Gaussian Process Reconstruction of Reionization History

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Epoch of Reionization

Universe expanding and cooling

Present Day

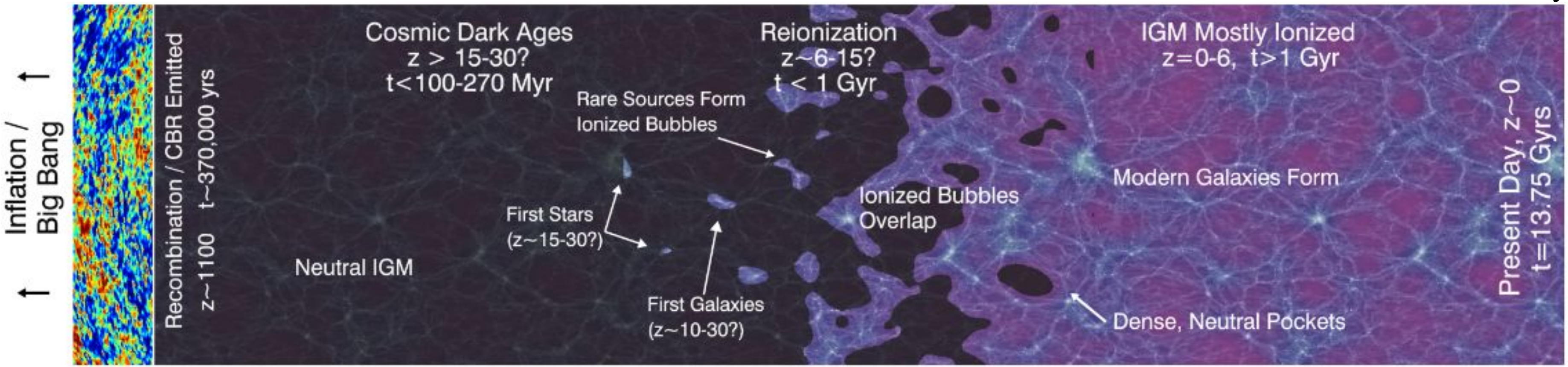


Image source: arXiv 1011.0727

Observational Probes of Reionization

QUASAR SPECTRA

Evidence of Reionization:

Lyman-α forest

Absorption of redshifted photons from the quasar by HI in the IGM (at 1215.67Å - corresponding to the Lyman-α transition of HI)

Lyman-α optical depth:

$$\tau_\alpha = \int_O^Q n_{\mathrm{HI}} \sigma_\alpha dl/(1+z)$$

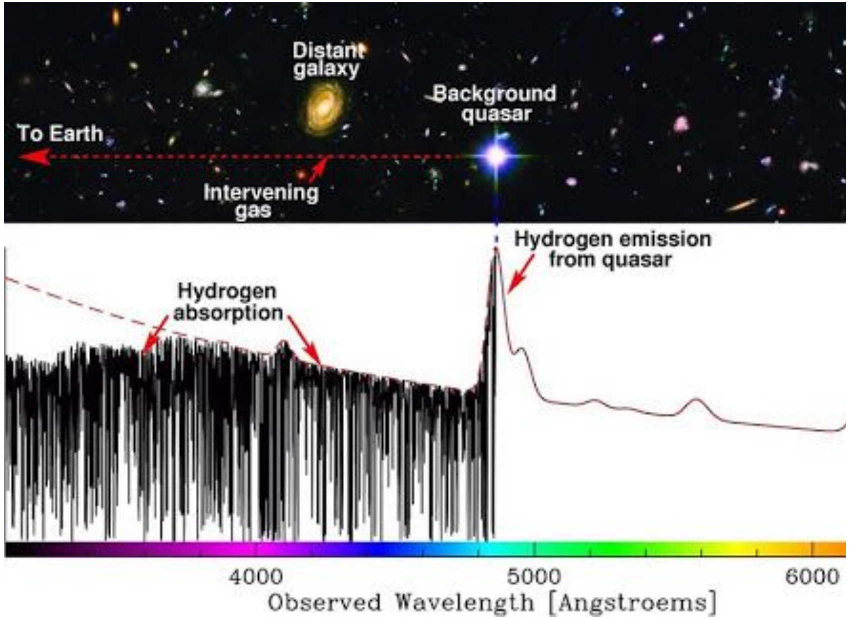


Image source: ESO

COSMIC MICROWAVE BACKGROUND

The CMB photons undergo Thomson scattering with the free electrons from reionization.

