Inflationary degeneracies, Reheating constraints and Gravitational Waves
Saturday Seminar 2021 @IIT-Madras

Swagat Saurav Mishra (Research Associate)
Inter-University Centre for Astronomy and Astrophysics(**IUCAA**), Pune.

Collaborators: **Prof. Varun Sahni (IUCAA) and Prof. Alexei Starobinsky (Landau Institute, Moscow)**

7th August 2021

Paper

PART-1 of an ongoing project

"***Curing inflationary degeneracies using reheating predictions and relic gravitational waves***",

S. S. Mishra, V. Sahni and A.A Starobinsky,
JCAP 05 (2021) 075

[arXiv:2101:00271 [gr-qc]]

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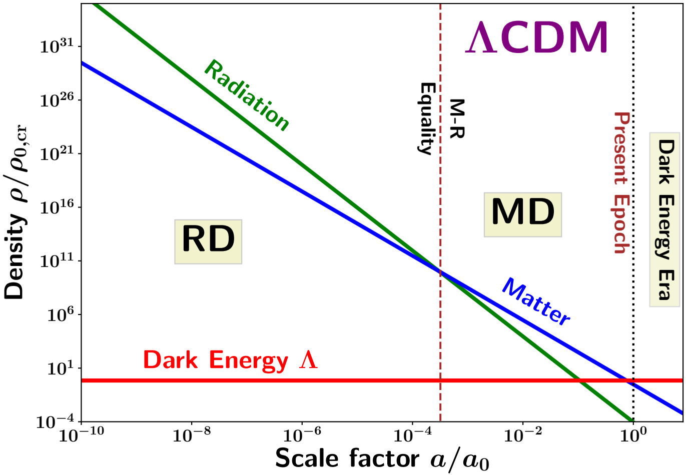
  Reduced planck mass $m_p = \frac{1}{\sqrt{8\,\pi\,G}} = 2.4 \times 10^{18}$ GeV

The Early Universe



Inflationary Dynamics of a Scalar Field

Standard – A single canonical scalar fie
minimally coupled to gravity

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi),\ p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

And Einstein's equations imply

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \left(\frac{8\pi G}{3}\right)\rho_\phi,$$

$$\dot{H} = -\frac{1}{2m_p^2}\dot{\phi}^2$$

$$\frac{\ddot{a}}{a} = -\left(\frac{4\pi G}{3}\right)\left(\rho_\phi + 3\,p_\phi\right)$$

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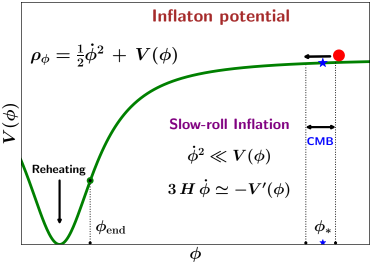

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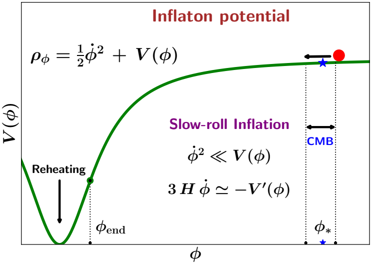

Condition for Inflation

$$\epsilon_H = -\frac{\dot{H}}{H^2} < 1 \Rightarrow \dot{\phi}^2 < V(\phi)$$

The dynamics of the scalar field is governed by

$$\ddot{\phi} + 3\,H\dot{\phi} + V'(\phi) = 0$$

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Slow-roll is an attractor trajectory [Mishra, Sahni, Toporensky 2018] – due to the role of **Hubble friction** at early times during inflation.

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2. **Tensor Perturbation** (Transverse, traceless h_{ij} – **relic Gravitational Waves**)

Equation of motion of each Fourier mode

$$\mathcal{R}''_k + 2\frac{z'}{z}\mathcal{R}'_k + k^2\mathcal{R}_k = 0, \quad \frac{z'}{z} = aH(1 + \epsilon_H - \eta_H)$$

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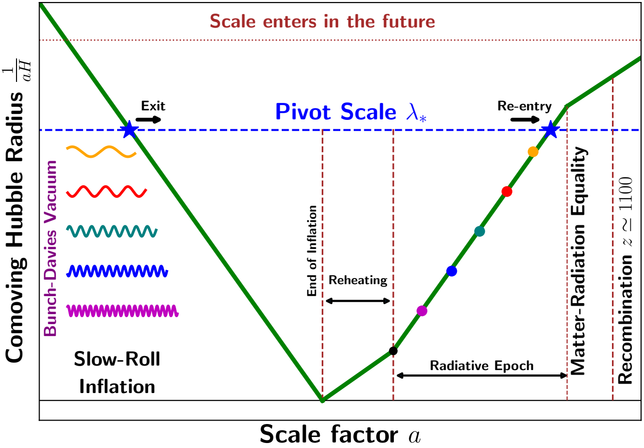
solution on **super-Hubble scales** $k \ll aH$ look like

$$\mathcal{R}_k(\tau) \simeq A + B\int\frac{d\tau}{z^2} = A + B\int d\tau\ e^{-\int d\tau\ 2aH(1+\epsilon_H-\eta_H)}$$

