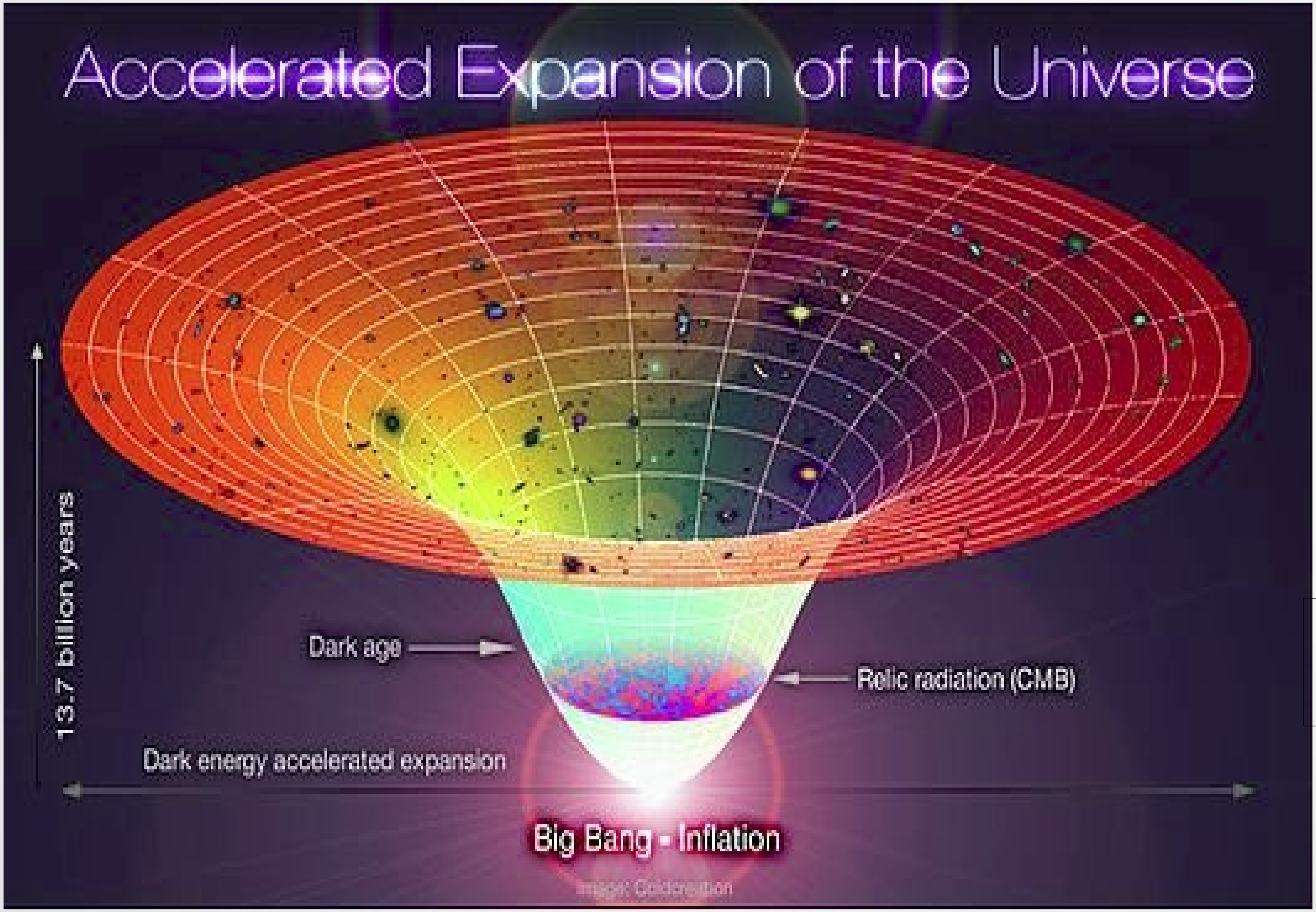
Probing the reheating phase of the universe

Debaprasad Maity@IIT Guwahati, India

In collaboration with R. Haque, Sourav Pal , P. Saha, Tanmoy Paul, L Sriramkumar

Time evolving cosmos



Cosmological constant

Matter

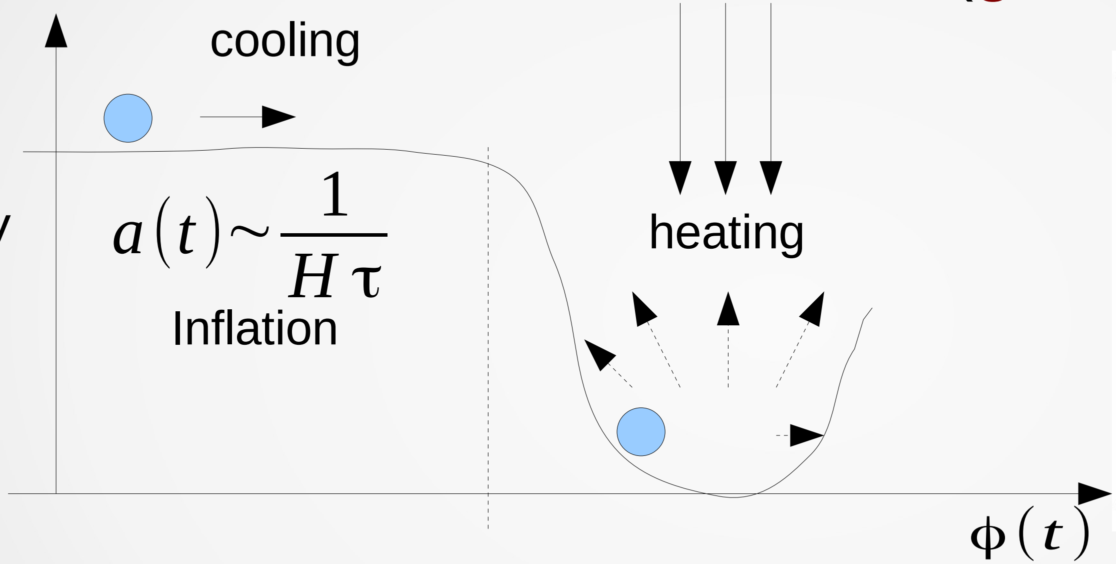
Radiation

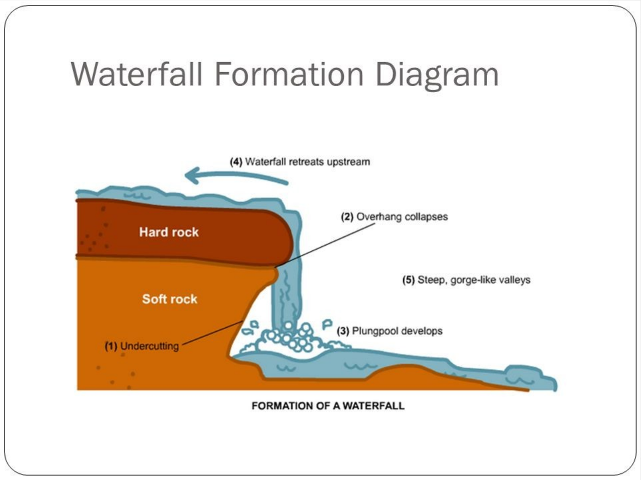
Reheating

Inflation

What is reheating phase?

- Natural consequence after inflation: creats huge empty space which needs to be filled with matters (**generate entropy)**.

