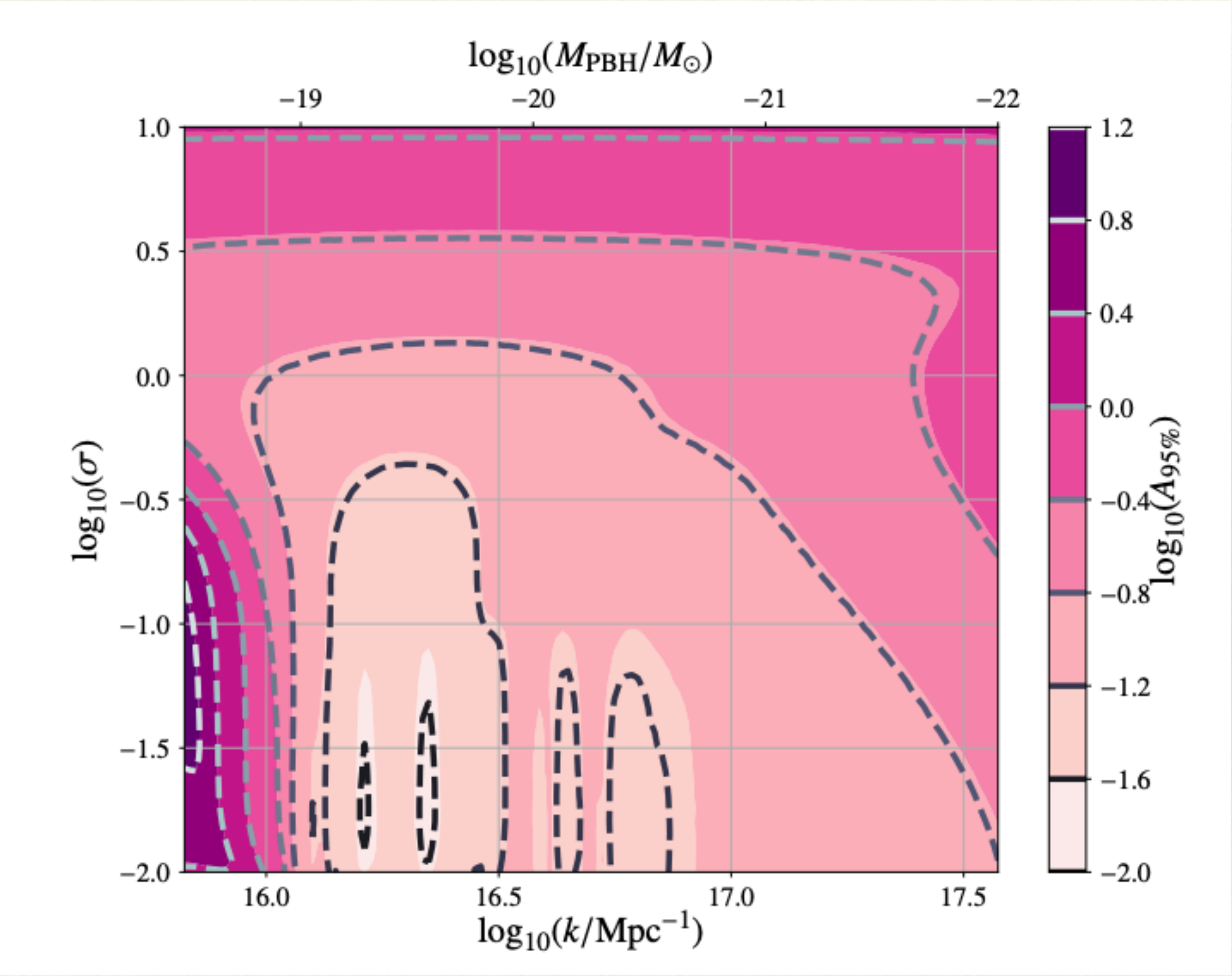

$\log_{10}(k/\mathrm{Mpc}^{-1})$
17.5
17.0
16.5
16.0
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
$\log_{10}(\sigma)$
−22
−21
−20
−19
$\log_{10}(M_{PBH}/M_{\odot})$
3.0
2.4
1.8
1.2
0.6
0.0
−0.6
−1.2
−1.8
SNR

