PBH formation from Non-Canonical Axion-Curvaton Scenario

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Growth of curvature perturbations for PBH formation & detectable GWs in non-minimal curvaton scenario revisited,
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Contents

Motivation for Curvaton

- An alternative method to generate adiabatic curvature perturbation where inflaton is not responsible for it [arXiv:hep-ph/0110002]
- Curvaton does not require any assumptions about the nature of inflation
- Curvaton can save some models that are ruled out by data [arXiv:1312.1353]
- Possibility of producing large non-Gaussianities

The Curvaton Picture

- The curvaton field is subdominant in energy during inflation
- Curvaton field is almost forzen during inflation
- Curvaton does not affect the background inflationary evolution
- After inflation the curvaton starts to dominate the energy density and produce observed curvature perturbation

Curvaton dynamics

- Background curvaton: $\bar{\chi}$; curvaton perturbation: $\delta\chi$
- The system

$$S = \int d^4x\sqrt{-g}\left(\frac{M_{\rm Pl}^2}{2}R - \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi - \frac{1}{2}\nabla_\mu\chi\nabla^\mu\chi - V(\phi,\chi)\right) \quad (1)$$

Inflaton and curvaton do not interact with each other

- The background and perturbed curvaton equation of motion:

$$\ddot{\bar{\chi}} + 3H\dot{\bar{\chi}} + V_{,\chi} = 0 \quad (2)$$

$$\delta\ddot{\chi}_k + 3H\delta\dot{\chi}_k + \left(\frac{k^2}{a^2} + V_{,\chi\chi}\right)\delta\chi_k \simeq 0 \quad (3)$$

- $\bar{\chi}$ is almost frozen during inflation
- Gaussian fluctuation: $\delta\chi \sim \dfrac{H_{inf}}{2\pi}$

Observable curvature perturbation gets generated after inflation

- The metric can be written as,

$$ds^2 = -dt^2 + a^2(1+\zeta)g^{ij}dx_i dx_j$$

- ζ at large scales can be written as,

$$\dot{\zeta} = -\frac{H}{\rho + P}\delta P_{nad}$$

$\delta P_{nad} \rightarrow$ Non-adiabatic perturbation

Amplification process

- After inflation we have two components: radiation (rad) and curvaton (χ)
- Gauge invariant perturbations : $\zeta_i = -\psi + \dfrac{\delta\rho_i}{\dot{\rho}_i}$
- The total curvature perturbation: $\zeta = (1-f)\zeta_{rad} + f\zeta_\chi$
  $f = \dfrac{3\rho_\chi}{3\rho_\chi + 4\rho_{rad}}$
- ζ_i's are conserved seperatly but the total curvature perturbation evolves as,

$$\dot{\zeta} = \dot{f}(\zeta_\chi - \zeta_{rad}) \tag{4}$$

- $\dot{f} > 0 \Rightarrow$ Growth of curvature perturbations
- The non-adaibatic pressure: $\delta P_{nad} \propto (\zeta_\chi - \zeta_{rad})$

δN formalism

- Motivation: Easier to study the superhorizon evolution of perturbations. All the informations are encoded in background evoltuion.
- If we follow the volume expansion rate starting from a flat hypersurface to uniform density hypersurface for any field ϕ curvature perturbation can be estimated as,

$$\zeta \sim \frac{\partial N(\phi)}{\partial \phi}\delta\phi$$

and the spectrum,

$$\langle\zeta\zeta\rangle \sim \left(\frac{\partial N}{\partial \phi}\right)^2 \langle\delta\phi\delta\phi\rangle$$

δN: Implications in curvaton

During inflation the N does not depend on χ
After inflation if χ startes dominating the energy density,

$$\frac{\partial N}{\partial \chi} \neq 0$$

We get the amplification of initial curvaton density fluctuations and can generate observable CMB fluctuations.