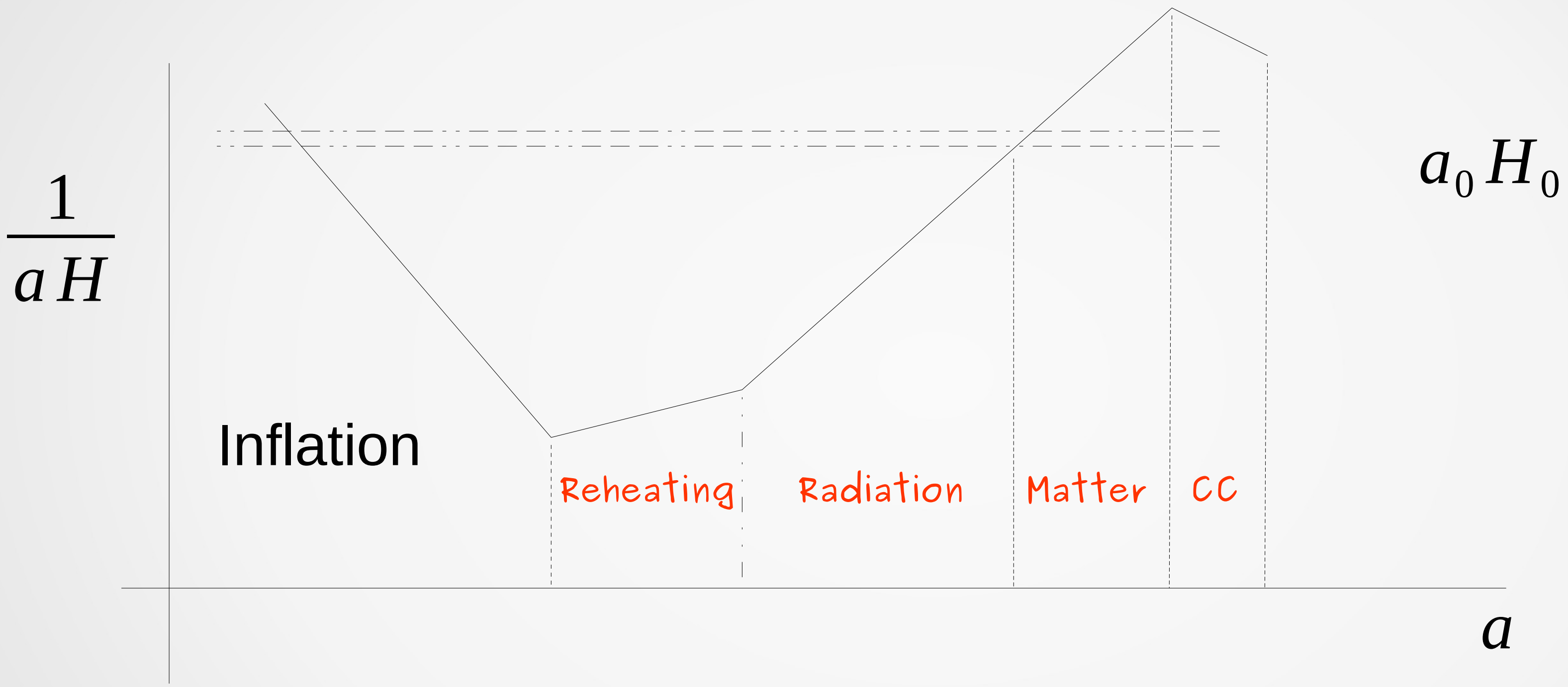




Background expansion + evolution of fluctuation

Reheating: Non-equilibrium decay of inflaton,
not well understood, observationally and theoreticly.

Cartoon diagram of evolving universe



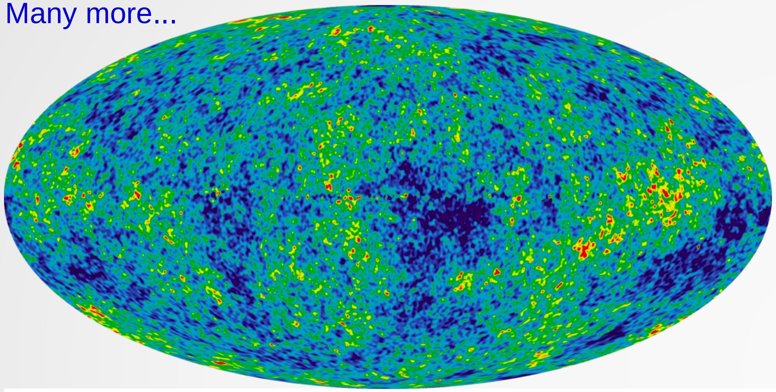
We need to understand all possible cosmological observables, where, effect of the reheating phase should be imprinted

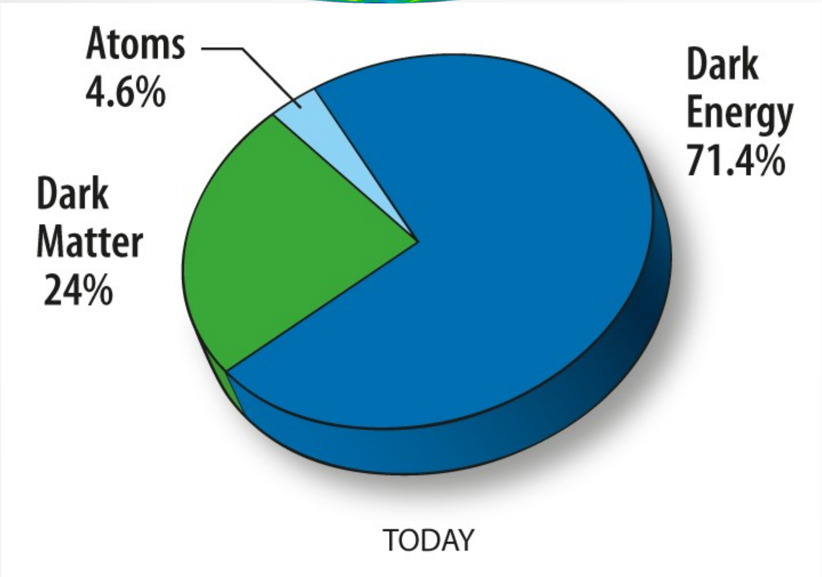
Given the inflationary phase:
What do we expect to observe today?

P.A.R Ade et. al. ArXiv:1502:01589

Extremely homogeneous Universe

Many more...





What inflation models?

Fluctuations of all the fundamental fields

Scalar type

Density(curavature) fluctuations,
Dark Matter, Dark energy...

FermionType

Baryonic Matter,
Dark energy, Dark matter ?

Vector type

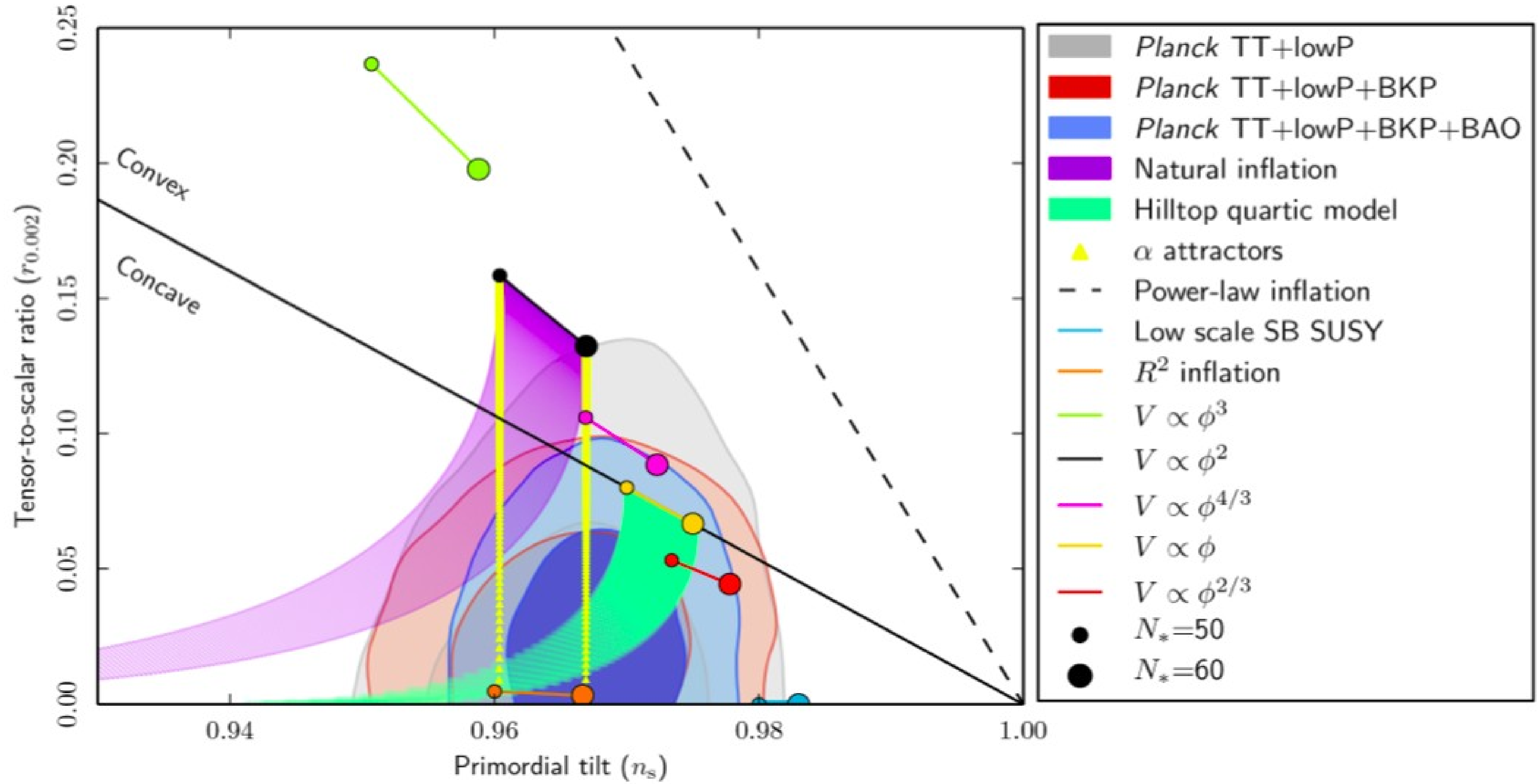
Primordial Large scale
Magnetic field, EM
radiation...