Causes amplification in the CMB polarization signal at large scale ("reionization bump")

Thomson scattering optical depth:

$$\tau = \int \sigma_T n_e dl$$

$$\tau = \int \sigma_T n_e \frac{cH_0^{-1} dz}{(1+z)\sqrt{\Omega_m (1+z)^3 + \Omega_\Lambda}}$$

Modelling Reionization

We look at two approaches of modelling the process of reionization:

1. Parametrizing the fraction of neutral hydrogen in the intergalactic medium as a function of redshift.

2. Solving the ionization equation for the intergalactic medium

Parametric Models

Free electron fraction per hydrogen ionization : $x_e(z) \equiv \frac{n_e(z)}{n_H(z)}$

$$\tau = n_{\rm H}(0) c \sigma_{\rm T} \int_0^{z_{\rm max}} dz \, x_{\rm e}(z) \frac{(1+z)^2}{H(z)}$$

Planck collaboration uses the following parametrizations of the free electron fraction in the IGM:

1. Redshift symmetric tanh parametrization
2. Redshift asymmetric parametrization

Redshift Symmetric Parametrization

Tanh reionization history (used since CAMB):

$$x_e(z) = \frac{f}{2}\left[1 + \tanh\left(\frac{y(z_{re}) - y(z))}{\delta y}\right)\right]$$

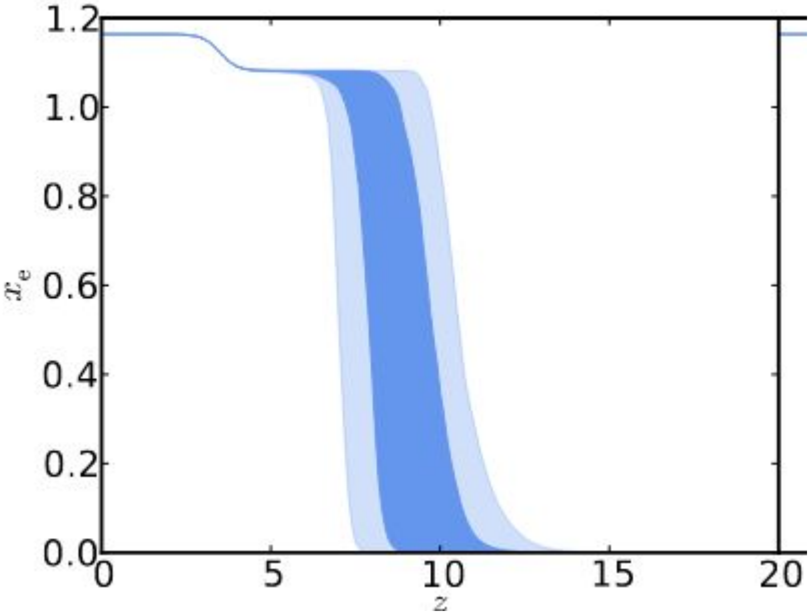
$$f = 1 + F_{He} = 1 + \frac{n_{He}}{n_H} \sim 1.08$$

$$\delta y = \frac{3}{2}\sqrt{1+z}\;\delta z$$

δz is the width of the hyperbolic tangent step.

z_{re} is the redshift where reionization is 50% complete, *i.e.* $x_e(z_{re}) = f/2$.