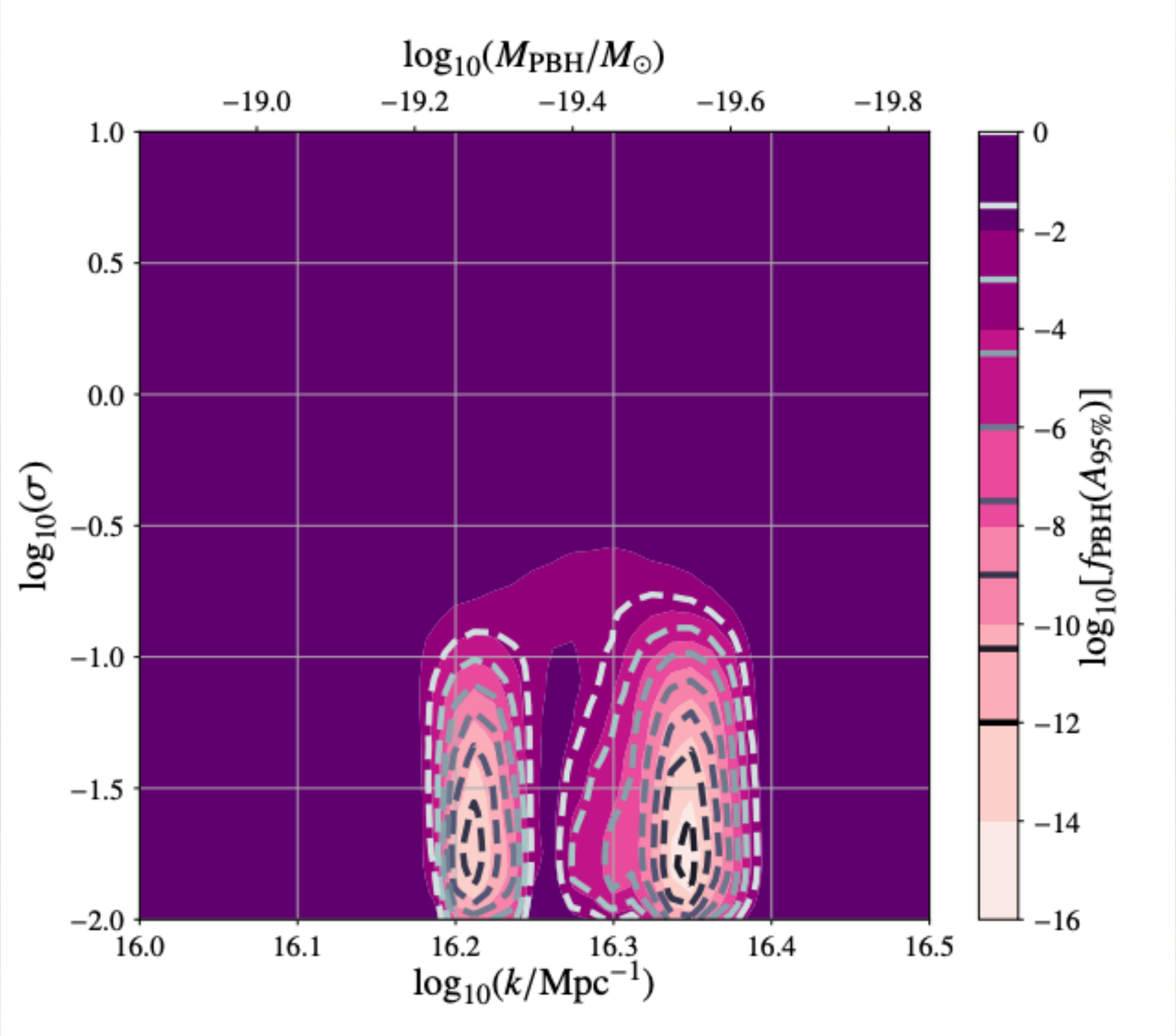
Sanity Check



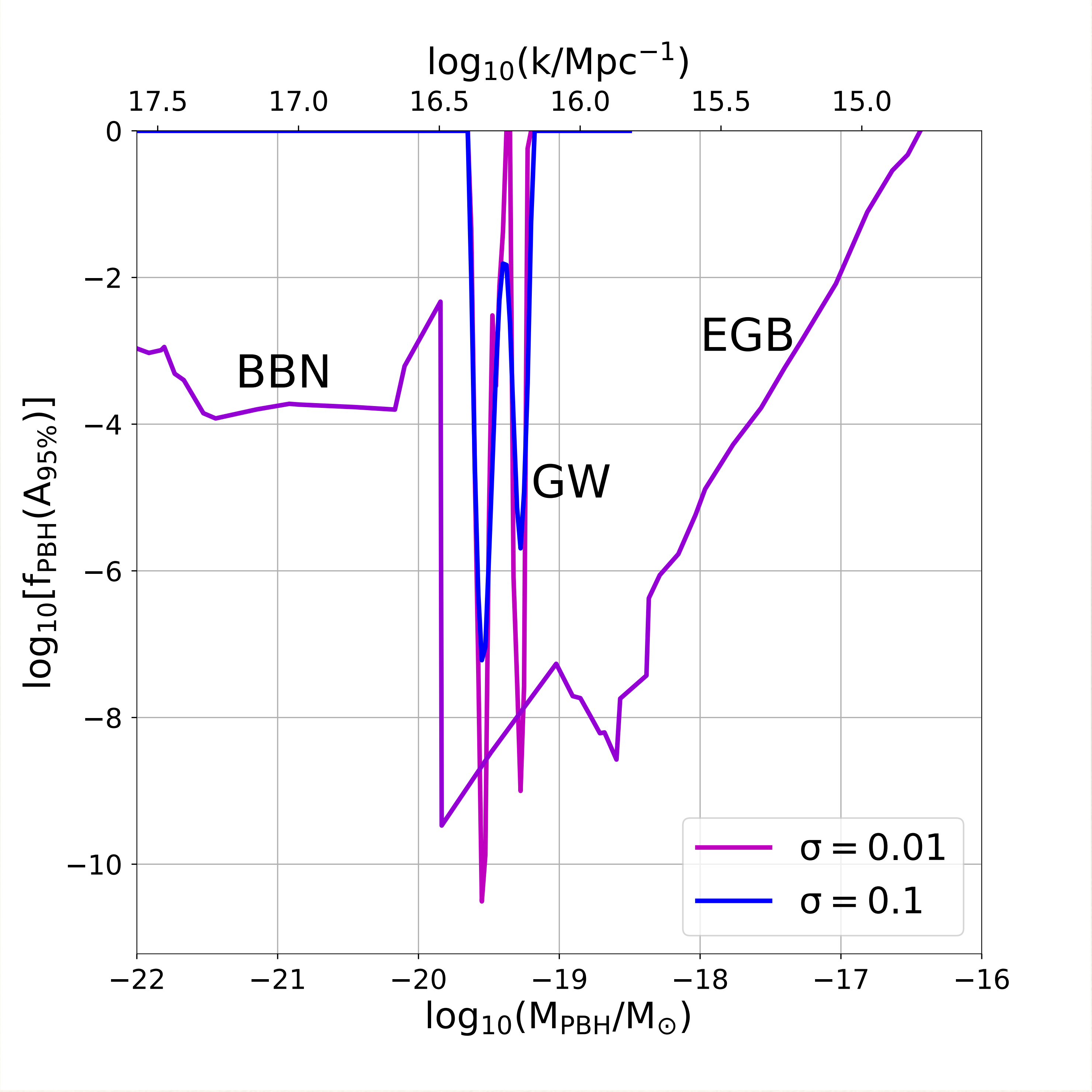


Amplitude and Abundance





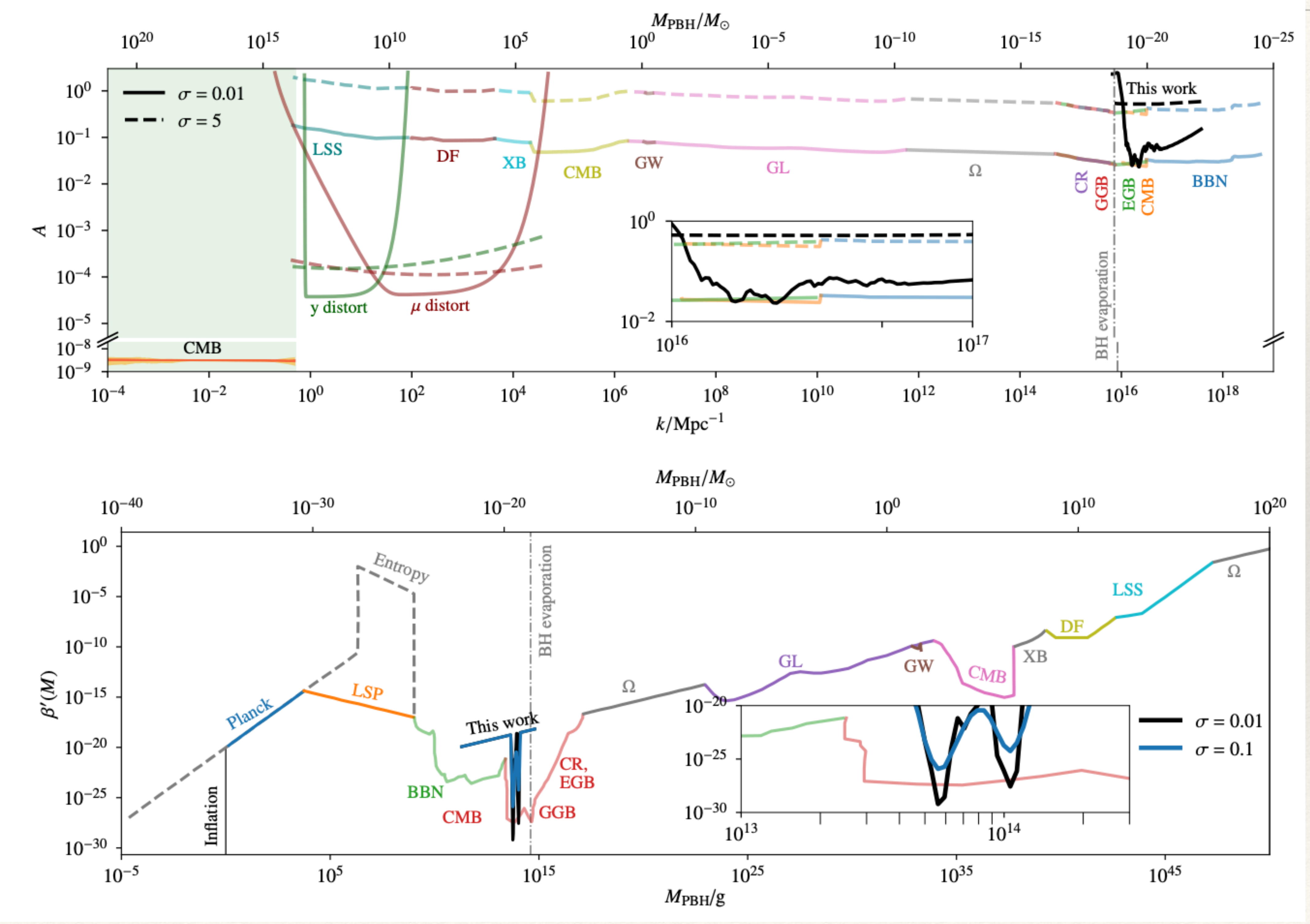
EM vs GW constraints



BBN, EGB:
Hawking Radiation
Based

GW:
Hawking Radiation
Independent

Global Perspective of our Constraints



Conclusions

- We search for stochastic background from primordial curvature perturbations
- We use O2 data cross-correlation data made publicly available
- No conclusive evidence of a signal
- Upper limits on A and f_{PBH}
    - Very first from GW search and PE
    - Should get significantly stronger with O3 and O4 data

