# Exotic Image Formation in Gravitational Lensing

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## This talk is based on the following papers:

- Meena, A. K., Bagla, J. S., “Finding Singularities in Gravitational Lensing”, MNRAS, 492, 3294 (2020).
- Meena, A. K., Bagla, J. S., “Exotic Image Formation in Strong Gravitational Lensing by Clusters of Galaxies. I: Cross-Section”, MNRAS, 503, 2097 (2021).
- Meena, A. K., Ghosh A., Bagla J. S., Williams, L. L. R., “Exotic Image Formation in Strong Gravitational Lensing by Clusters of Galaxies. II: Uncertainties”, MNRAS, 506, 1526 (2021).
- Meena, A. K., Bagla, J. S., “Exotic Image Formation in Strong Gravitational Lensing by Clusters of Galaxies. III: Statistics with HUDF”, arXiv:2107.11955

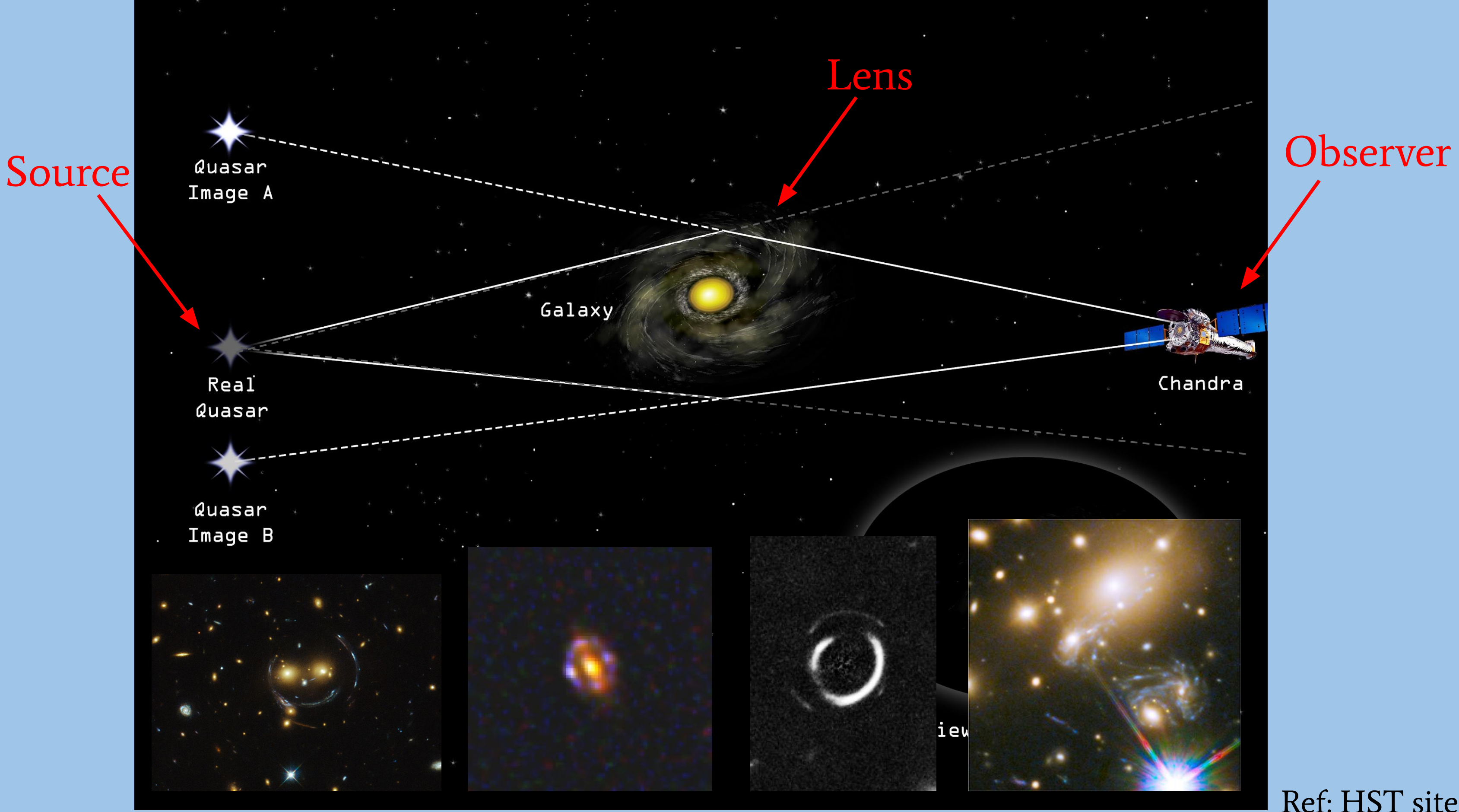
## Collaborators:

- Prof. Jasjeet Singh Bagla, IISER Mohali
- Agniva Ghosh, UMN, USA
- Prof. Liliya Williams, UMN, USA

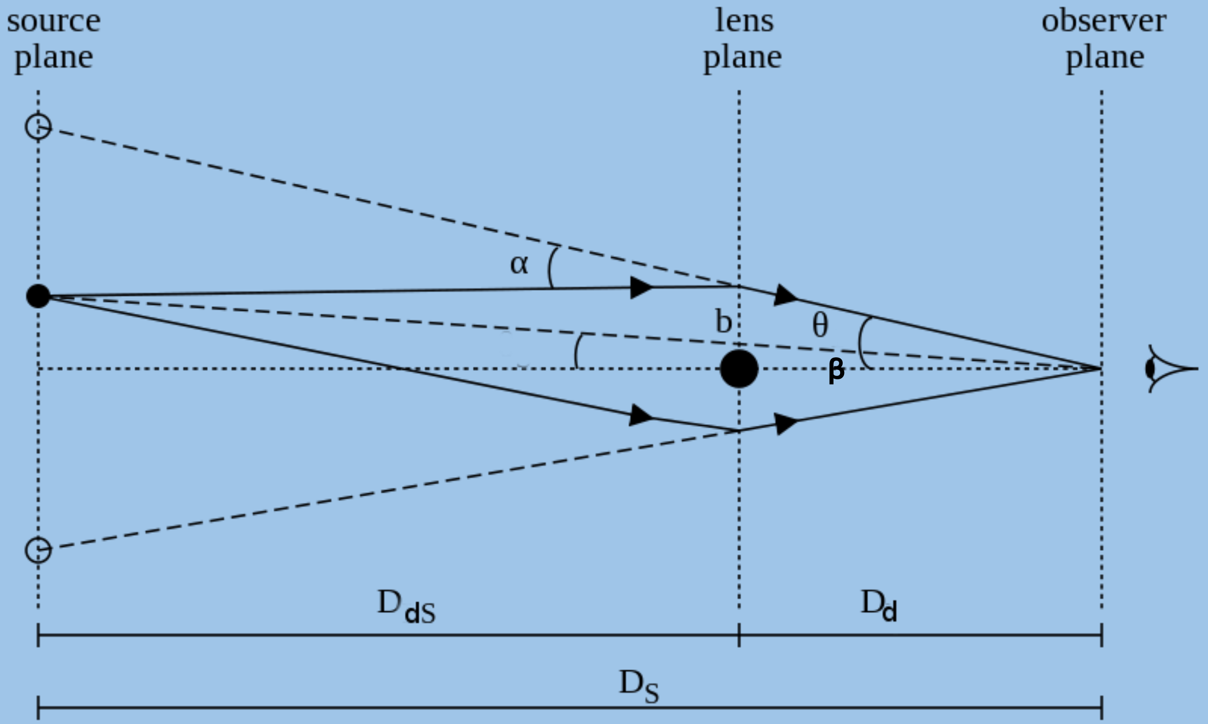
# Exotic Image Formation in Gravitational Lensing

- Basic Gravitational Lensing
- Exotic Image Formation
  - Finding Exotic Images
  - Cross-Section of Exotic Images
  - Effect of Statistical Uncertainties
  - Exotic Images in Simulated Sky
- Summary & Conclusion (& Questions)

# What is gravitational lensing?




Ref: HST site


source plane
lens plane
observer plane
α
b
θ
β
$D_{dS}$
$D_d$
$D_S$

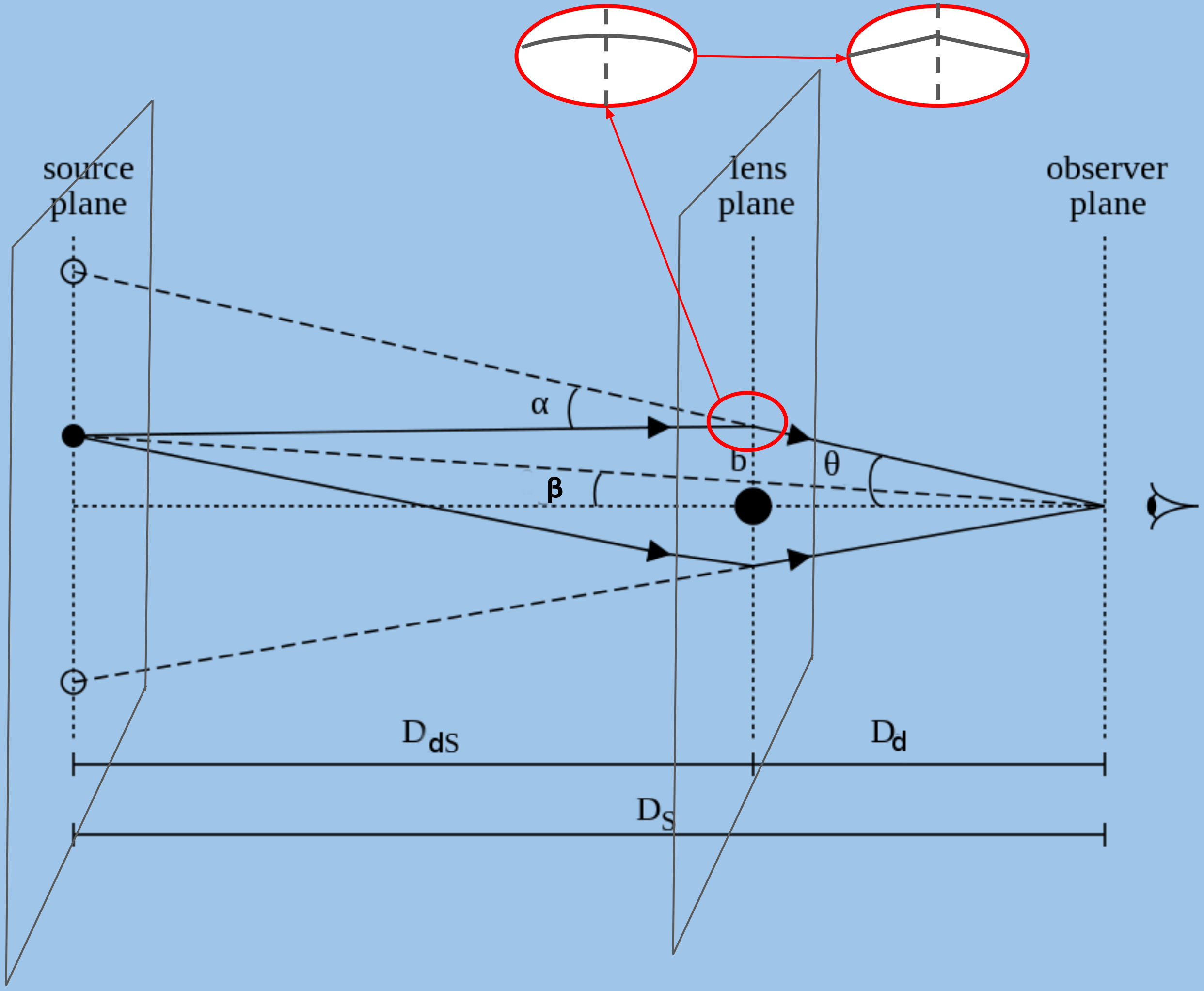
- Thin lens approximation
- Small angle approximation