Tensor type

Primodial Graviational
wave, Kalb Ramond...

Where do we stand?

P. A. R Ade et. al. ArXiv:1502:01589



Planck-2015

Plan

- Set up and goal
- Probing the reheating phase by
  1. CMB spectrum (Scalar Type)
  2. Dark matter abundance (Fermion type)
  3. Primordial magnetic field (PMF) spectrum (Vector)
  4. Primordial gravitational waves(PGW) spectrum (Tensor)
- Conclusions

Motivation and **Goal**

- **Reheating is an unavoidable phase of the universe,**
- **Very high energy pheneomena which may/should contain new physics (NP):**
- **Obvious NP: Nature of inflation, its interaction with new types of matters**
- **Dark matter, baryogenesis, nutrino mass...**

Establish the relation with the present day observables such as

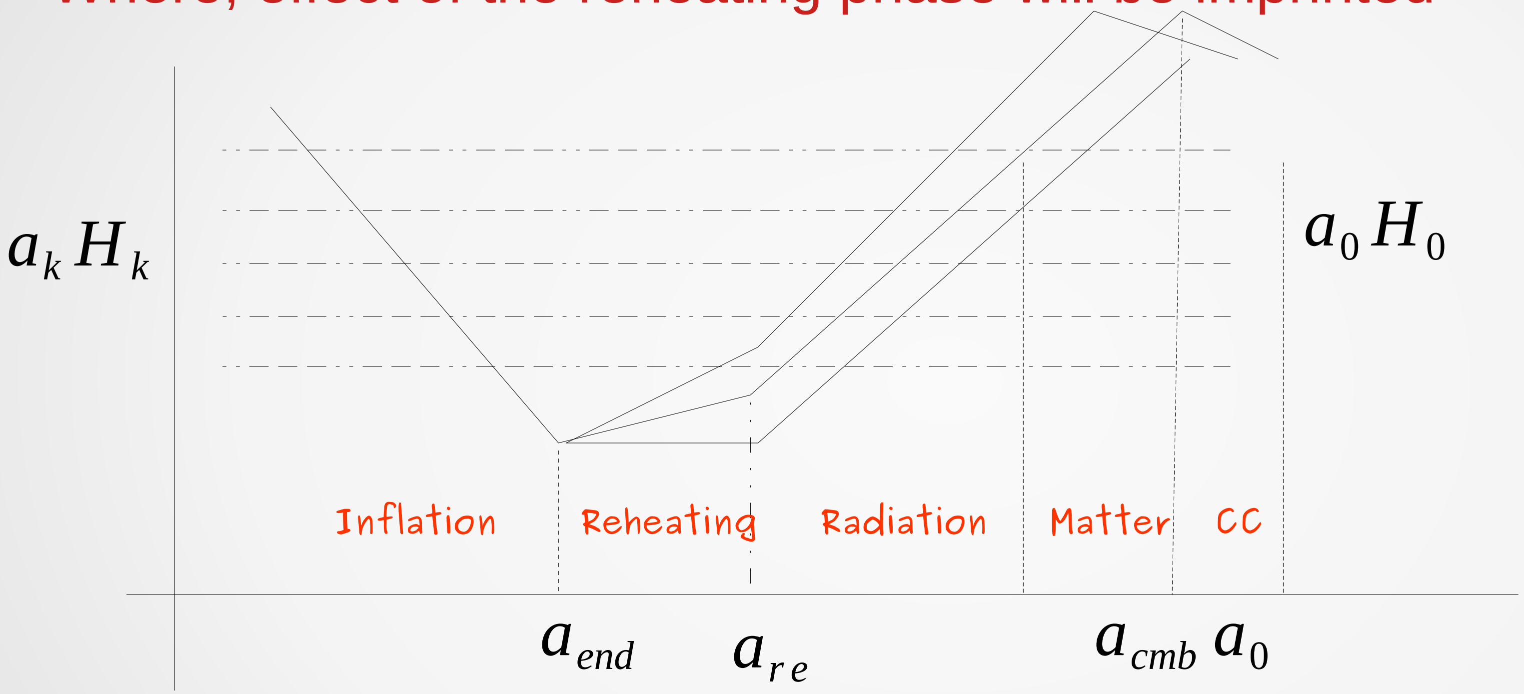
CMB power spectrum, Dark matter abundance, Power spectrum of Large scale magnetic field, GW power specturm

With the reheating parameters

$$N_{re}, T_{re},$$
$$(\omega_{eff}, \omega_{\phi}), its\ time\ variation$$

Goal

Understanding all possible cosmological observables,
Where, effect of the reheating phase will be imprinted



Try to understand in terms of inflationary and conventional reheating parameters

$$n_s, r, N_{re}, T_{re},$$
$$(\omega_{eff}, \omega_\phi), its\ time\ variation$$

Important analytical results

DM and P. Saha, PRD 98, 103525 (2018), L. Dai, M. Kamionkowski and J. Wang, PRL. 113, 041302 (2014)

- Reheating and CMB:

$$\log_{10}(T_{\rm re}\ \mathrm{GeV}) \simeq Q_p\left[A + B(n_s - 0.962) + C(n_s - 0.962)^2\right]$$

- Reheating and Dark matter:

DM and P. Saha, PRD 98, 103525 (2018),
Phys.Dark Univ. 25 (2019) 100317,
CQG 36 (2019) 045010

$$\Omega_X h^2 \propto \langle\sigma|v|\rangle M_X^4 \exp\left[-\frac{(17+w)M_X}{(1+w)T_{\rm max}}\right] \quad \text{for } M_X \gtrsim T_{\rm max}$$

$$\Omega_X h^2 \propto \langle\sigma v\rangle \frac{T_{re}^{\frac{7+3w_\phi}{1+w_\phi}}}{M_X^{\frac{9-7w_\phi}{2(1+w_\phi)}}} \propto \frac{\langle\sigma v\rangle}{M_X^{\frac{9-7w_\phi}{2(1+w_\phi)}}}\left[\left(\frac{a_0 T_0}{k}\right) H_k e^{-N_k} e^{-N_{\rm re}}\right]^{\frac{7+3w_\phi}{1+w_\phi}} \quad \text{for } T_{\rm max} > M_X > T_{\rm re}$$

$$\Omega_X h^2 \propto \langle\sigma v\rangle M_X\, T_{re} \propto \langle\sigma v\rangle M_X \left(\frac{a_0 T_0}{k}\right) H_k e^{-N_k} e^{-N_{re}} \quad \text{When } M_X < T_{\rm re}. \qquad (24)$$

- Reheating and PMF:

R. Haque, DM, S pal
PRD, 103 (2021) 10, 103540
DM, S. Pal, T Paul, JCAP, 05 (2021) 045

$$\mathcal{P}_{B0}(k) \simeq \frac{\Gamma(n-\frac{1}{2})^2}{\pi^3}\frac{2^{2n-3}\,(2.6\times 10^{39})}{(6.4\times 10^{-39})^{2n-6}}\left(\frac{k}{a_0}M_{pc}\right)^{-2(n-3)}\left(\frac{11 g_{s,re}}{43}\right)^{\frac{2-2n}{3}}\left(\frac{T_{re}}{T_0}\right)^{2-2n}\left(\frac{H_{inf}}{GeV}\frac{1}{A_{re}}\right)^{2(n-1)}$$
$$\left\{1 + (2n-1)\left[\frac{2}{3\omega_\phi+1}\left(\eta^{\frac{3\omega_\phi+1}{3(1+\omega_\phi)}} - 1\right) + \frac{4}{3\omega_\phi - 1}\eta^{1/2}\left(A_{re}^{\frac{3\omega_\phi-1}{4}} - \eta^{\frac{3\omega_\phi-1}{6(1+\omega_\phi)}}\right)\right]\right\}^2 G^2 .$$

- Reheating and PGW:

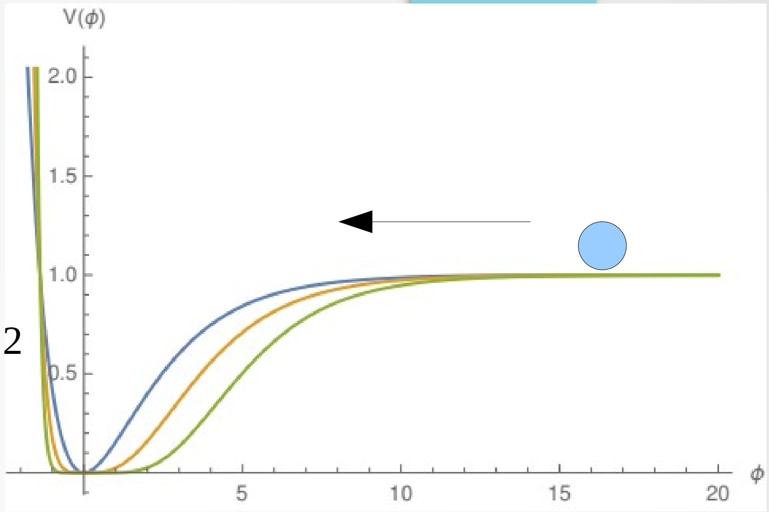
V. Sahni, PRD 42, 453 (1990),
R.Haque, et al, PRD, 104 (2021) 6, 063513

$$\Omega_{\rm GW}(k)\,h^2 \simeq \Omega_{\rm R}\,h^2\,\frac{H_{\rm I}^2}{12\,\pi^2 M_{\rm Pl}^2}\,\frac{4\,\gamma^2}{\pi}\,\Gamma^2\left(1+\frac{\nu}{\gamma}\right)\left(\frac{k}{2\,\gamma\,k_{\rm re}}\right)^{n_{\rm GW}}$$