Planck constraints



[Source: Planck intermediate results. XLVII: Planck constraints on reionization history]

Redshift Asymmetric Parametrization

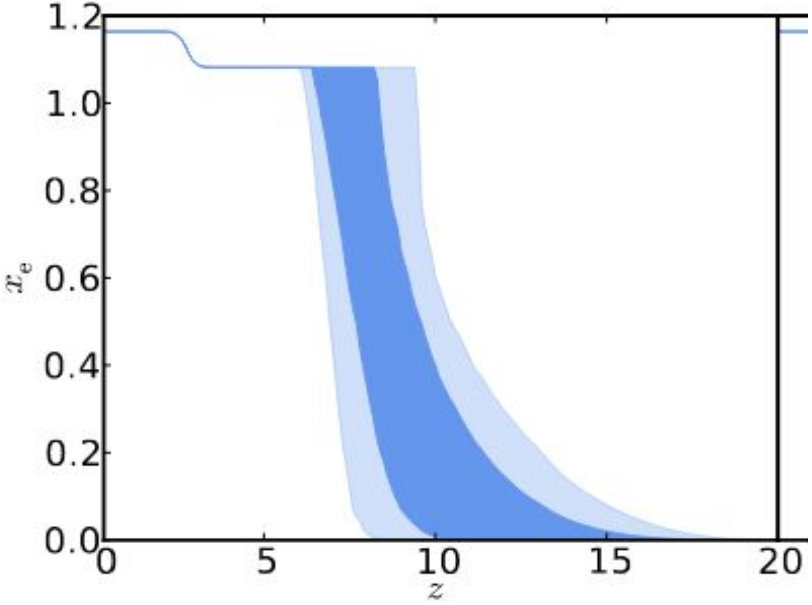
A power law form of this parametrization is used in Planck XLVII:

$$x_e(z) = \begin{cases} f & \text{for } z < z_{end} \\ f\left(\frac{z_{early}-z}{z_{early}-z_{end}}\right)^{\alpha} & \text{for } z \geq z_{end} \end{cases}$$

z_{early} is taken to be z=20 here.

model	z_{re}	Δz	z_{end}	z_{beg}
redshift-symmetric . .	8.8 ± 0.9	< 4.6	< 8.6	9.4 ± 1.2
redshift-asymmetric . .	8.5 ± 0.9	< 6.8	< 8.9	10.4 ± 1.8

Planck constraints



[Source: Planck intermediate results. XLVII: Planck constraints on reionization history]

Ionization Equation

The volume filling factor of ionized hydrogen, $Q_{H_{II}}$ is given by:
(dotted quantities represent time derivatives)

$$\frac{dQ_{H_{II}}}{dt} = \frac{\dot{n}_{ion}}{\langle n_H \rangle} - \frac{Q_{H_{II}}}{t_{rec}}$$

$\dot{n}_{ion}$: ionizing photon production rate

$$\dot{n}_{ion} = \rho_{UV} \langle \xi_{ion} f_{esc} \rangle$$

ρ_{UV}: UV luminosity density

ξ_{ion} : photon production efficiency

f_{esc} : escape fraction

$\langle n_H \rangle$: avg density of H atoms; $\langle n_H \rangle = \frac{X_p \Omega_b \rho_c}{m_H}$

Recombination time

$$t_{rec}(z) = \frac{1}{C_{H_{II}} \alpha(T_{IGM}) \left(1 + \frac{Y_p}{4X_p}\right) \langle n_H \rangle (1+z)^3}$$

Clumping factor: $C_{H_{II}} \equiv \frac{\langle n_{H_{II}}^2 \rangle}{\langle n_{H_{II}} \rangle^2}$

In terms of redshift, we can write the ionization equation as:

$$\frac{dQ_{H_{II}}(z)}{dz} = -\frac{1}{(1+z)H(z)}\left(\frac{\rho_{UV}(z)\langle\xi_{ion}f_{esc}\rangle}{\langle n_H\rangle} - \frac{Q_{H_{II}}(z)}{t_{rec}(z)}\right)$$

The Thomson optical depth expression then becomes

$$\tau = \int \frac{c(1+z)^2}{H(z)} Q_{H_{II}}(z) n_H(0) \sigma_T \left(1 + \eta \frac{Y_p}{4X_p}\right) dz$$

where,

$$\eta \equiv \begin{cases} 0 & \text{if only H ionized} \\ 1 & \text{if H ionized and He singly ionized} \\ 2 & \text{if H ionized and He doubly ionized} \end{cases}$$

Datasets

1. Optical depth constraints from Planck 2018 release ($\tau = 0.054 \pm 0.007$)
2. The derived UV luminosity density data [in Ishigaki et. al. (2018)] analysis, from HFF observations. Here we use the luminosity density measurements with truncation magnitudes of -17 and -15 (labelled ahead as UV17 and UV15 respectively)
3. We use the measurements of neutral hydrogen fractions from the Lyman-α emission from galaxies, damping wings of Gamma Ray Bursts and Quasars spectra.