Behaviour of fluctuations



Inflationary Power-spectrum

During slow-roll $\epsilon_H, \eta_H \ll 1$
Primordial Power-spectra at least at large scales –

$$P_{\mathcal{R}} = A_s \left(\frac{k}{k_*}\right)^{n_S - 1}$$

$$P_{\mathcal{T}} = A_t \left(\frac{k}{k_*}\right)^{n_t}$$

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$$A_s = \frac{1}{8\pi^2}\left(\frac{H_*}{m_p}\right)^2 \frac{1}{\epsilon_{_H}^*}$$

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Scalar Spectral Index

$$n_{_S} = 1 + 2\eta_{_H}(k_*) - 4\epsilon_{_H}(k_*)$$

Tensor to Scalar Ratio

$$r = 16\epsilon_{_H}(k_*)$$

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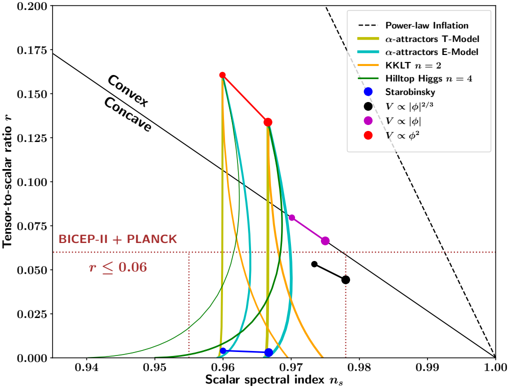

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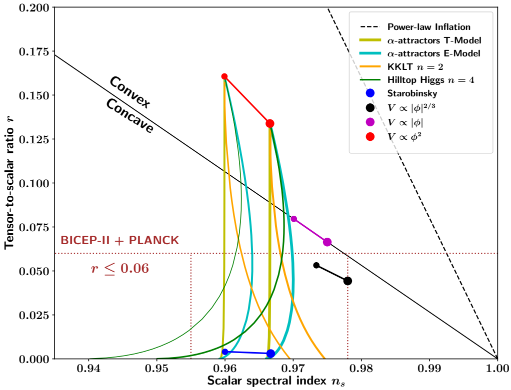

$r \leq 0.06 \Rightarrow$ **Asymptotically flat** Concave potentials, nearly scale-invariant but slightly red tilted scalar power spectrum with $n_S \simeq 0.967$ near the CMB pivot scale.

$$H_k^{\rm inf} \leq 2.5 \times 10^{-5}\ m_p = 6.1 \times 10^{13}\ \text{GeV}$$

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- $\Rightarrow$ CMB observations can not constitute a unique probe of the physics during inflation.
- Let's see a demonstration of such degeneracies.

Degeneracy in the T-model

T-model potential $V(\phi) = V_0 \tanh^{2p}(\lambda\phi/m_p)\ ; \quad p = 1, 2, 3....$

$$V(\phi) \simeq V_0,\ |\lambda\phi| \gg m_p$$

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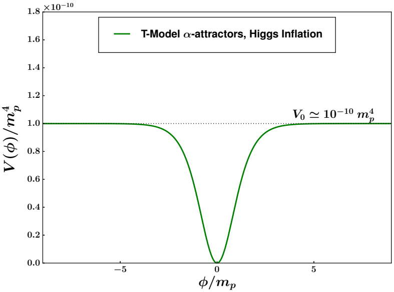

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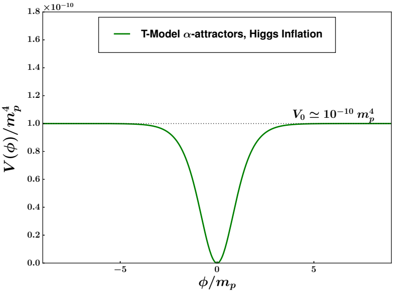

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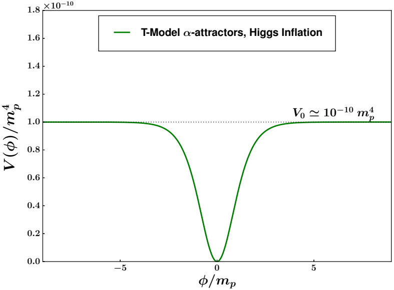