Start with Inflation

Inflaton potential

$$V(\phi) = \Lambda^4 \left[1 - e^{-\sqrt{\frac{2}{3\alpha}}\frac{\phi}{M_p}}\right]^{2p} .$$



Slow roll parameters $\eta \sim M_p^2 \frac{V''(\phi)}{V(\phi)}$ $\quad \epsilon \sim M_p^2 \left(\frac{V'(\phi)}{V(\phi)}\right)^2$

Spectral index

$$n_s^k = 1 - 6\epsilon(\phi_k) + 2\eta(\phi_k) \ , \ r_k = 16\epsilon(\phi_k)$$

$$ds^2 = a(\tau)^2 \left(-d\tau^2 + d\mathbf{x}^2\right)$$

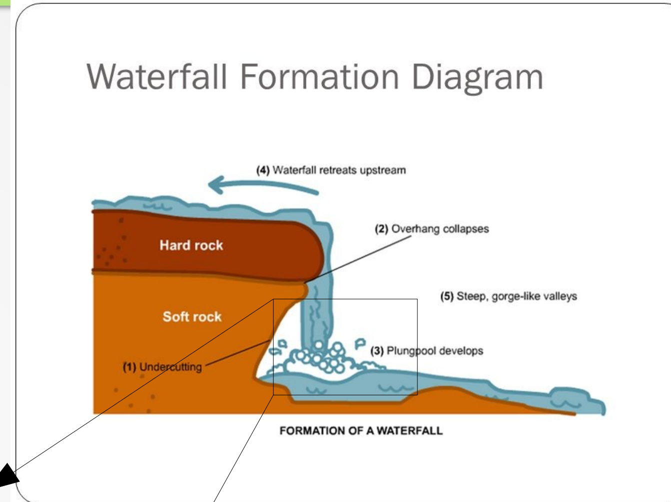
E-folding no. & Energy scale

$$N_k = \log\left(\frac{a_{end}}{a_k}\right) = \int_{\phi_k}^{\phi_{end}} \frac{|d\phi|}{\sqrt{2\epsilon_v} M_p} \ , \quad H_k = \frac{\pi M_p \sqrt{r_k A_s}}{\sqrt{2}} \ ,$$

Perturbative set up of reheating phase

DM, arXiv:1709.00251; DM, P. Saha, PRD 2018

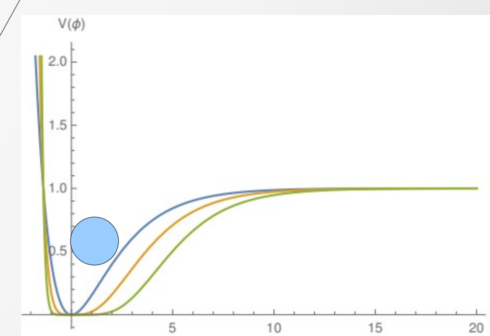
Inflaton + Radiation + Dark matter



$$\Phi = \frac{\rho_\phi a^{3(1+w_\phi)}}{m_\phi^{(1-3w_\phi)}}; \qquad R = \rho_R a^4; \qquad X = n_X a^3. \qquad A = a/a_I$$

$$\mathbb{H} = \left(\Phi/A^{3w_\phi} + R/A + X\langle E_X\rangle/m_\phi\right)^{1/2}$$

$$\begin{aligned}
\frac{d\Phi}{dA} &= -c_1(1+w_\phi)\frac{A^{1/2}\Phi}{\mathbb{H}}; \\
\frac{dR}{dA} &= c_1(1+w_\phi)\frac{A^{3(1-2w_\phi)/2}}{\mathbb{H}}\Phi + c_2\frac{A^{-3/2}2\langle E_X\rangle\langle\sigma v\rangle M_{pl}}{\mathbb{H}}\left(X^2 - X_{eq}^2\right) \\
\frac{dX}{dA} &= -c_2\frac{A^{-5/2}2\langle E_X\rangle\langle\sigma v\rangle M_{pl}}{\mathbb{H}}\left(X^2 - X_{eq}^2\right);
\end{aligned}$$



$$\begin{aligned}
\Omega_X h^2 &= \frac{\rho_X(T_F)}{\rho_R(T_F)}\frac{T_F}{T_{now}}\Omega_R h^2, \\
&= \langle E_X\rangle\frac{X(T_F)}{R(T_F)}\frac{T_F}{T_{now}}\frac{A_F}{m_\phi}\Omega_R h^2
\end{aligned}$$

$$c_1 = \frac{\sqrt{\frac{3}{8\pi}}M_{pl}\Gamma_\phi}{m_\phi^2}, \quad c_2 = \sqrt{\frac{3}{8\pi}}$$

$$\langle E_X\rangle = \sqrt{M_X^2 + 9T^2}$$

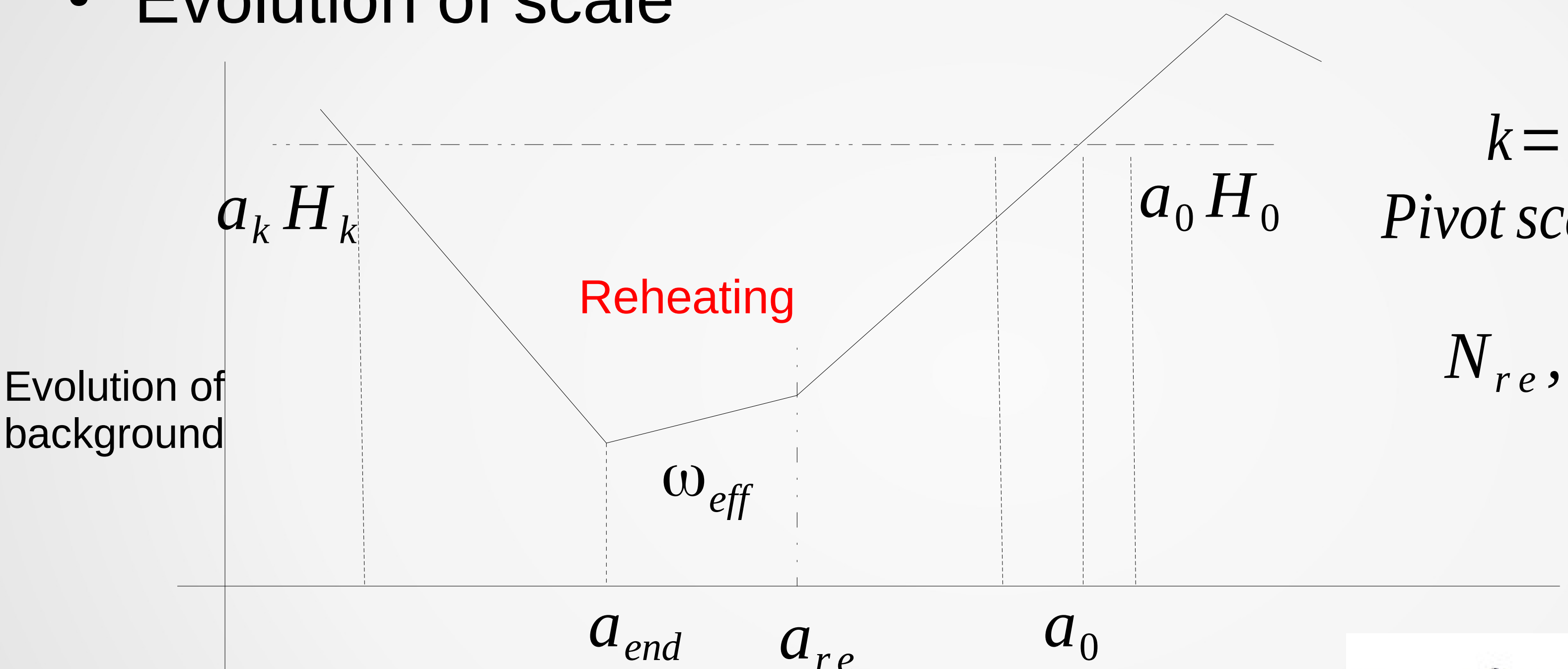
Parameters

$T_{re} = \Gamma, M_X, <\sigma v>$

Reheating: CMB and DM (scalar, Fermion)