UV Luminosity Density

To obtain solutions to the ionization equation, we need to assume some form for the UV luminosity density.

The evolution of the UV luminosity density with redshift can be obtained by parametric and non-parametric methods.

We consider commonly used parametric methods which assume the density to be described by single power-law [Yu et al. (2012);Bouwens (2016)] and double power-law [Ishigaki et al. (2015); Ishigaki et al. (2018)] forms:

$$\rho_{UV}(z) = \rho_{UV_1} \cdot 10^{-az}$$

$$\rho_{UV}(z) = \frac{2\rho_{UV,z=z_1}}{10^{a(z-z_1)} + 10^{b(z-z_1)}}$$

Gaussian Process Regression

- A *Gaussian process* is a collection of random variables, any finite number of which have a joint Gaussian distribution.
- A Gaussian process is completely specified by its mean function and covariance function

for a real process $f(\mathrm{x})$, we have

Mean function: $\mu(\mathrm{x}) = \mathbb{E}[f(\mathrm{x})]$

Covariance function: $k(\mathrm{x}, \mathrm{x}') = \mathbb{E}[(f(\mathrm{x}) - \mu(\mathrm{x}))(f(\mathrm{x}') - \mu(\mathrm{x}'))]$

Ref: Rasmussen and Williams 2006

Gaussian Process Regression

Consider a finite set of training points $\mathrm{x} = \{x_i\}$. A function $f(\mathrm{x})$ evaluated at each x_i is a random variable with a Gaussian distribution, such that the vector $\mathrm{f} = \{f_i\}$ has a multivariate Gaussian distribution given as:

$$\mathrm{f} \sim \mathcal{N}\left(\mu(\mathrm{x}), C(\mathrm{x}, \mathrm{x})\right)$$

Where C is the covariance matrix characterized by the covariance function k, which gives the covariance between two random variables,

$$[C(\mathrm{x}, \mathrm{x})]_{ij} = \mathrm{cov}(f_i, f_j) = k(x_i, x_j)$$

Radial Basis Function (RBF) kernel:

$$k(x_i, x_j) = \exp\left(-\frac{(x_i - x_j)^2}{2l^2}\right) ,$$

l : correlation length (kernel hyperparameter)

Ref: Rasmussen and Williams 2006

Gaussian Process Regression

Let $\{x_i^*, f_i^*\}$ be a finite set of test points, then,

$$\mathrm{f}^* \sim \mathcal{N}\left(\mu(\mathrm{x}^*), C(\mathrm{x}^*, \mathrm{x}^*)\right)$$

The joint distribution of f and f^* is given by:

$$\begin{bmatrix} \mathrm{f} \\ \mathrm{f}^* \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \mu(\mathrm{x}) \\ \mu(\mathrm{x}^*) \end{bmatrix}, \begin{bmatrix} C(\mathrm{x}, \mathrm{x}) & C(\mathrm{x}, \mathrm{x}^*) \\ C(\mathrm{x}^*, \mathrm{x}) & C(\mathrm{x}^*, \mathrm{x}^*) \end{bmatrix} \right)$$

To get the posterior distribution over functions we need to restrict this joint prior distribution to contain only those functions which agree with the observed data points.