δN : Basic formulas

The powerspectrum:

$$P_\zeta = \left(\frac{\partial N}{\partial \phi}\right)^2 \langle \delta\phi\delta\phi \rangle = \left(\frac{H_{inf}}{2\pi}\right)^2 \left(\frac{\partial N}{\partial \phi}\right)^2$$

The bispectrum:

$$f_{NL} = \frac{\partial^2 N}{\partial \phi^2} / \left(\frac{\partial N}{\partial \phi}\right)^2$$

A smaller powerspectrum leads to large non-Gaussianity

The vanilla curvaton

The Lagrangian:

$$\mathcal{L} = -\frac{1}{2}(\partial_\mu \chi)^2 - m^2\chi^2$$

- Conisderation: $m \ll H_{inf}$
- After $H \sim m$, χ starts to dominate the energy density it's energy density redshifs as, a^{-3} as opposed to the radiation behavior a^{-4}.
- A small decay width of $\chi \rightarrow$ dominate the energy density of the Universe $\rightarrow$ can generate sufficient perturbation

Start with energy equation at the time of χ decay with Γ as decay width:

$$3M_p^2\Gamma^2 = \rho_{rad,0}e^{-4N} + \rho_{\chi,0}e^{-3N}$$

Differentiating w.r.t χ

$$\frac{\partial N}{\partial \chi} = \frac{2r}{4+3r}\frac{1}{\chi_*}$$

with, $r = \dfrac{\rho_{\chi,decay}}{\rho_{rad,decay}}$

- For sufficient curvaton domination, $r \to \infty \Rightarrow \dfrac{\partial N}{\partial \chi} \sim \dfrac{2}{3\chi_*}$
- For $\to 0 \Rightarrow \dfrac{\partial N}{\partial \chi} = \dfrac{r}{2}\dfrac{1}{\chi_*}$
- Non-Gaussianity: $f_{NL} = \dfrac{5}{12}(-3 + \dfrac{4}{r} + \dfrac{8}{4+3r})$;

  For $r \to \infty \Rightarrow f_{NL} = -\dfrac{15}{12} \Rightarrow$ constant and negative

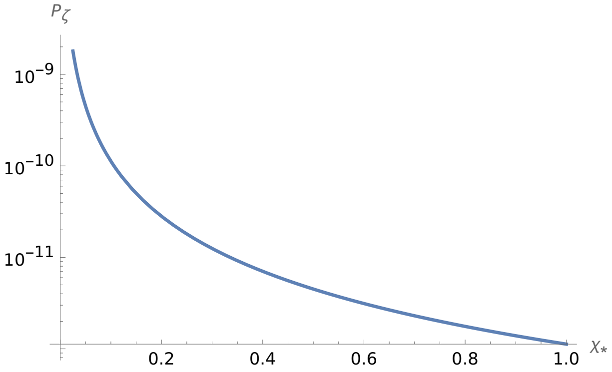


Figure: P_ζ for $r \to \infty$

- The powerspectrum, $P_\zeta = \left(\frac{H}{2\pi}\right)^2 \frac{4}{9\chi_*^2}$
- Spectral Index: $n_s - 1 = \frac{2}{3}\frac{V''(\phi)}{H_{inf}^2} + \frac{2\dot{H}_{\rm inf}}{H_{inf}} = \frac{2}{3}\frac{m^2}{H_{inf}^2} - \epsilon$

For $m^2 \ll H_{inf}^2$, ϵ controls n_s

The system:

$$\mathcal{L} = -\frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \Lambda^4 \left[1 - \cos\left(\frac{\phi}{f}\right)\right]$$

$\Lambda^4 = m^2 f^2$ with f^2 is the decay constant

- Analytical estimation:
  $$P_\zeta = \left(\frac{H_{inf}}{2\pi}\right)^2 \left(\frac{r}{4+3r}\right)^2 \left(\frac{1+\cos\theta_*}{\sin\theta_*}\right)^2$$ with,
  $\theta = \frac{\phi}{f}$, [arXiv:2007.01741, T. Kobayashi]
- At $\theta \to 0$ limit the axion potential behaves like vanilla curvaton and the powerspectrum follows the same form
- Spectral index: $n_s - 1 = \frac{2}{3}\frac{m^2}{H_{inf}^2}\cos\theta_* - \epsilon$; [arXiv:2007.01741, T. Kobayashi]
- Axion curvaton can generate the correct amplitude of the powerspectrum and spectral tilt without ϵ

Numerical analysis

- Any potential beyond quadratic $(m^2\phi^2)$ needs to be analyzed numerically. Reason: Analytical computation assumes energy of the curvaton redshifts as a^{-3} irrespective of the potential's nature
- We need to numerically solve (post-inflation):

$$\frac{dN}{dx} = \left[\alpha e^{-4N} + \frac{1}{3M_P^2}\left\{\frac{1}{2}\left(\frac{d\phi}{dx}\right)^2 + V(\phi)\right\}\right]^{\frac{1}{2}}$$
$$\frac{d^2\phi}{dx^2} = -3\frac{dN}{dx}\frac{d\phi}{dx} - \frac{dV(\phi)}{d\phi} \tag{5}$$