Cross Correlation Statistic

$C(f,t)$ is

$$\begin{aligned}\langle C(f,t)\rangle &\equiv \frac{2}{T}\langle \tilde{d}_I^\star(f)\tilde{d}_J(f,t)\rangle \\ &= \frac{2}{T}\left(\langle \tilde{h}_I^\star(f,t)\tilde{h}_J(f,t)\rangle + \langle \tilde{h}_I^\star(f,t)\tilde{n}_J(f,t)\rangle + \langle \tilde{n}_I^\star(f,t)\tilde{h}_J(f,t)\rangle + \langle \tilde{n}_I^\star(f,t)\tilde{n}_J(f,t)\rangle\right) \\ &\propto \Omega_{\rm GW}(f) \qquad (18)\end{aligned}$$

$$S_h(f) = \frac{3H_0^2}{2\pi^2}\frac{\Omega_{\rm GW}(f)}{f^3}$$

Variance in the Press-Schechter Distribution

$$\sigma_c^2(k) = \frac{16}{81} \int_{-\infty}^{+\infty} \frac{q^4}{k^4} \mathcal{T}^2(q, 1/k) P_\zeta(q) d\log q\, w^2(q/k) \quad (3.8)$$

Time Averaged Power Spectrum

$$\left(\frac{k}{aH}\right)^2 \overline{\mathrm{P}_h(\tau,k)} = 4\int_0^\infty dv \int_{-|1-v|}^{1+v} du \left[\frac{4v^2-(1+v^2-u^2)^2}{4uv}\right]^2 (k\tau)^2 \overline{I^2(v,u,k\tau \gg 1)} \mathcal{P}_\zeta(kv)\mathcal{P}_\zeta(ku)\ . \tag{D2}$$

$$(k\tau)^2\overline{I^2(v,u,k\tau\gg 1)} = \frac{1}{2}\left(\frac{3(u^2+v^2-3)}{4u^3v^3}\right)^2 \left(\left(-4uv+(u^2+v^2-3)\ln\left|\frac{3-(u+v)^2}{3-(u-v)^2}\right|\right)^2 + \pi^2(u^2+v^2-3)^2\Theta(v+u-\sqrt{3})\right). \tag{D3}$$