Conditional distribution gives

$$\begin{aligned} \mathrm{f}^* | \mathrm{x}^*, \mathrm{x}, \mathrm{f} \quad \sim \quad & \mathcal{N}\Big(\mu(\mathrm{x}^*) + C(\mathrm{x}^*, \mathrm{x}) C(\mathrm{x}, \mathrm{x})^{-1} (\mathrm{f} - \mu(\mathrm{x})), \\ & \qquad C(\mathrm{x}^*, \mathrm{x}^*) - C(\mathrm{x}^*, \mathrm{x}) C(\mathrm{x}, \mathrm{x})^{-1} C(\mathrm{x}, \mathrm{x}^*)\Big) \end{aligned}$$

Gaussian Process Regression

If we have noisy data $\{x_i, y_i\}$ (with variance σ_i^2) as training points, then

$$\begin{bmatrix} \mathrm{y} \\ \mathrm{f}^* \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \mu(\mathrm{x}) \\ \mu(\mathrm{x}^*) \end{bmatrix}, \begin{bmatrix} C(\mathrm{x},\mathrm{x}) + \sigma_i^2 I & C(\mathrm{x},\mathrm{x}^*) \\ C(\mathrm{x}^*,\mathrm{x}) & C(\mathrm{x}^*,\mathrm{x}^*) \end{bmatrix} \right)$$

we can find the joint posterior distribution ($\mathrm{f}^* | \mathrm{x}^*, \mathrm{x}, \mathrm{y}$) by conditioning this joint Gaussian prior distribution on the observations.

The kernel hyperparameter l can be trained using the data points by marginalizing over all functions f at x, and maximizing the marginal likelihood $P(\mathrm{y}|\mathrm{x}, l) = \int P(\mathrm{y}|\mathrm{f}) P(\mathrm{f}|\mathrm{x}, l) d\mathrm{f}$

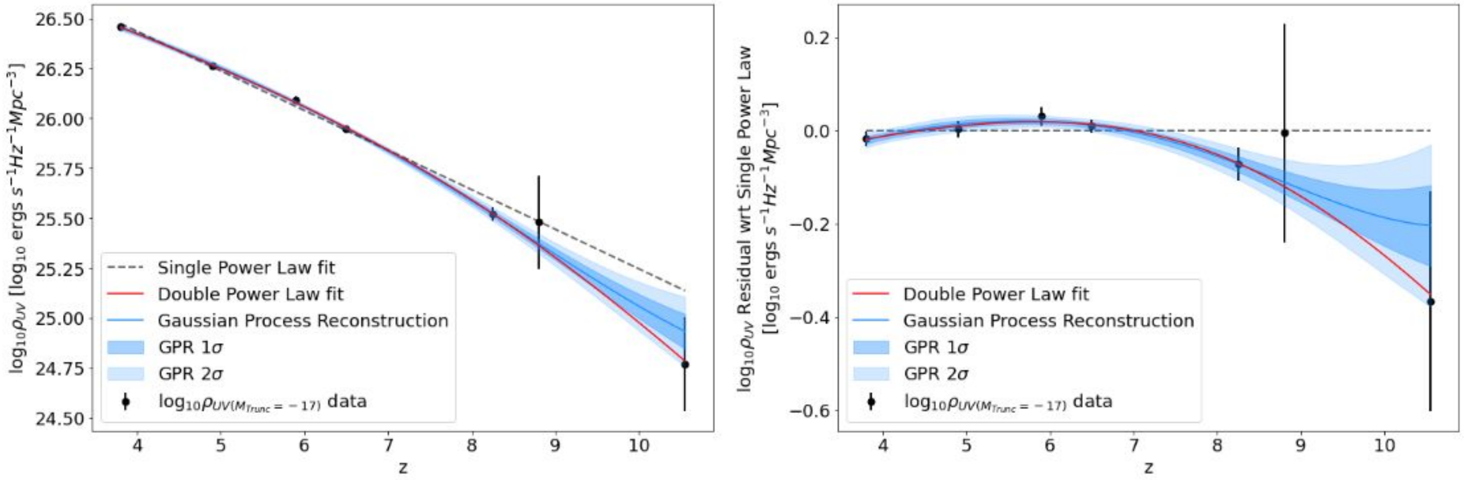
$$\Longrightarrow \ln P(\mathrm{y}|\mathrm{x}, l) = -\frac{1}{2}(\mathrm{y}-\mu(\mathrm{x}))^T (C(\mathrm{x},\mathrm{x}) + \sigma_i^2 I)^{-1} (\mathrm{y}-\mu(\mathrm{x}))^T - \frac{1}{2} \ln |C(\mathrm{x},\mathrm{x})| - \frac{n}{2} \ln 2\pi$$

Once the value of the correlation length hyperparameter is obtained by maximizing the above equation, we can predict the values for the test points f* at the locations x*.