$$\vec{\beta} = \vec{\theta} - \vec{\alpha}(\vec{\theta}) = \vec{\theta} - \frac{D_{ds}}{D_s}\vec{\hat{\alpha}}(\overrightarrow{D_d\theta})$$

$$A_{ij} = \frac{\partial \beta_i}{\partial \theta_j} = \delta_{ij} - \frac{\partial \alpha_i}{\partial \theta_j} = \delta_{ij} - \frac{\partial^2 \psi}{\partial \theta_i \partial \theta_j}$$

$$\mu = \frac{1}{\det A} = \frac{1}{[(1-\kappa)^2 - \gamma^2]}$$

$\mu = \infty$ → Singularity



- Thin lens approximation
- Small angle approximation

$$\vec{\beta} = \vec{\theta} - \vec{\alpha}(\vec{\theta}) = \vec{\theta} - \frac{D_{ds}}{D_s}\vec{\hat{\alpha}}(\overrightarrow{D_d\theta})$$

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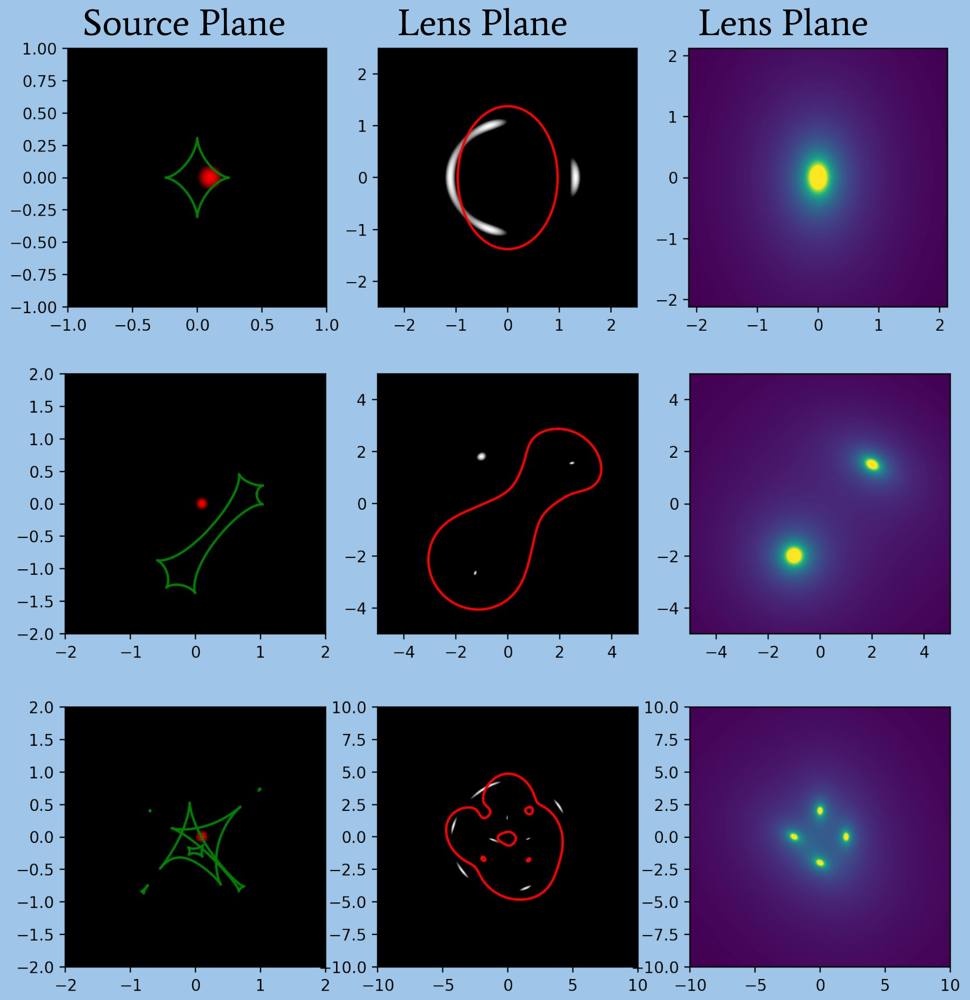
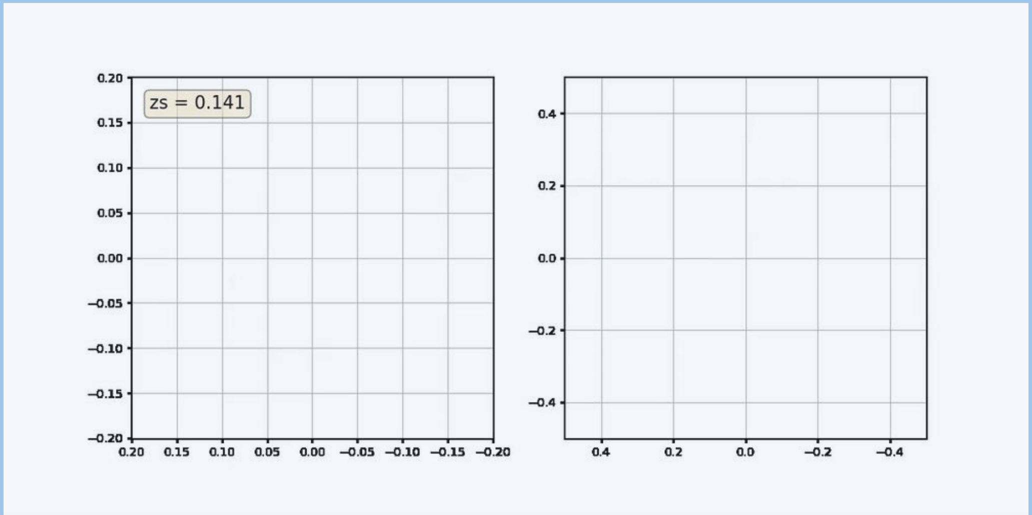
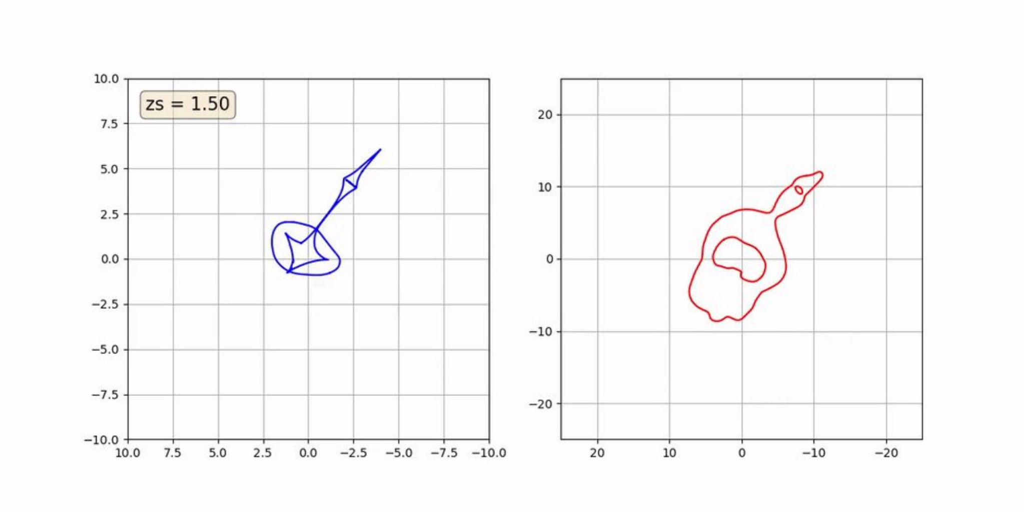
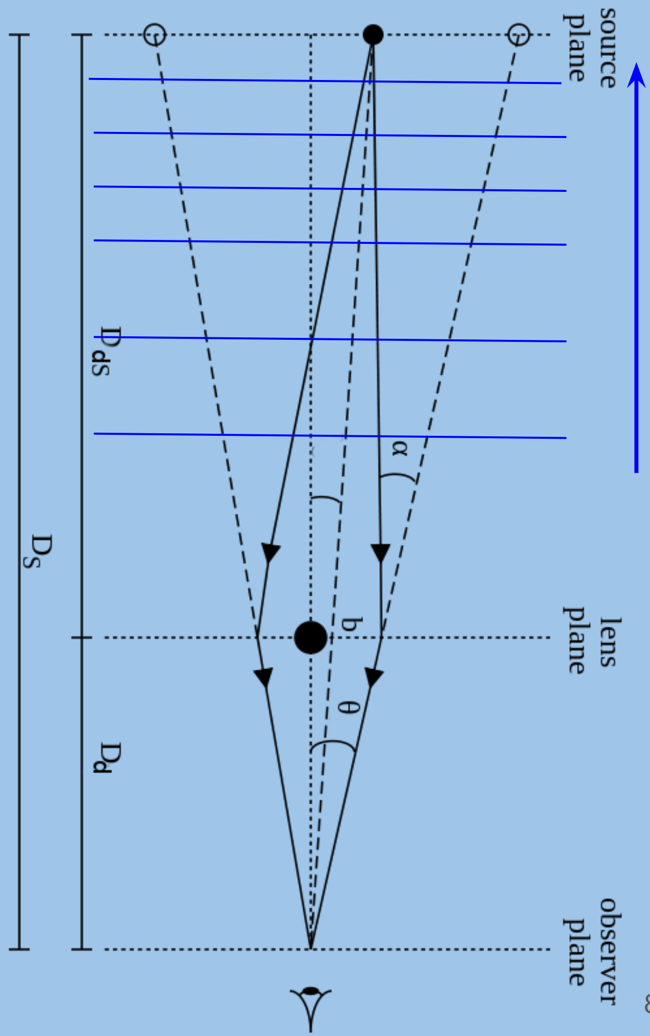
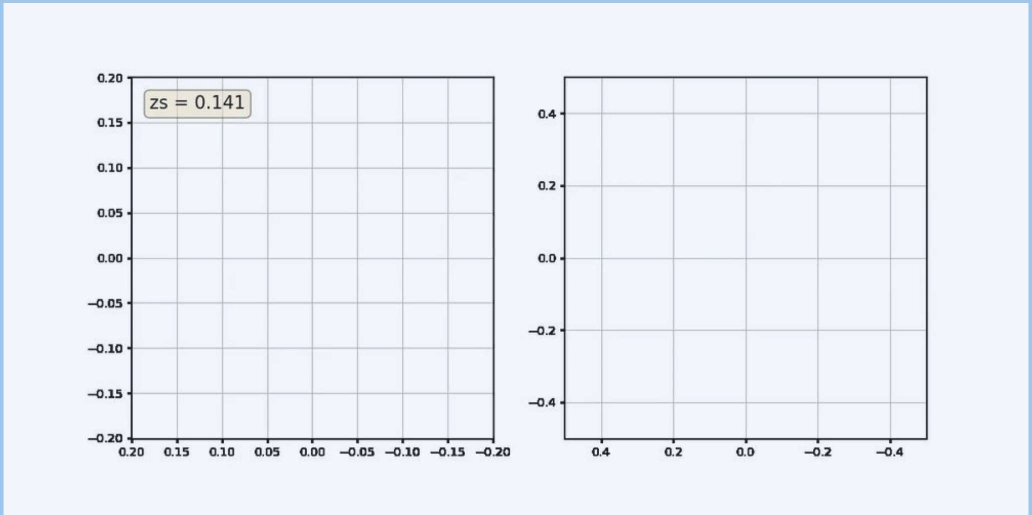


