Helical magnetic fields lead to baryogenesis

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Based on the works with **S. Shankaranarayanan**,
PRD 102,103528 (2020) [arXiv:2008.10825] and [2103.05339]

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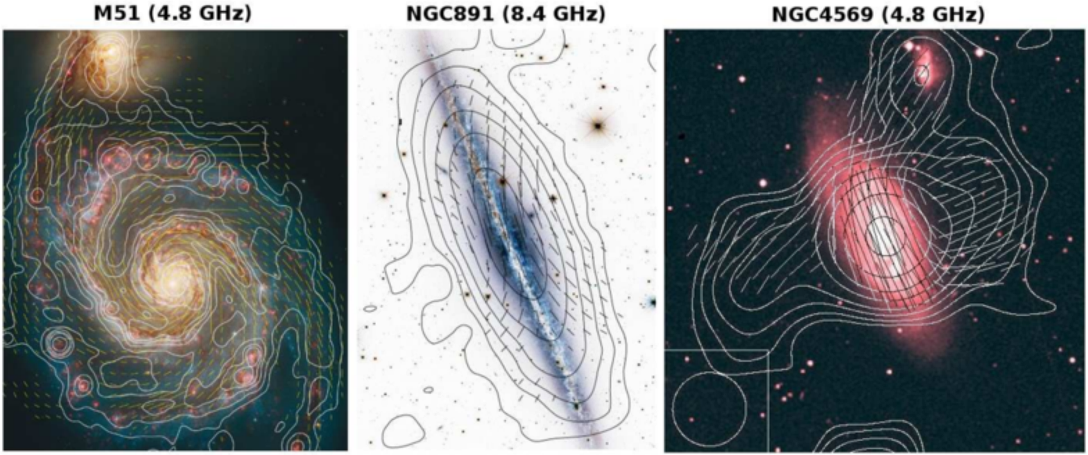
April 17, 2020

Outline

- Two unresolved problems
  - Origin of cosmological magnetic fields
  - Matter-antimatter asymmetry
- Helical magnetic fields
- Problem with magnetic field generation during inflation
- Helical magnetic fields from Riemann coupling
- Baryogenesis
- Conclusion

Two unresolved problems

Observations of magnetic fields in the universe

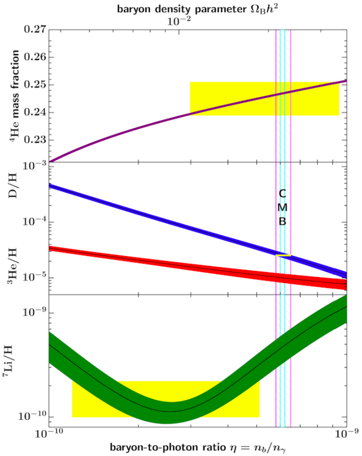


Max Planck Institute for Radio Astronomy

Micro-Gauss strength magnetic field over 10kpc coherence length scale is present in galaxies.

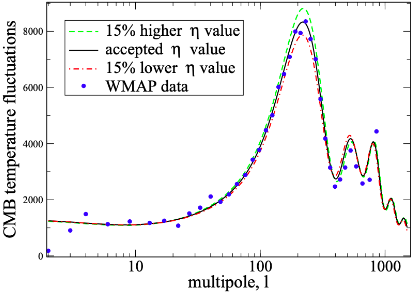
Origin of cosmological magnetic fields is still an unresolved problem.

Matter-Antimatter asymmetry

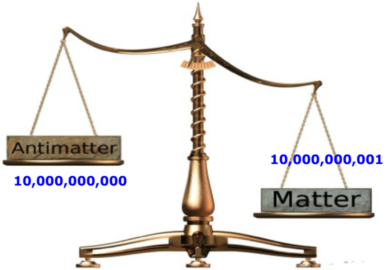


Particle Data Group (2019)

$$\eta_B \equiv \frac{n_B - n_{\bar{B}}}{n_\gamma} \simeq 6.1 \times 10^{-10}$$



J. Cline (2006)



Sakharov's conditions

In 1967, Sakharov proposed three necessary conditions for creating the baryon asymmetry

- Baryon number violation
- Charge (C) and charge parity (CP) violation
- Departure from thermal equilibrium

Davidson's conditions

Davidson, PLB (1996)

In 1996, Davidson pointed out an interesting relation between the primordial magnetic field and Sakharov's conditions

- There should be some out of thermal equilibrium dynamics because in equilibrium, the photon distribution is thermal, and there are no particle currents to sustain a "long-range" field
- Since $\overrightarrow{B}$ is odd under C and CP, the presence of magnetic field will lead to CP violation.
- Since the magnetic field is a vector quantity, it chooses a particular direction hence breaks the isotropy (rotational invariance).

Davidson's conditions are necessary but not sufficient. There is a key missing ingredient.