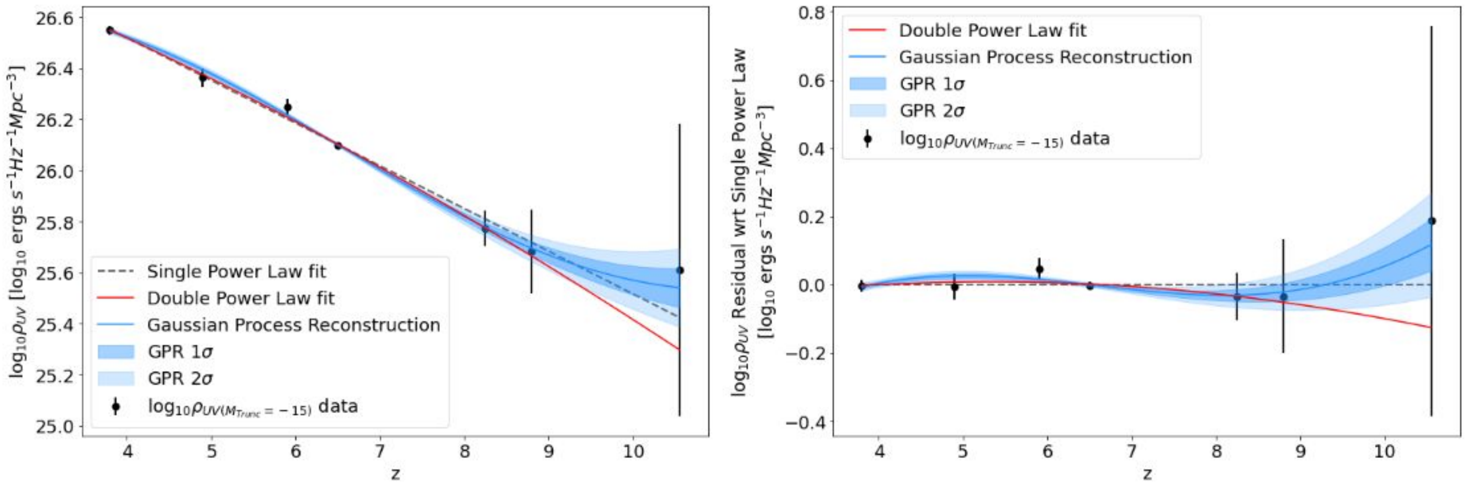
Ref: Rasmussen and Williams 2006

Can a power-law explain the UV luminosity density data?