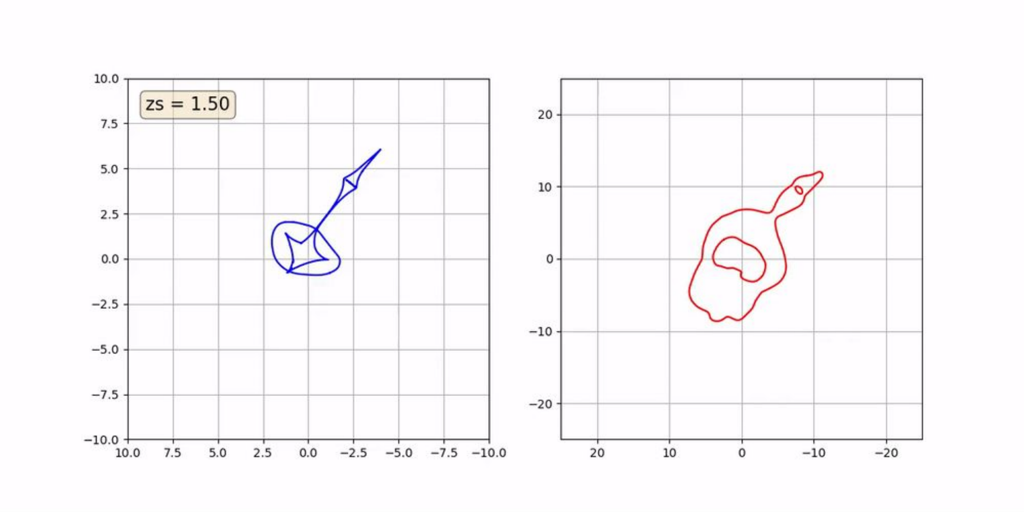

Ref: PYGRALE


zs = 0.141


zs = 1.50


source plane
lens plane
observer plane
D_ds
D_s
D_d
α
b
θ




- Stable singularities:
    - Fold and cusp
    - Present for all source redshifts in strong lensing.
- Unstable singularities:
    - Swallowtail, umbilics,....
    - Present for only specific geometries in a given lens system.
    - Highly sensitive to the lens parameters.
    - Look for cusp.

## Recap:

- Lens → Lens plane, Source → Source plane.
- Lens Equation: Relation between lens and source plane.
- Deflection is of the order of arcseconds.
- Singularities are points where magnification is infinite.
    - Critical curves in lens plane and caustics in the source plane.
    - Stable: Fold and cusp; Unstable: Swallowtail, umbilics.
    - Structure of critical lines and caustics depend on the system geometry.

# How to find point (unstable) singularities?

$$\vec{\beta} = \vec{\theta} - \vec{\alpha}(\vec{\theta}) = \vec{\theta} - \frac{D_{ds}}{D_s}\vec{\hat{\alpha}}(\overrightarrow{D_d\theta})$$

$$\psi_{ij} = \frac{\partial^2 \psi}{\partial x_i \partial x_j}.$$

$$A_{ij} = \frac{\partial \beta_i}{\partial \theta_j} = \delta_{ij} - \frac{\partial \alpha_i}{\partial \theta_j} = \delta_{ij} - \frac{\partial^2 \psi}{\partial \theta_i \partial \theta_j}$$

$$\psi_{ij} = \begin{pmatrix} \kappa + \gamma_1 & \gamma_2 \\ \gamma_2 & \kappa - \gamma_1 \end{pmatrix}$$

$$\mu = \frac{1}{\det A} = \frac{1}{[(1-\kappa)^2 - \gamma^2]}$$

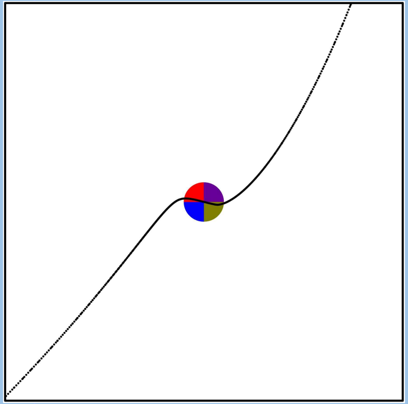
$$\mu(\mathbf{x}) \equiv \frac{1}{\det A(\mathbf{x})} = \frac{1}{(1-a\alpha)(1-a\beta)}$$

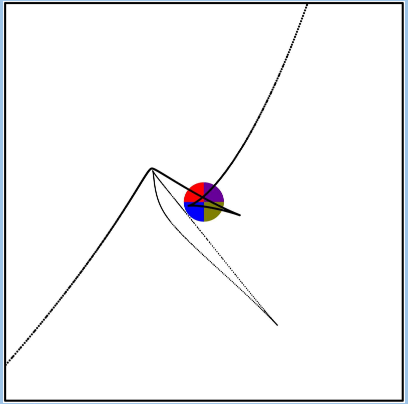
$$n_\lambda . \nabla_x \lambda = 0.$$

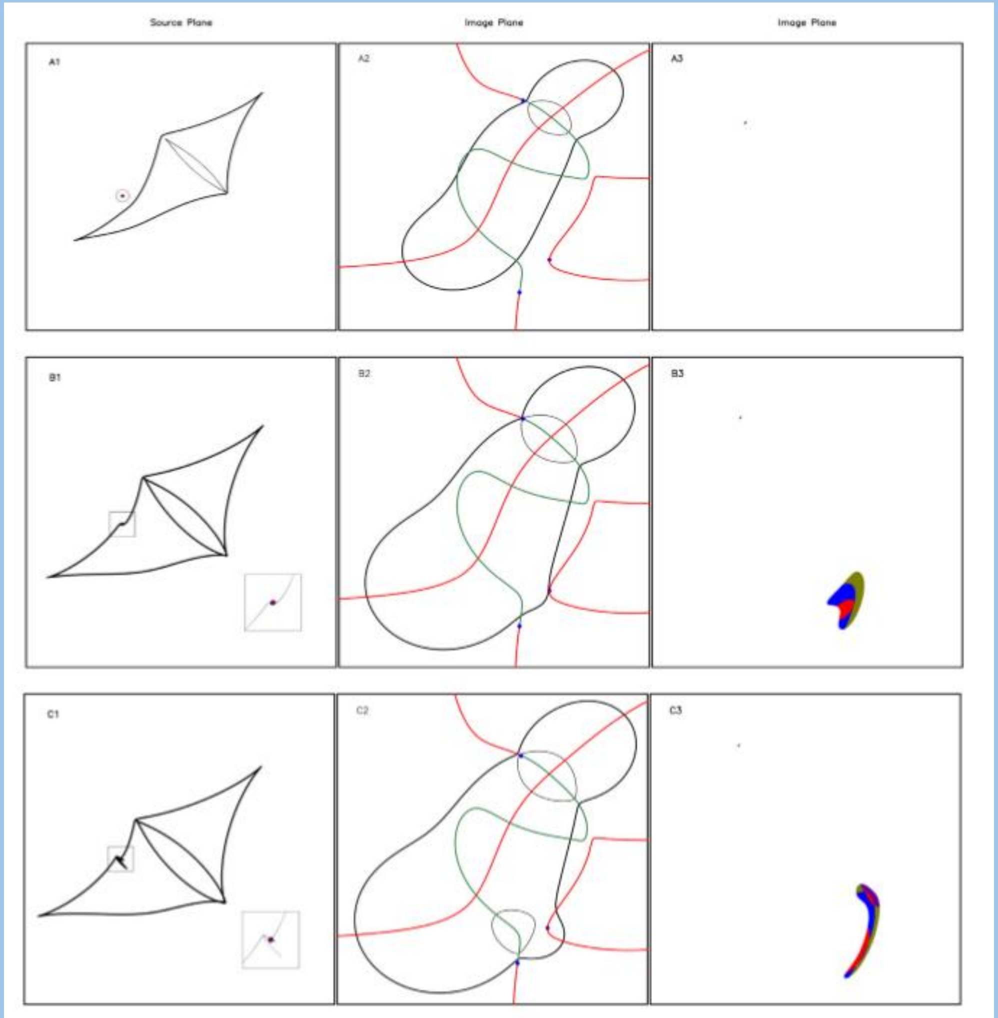
$D_4$-points: $\boldsymbol{\alpha} = \boldsymbol{\beta}$