Helical magnetic fields

Two kinds of fields

- Electromagnetic field has two transverse degrees of freedom which can be associated with Left circular and right circular polarization.
- For massless particle helicity is the projection of the direction of spin (clockwise or anti-clockwise) along the direction of propagation. Hence giving $+1, -1$ **for right handed and left handed helicity modes.**
- Same propagation (**speed or dispersion relation**) of both polarization modes lead to non-helical, and differently propagating modes lead to helical fields.
- If both the polarization modes propagate differently $\rightarrow$ **Helicity imbalance**

How to create helicity imbalance?

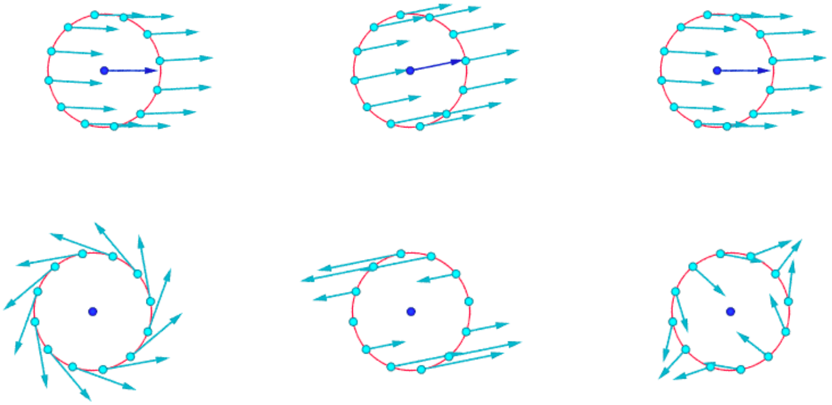
Helical magnetic fields

- Lorentz force, $\overrightarrow{F} = m\frac{d\overrightarrow{v}}{dt} = \overrightarrow{E} + \overrightarrow{v} \times \overrightarrow{B}$ implies that under parity transformation (changing the sign of coordinate system): $\overrightarrow{E} \longrightarrow -\overrightarrow{E}, \overrightarrow{B} \longrightarrow \overrightarrow{B}$.
- Because standard EM action, $F_{\mu\nu}F^{\mu\nu} \propto B^2 - E^2$, is quadratic in $\overrightarrow{E}$ and $\overrightarrow{B}$, it is invariant under parity symmetry.
- $F_{\mu\nu}\tilde{F}^{\mu\nu} = -4\overrightarrow{E} \cdot \overrightarrow{B}$ is parity non-invariant, where $\tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$.

Hence $F_{\mu\nu}\tilde{F}^{\mu\nu}$ can create the **Helicity imbalance**.

A simple example: Vorticity

Vorticity is defined as $\vec{\Omega} = \vec{\nabla} \times \vec{v}$, where $\vec{v}$ is velocity field.



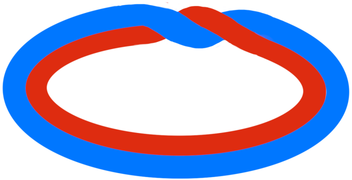
Vorticity ≠ 0 Vorticity ≠ 0 Vorticity = 0

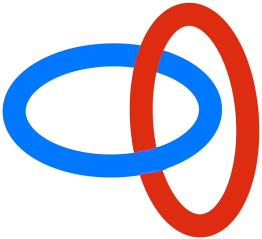
Wikipedia

Magnetic helicity

- Magnetic helicity ($\mathcal{H}_M$) is defined as: $\int d^3x \overrightarrow{A} \cdot \overrightarrow{B}$ and $\overrightarrow{B} \cdot \overrightarrow{\nabla} \times \overrightarrow{B}$.

- It is a measure of twist and linkage of magnetic field lines.





Grasso and Rubinstein (2001), Blackman (2014)

$$\mathcal{H}_M = \int d^3x \overrightarrow{A} \cdot \overrightarrow{B} = 2 V_1 \cdot V_2$$

Why helical magnetic fields are interesting?

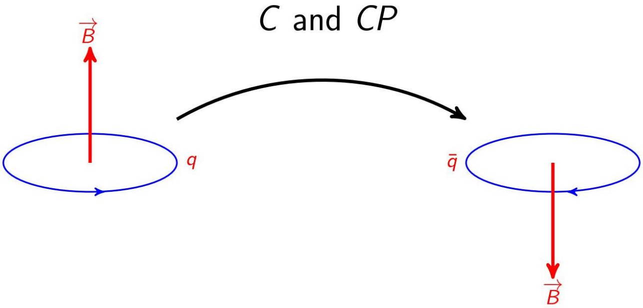
- Helical magnetic fields leave a very distinct signature as they **violate parity symmetry** which leads to observable effects, e.g. correlations between the anisotropies in the temperature and B-polarisation or in the E- and the B-polarisations in the CMB.
  Kahniashvili (2006)