L. Dai, M. Kamionkowski and J. Wang, PRL. 113, 041302 (2014), J. L. Cook, etal JCAP 1504 (2015) 047; J. Ellis etal, JCAP 1507 (2015,, 050; Y. Ueno and K. Yamamoto, PRD 93 (2016), 083524; M. Eshaghi etal, PRD 93 (2016), 123517, A. Di Marco, etal, PRD 95 (2017),, 103502, S. Bhattacharya etal, PRD 96 (2017), 083522, …

- Evolution of scale



$$k = 0.02\, Mpc^{-1}$$

Pivot scale of PLANCK

$$N_{re}, T_{re}, \bar{\omega}_{re}$$

Minimal inflation

$$ln\left(\frac{a_k H_k}{a_0 H_0}\right) = -N_k - N_{re} - ln\left(\frac{a_{re} H_k}{a_0 H_0}\right)$$

$$g_{re} T_{rad}^3 = \left(\frac{a_0}{a_{re}}\right)^3 \left(2T_0^3 + 6 \times \frac{7}{8} T_{\nu 0}^3\right)$$

$$T_{re} = \left(\frac{43}{11 g_{re}}\right)^{\frac{1}{3}} \left(\frac{a_0 T_0}{k}\right) H_k e^{-N_k} e^{-N_{re}}$$

$$H_k = \frac{1}{3M_p^2} V(\phi_k) = \frac{\pi M_p \sqrt{r_k A_s}}{\sqrt{2}}$$

$$N_k = \log\left(\frac{a_{end}}{a_k}\right) = \int_{\phi_{end}}^{\phi_k} \frac{3}{2} \frac{V(\phi)}{V'(\phi)} d\phi = \int_{\phi_k}^{\phi_{end}} \frac{|d\phi|}{\sqrt{2\epsilon_v} M_p}$$

Parameter counting

Unique Initial conditions:

$$\Phi(1) = \frac{3}{8\pi}\frac{M_{pl}^2 H_I^2}{m_\phi^4}; \qquad R(1) = X(1) = 0.$$

Constraint conditions:

$$T_{re} = \left(\frac{43}{11 g_{re}}\right)^{\frac{1}{3}} \left(\frac{a_0 T_0}{k}\right) H_k e^{-N_k} e^{-N_r e}$$

$$T_{\rm re} \equiv T_{\rm rad}^{\rm end} = \left[30/\pi^2 g_*(T)\right]^{1/4} \rho_R(\Gamma, n_s, M_X)^{1/4}.$$

$$\Omega_X h^2 = 0.12 \qquad n_s = 0.9659 \pm 0.0082$$

$$3 \quad Parameters$$

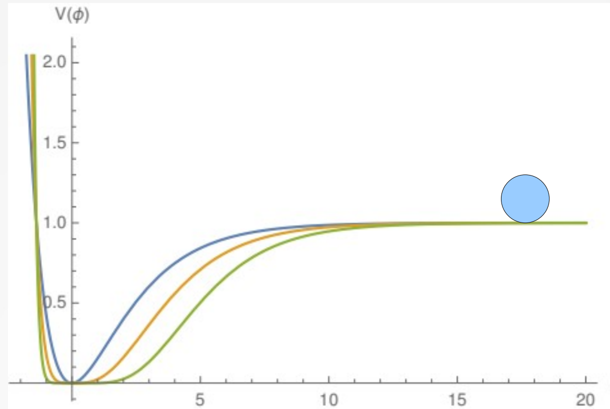
$$T_{re} \approx \Gamma, M_X, <\sigma v>$$

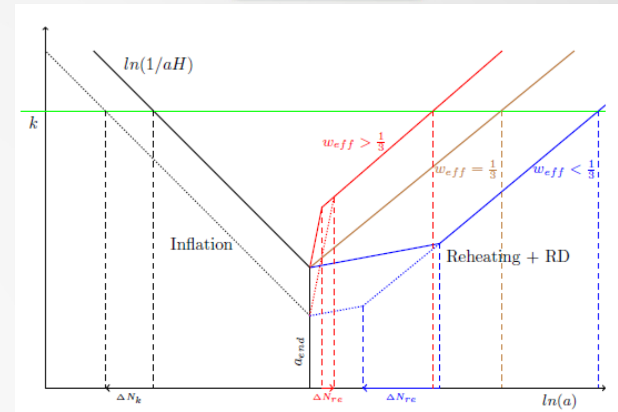
Therefore GIVEN a dark matter mass, all other parameters are uniquely fixed: Therefoe, we can successfully establish the connection we were looking for.

Alpha-attractor potential: CMB vs Reheating

PRD98, 103525 (2018)
Phy.Dark.Univ. 25, 100317(2019)

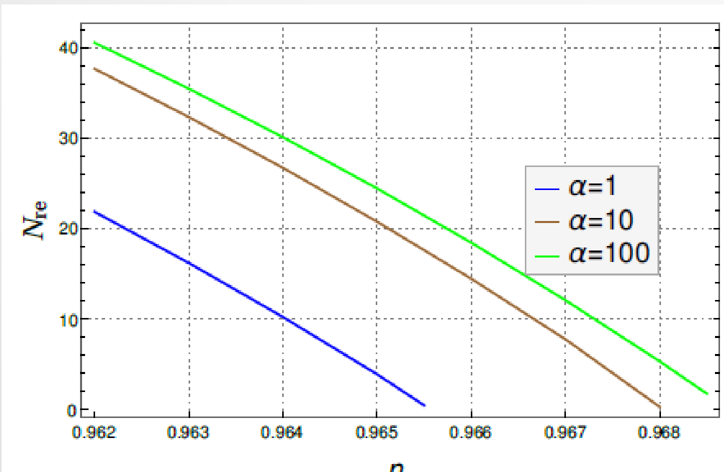
$$V(\phi) = \Lambda^4 \left[1 - e^{-\sqrt{\frac{2}{3\alpha}}\frac{\phi}{M_p}}\right]^{2p}.$$

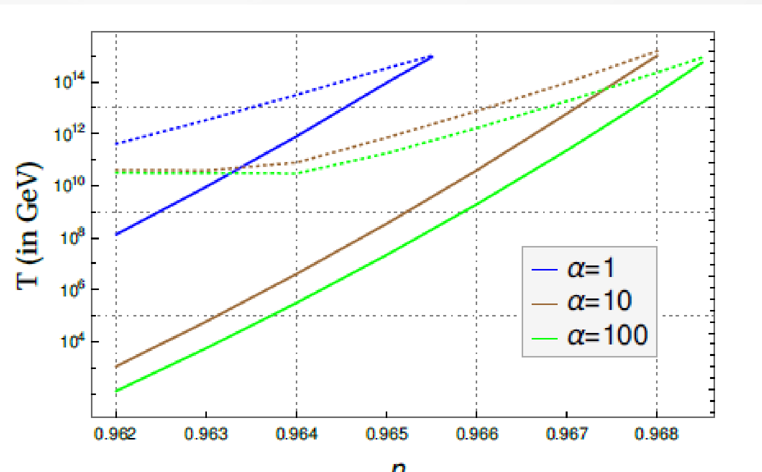




Effect of different inflaton equation of state during reheating

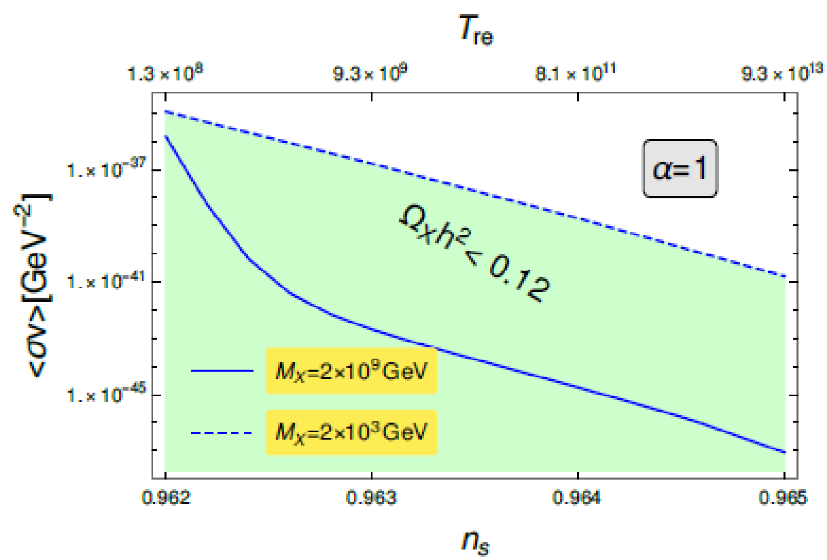
$$\omega_\phi = \frac{P_\phi}{\rho_\phi} = \frac{\langle \phi v'(\phi)\rangle - \langle 2V\rangle}{\langle \phi v'(\phi)\rangle + \langle 2V\rangle} = \frac{(p-1)}{(p+1)}$$