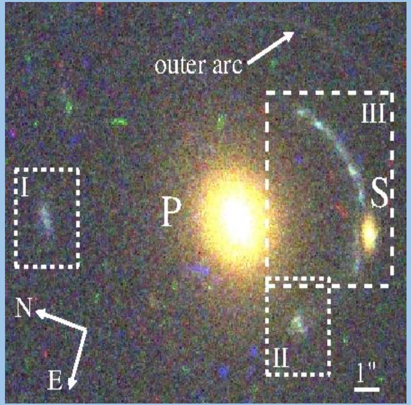
Swallowtail: A4-points, Umbilics: D4-points

# Swallowtail





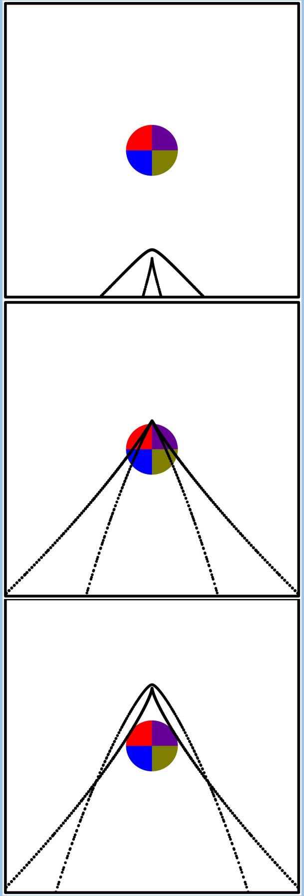


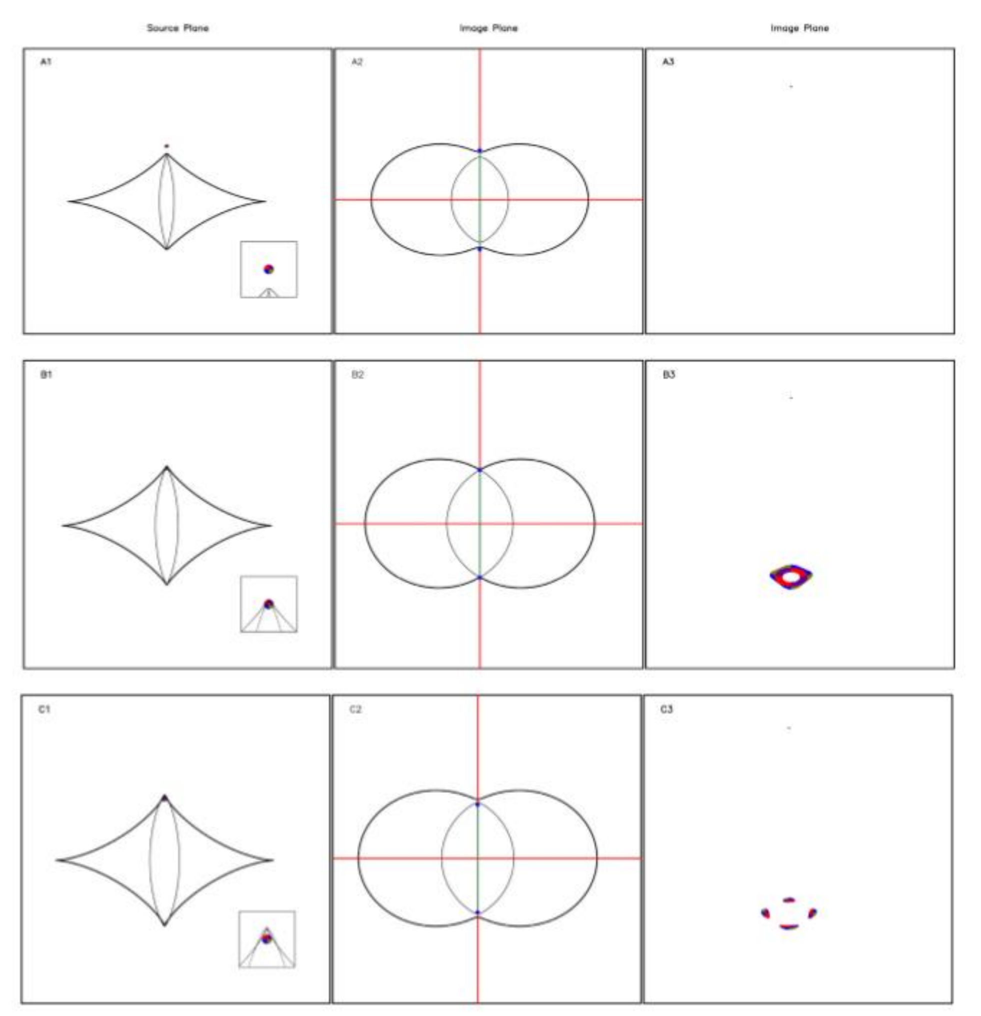




No. of systems > 1

Ref: ESA/HUBBLE
Ref: Suyu et al 2010

# Purse

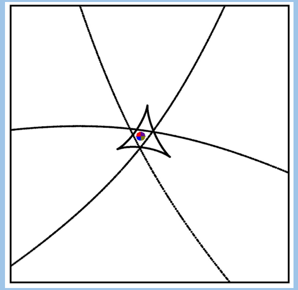


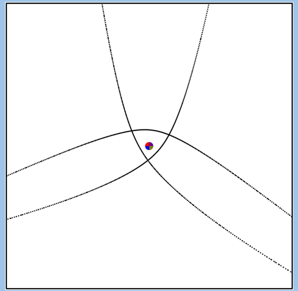


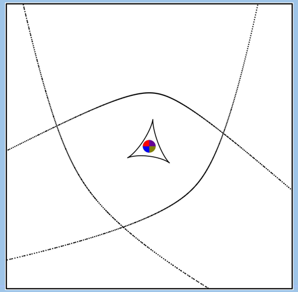

No. of systems = 1

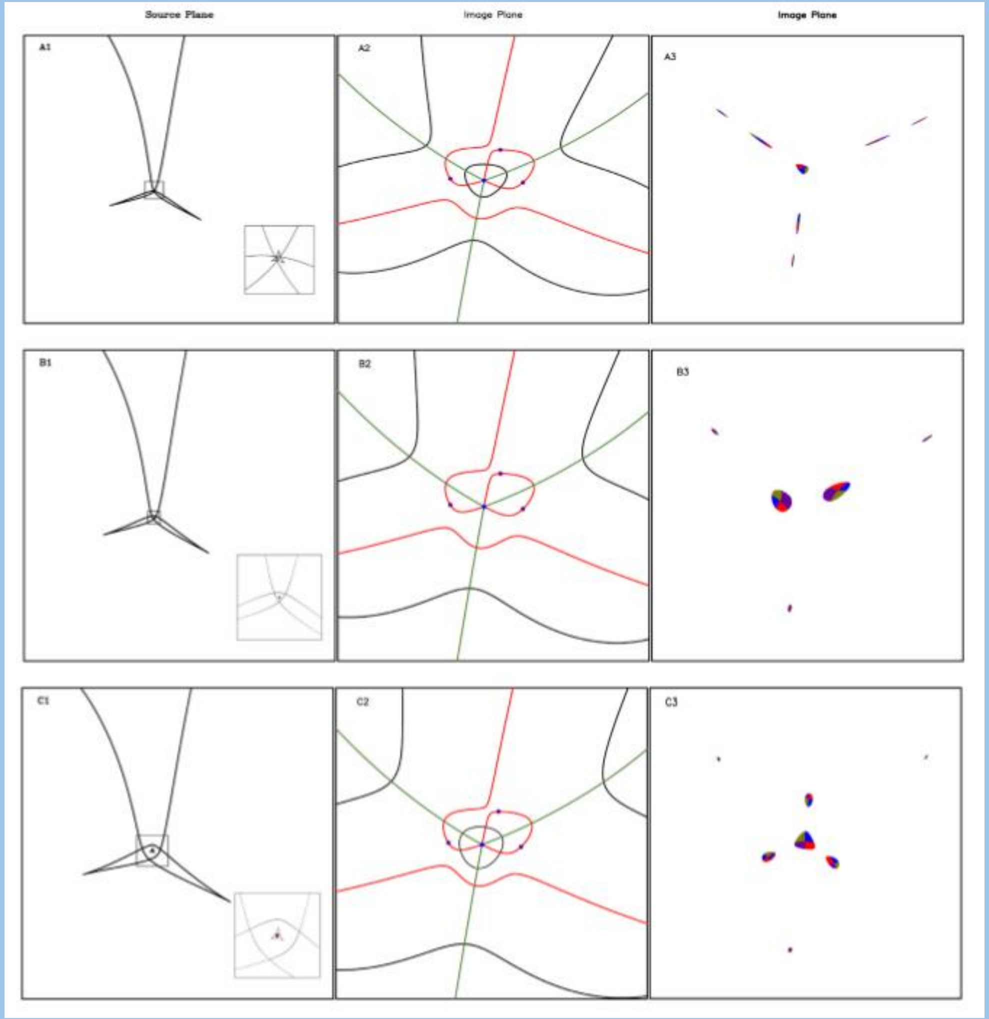
Ref: ESA/HUBBLE

# Pyramid



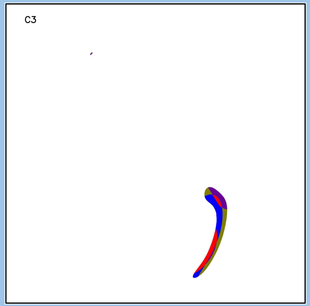


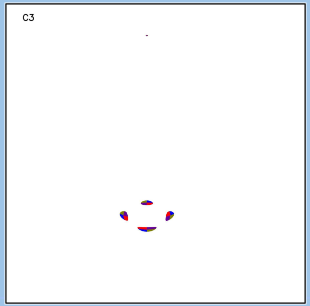


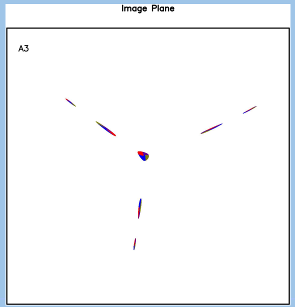



No. of systems = 0

- Meena, A. K., Bagla, J. S., "Finding Singularities in Gravitational Lensing", MNRAS, 492, 3294 (2020).
    - In this work, we developed algorithm to locate these point singularities. The final outcome is a singularity map containing $A_3$-lines, swallowtail, purse, and pyramid.
    - We also studied the effect of external perturbations on the stability of these point singularities.
    - We validated our algorithm by applying it in ideal lenses, simulated lenses and Abell 697.
    - A singularity map allows us to estimate the probability of observing such image formations with the upcoming facilities.