- One of the interests in primordial magnetic helicity is that it can be a direct indication of parity violation (CP violation) in the early Universe. Vachaspati (2001)

- The decay rate of energy density and coherence length is slower for helical magnetic fields due to inverse cascade (**transfer power from small to large scales so that even blue spectra can lead to significant power on large scales**). Durrer etal.(2011)

Missing link between Sakharov and Davidson's conditions

Broken symmetries in the presence of magnetic field



Davidson's conditions :

- There should be some out-of-thermal-equilibrium dynamics
- Breaks C, CP and $SO(3)$

Presence of magnetic fields satisfy only two of Sakharov's conditions

Missing ingredient: Helical magnetic fields

- The presence of helical fields leads to **non-zero Chern-Simons number density** and, eventually, the change in the Fermion number.

- In the presence of an electromagnetic field in curved space-time, the chiral anomaly is given by the following equation

$$\nabla_\mu J^\mu_A = -\frac{1}{384\pi^2}\epsilon^{\mu\nu\rho\sigma}R_{\mu\nu\alpha\beta}R^{\alpha\beta}{}_{\rho\sigma} + \frac{e^2}{16\pi^2}\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta} \quad (1)$$

  where J^μ_A is the chiral current.

- **First term on RHS vanishes (up to first order) in flat FRW universe**, however due to the presence of the antisymmetric tensor, the gravitational fluctuations lead to gravitational birefringence and can lead to net chiral current. Alexander et al. (2006)

- The second term on RHS, in magnetic field background is non-zero and hence leads to a net chiral current.

- Baryon number density $n_B = n_b - n_{\bar{b}} = a(\eta)\langle 0|J_A^0|0\rangle = \frac{e^2}{4\pi^2} a(\eta) n_{CS}$, where Chern Simon number density is

$$n_{CS} = \frac{1}{a^4}\int_{\mu}^{\Lambda} \frac{dk}{k} \frac{k^4}{2\pi^2}\left(|A_+|^2 - |A_-|^2\right) \tag{2}$$

- For **non-helical fields** $|A_+| = |A_-|$ which implies $n_{CS} = 0$.
- For **helical fields**, $n_{CS} \neq 0$ implies an imbalance between baryons and anti-baryons $\longrightarrow$ **Baryon number violation**

Hence the requirement of helical magnetic fields
to have non-zero n_{CS} is missing in Davidson's conditions.

How to generate magnetic fields ?

Problem with magnetic field generation during inflation

- EM action for an arbitrary 4-D metric

$$S_{\mathrm{EM}} = -\frac{1}{4}\int d^4x\sqrt{-g}\,g^{\alpha\mu}g^{\beta\nu}F_{\alpha\beta}F_{\mu\nu}$$

  where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, and A^μ is electromagnetic four vector.
- Under conformal transformation $\tilde{g}_{\mu\nu} = \omega^2(x)g_{\mu\nu}$

$$\tilde{S}_{\mathrm{EM}} = -\frac{1}{4}\int d^4x\sqrt{-\tilde{g}}\,\tilde{g}^{\alpha\mu}\tilde{g}^{\beta\nu}F_{\alpha\beta}F_{\mu\nu} = S_{\mathrm{EM}}$$

- EM action is conformally invariant, and hence equations of motion for magnetic fields

- Flat FRW line element:

$$ds^2 = dt^2 - a^2(t)\left[dx^2 + dy^2 + dz^2\right] \tag{3}$$

with $dt = a(\eta)d\eta$ is

$$ds^2 = \underbrace{a^2(\eta)[d\eta^2 - dx^2 - dy^2 - dz^2]}_{\text{conformally flat}} \tag{4}$$

- FRW models are conformally flat: $g_{\mu\nu} = a^2(\eta)\eta_{\mu\nu}$
- EM action in conformal FRW metric is

$$\mathcal{S} = -\frac{1}{4}\int d^4x \eta^{\alpha\mu}\eta^{\beta\nu}F_{\alpha\beta}F_{\mu\nu}$$

Which is same as in Minkowski space-time.

- Hence in conformally flat FRW background, $B \sim \frac{1}{a^2}$
- for inflation $a(t) = e^{Ht}$, so after the end of inflation (for 60 e-foldings), $B \sim e^{-120}$

We need to break the conformal invariance of EM action

(Helical fields) Models in the literature

Scalar field coupled models: $f(\phi)F_{\mu\nu}\tilde{F}^{\mu\nu}$ where $f(\phi)$ is time-dependent coupling function.