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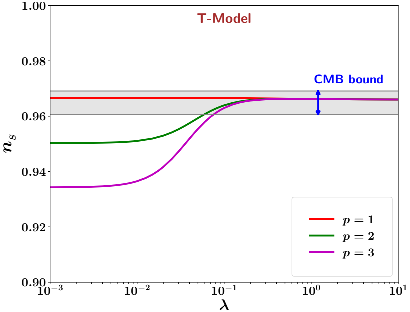

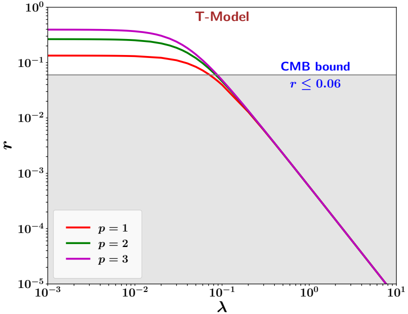

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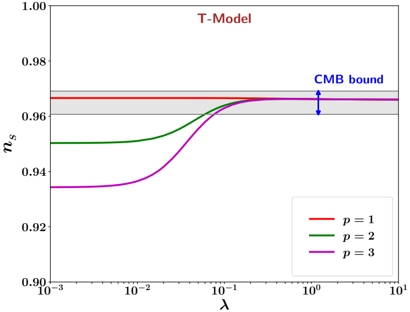

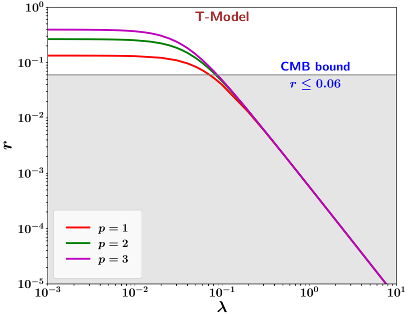

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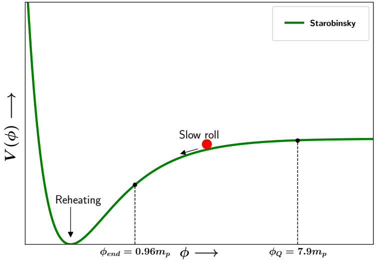

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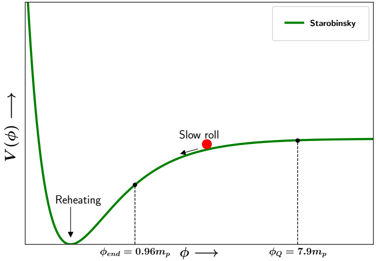

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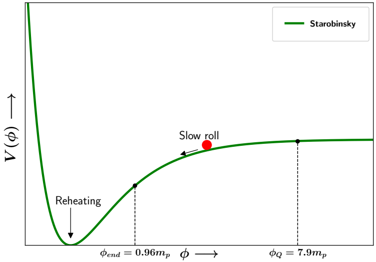

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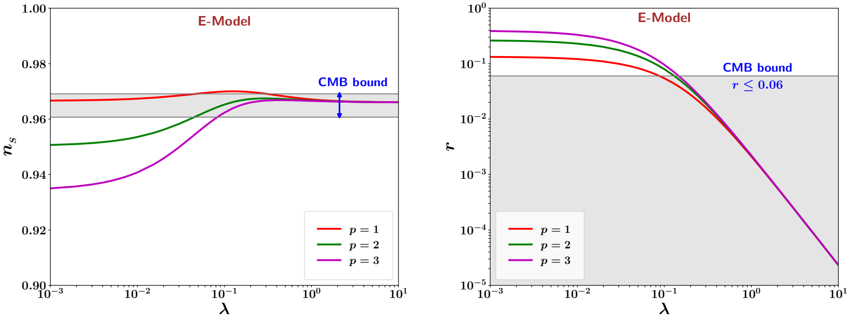

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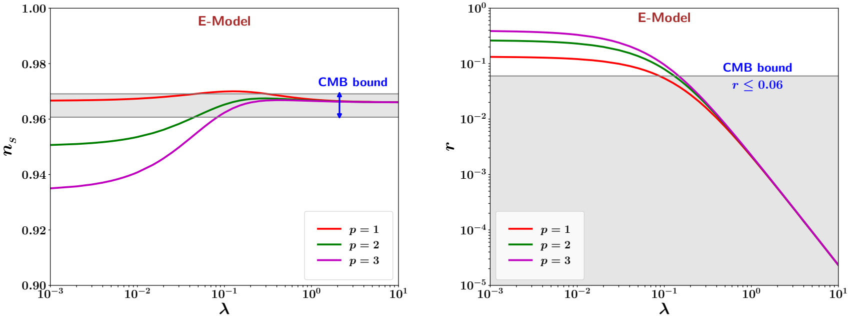