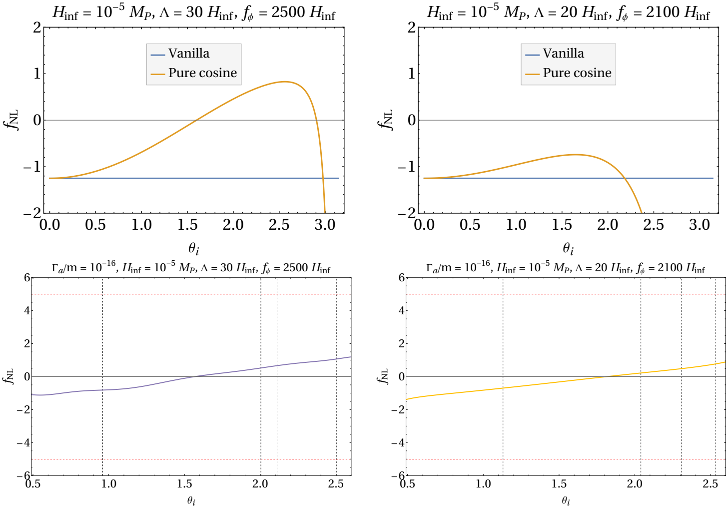
[arXiv: 0902.2619; P. Chingangbam, Q. Huang]

- $x = mt$; The integration has to be done from $x = 1$ $(H = m)$ to $x = \frac{m}{\Gamma}$ $(H = \Gamma)$
- $r \to \infty$ corresponds to $x > 10^{12}$

- Solve the axion equation of motion during inflation even if the axion evolution is neglible
- The coupled equation in post-inflationary evolution is hard to handle; after certain x the code does not behave well
  - We solve the coupled equation upto some $x = x_1$ where the code behaves well $\Rightarrow$ This is not $r \to \infty$ limit
  - After this x_1 we assume the curvaton potential behaves like quadratic one and hence its energy density evolves as a^{-3}
  - We solve

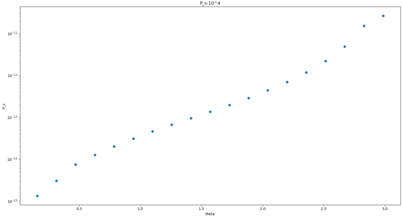
$$\frac{dN}{dx} = \left[\alpha e^{-4N} + \rho_{\theta,x_1} e^{-3N}\right]^{1/2}$$

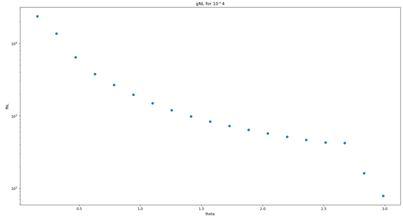
from x_1 to $x_{decay} = \frac{m}{\Gamma}$



[arXiv: 2302.00668; A. Ghoshal, A.N.]

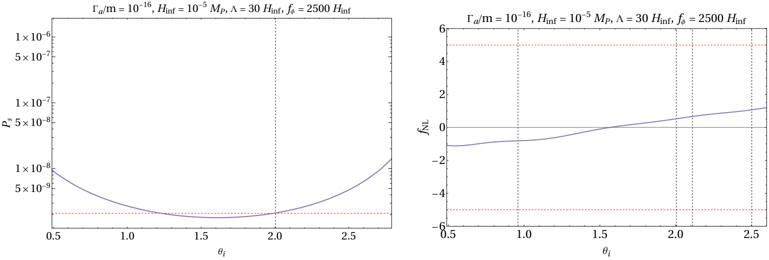
The parameters of the theory: $H_{inf},\ \Lambda,\ f, \Gamma, \phi_*$





$x = 10^4,\ H_{inf} = 10^{-5},\ \Lambda = 30H_{inf},\ f = 2500H_{inf}$

The parameters of the theory: H_{inf}, Λ, f, Γ, ϕ_*



- Varying H_{inf}, Λ, f, Γ would shift the normalisation of powerspectrum along the θ_i axis
- As the powerspectrum is conninciding the CMB normalisation at two different θ_i it is possible to find a suitable parameter space where both amplitude and n_s satisfy observation only from axion contribution