- **Problems with these models :**
  - **Strong coupling** - Coupling between charged particles and the EM field is so strong that theory can not be treated perturbatively.
  - **Back-reaction** - Overproduction of gauge fields affect the background inflationary dynamics
  - Because magnetic fields are produced near the end of inflation, strength of the **fields generated depends on the reheating scale.**

To resolve strong coupling and back-reaction problem $f(\phi)$ is assumed to increase during inflation and decrease back to its initial value post inflation.
Durrer et al.(2011), Sharma et al.(2018)

Helical magnetic fields from Riemann coupling

Motivation

- Non-minimal coupling to the Riemann tensor generates sufficient primordial helical magnetic fields at **all observable scales.**
- **Necessary condition :** Conformal invariance breaking + parity violation

$$
\begin{aligned}
S = & \overbrace{-\frac{M_{\mathrm{P}}^2}{2}\int d^4x\sqrt{-g}\,R}^{\text{Einstein-Hilbert term}} + \overbrace{\int d^4x\sqrt{-g}\left[\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right]}^{\text{Scalar field}} \\
& -\frac{1}{4}\int d^4x\,\sqrt{-g}\,F_{\mu\nu}F^{\mu\nu} \underbrace{-\frac{1}{M^2}\int d^4x\,\sqrt{-g}\,R_{\mu\nu}{}^{\alpha\beta}F_{\alpha\beta}\tilde{F}^{\mu\nu}}_{\text{Conformal breaking}}
\end{aligned} \tag{5}
$$

where M is the energy scale, which sets the scale for the breaking of conformal invariance.

In Flat FRW universe : $ds^2 = a^2(\eta)\left(d\eta^2 - \delta_{ij}dx^i dx^j\right)$. In the Coulomb gauge $(A^0 = 0, \partial_i A^i = 0)$, equation of motion is

$$A_i'' + \frac{4\,\epsilon_{ijl}}{M^2}\left(\frac{a'''}{a^3} - 3\frac{a''a'}{a^4}\right)\partial_j A_l - \partial_j \partial_j A_i = 0 \tag{6}$$

Which in helicity basis can be written as:

$$A_h'' + \left[k^2 - \frac{4kh}{M^2}\Gamma(\eta)\right]A_h = 0 \tag{7}$$

where,

$$\Gamma(\eta) = \frac{a'''}{a^3} - 3\frac{a''a'}{a^4} = \frac{1}{a^2}\left(\mathcal{H}'' - 2\mathcal{H}^3\right) \tag{8}$$

which vanishes for de-sitter case.

Helical magnetic field generation

- For power law inflation: $a(\eta) = \left(-\frac{\eta}{\eta_0}\right)^{\beta+1}$, de-sitter $\beta = -2$, we have

$$A_h'' + \left[k^2 - \frac{8kh}{M^2}\frac{\beta(\beta+1)(\beta+2)}{\eta_0^3}\left(\frac{-\eta_0}{\eta}\right)^{(2\beta+5)}\right] A_h = 0 \tag{9}$$

- Sub-horizon mode $|-k\eta| >> 1$ solution is: $A_h = \frac{1}{\sqrt{k}} e^{-ik\eta}$
- For super-horizon mode $|-k\eta| << 1$, with dimensionless variable, $\tau = \left(-\frac{\eta_0}{\eta}\right)^{\alpha}$ and $\alpha = \beta + \frac{3}{2}$

$$A_+(\tau, k) = \tau^{-\frac{1}{2\alpha}} J_{\frac{1}{2\alpha}}\left(\frac{\varsigma\sqrt{k}}{\alpha}\tau\right) C_1 + \tau^{-\frac{1}{2\alpha}} Y_{\frac{1}{2\alpha}}\left(\frac{\varsigma\sqrt{k}}{\alpha}\tau\right) C_2 \tag{10a}$$

$$A_-(\tau, k) = \tau^{-\frac{1}{2\alpha}} J_{\frac{1}{2\alpha}}\left(-i\frac{\varsigma\sqrt{k}}{\alpha}\tau\right) C_3 + \tau^{-\frac{1}{2\alpha}} Y_{\frac{1}{2\alpha}}\left(-i\frac{\varsigma\sqrt{k}}{\alpha}\tau\right) C_4 \tag{10b}$$

Taking $\mathcal{H} \sim {\eta_0}^{-1} \sim 10^{14}\mathrm{GeV}$, and $M \sim 10^{17}\mathrm{GeV}$ gives

$$|C_1| \approx |C_3| \approx 10^{-17/2}\mathrm{GeV}^{-\frac{1}{2}}, \quad \text{and} \quad |\mathrm{C}_2| \approx |\mathrm{C}_4| \approx 10^{-11/2}\mathrm{GeV}^{-\frac{1}{2}}. \tag{11}$$