UV17



UV15



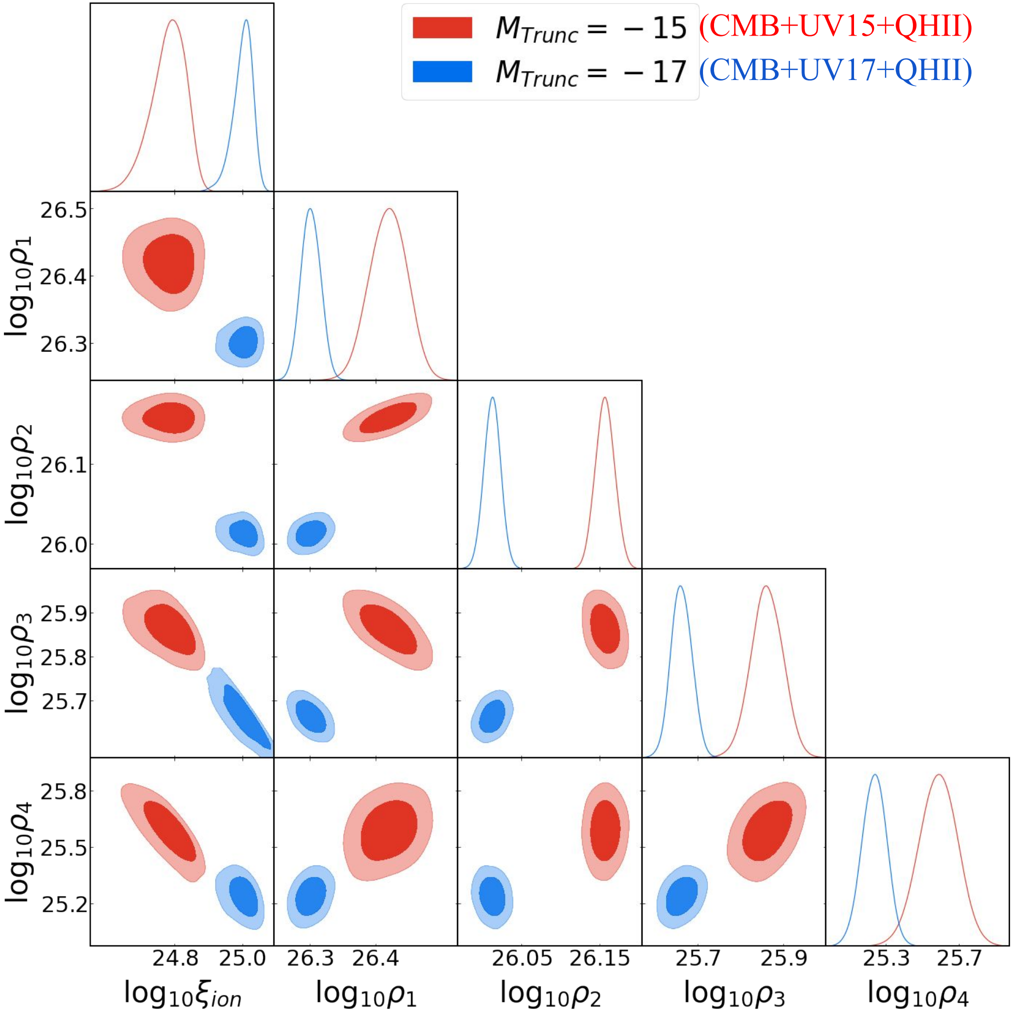
arXiv:2106.01728

Constraints on Reionization History

$$\frac{dQ_{H_{II}}(z)}{dz} = -\frac{1}{(1+z)H(z)}\left(\frac{\rho_{UV}(z)\langle\xi_{ion}f_{esc}\rangle}{\langle n_H\rangle} - \frac{Q_{H_{II}}(z)}{t_{rec}(z)}\right)$$

We now obtain joint constraints on reionization history using all the 3 data sets described earlier: Planck optical depth, QHII data and UV luminosity density data for $M_{\text{trunc}} = -17$ and -15.

- 4 equidistant nodes between redshifts 4-10 to define the UV luminosity densities.
- The values of UV luminosity density at the redshift nodes are taken as free parameters for MCMC sampling, and at each step these points are used as training points for GPR
- Solve the ionization equation to get the reionization history



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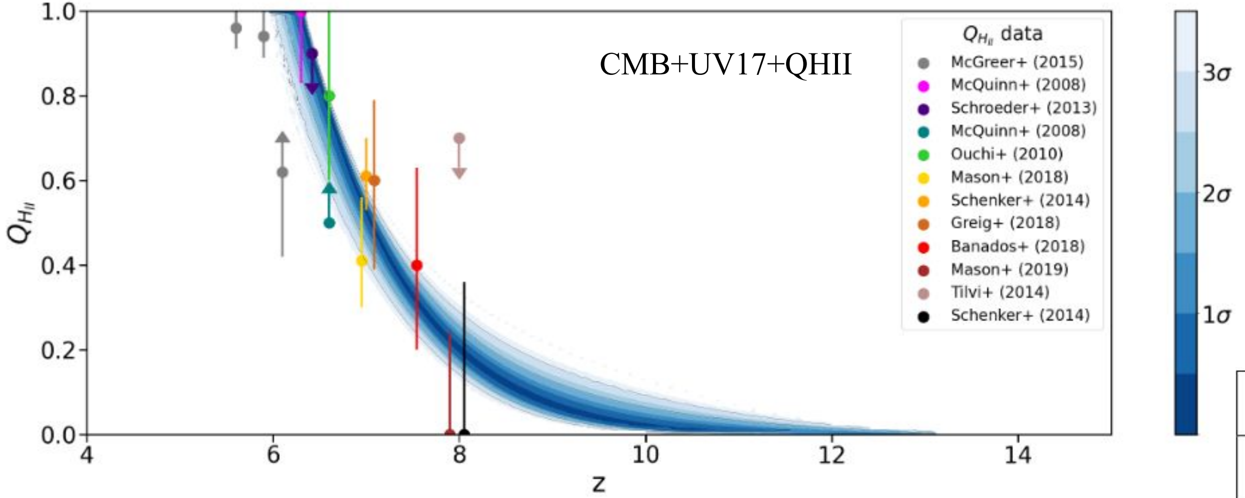