Summary of first part

- Curvaton mechanism can explain observed CMB fluctuations
- The spectral index can be attributed to inflation in most cases but axion curvaton can explain n_s itself
- Analytical estimation is not possible beyond vanilla curvaton (quadratic potential) scenario

Non-Canonical axion curvaton

The system:

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R - \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi - \frac{1}{2}\lambda^2(\phi)\nabla_\mu\chi\nabla^\mu\chi - V(\phi,\chi)\right]$$

The Friedmann Equations and Equation of motion of the fields:

$$3M_{\rm Pl}^2H^2 = \left[\frac{1}{2}\dot{\bar{\phi}}^2 + \frac{\lambda^2(\bar{\phi})}{2}\dot{\bar{\chi}}^2 + V(\bar{\phi},\bar{\chi})\right] , \qquad (6)$$

$$\ddot{\bar{\phi}} + 3H\dot{\bar{\phi}} + V_{,\phi} = \lambda\lambda_{,\phi}\dot{\bar{\chi}}^2 , \qquad (7)$$

$$\ddot{\bar{\chi}} + \left(3H + 2\frac{\lambda_{,\phi}}{\lambda}\dot{\bar{\phi}}\right)\dot{\bar{\chi}} + \frac{V_{,\chi}}{\lambda^2} = 0 , \qquad (8)$$

$\dot{\bar{\chi}} \to 0$ ensures χ does not backreact inflationary dynamics.

Fluctuations

Fluctuation equation of motion

$$\delta\ddot{\chi}_k + \left(3 + 2\frac{\lambda_{,\phi}}{\lambda}\frac{\dot{\phi}}{H}\right) H\delta\dot{\chi}_k + \left(\frac{k^2}{a^2} + \frac{V_{,\chi\chi}}{\lambda^2}\right)\delta\chi_k \simeq 0$$

Assuming $\lambda_{,\phi}$ does not affect $\delta\chi$ the spectrum is,

$$\mathcal{P}_{\delta\chi}(k) \equiv \frac{k^3}{2\pi^2}|\delta\chi_k|^2 \simeq \left(\frac{H_{\rm inf}}{2\pi\lambda(\phi)}\right)^2\Bigg|_{k=aH} . \quad (9)$$

A small $\lambda(\phi)$ is responsible for the enhancement in χ spectrum

The Non-Canonical term we consider has a gaussian-dip

$$\lambda(\phi) = \lambda_c \left\{ 1 - A \exp\left[-\frac{(\phi - \phi_{\rm dip})^2}{2\sigma_\lambda} \right] \right\} , \qquad (10)$$

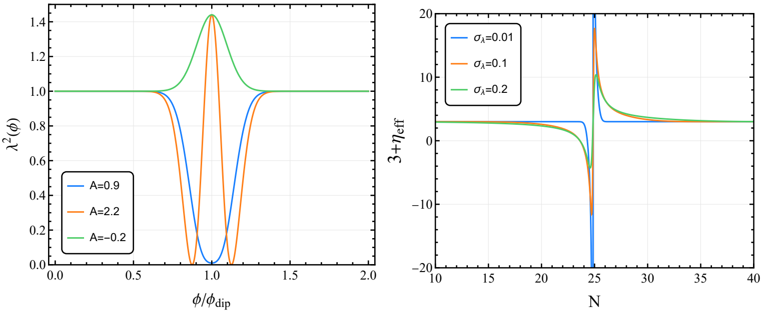
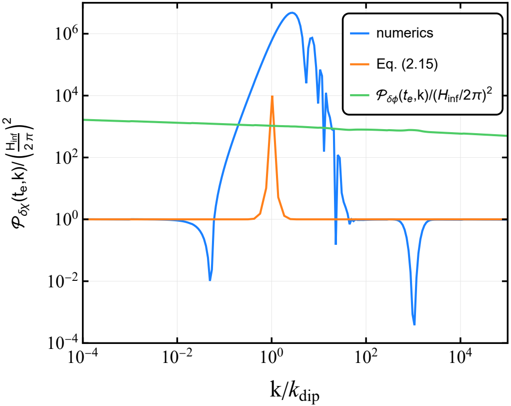


Figure: **Left:**$\sigma = 0.1$; **Right:** $\phi_{\rm ini}/M_{\rm Pl} = 5.5$, $\phi_{\rm dip}/M_{\rm Pl} = 4.8$, $A = 0.995$,