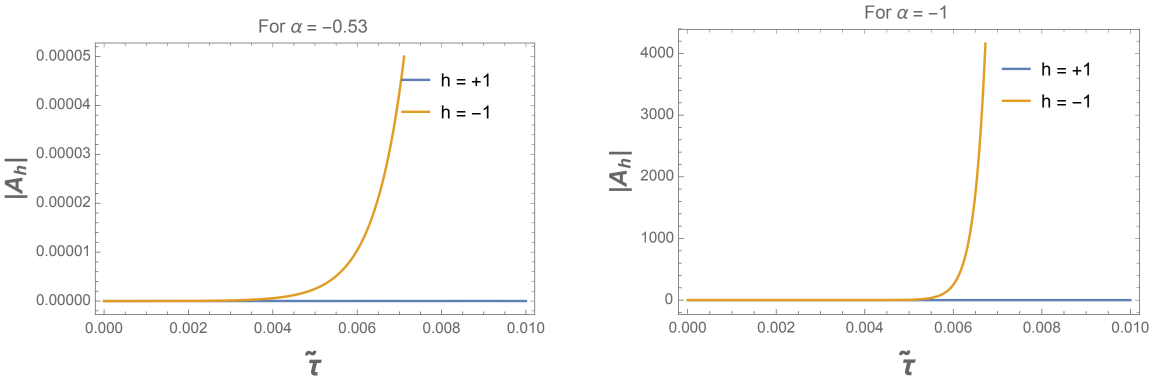
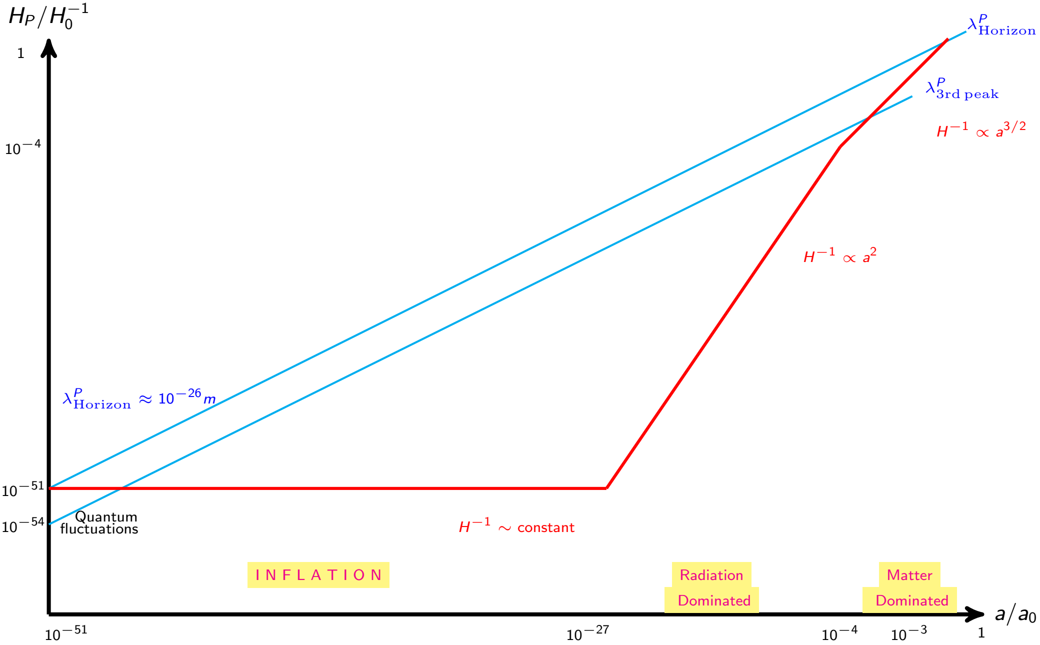


Figure: Figure showing the behaviour of positive and negative helicity mode for $\alpha = -0.53$ and $\alpha = -1$. $\tilde{\tau} = 10^{-\frac{63}{2}}\,\tau$ and the vertical axis is in $\mathrm{GeV}^{-1/2}$.

We can ignore the negative helicity mode.



H_P/H_0^{-1}
1
10^{-4}
10^{-51}
10^{-54}
$\lambda^P_{\rm Horizon}$
$\lambda^P_{\rm 3rd\ peak}$
$H^{-1} \propto a^{3/2}$
$H^{-1} \propto a^2$
$\lambda^P_{\rm Horizon} \approx 10^{-26} m$
Quantum fluctuations
$H^{-1} \sim$ constant
I N F L A T I O N
Radiation Dominated
Matter Dominated
a/a_0
10^{-51}
10^{-27}
10^{-4}
10^{-3}
1

Electromagnetic energy density

To identify whether these modes lead to back-reaction on the metric, we define R, which is the ratio of the total energy density of the fluctuations and background energy density during inflation: Talebian et al.(2020)

$$R = \frac{(\rho_B + \rho_E)|_{k_* \sim \mathcal{H}}}{6 M_P^2 H^2} \tag{12}$$

α	ρ (in GeV^4)	R
$-\frac{1}{2} - \epsilon$	$\sim 10^{64}$	$\sim 10^{-4}$
$-\frac{3}{4}$	$\sim 10^{62}$	$\sim 10^{-6}$
-1	$\sim 10^{61}$	$\sim 10^{-7}$
-3	$\sim 10^{59}$	$\sim 10^{-9}$

No back-reaction on the background metric.