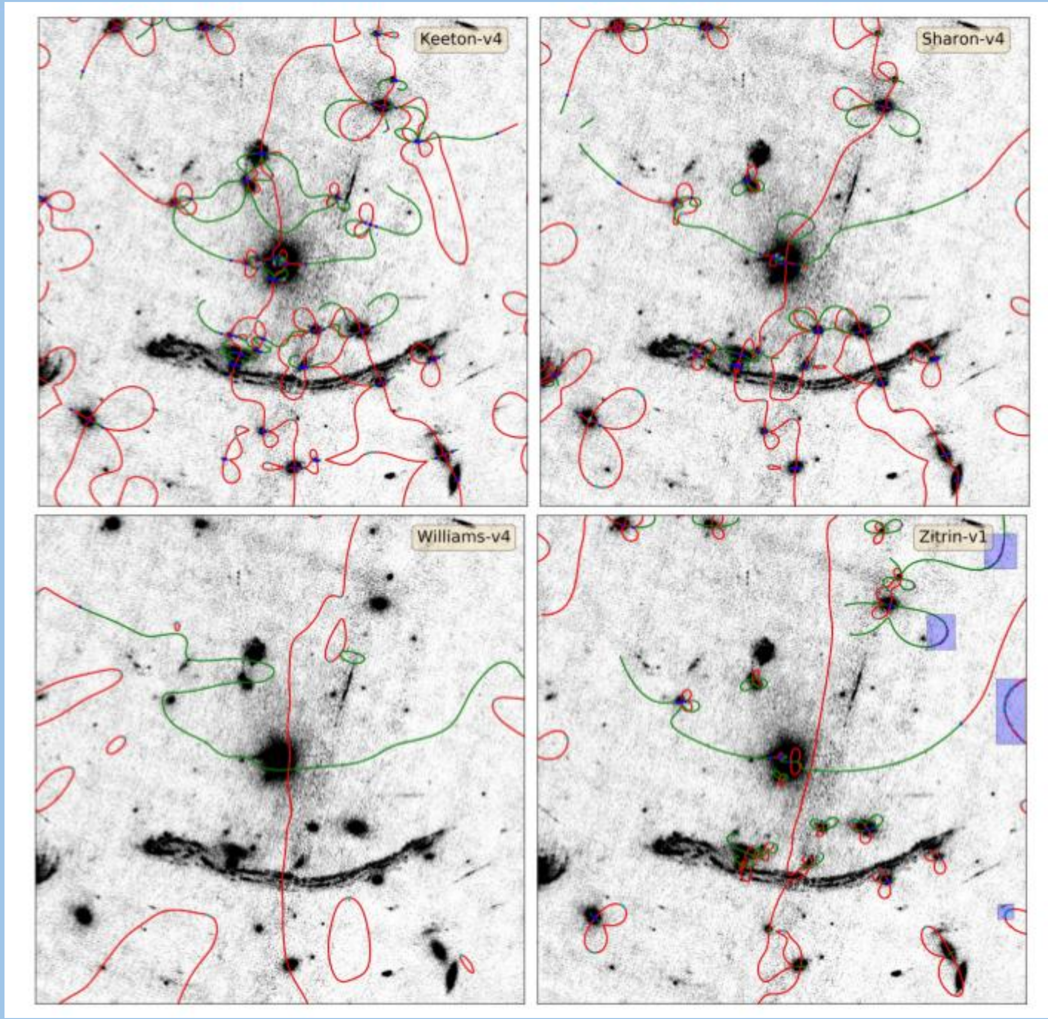



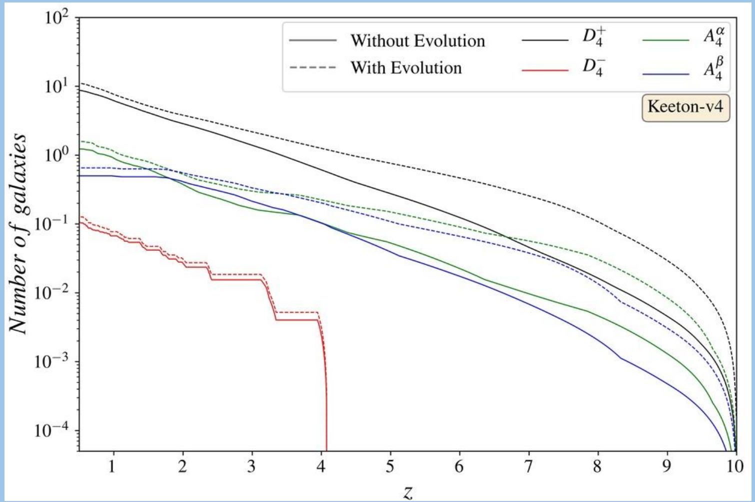
| HFF Clusters | | | | |
|---|---|---|---|---|
| Cluster Name | Keeton | Sharon | Williams | Zitrin |
| A370 | 0.06 arcsec | 0.05 arcsec | 0.05 arcsec | 0.05 arcsec |
| A2744 | 0.06 arcsec | 0.05 arcsec | 0.05 arcsec | 0.06 arcsec |
| AS1063 | 0.06 arcsec | 0.05 arcsec | 0.05 arcsec | 0.065 arcsec |
| MACS0416 | 0.06 arcsec | 0.05 arcsec | 0.05 arcsec | 0.06 arcsec |
| MACS1149 | 0.06 arcsec | 0.05 arcsec | 0.05 arcsec | |
| **RELICS Clusters** | | | | |
| MACS0159 | | | | 0.06 arcsec |
| MACS0308 | | | | 0.06 arcsec |
| PLCKG171 | | | | 0.06 arcsec |
| PLCKG287 | | | | 0.06 arcsec |
| SPT0615 | | | | 0.06 arcsec |

**Cluster Name**: Abell 370

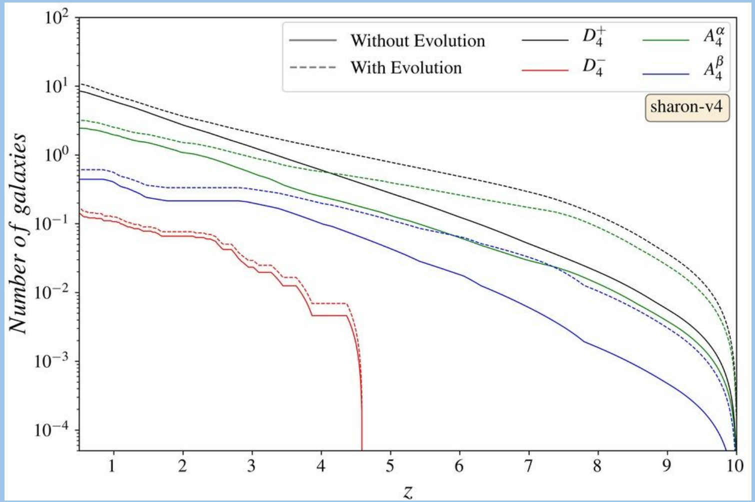
**Central region**:
40 arcsec x 40 arcsec

**Keeton**: parametric model
**Sharon**: parametric model
**Williams**: Non-parametric
**Zitrin**: parametric

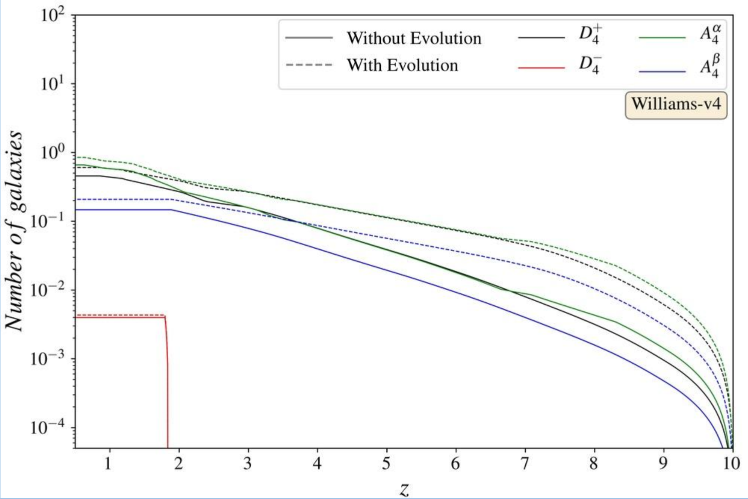




Without Evolution
With Evolution
$D_4^+$
$D_4^-$
$A_4^\alpha$
$A_4^\beta$
Keeton-v4
Number of galaxies
z

Without Evolution
With Evolution
$D_4^+$
$D_4^-$
$A_4^\alpha$
$A_4^\beta$
sharon-v4
Number of galaxies
z