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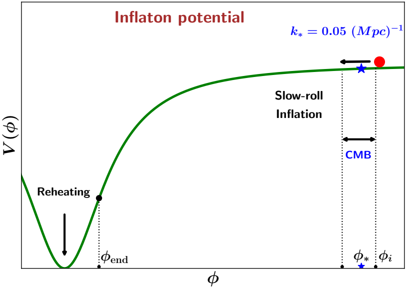
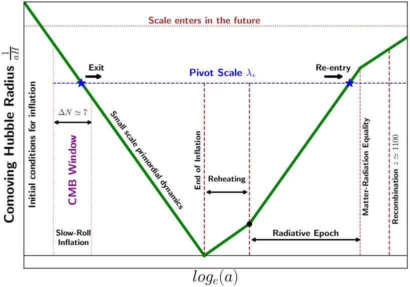
Post-inflationary Evolution





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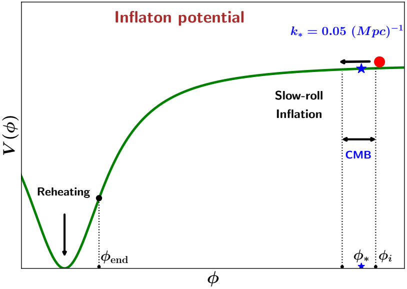

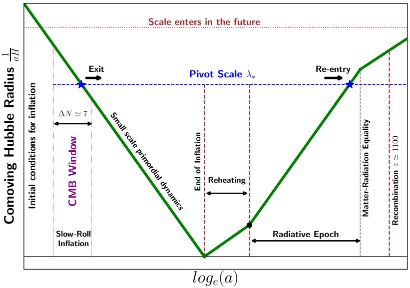

Inflation $\rightarrow$ Reheating $\rightarrow$ hot Big Bang

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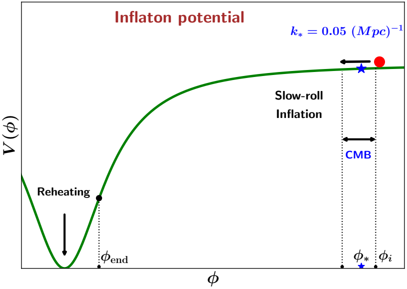

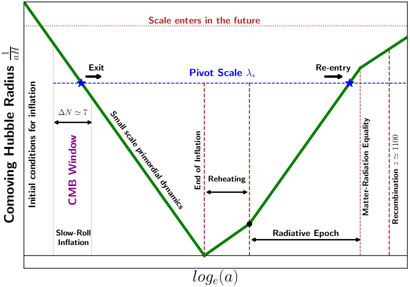

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Two implications

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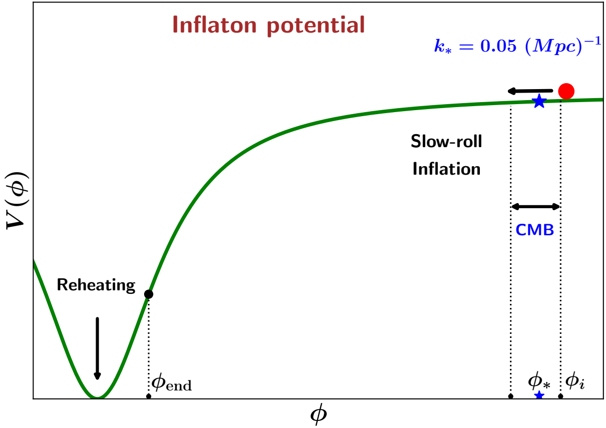

1) Where is ϕ_*

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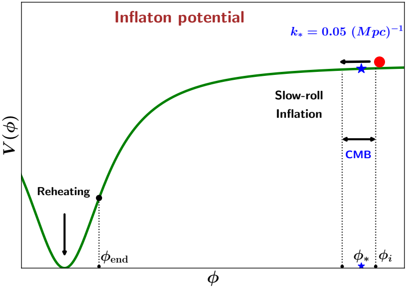

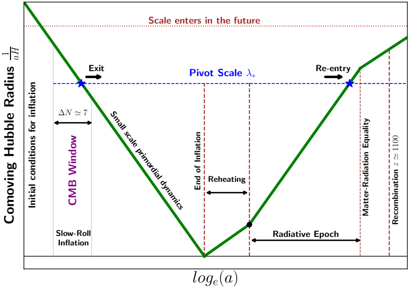

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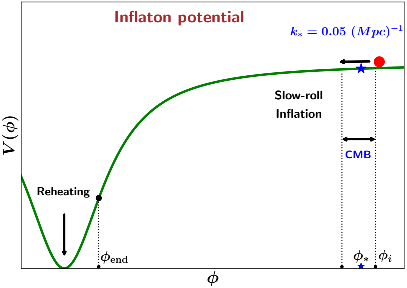