Estimating the strength of helical magnetic fields

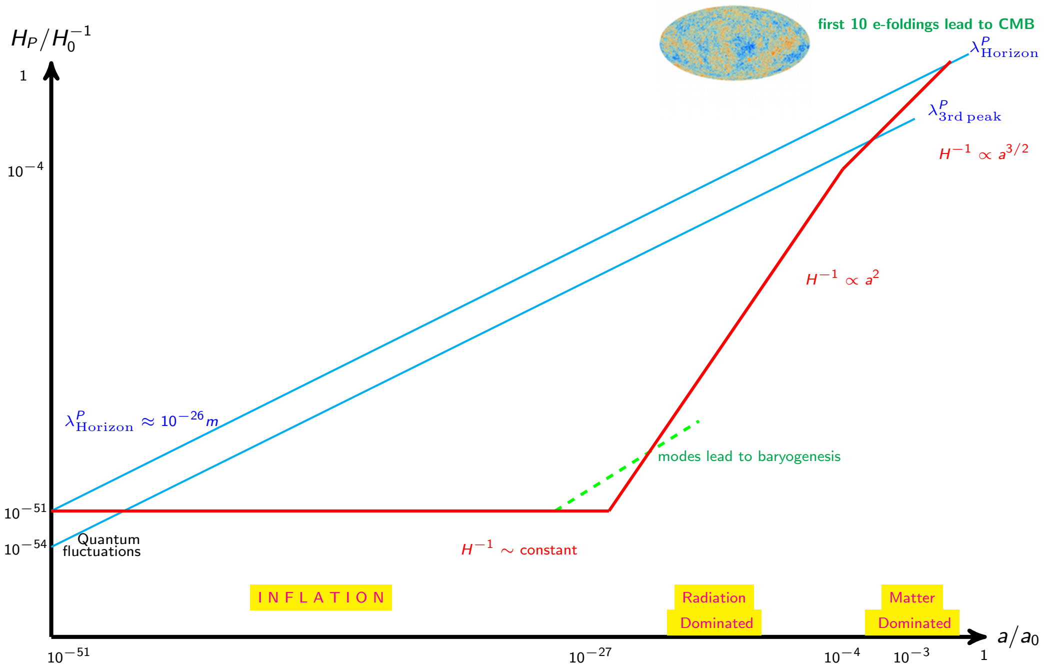
- Assuming instantaneous reheating, and the Universe becomes radiation dominated after inflation. Due to flux conservation, the magnetic energy density will decay as $1/a^4$: Subramanian (2016)
- Using the fact that the relevant modes exited Hubble radius around 30 e-foldings of inflation, with energy density $\rho_B \approx 10^{64}\mathrm{GeV}^4$, the primordial helical fields at GPc scales is:

$$B_0 \approx 10^{-20}\mathrm{G} \tag{13}$$

- Helical magnetic fields that re-entered the horizon at two different epochs:

$$B|_{50\ \mathrm{MPc}} \sim 10^{-18}\ G\ (z \sim 20)\ ;\quad B|_{1\ \mathrm{MPc}} \sim 10^{-14}\ G\ (z \sim 1000)$$

Baryogenesis from helical magnetic fields


first 10 e-foldings lead to CMB
H_P/H_0^{-1}
1
10^{-4}
10^{-51}
10^{-54}
$\lambda^P_{\rm Horizon}$
$\lambda^P_{\rm 3rd\ peak}$
$H^{-1} \propto a^{3/2}$
$H^{-1} \propto a^2$
$\lambda^P_{\rm Horizon} \approx 10^{-26} m$
modes lead to baryogenesis
Quantum fluctuations
$H^{-1} \sim$ constant
I N F L A T I O N
Radiation Dominated
Matter Dominated
a/a_0
10^{-51}
10^{-27}
10^{-4}
10^{-3}
1

Baryon asymmetry parameter

- The modes that **re-enter very early** during the radiation-dominated epoch are responsible for the generation of baryon asymmetry.
- Therefore we consider the modes which left the horizon around 5 to 10 e-foldings, Chern-Simon number density is

$$n_{CS} = \frac{1}{2\pi^2\, a^4(\eta)} \int_{\mu}^{\Lambda} dk \left(|C|^2\, k^{3+\frac{1}{2\alpha}} + \left| C_2 \frac{\mathcal{F}^{-1}}{\pi} \Gamma\left(\frac{1}{2\alpha}\right) \right|^2 k^{3-\frac{1}{2\alpha}} \tau^{-\frac{2}{\alpha}} \right). \tag{14}$$