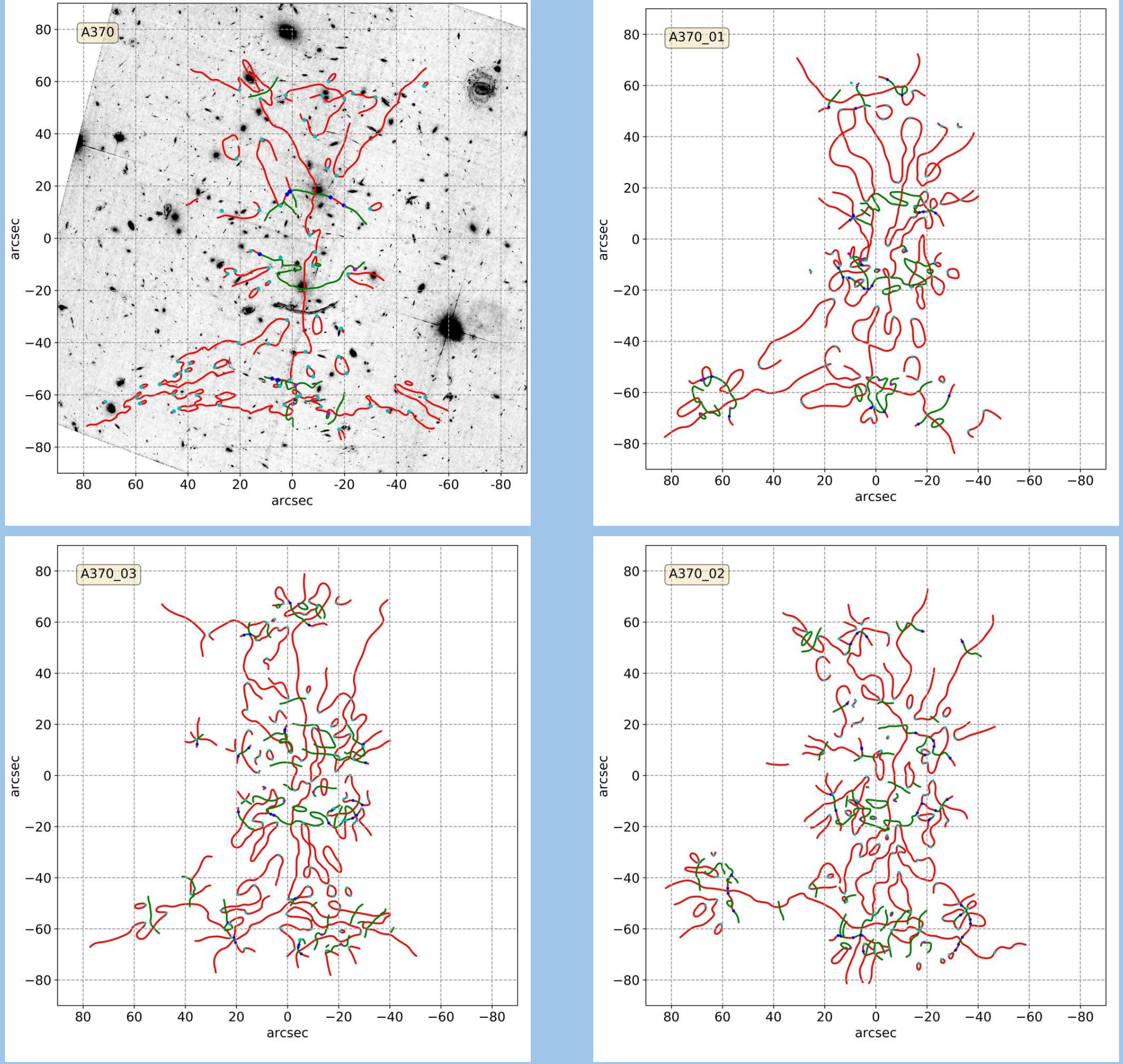
Without Evolution
With Evolution
$D_4^+$
$D_4^-$
$A_4^\alpha$
$A_4^\beta$
Williams-v4
Number of galaxies
z

- Meena, A. K., Bagla, J. S., "Exotic Image Formation in Strong Gravitational Lensing by Clusters of Galaxies. I: Cross-Section", MNRAS, 503, 2097 (2021).

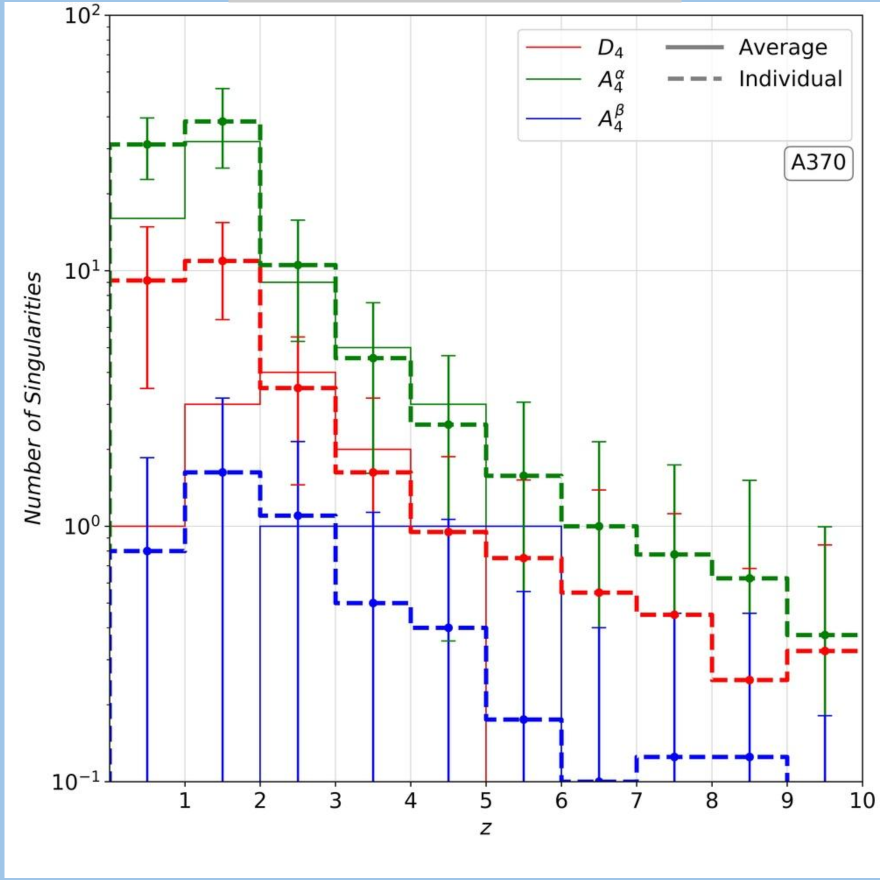
    - Applied our algorithm in ten different galaxy clusters using both parametric and non-parametric mass models.
    - We found that the non-parametric (free-form) method of lens mass reconstruction yields the least number of point singularities.
    - We also estimated the number of galaxies lying near these unstable (point) singularities, which can be observed with the JWST. We find that we expect to get at least one purse and one swallowtail image formation for a source at $z > 1$ for every five clusters with JWST.

- Meena, A. K., Bagla, J. S., “Exotic Image Formation in Strong Gravitational Lensing by Clusters of Galaxies. I: Cross-Section”, MNRAS, 503, 2097 (2021).

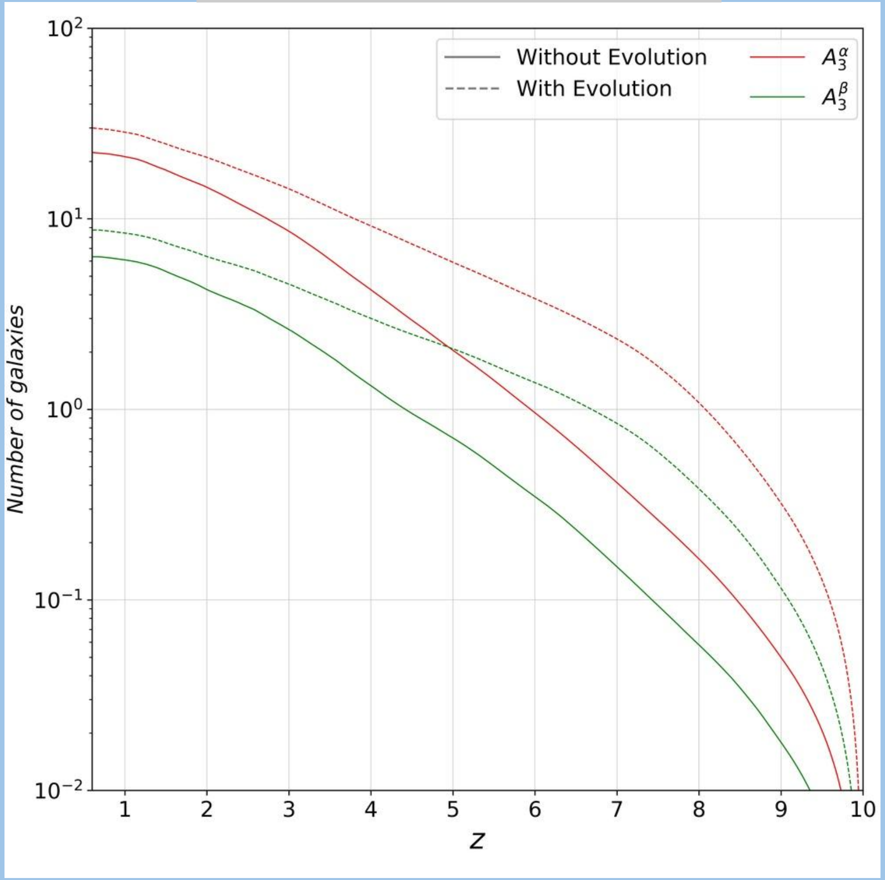
    - The lensing data is finite. So there are multiple lens models that fit the same data (as shown in the last slide). Our estimations are based on the best-fit mass models.
    - But even the best fit mass models have uncertainties associated with them.
      $\Rightarrow$ A range of lens parameters value around the best-fit values.
      $\Rightarrow$ Different cross-section.
    - We considered Grale mass modeling of simulated (from Ghosh et al 2020) and HFF cluster mass models.
    - For simulated cluster three different sets of images were used in reconstruction: 150, 500, 1000.