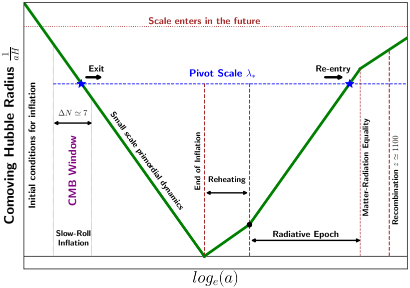

Two implications **1) Where is ϕ_* 2) Breaking degeneracies**

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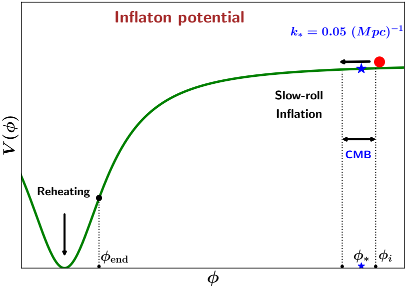

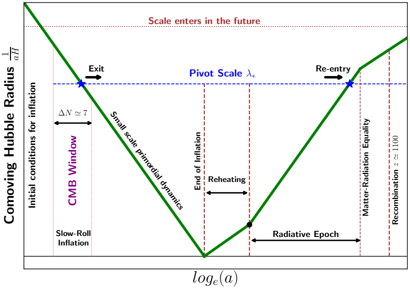

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- **Non-perturbative Reheating**
  Parametric Resonance,
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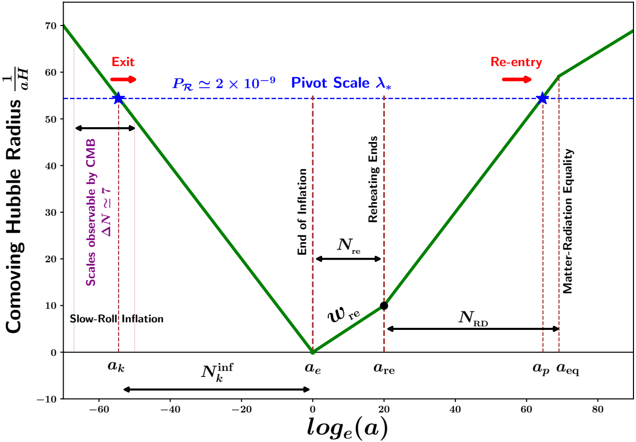
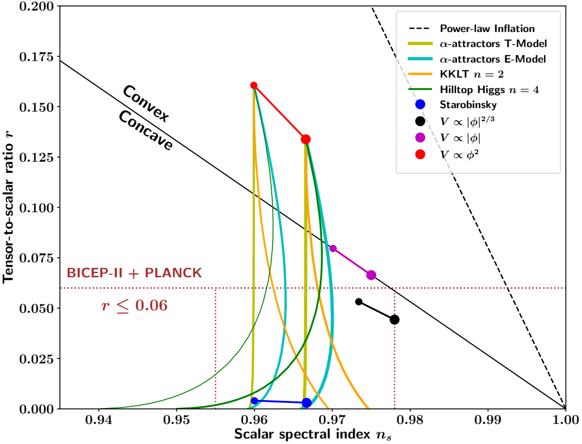


Reheating parameters $\{w_{re}, N_{re}, T_{re}\}$

The usual 50–60 e-folds



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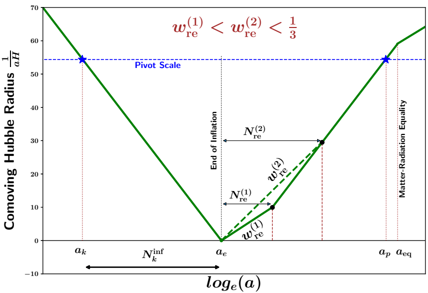
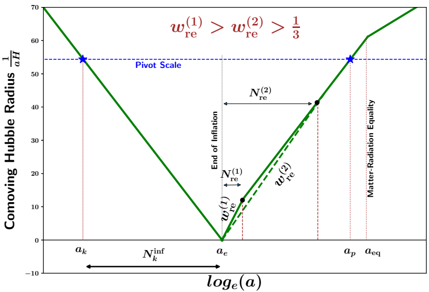
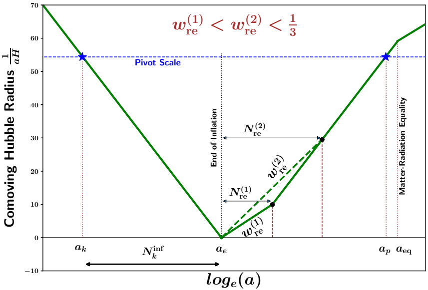
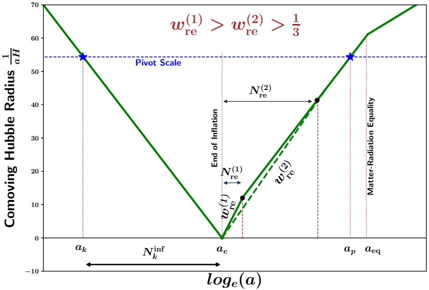
$\Rightarrow$ **sharp predictions for** $\{n_S, r\}$