- Since entropy density per comoving volume is conserved, the quantity n_B/s is better suited for theoretical calculations.
- Assuming that there was no significant entropy production after reheating phase, entropy density in the radiation-dominated epoch is:

$$s \simeq \frac{2\pi^2}{45} g\, T_{\mathrm{RH}}^3 \tag{15}$$

- Baryon asymmetry parameter

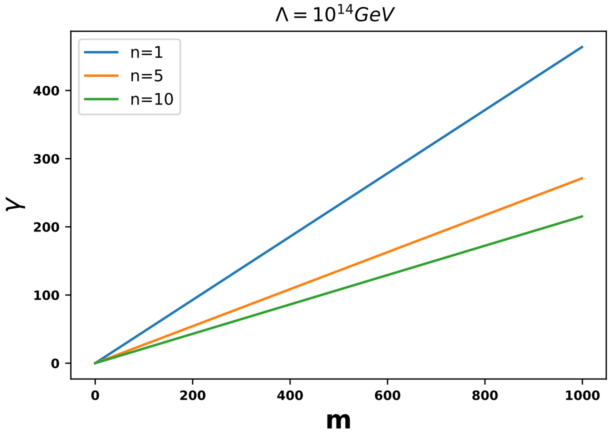
$$\eta_B = \frac{n_B}{s} \approx 10^{-2} \left(\frac{M}{M_P}\right)^3 \left(\frac{\Lambda}{T_{\rm RH}}\right)^3 \tag{16}$$

- Using the parametrization

$$\eta_B = n \times 10^{-10},\; M = m \times 10^{14}\, GeV,\; \Lambda = \delta \times 10^{12}\, GeV,\; T_{RH} = \gamma \times 10^{12}\, GeV \tag{17}$$

equation for baryon asymmetry parameter becomes:

$$\boxed{\frac{m^3 \times \delta^3}{\gamma^3} \approx n\, 10^7} \tag{18}$$



- For a range of values of γ, δ, and m, BAU can have values between 10^{-10} to 10^{-9}.
- The analysis shows that $M \sim 10^{17}\, GeV$ is consistent with baryogenesis.

Conclusion and Future work

- Our model does not require the coupling of the electromagnetic field with the scalar field. Hence, there are no extra degrees of freedom and will not lead to a strong-coupling problem.
- Since the curvature is large in the early Universe, the coupling term will introduce non-trivial corrections to the electromagnetic action.
- We have explicitly shown that Davidson's conditions are necessary but not sufficient. The key missing ingredient is the requirement of helical magnetic fields.
- The BAU parameter predicted by our model is independent of any specific inflation model and reheating dynamics; however, it depends on the scale at which inflation ends and reheating temperature.
- Currently, we are studying the effects on the asymmetry generated in **quarks** and **leptons**.

Thank you

Backup slides

Conformal transformation

$$\tilde{g}_{\mu\nu} = \omega^2(x) g_{\mu\nu} \implies \tilde{\Gamma}^\lambda_{\mu\nu} = \Gamma^\lambda_{\mu\nu} + C^\lambda_{\mu\nu} \tag{19}$$

where $C^\lambda_{\mu\nu} = \omega^{-1}\left(\delta^\lambda_\mu \nabla_\nu \omega + \delta^\lambda_\nu \nabla_\mu \omega - g_{\mu\nu} g^{\rho\lambda} \nabla_\rho \omega\right)$

$$F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu = \partial_\mu A_\nu - \Gamma^\lambda_{\mu\nu} A_\lambda - \partial_\nu A_\mu + \Gamma^\lambda_{\nu\mu} A_\lambda = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{20}$$

$$\tilde{R}^\lambda_{\sigma\mu\nu} = R^\lambda_{\sigma\mu\nu} + \nabla_\mu C^\lambda_{\nu\sigma} - \nabla_\nu C^\lambda_{\mu\sigma} + C^\lambda_{\mu\rho} C^\rho_{\nu\sigma} - C^\lambda_{\nu\rho} C^\rho_{\mu\sigma} \tag{21}$$

$$\begin{aligned} \tilde{R}_{\mu\nu} = R_{\mu\nu} &- [2\delta^\alpha_\mu \delta^\beta_\nu + g_{\mu\nu} g^{\alpha\beta}] \omega^{-1} (\nabla_\alpha \nabla_\beta \omega) \\ &+ [4\delta^\alpha_\mu \delta^\beta_\nu - g_{\mu\nu} g^{\alpha\beta}] \omega^{-2} (\nabla_\alpha \omega)(\nabla_\beta \omega) \end{aligned} \tag{22}$$

$$\tilde{R} = \omega^{-2} R - -6 g^{\alpha\beta} \omega^{-3} (\nabla_\alpha \nabla_\beta \omega) \tag{23}$$

$$\tilde{\nabla}_\mu \tilde{\nabla}_\nu \phi = \nabla_\mu \nabla_\nu \phi - (\ \delta^\alpha_\mu\ \delta^\beta_\nu\ +\ \delta^\beta_\mu\ \delta^\alpha_\nu\ )\ \omega^{-1}\ (\nabla_\alpha \omega)(\nabla_\beta \omega) \tag{24}$$