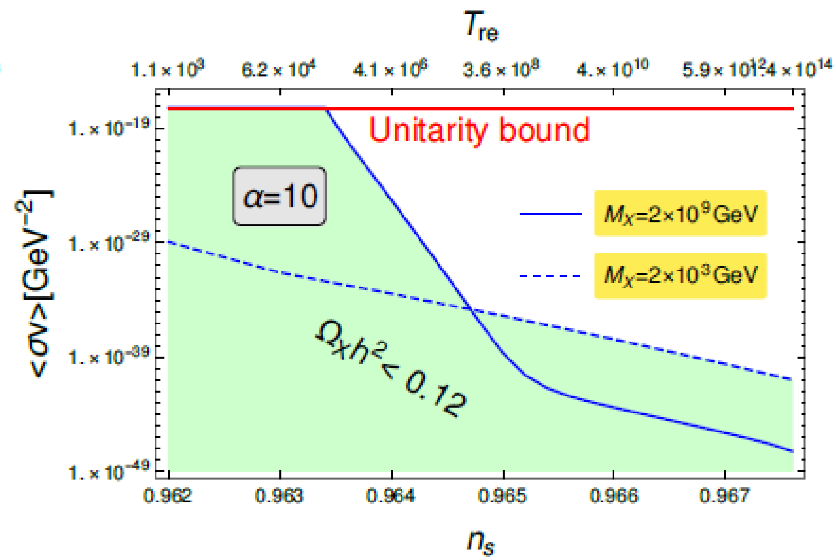


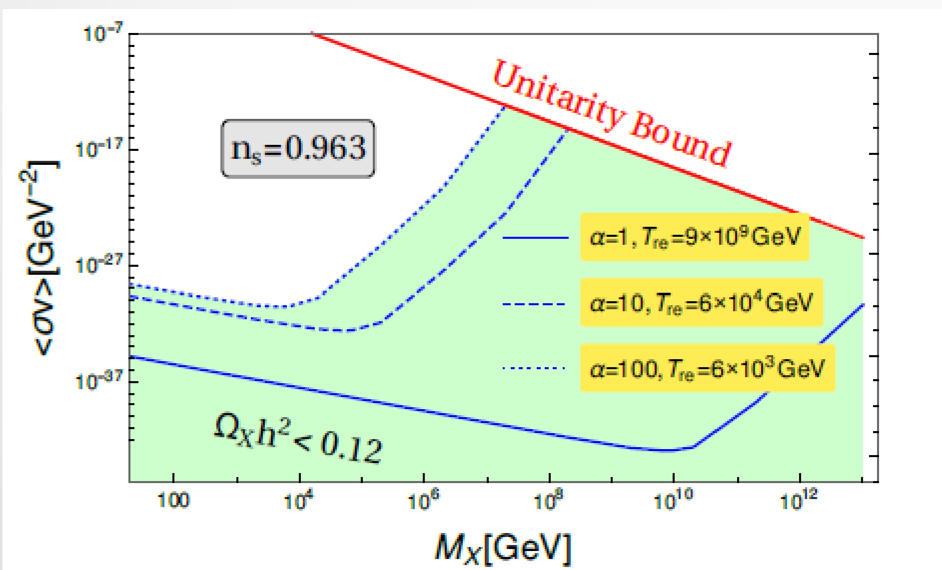


$p = 1$

Dark matter vs reheating

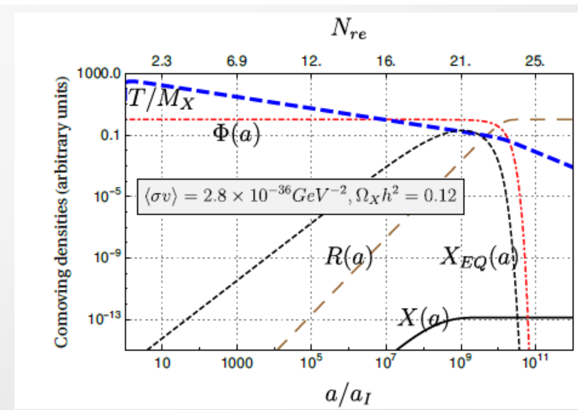






$$\langle \sigma v \rangle \Big|_{M_X > T_{re}} \propto 10^{-7A-7B(n_s-0.962)-7C(n_s-0.962)^2}.$$

$$\langle \sigma v \rangle \Big|_{M_X < T_{re}} \propto 10^{-A-B(n_s-0.962)-C(n_s-0.962)^2}.$$



Dark matter abundance & CMB anisotropy

For general inflaton equation of state, DM, P. Saha, Phy.Dark.Univ. 2019; CQG 2019;

$$\Omega_X h^2 \propto \langle\sigma|v|\rangle M_X^4 \exp\left[-\frac{(17+w)M_X}{(1+w)T_{\max}}\right] \quad \text{for } M_X \gtrsim T_{\max}$$

$$\Omega_X h^2 \propto \langle\sigma v\rangle \frac{T_{re}^{\frac{7+3w_\phi}{1+w_\phi}}}{M_X^{\frac{9-7w_\phi}{2(1+w_\phi)}}} \propto \frac{\langle\sigma v\rangle}{M_X^{\frac{9-7w_\phi}{2(1+w_\phi)}}}\left[\left(\frac{a_0 T_0}{k}\right) H_k e^{-N_k} e^{-N_{\rm re}}\right]^{\frac{7+3w_\phi}{1+w_\phi}} \quad \text{for } T_{\max} > M_X > T_{\rm re}$$

$$\Omega_X h^2 \propto \langle\sigma v\rangle M_X\, T_{re} \propto \langle\sigma v\rangle M_X \left(\frac{a_0 T_0}{k}\right) H_k e^{-N_k} e^{-N_{re}} \quad \text{When } M_X < T_{\rm re}. \tag{24}$$

$$\log_{10}(T_{\rm re}\ {\rm GeV}) \simeq Q_p \left[A + B(n_s - 0.962) + C(n_s - 0.962)^2\right]$$

$$A = 8,\ B = 1.8\times 10^3 \text{ and } C = 5.5\times 10^4$$

Alpha-attractor

$$Q_p \sim \log_{10}(\alpha)/\alpha^{1/2}$$

$$C_\ell^{TT} = \frac{2}{\pi}\int k^2 {\rm d}k \underbrace{P_\zeta(k)}_{\text{Inflation}} \underbrace{\Delta_{T\ell}(k)\Delta_{T\ell}(k)}_{\text{Anisotropies}}$$

Axion

$$Q_p \propto 1/f_{\ell}$$

Reheating: primordial magnetic field (vector)

- Magnetic fields are ubiquitous on all scales from the surface of stars to galaxies to the voids in the large-scale structure of the Universe.
- For galactic scale magnetic field (micro gauss), the seed fields with a present strength of only $10^{-22} - 10^{-16}$ G is required. The origin of these magnetic fields is still not well understood.
- Observations suggests that even the intergalactic medium (IGM) in voids can host a weak $\sim 10^{-20}$ Gauss magnetic field, coherence length as large as Mpc scales

- Origing of such large scale MF: Inflationary magnetogenesis (unified mechanism)

A. Neronov and I. Vovk, Science 328 (2010) 73 doi:10.1126/science.1184192; K. Dolag, M. Kachelriess, S. Ostapchenko and R. Tomas, Astrophys. J. 727 (2011) L4 ; M. Ackermann et al. [Fermi-LAT Collaboration], Astrophys. J. Suppl. 237 (2018) no.2, 32

Inflationary magnetogenesis

DM, R. Haque, S pal, arXiv:2012.10859; DM, S. Pal, T. Paul, 2103.02411

Inflation + Evolution of electromagnetic field

- Conformal invariance must be broken, either by inflaton or some other field

$$S = -\frac{1}{4}\int d^4x\sqrt{-g}I(\tau)^2 F_{\mu\nu}F^{\mu\nu} \qquad I(\tau) = \begin{cases} \left(\frac{a_{end}}{a}\right)^n & a \le a_{end} \\ 1 & a \ge a_{end}, \end{cases}$$

- Irreducible field decomposition

$$A_\mu = (A_0, \partial_i S + v_i) \quad \text{with} \quad \partial_i V_i = 0$$

- Mode decomposition:

$$V_i(\tau, x) = \sum_{p=1,2}\int \frac{d^3k}{(2\pi)^3}\epsilon_i^{(p)}(\mathbf{k})\left\{e^{i\mathbf{k}.\mathbf{x}}a_k^{(p)}u_k^{(p)}(\tau) + e^{-i\mathbf{k}.\mathbf{x}}a_k^{\dagger(p)}u_k^{*(p)}(\tau)\right\}$$