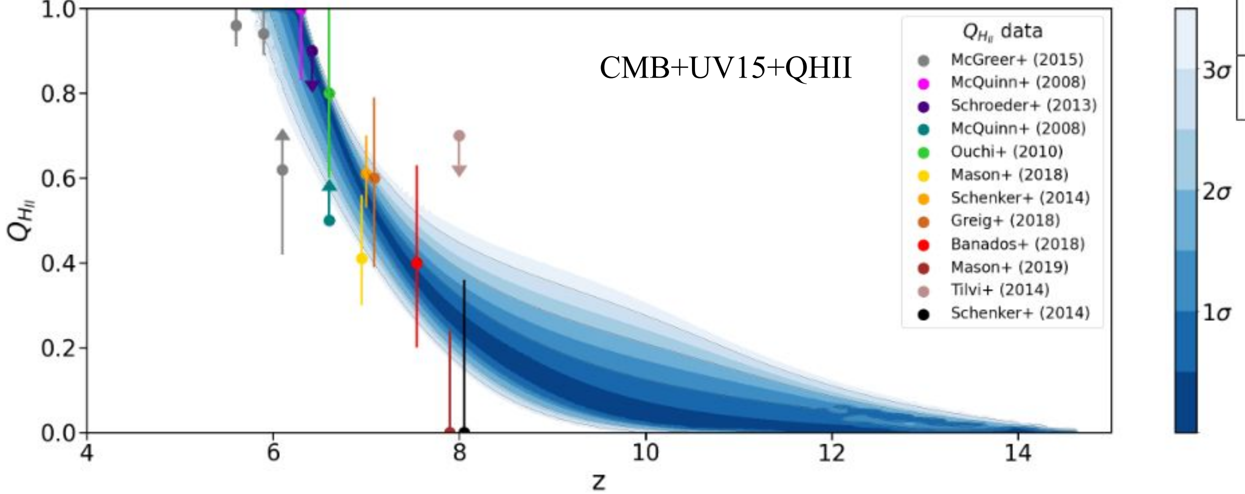

	CMB+UV17+QHII	CMB+UV15+QHII
τ	$0.052 \pm 0.001 \pm 0.002$	$0.056^{+0.002+0.006}_{-0.004-0.005}$
Δz	$2.05^{+0.11+0.37}_{-0.21-0.30}$	$2.98^{+0.30+1.8}_{-1.0-1.1}$
z_{re}	$6.93^{+0.08+0.11}_{-0.04-0.13}$	$7.00^{+0.11+0.22}_{-0.10-0.22}$

arXiv:2106.01728

Conclusions

- While the commonly used logarithmic double power-law model of UV luminosity density evolution agrees well with the data, the single power-law is ruled out.
- Using the reconstructed UV luminosity density evolution, we reconstruct the reionization history using the optical depth from the CMB observation, UV luminosity data from HFF observation, and neutral hydrogen fraction data from galaxy, quasar and gamma ray burst observations.

  For CMB+UV17+QHII, we get optical depth $0.052 \pm 0.001 \pm 0.002$ (agrees with 1σ optical depth from Planck results)
- High redshift observations with JWST and THESEUS will definitely be helpful in providing better constraints, particularly around the tail