Back

Energy densities

Gauge field decomposition:

$$A^i(\vec{x},\eta) = \int \frac{d^3k}{(2\pi)^3} \sum_{\lambda=1,2} \varepsilon^i_\lambda \left[A_\lambda(k,\eta) b_\lambda(\vec{k}) e^{ik\cdot x} + A^*_\lambda(k,\eta) b^\dagger_\lambda(\vec{k}) e^{-ik\cdot x}\right] \tag{25}$$

The EM energy densities with respect to the comoving observer are:

$$\rho_B(\eta,k) \equiv -\frac{1}{2}\langle 0|B_\mu B^\mu|0\rangle = \int \frac{dk}{k} \frac{1}{(2\pi)^2} \frac{k^5}{a^4} \left(|A_+(\eta,k)|^2 + |A_-(\eta,k)\ |^2 \right) \tag{26}$$

$$\rho_E(\eta,k) \equiv -\frac{1}{2}\langle 0|E_\mu E^\mu|0\rangle = \int \frac{dk}{k} \frac{1}{(2\pi)^2} \frac{k^3}{a^4} \left(|A'_+(\eta,k)|^2 + |A'_-(\eta,k)|^2 \right) \tag{27}$$

$$\rho_h(\eta,k) \equiv -\langle 0|A_\mu B^\nu|0\rangle = \int \frac{dk}{k} \frac{1}{2\pi^2} \frac{k^4}{a^3} \left(|A_+(\eta,k)|^2 - |A_-(\eta,k)|^2 \right). \tag{28}$$

where spectral energy density is given by $\frac{d\rho_\Upsilon}{d\ln k}$ for $\Upsilon \in (B, E, h)$

Using the fact that we can approximate the super-horizon modes by power law, we have

$$A_+(\tau, k) = C\, k^{\frac{1}{4\alpha}} - C_2 \frac{\mathcal{F}^{-1}}{\pi} \Gamma\left(\frac{1}{2\alpha}\right) k^{-\frac{1}{4\alpha}} \tau^{-\frac{1}{\alpha}} \tag{29}$$

where

$$\mathcal{F}(\tau) = F(\tau) \left(\frac{\varsigma}{2\alpha}\right)^{\frac{1}{2\alpha}}, \tag{30}$$

$$C(\tau) = F(\tau) \left(\frac{\varsigma}{2\alpha}\right)^{\frac{1}{2\alpha}} \left[\frac{C_1}{\Gamma\left(1 + \frac{1}{2\alpha}\right)} - \frac{C_2}{\pi} \Gamma\left(-\frac{1}{2\alpha}\right) \cos\left(\frac{\pi}{2\alpha}\right)\right], \tag{31}$$

and the approximate values are $|\mathcal{F}| \sim 10^{-\frac{5}{\alpha}}\ \mathrm{GeV}^{-1/4\alpha}$, $|\mathrm{C}| \sim 10^{-\frac{5}{\alpha} - \frac{11}{2}} \mathrm{GeV}^{-\frac{1}{4\alpha} - \frac{1}{2}}$.

Power spectrum

$$\left.\frac{d\rho_B}{d\ln k}\right|_{k_*\sim\mathcal{H}} \propto |C|^2\, k_*^{3+4\alpha+\frac{1}{2\alpha}} + \left|C_2\frac{\mathcal{F}^{-1}}{\pi}\Gamma\left(\frac{1}{2\alpha}\right)\right|^2 \frac{(2\alpha-1)^2}{4\eta_0^2}\, k_*^{1+4\alpha-\frac{1}{2\alpha}} \tag{32}$$

- It has two branches
  - The first branch (setting $C_2 = 0$) has **scale-invariant spectrum** for $\alpha = -\frac{1}{2}, -\frac{1}{4}$.
  - Second branch (setting $C = 0$) has **scale invariant spectrum** for $\alpha = -\frac{1}{2}, \frac{1}{4}$.
- Physically allowed values of $\alpha \leq -1/2$. Hence, $\alpha = \pm 1/4$ **is ruled out.**
- For slow-roll inflation ($\alpha = -\frac{1}{2} - \epsilon$), the two branches scale differently: $k_*^{-2\epsilon}$ (first branch) and $k_*^{-6\epsilon}$ (second branch).
- Since ϵ is positive, this implies that our model produces **more power on the large scales.**$\longrightarrow$ **Red spectrum for slow roll inflation**

Plots for lower energy scales of Λ and μ