A370
A370_01
A370_03
A370_02
arcsec
arcsec

## Best-fit → lower limit

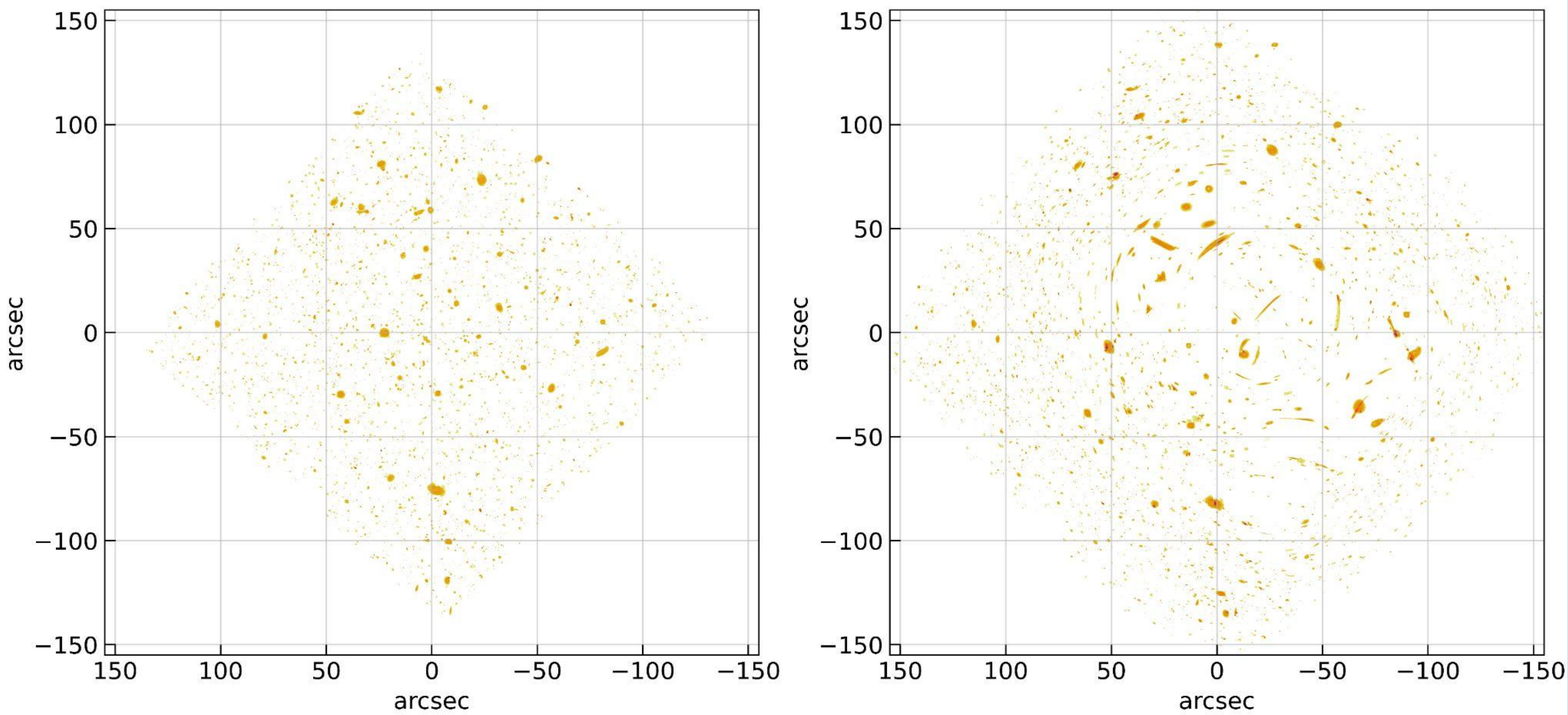



## Cumulative cusp cross-section

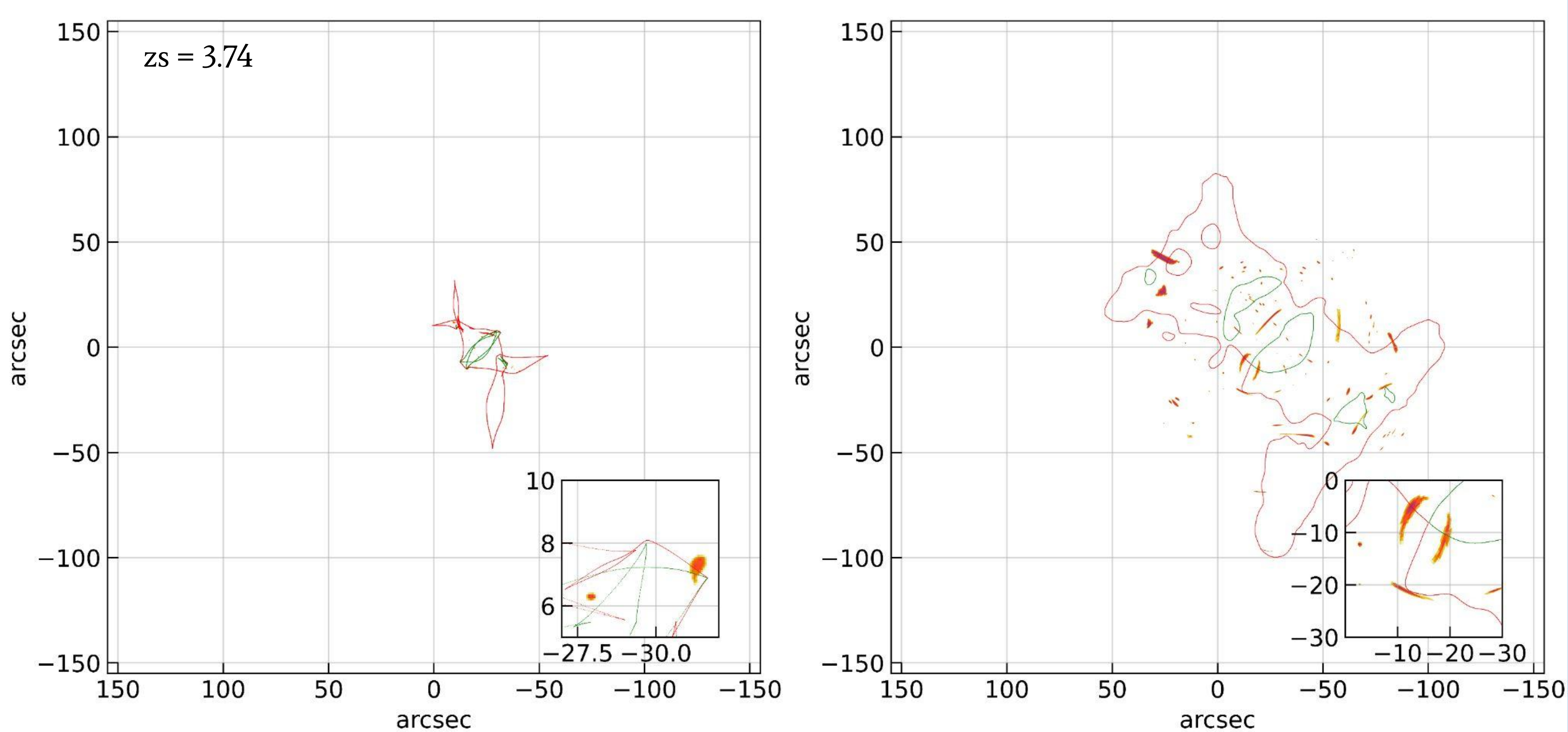


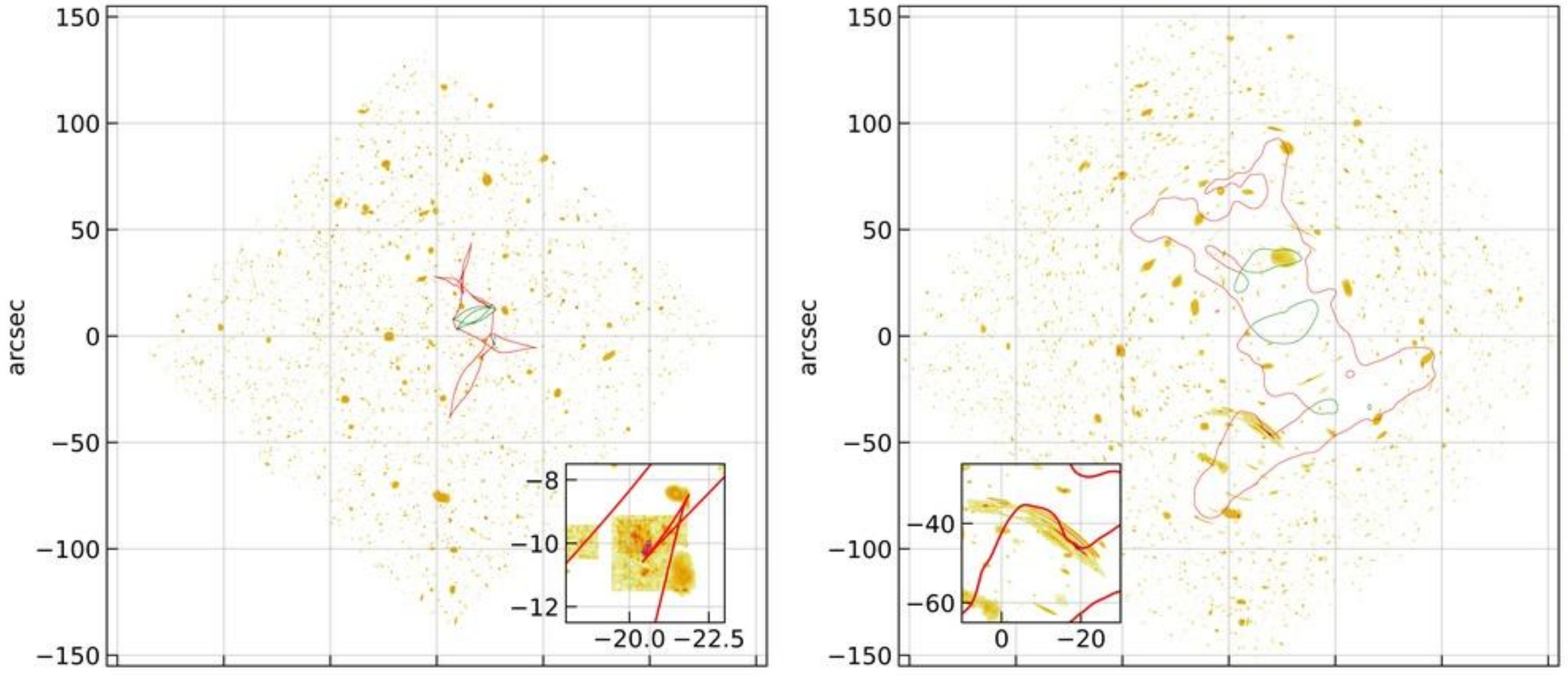
- Meena, A. K., Ghosh, A., Bagla, J. S., Williams, L. L. R., “Exotic Image Formation in Strong Gravitational Lensing by Clusters of Galaxies. II: Uncertainties”, MNRAS, 506, 1526 (2021).
    - Grale best-fit mass models give the lower limit on the cusp and point singularity cross-section.
    - We find that we expect to observe at least 20-30 tangential and 5-10 radial three-image arcs in the HFF cluster lenses with JWST.