- Dynamics:

$$u_k^{(p)\prime\prime} + 2\frac{I'}{I} + k^2 u_k^{(p)\prime} = 0$$

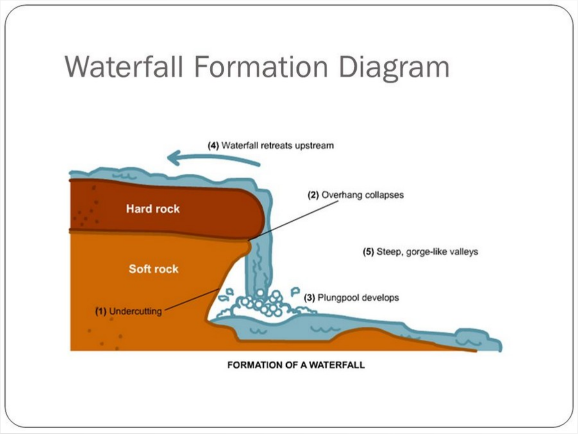
Bunch-Davis vacuum

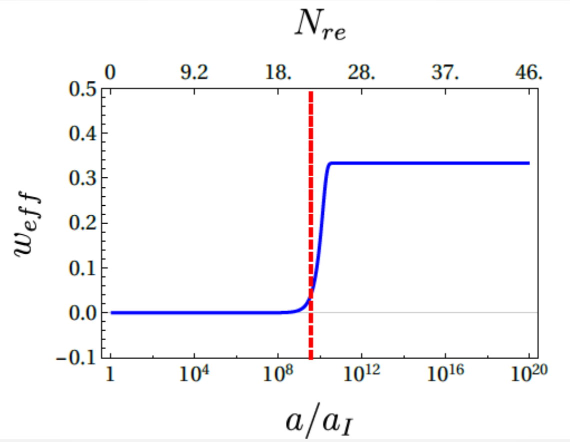
- Inflatinoary Spectrum:

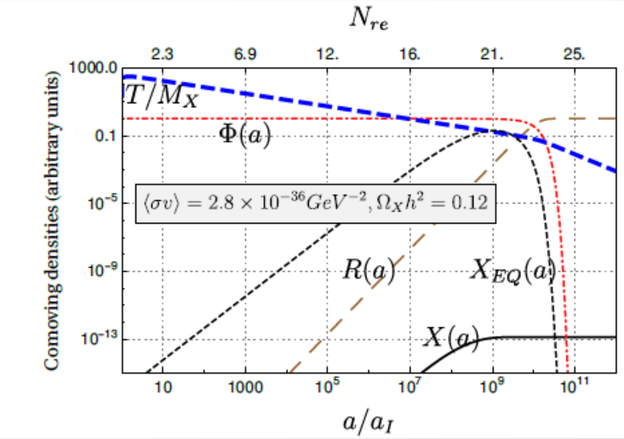
$$P_E(k) = \frac{k^3}{2\pi^2 a^4}\sum_{p=1,2}|u_k^{(p)\prime}|^2 \; ; \; P_B(k) = \frac{k^5}{2\pi^2 a^4}\sum_{p=1,2}|u_k^{(p)}|^2$$

Reheating

- Usual approach: Instantaneous reheating, Large conductivity
- Let us go little deeper into the reheating phase
- Inflaton equation of state evolves into radiation equation of state not instantaneously



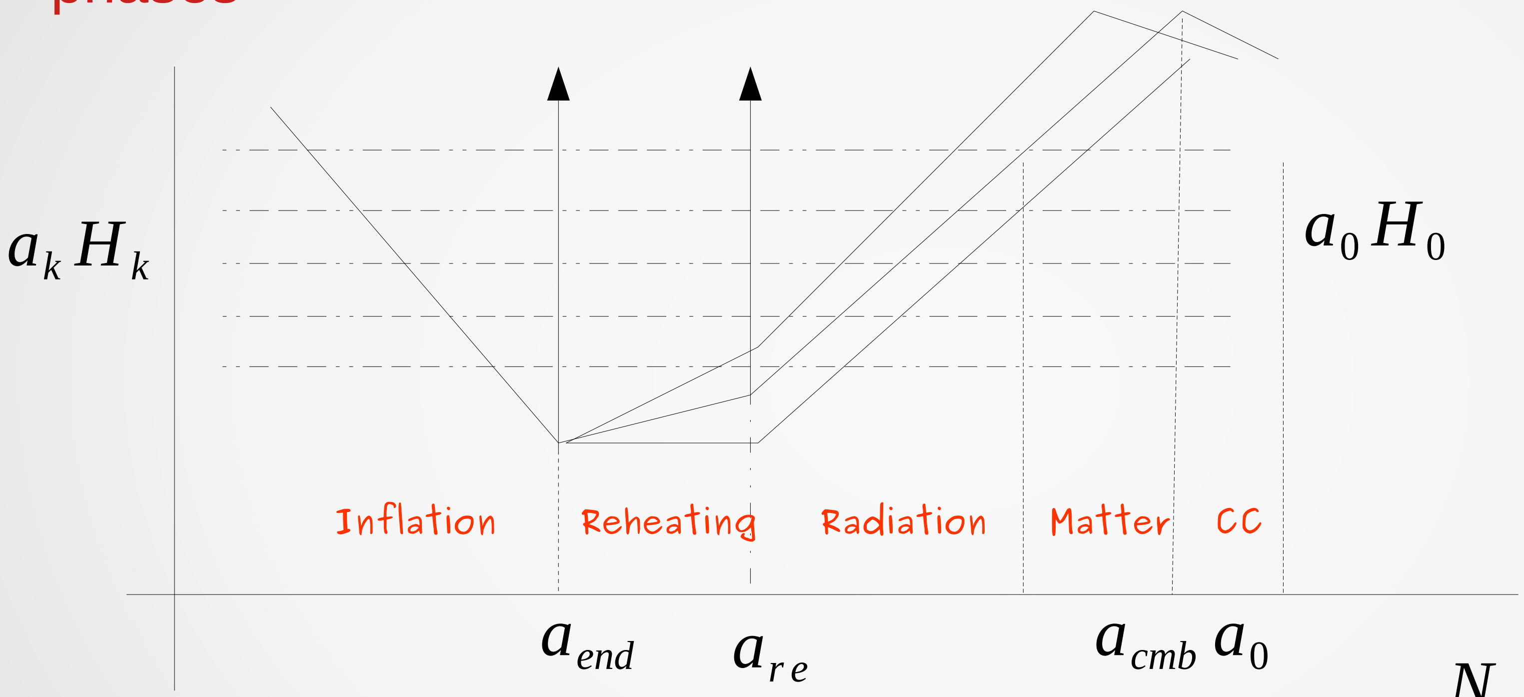




$$\omega_{eff} \approx \omega_\phi$$

Evolution through subsequent phase

We need to evolve the power spectrum through the subsequent phases



$N_{re}, T_{re},$

$(\omega_{eff}, \omega_{\phi})$, *its time variation*

Evolution is same as during inflation, the boundary conditions are different.

How reheating plays the role

Electric to magnetic conversion and viceversa will act, which leads to different evolution properties of electric and magnetic field

- Conformal invariance is restored
- $$S = -\frac{1}{4}\int d^4x\sqrt{-g}I(\tau)^2 F_{\mu\nu}F^{\mu\nu}$$ $$I(\tau) = \begin{cases} \left(\frac{a_{end}}{a}\right)^n & a \leq a_{end} \\ 1 & a \geq a_{end}, \end{cases}$$
- Free EM solution

$$u_k^{(p)} = \frac{1}{\sqrt{2k}}\{\alpha_k^{(p)}(z_{end})e^{-ik(\tau-\tau_{end})} + \beta_k^{(p)}(z_{end})e^{-ik(\tau-\tau_{end})}\}$$

$$P_E(k) = \frac{k^3}{2\pi^2 a^4}\sum_{p=1,2}|u_k^{(p)\prime}|^2\ ;\ P_B(k) = \frac{k^5}{2\pi^2 a^4}\sum_{p=1,2}|u_k^{(p)}|^2$$

Boundary condition set
at the end of inflation

Reheating: Primordial magnetic field

DM, R. Haque, S pal, arXiv:2012.10859; DM, S. Pal, T. Paul, 2103.02411

- Electric and magnetic power spectrum during reheating for large scale

$$\mathcal{P}_E(k) \simeq \frac{8\Gamma(n+\frac{1}{2})^2}{\pi^3} H_{inf}^4 \left(\frac{k}{2a_{end}H_{inf}}\right)^{-2(n-2)} \left(\frac{a_{end}}{a}\right)^4$$