$\eta_{eff} = 2\dfrac{\lambda_{,\phi}}{\lambda}\dfrac{\dot{\phi}}{H^2}$ → A friction term

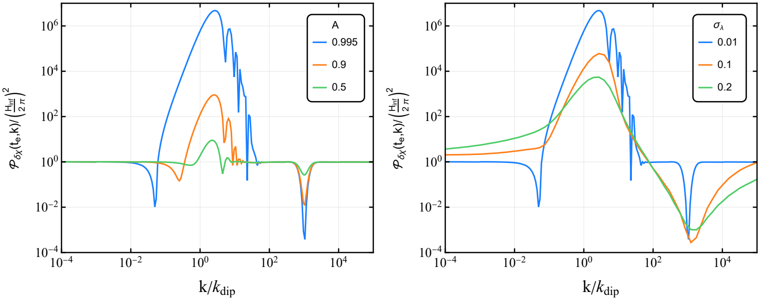
Methodology

- During inflation solve the coupled equation of inflaton and curvaton background and perturbation equations
- Sort out $\langle \delta\chi\delta\chi \rangle$ produced during inflation
- Apply δN formalism to study post inflationary evolution



$\lambda'(\phi)$ non-trivially affect the fluctuations$\rightarrow$ **Larger amplification and two dips before and after the growth;** $A \rightarrow 0.995$, $\sigma_\lambda = 0.001$

Effect of A and σ on the powerspectrum



- Larger $A \to$ Larger amplitude
- Larger $\sigma \to$ amplification for more modes but less amplification as λ' is small

PBH Abundance

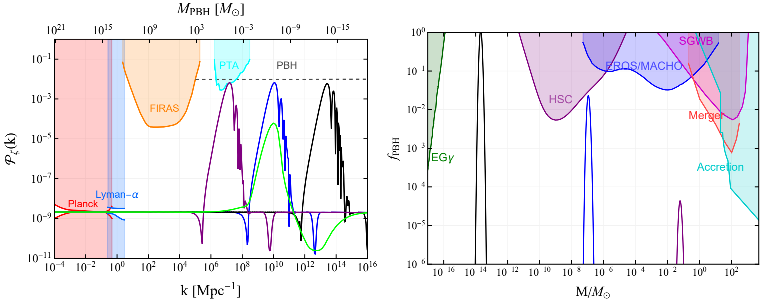


Figure: $\{\phi_{dip}/M_{Pl} = 4.8, \sigma_\lambda = 0.01\}$ (the blue curve);
$\{\phi_{dip}/M_{Pl} = 4.5, \sigma_\lambda = 0.01\}$ (the black curve);
$\{\phi_{dip}/M_{Pl} = 5.0, \sigma_\lambda = 0.01\}$ (the purple curve);
$\{\phi_{dip}/M_{Pl} = 4.8, \sigma_\lambda = 0.1\}$ (the green curve)

Summary

- An axion coupled kinetically with inflaton can lead to large amplification of the curvature perturbation
- We studied a coupling $\lambda(\phi)$ that has a Gaussian dip
- The quantity $\lambda'(\phi)$ is non-trivially responsible for the enhancement of the perturbations as the dip
- By tuning our parameters we can explain production of PBH in different mass range

PBH Mechanism

- Density contrast:

$$\frac{\delta\rho}{\rho} = -\frac{4(1+\omega)}{5+3\omega}\left(\frac{1}{aH}\right)^2 e^{-5\zeta/2}\nabla^2 e^{\zeta/2}\,, \tag{11}$$

- Non-linear density contrast is related to linear density contrast

$$\delta_{\rm nl} = \delta_l - \frac{3}{8}\delta_l^2\,, \tag{12}$$

- PDF: $P(\delta_l) = \frac{1}{\sqrt{2\pi}\sigma_l}\exp\left(-\delta_l^2/(2\sigma_l^2)\right)$;

$$\sigma_l^2 = \frac{1}{(2\pi)^3}\frac{16}{81}\int {\rm d}\ln k W(kR)^2\left(\frac{k}{aH}\right)^4\mathcal{P}_\zeta(k)$$

- Mass fraction:

$$\beta(M) = 2\int_{\delta_c}^{\infty} P(\delta_{\rm nl}){\rm d}\delta_{\rm nl}\,. \tag{13}$$