Geometrically Understanding









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and

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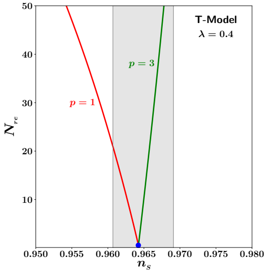
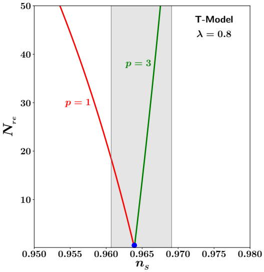
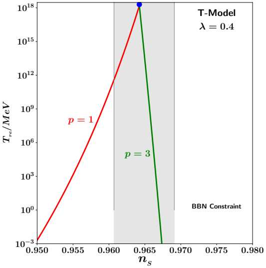
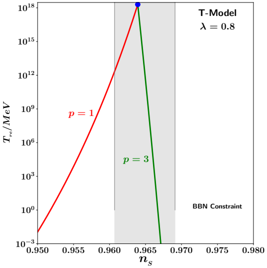
Good enough to break inflationary degeneracies

Breaking degeneracies in the T-Model







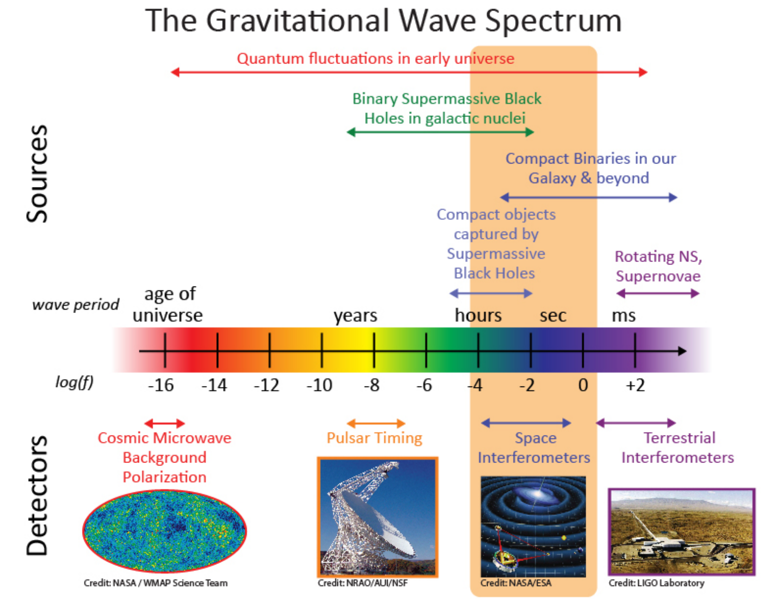


Segregation of Inflationary Observables

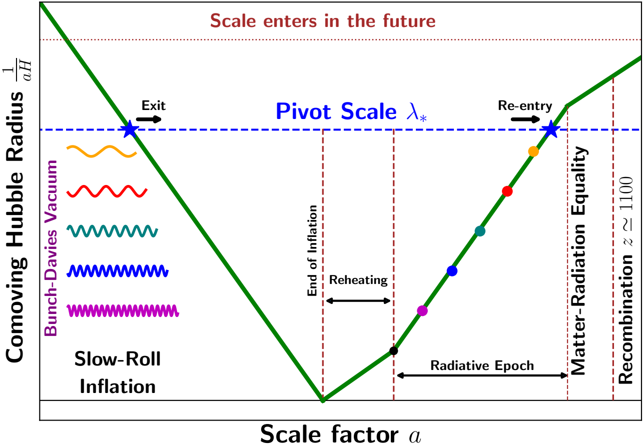
λ	Observables	$p = 1,\, w_{re} = 0$	$p = 2,\, w_{re} = 1/3$	$p = 3,\, w_{re} = 1/2$
0.4	$N_k^{\rm inf}$	[50.4931, 55.686]	55.72	[55.724, 60.383]
	n_s	[0.9607, 0.9643]	0.9643	[0.96435, 0.967]
	r	[0.003867, 0.004683]	0.003899	[0.003326, 0.003896]
	N_{re}	[0, 20.9624]	0	[0, 37.5836]
	T_{re}	$[3.9 \times 10^8,\, 2.4 \times 10^{15}]$ GeV	2.4×10^{15} GeV	$[10^{-3},\, 2.4 \times 10^{15}]$ GeV
0.8	$N_k^{\rm inf}$	[50.6026, 55.17]	55.1749	[55.1761, 59.8198]
	n_s	[0.9607, 0.9639]	0.9639	[0.9639, 0.966706]
	r	[0.001009, 0.001198]	0.00101	[0.0008604, 0.00101]
	N_{re}	[0, 18.4401]	0	[0, 37.4696]
	T_{re}	$[2.2 \times 10^9,\, 2.1 \times 10^{15}]$ GeV	2.1×10^{15} GeV	$[10^{-3},\, 2.1 \times 10^{15}]$ GeV