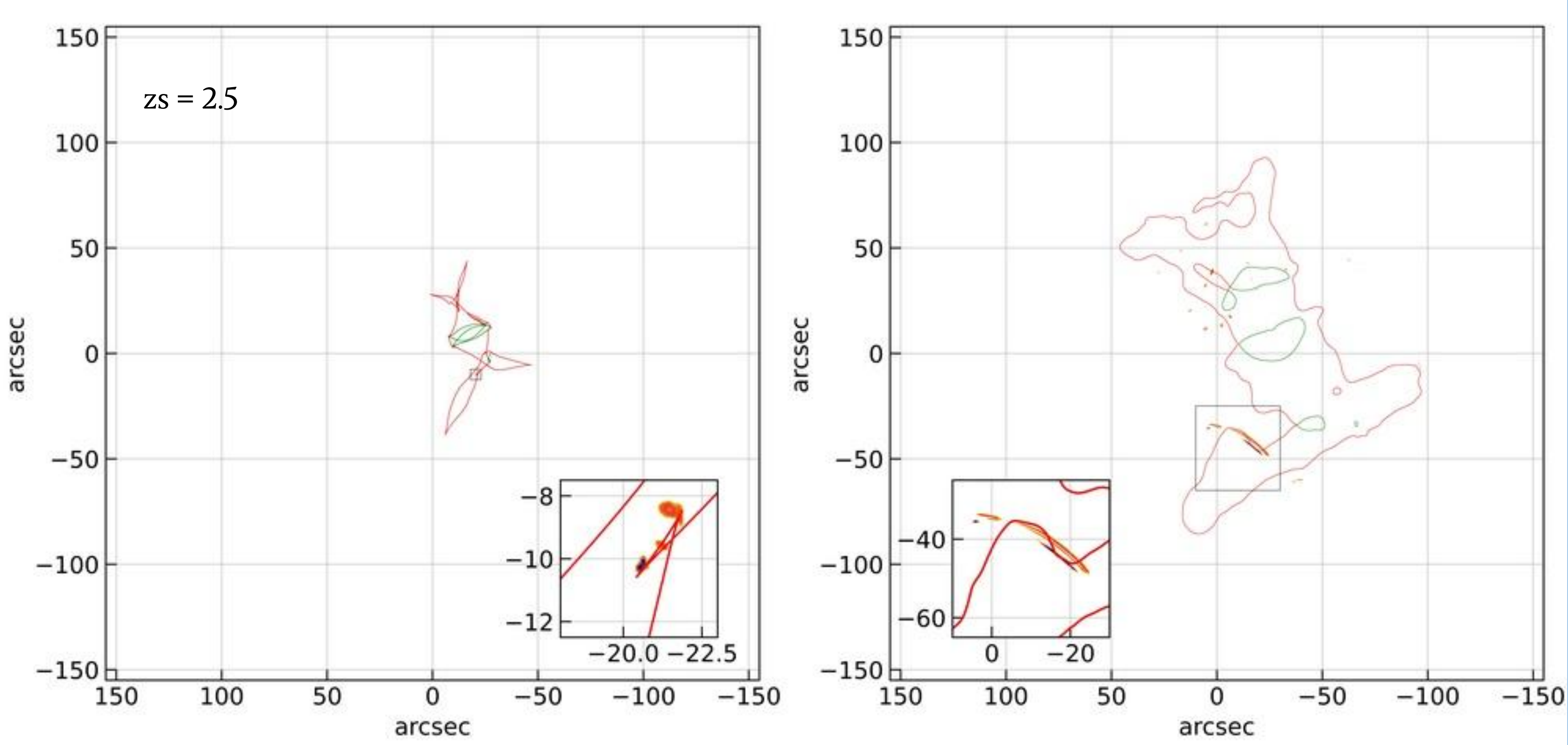
- In above works, we have estimated the exotic image formation cross-section in HFF clusters.
- The other way to estimate these numbers is to simulate lensed sky:
  ⇒ Simulate unlensed sky. ⇒ Lens it. ⇒ Find the exotic images.
- This is what we did in our next work:
  Meena, A. K., Bagla, J. S., "Exotic Image Formation in Strong Gravitational Lensing by Clusters of Galaxies. III: Statistics with HUDF", arXiv:2107.11955
- However, instead of simulating the unlensed sky, we used the HUDF with only half of the sources.



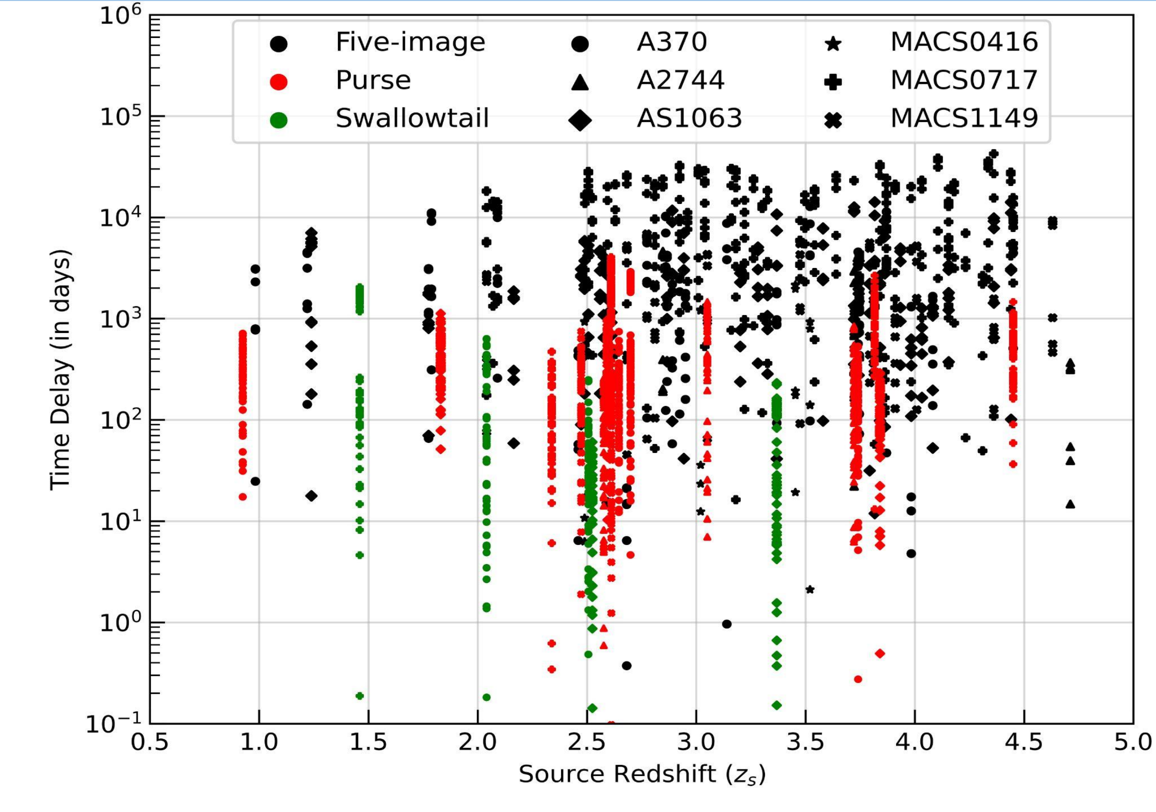
150
100
50
0
−50
−100
−150
arcsec
150
100
50
0
−50
−100
−150
arcsec
150
100
50
0
−50
−100
−150
arcsec
150
100
50
0
−50
−100
−150
arcsec

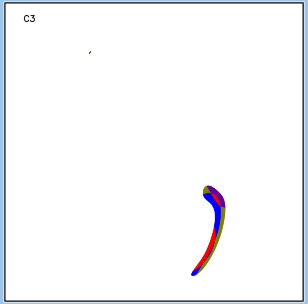

zs = 3.74
arcsec
arcsec
arcsec
arcsec

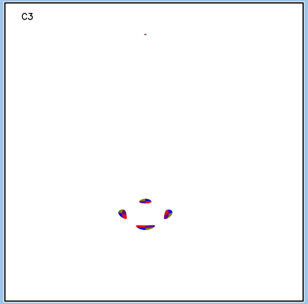

150
100
50
0
−50
−100
−150
arcsec
−8
−10
−12
−20.0
−22.5
−40
−60
0
−20



zs = 2.5
150
100
50
0
−50
−100
−150
arcsec
arcsec
−8
−10
−12
−20.0
−22.5
−40
−60
−20


Five-image
Purse
Swallowtail
A370
A2744
AS1063
MACS0416
MACS0717
MACS1149
Time Delay (in days)
Source Redshift ($z_s$)
$10^{-1}$
$10^0$
$10^1$
$10^2$
$10^3$
$10^4$
$10^5$
$10^6$
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0


C3


C3

# Summary

- **Part I:** A singularity map locates all the high magnification regions with three or more images and very sensitive to the lens mass distribution. Hence, can be very helpful for looking at the sources at very high redshifts.

- **Part II:** Grale (best-fit) mass models give the lower limit:
    - 20-30 tangential three image arcs in HFF clusters
    - 5-10 radial three image arcs in HFF clusters
    - 1→Swallowtail, 1→Purse in HFF clusters

- **Part III:** Above estimates are valid even if we include the statistical uncertainties.

- **Part IV:** Looked for these exotic images in simulated sky. The initial results are quite encouraging and need further investigation.