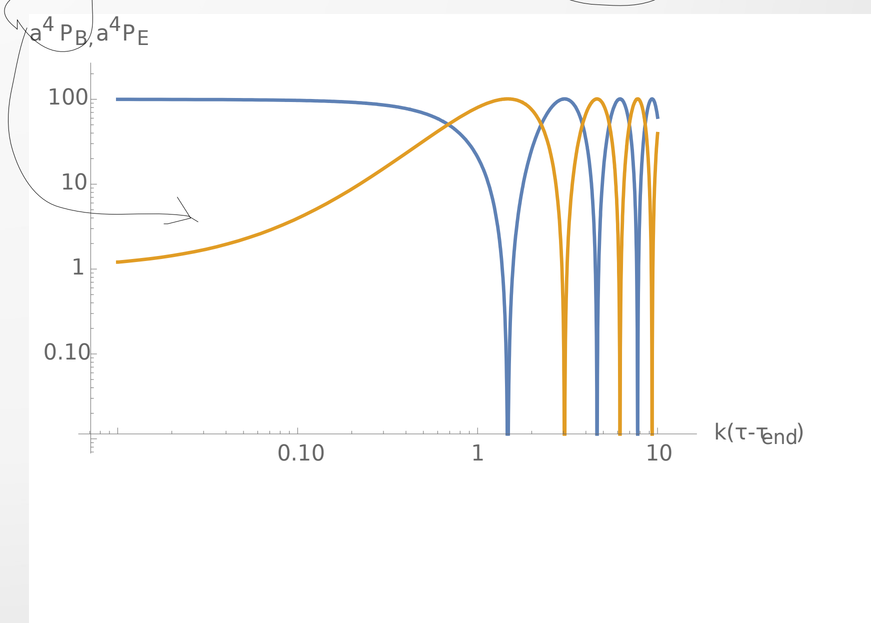
$$\mathcal{P}_{B_{re}}(k) \simeq \frac{8\Gamma(n-\frac{1}{2})^2}{\pi^3 H_{Inf}^{-4}} \left(\frac{k}{2a_{end}H_{inf}}\right)^{-2(n-3)} \left(\frac{a_{end}}{a}\right)^4 \left\{1+\left(\frac{4n-2}{3\omega_{eff}+1}\right)\left(\frac{a_{end}H_{inf}}{aH}-1\right)\right\}^2$$

Simple illustration of evolution of electric and magnetic power spectrum for large scale during reheating

$$k(\tau-\tau_{end}) \ll 1$$

$$\wp_E \propto \frac{1}{a^4}$$

$$\wp_B \propto \frac{1}{H^2 a^4}$$



Spectrum at present: Primordial magnetic field

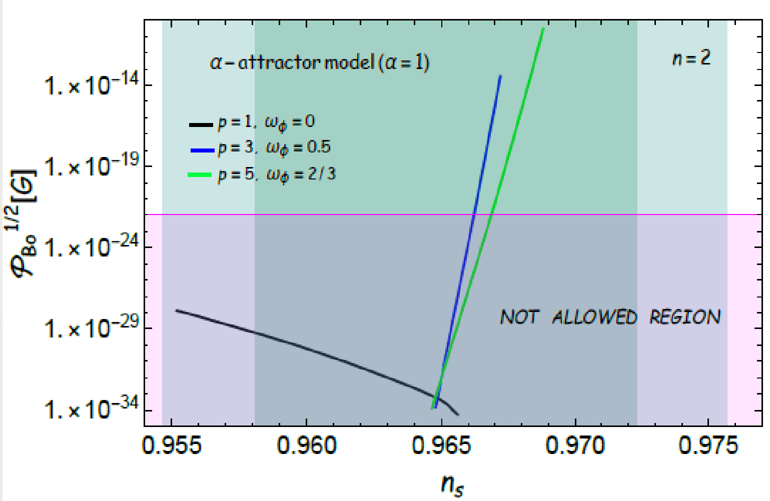
$$\mathcal{P}_{B0}(k) \simeq \frac{\Gamma(n-\frac{1}{2})^2}{\pi^3}\frac{2^{2n-3}\,(2.6\times 10^{39})}{(6.4\times 10^{-39})^{2n-6}}\left(\frac{k}{a_0}M_{pc}\right)^{-2(n-3)}\left(\frac{11 g_{s,re}}{43}\right)^{\frac{2-2n}{3}}\left(\frac{T_{re}}{T_0}\right)^{2-2n}\left(\frac{H_{inf}}{GeV}\frac{1}{A_{re}}\right)^{2(n-1)}$$
$$\left\{1+(2n-1)\left[\frac{2}{3\omega_\phi+1}\left(\eta^{\frac{3\omega_\phi+1}{3(1+\omega_\phi)}}-1\right)+\frac{4}{3\omega_\phi-1}\eta^{1/2}\left(A_{re}^{\frac{3\omega_\phi-1}{4}}-\eta^{\frac{3\omega_\phi-1}{6(1+\omega_\phi)}}\right)\right]\right\}^2 G^2 .$$

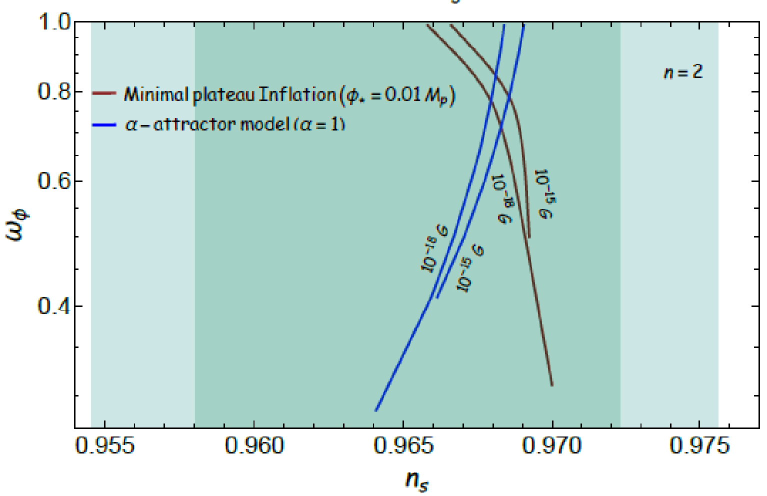
$$V \sim \phi^{2p}$$

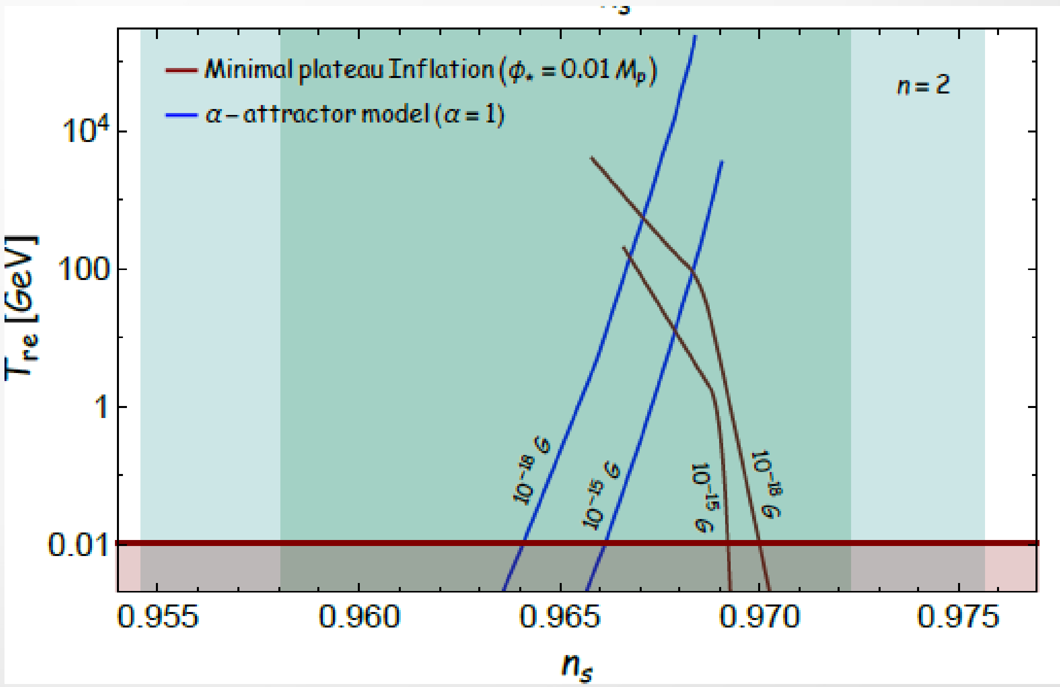
$$\omega_{eff} \approx \omega_\phi = \frac{(p-1)}{(p+1)}$$

$$\mathcal{P}_{B0}^{\frac{1}{2}} \gtrsim 10^{-18}\ \mathrm{G}.$$

$$\omega_{eff} > 0.28 \quad \Rightarrow \quad p > 1.8$$







Conclusions

- Reheating is a poorly understood phase
- It can give a new physics which heppens at very high energy scale beyond the scope of laboratory experiments
- Cosmology behaves as laboratoy system where experiments has already been performed, observables need to be explained.
- **CMB fluctuation, Dark matter, PMF fluctuation can ecode imprints of reheating which can help us understand better inflation and rehating.**
- PGW: Continue in the next talk.