$\Rightarrow$ **Degeneracies Broken**

Primordial Gravitational Waves



Tensor fluctuations → GWs (Physically)



Present-day Frequency of GWs

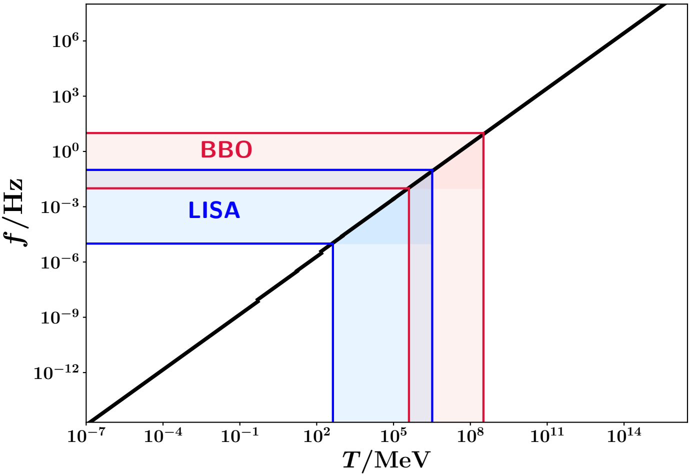
$$f = \frac{1}{2\pi}\left(\frac{k}{a_0}\right) = \frac{1}{2\pi}\left(\frac{a}{a_0}\right) H$$

$$f = 7.36 \times 10^{-8}\,\mathrm{Hz}\left(\frac{g_0^s}{g_T^s}\right)^{\frac{1}{3}}\left(\frac{g_T}{90}\right)^{\frac{1}{2}}\left(\frac{T}{\mathrm{GeV}}\right)$$

Epoch	**Temperature T**	**GW f (in Hz)**
Matter-radiation equality	~ 1 eV	1.7×10^{-17}
CMB pivot scale re-entry	~ 5 eV	8.5×10^{-17}
Big Bang Nucleosynthesis	~ 1 MeV	1.8×10^{-11}
Electro-weak symmetry breaking	~ 100 GeV	2.7×10^{-6}

Table:

Present-day Frequency of GWs



Gravitational Wave Spectrum

Definition:

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During reheating: $\Omega^{(\rm re)}_{_{\rm GW}}(f) = \Omega^{(\rm RD)}_{_{\rm GW}}(f) \left(\frac{f}{f_{\rm re}}\right)^{2\left(\frac{w-1/3}{w+1/3}\right)}\,, \;\; f_{\rm re} < f \leq f_{\rm e}\;,$

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$$n^{\rm PI}_{_{\rm GW}} = \frac{d\log\Omega_{_{\rm GW}}(k)}{d\log k} = \frac{d\log\Omega_{_{\rm GW}}(f)}{d\log f}$$

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GW Spectrum – General Behaviour

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GW Spectrum – General Behaviour

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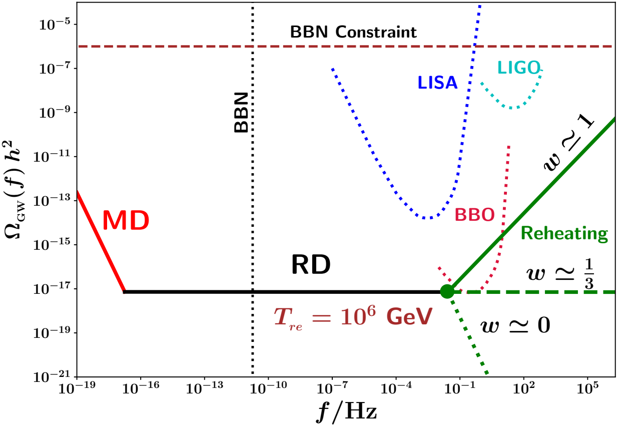

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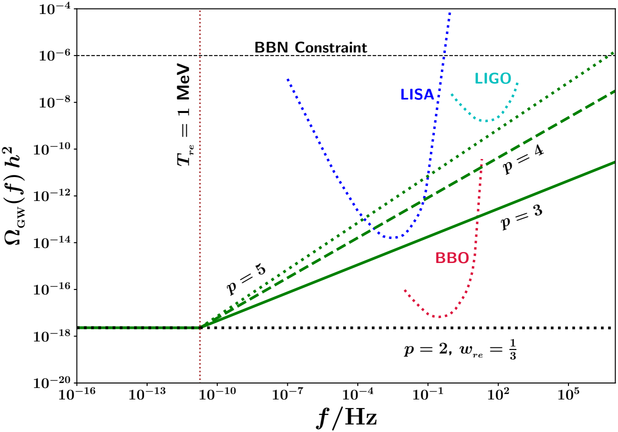

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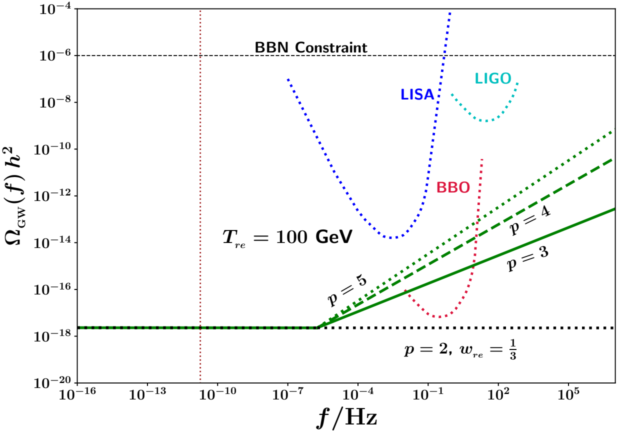

GW Spectrum – Breaking Degeneracies

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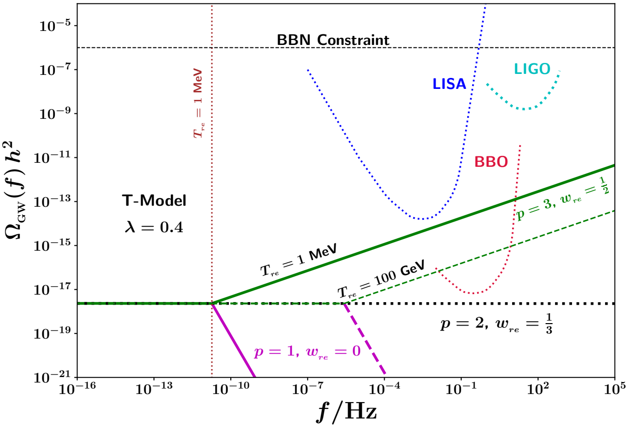

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- Relic Gravitational Waves as the cleanest probe of the Early Universe including the epoch of reheating

Extra Slides

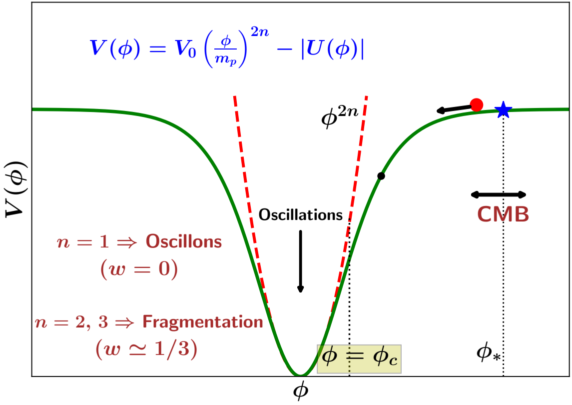
Non-perturbative Effects

Non-perturbative Self-interactions

Scalar Field Fragmentation/Oscillon Formation

During Reheating $V(\phi) \simeq V_0 \left(\phi/m_p\right)^{2p} - |U(\phi)|$

Fragmentation for $\lambda \gg 1 \qquad (r \ll 0.001)$



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Preheating $\longrightarrow$ **Scattering** $\longrightarrow$ **Perturbative Decay** $\longrightarrow$ **Thermalization**

Fully Non-linear Treatment: Numerically

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$$H^2 = \frac{1}{3\,m_p^2}\left[\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\left(\nabla\phi\right)^2 + \frac{1}{2}\dot{\chi}^2 + \frac{1}{2}\left(\nabla\chi\right)^2 + U(\phi,\chi)\right]\ ,$$

with

$$U(\phi,\chi) = V(\phi) + \frac{1}{2}g^2\phi^2\chi^2\ .$$

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Lattice Calculations $\Rightarrow \mathcal{W}_{\rm re}$

Non-perturbative Instant Preheating

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χ fattens and decays very effectively within an oscillation!!