Effective Theory of Inflationary Magnetogenesis and Constraints on Reheating

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by D. Maity, S. Pal and T. Paul

Outline

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Introduction

- Magnetic fields has been observed in galactic scales ($\sim$ Kpc scale) with the stength of 10 μG.
- Magnetic fields with coherence length above 1 Mpc is observed in the intergalactic medium.
- Strength of the IGMF is $\geq 10^{-16}$ G.[1]
- Among the several mechanisms proposed, inflationary magnetogenesis is the most accepted and viable method.
- Effective Field Theory is often used to study inflation, bounce and several other cosmological phenomena.
- The inflationary fluctuations can couple with the electromagnetic field.

[1] Neronov & Vovk, Science 10

Motivation

- Whether the extra couplings with the inflationary fluctuations help us to get more strength of the magnetic field at the end of inflation.
- The importance of the reheating phase.
- Indirect bounds on cosmological parameters such as the scalar spectral index n_s, the equation of state ω.

The model

- FLRW spacetime in conformal coordinates

$$ds^2 = a^2(\eta)(-d\eta^2 + \delta_{ij}dx^i dx^j)$$

- The total effective action

$$\mathcal{S} = \mathcal{S}_{bg} + \mathcal{S}_{em} + \mathcal{S}_{sp} + \mathcal{S}_{int}$$

With

$$\mathcal{S}_{bg} = \int d^4x\sqrt{-g}\left[\frac{1}{2}M_{\rm Pl}^2 R - \Lambda(\eta) - c(\eta)g^{00}\right]$$

$$\mathcal{S}_{em} = \int d^4x\sqrt{-g}\left[-\frac{1}{4}f_1(\eta)F_{\mu\nu}F^{\mu\nu} + f_2(\eta)F_i^0 F^{0i} + f_3(\eta)\epsilon^{ijk}F_i^0 F_{jk} + f_4(\eta)\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}\right]$$

$$\mathcal{S}_{int} = \int d^4x\sqrt{-g}\left[h(\eta)\left(\partial_i\delta g^{00}\right)F^{0i}\right.$$

The model

$$\begin{aligned}
\mathcal{S}_{sp} &= \int d^4x\sqrt{-g}\Bigg[\frac{M_2^4(\eta)}{2}(\delta g^{00})^2 - \frac{m_3^3(\eta)}{2}\delta K\delta g^{00} \\
&- m_4^2(\eta)\left(\delta K^2 - \delta K_{\mu\nu}\delta K^{\mu\nu}\right) + \frac{\tilde{m}_4^2(\eta)}{2}R^{(3)}\delta g^{00} \\
&- \bar{m}_4^2(\eta)\delta K^2 - \frac{m_5(\eta)}{2}R^{(3)}\delta K - \frac{\lambda_1(\eta)}{2}(R^{(3)})^2 \\
&- \frac{\lambda_2(\eta)}{2}\nabla_i R^{(3)}\nabla^i R^{(3)}\Bigg]
\end{aligned}$$

$$\delta K_{\mu\nu} = K_{\mu\nu} - H\sigma_{\mu\nu} \ ; \ \delta K = K - 3H$$

$$\delta g^{00} = \frac{4\left(\frac{1}{\kappa^2} + 2m_4^2\right)}{a(\eta)\left(\frac{2H}{\kappa^2} + 4Hm_4^2 - m_3^3\right)}\frac{\partial\Psi}{\partial\eta} \ ; \ \frac{1}{\kappa^2}$$

Inflationary Magnetogenesis

- $A_0 = 0$ and $\partial^i A_i = 0$
- Eq. of motion for A_i turns out as

$$
\begin{aligned}
f_1(\eta) &\quad \left[A_i'' + \frac{f_1'}{f_1} A_i' - \partial_l \partial_l A_i\right] + \frac{2}{a^2(\eta)} f_2(\eta) \left[A_i'' + \frac{f_2'}{f_2} A_i' - \frac{2a'}{a} A_i'\right] \\
+ &\quad \frac{2}{a^2} f_3(\eta) \epsilon_{ijk} \left[\frac{f_3'}{f_3} - \frac{2a'}{a}\right] \partial_j A_k - 8 f_4'(\eta) \epsilon_{ijk}\, \partial_j A_k \\
- &\quad h(\eta)\, \partial_0 \partial_i \left(\delta g^{00}\right) - h'(\eta)\, \partial_i \left(\delta g^{00}\right) = 0
\end{aligned}
$$

- Eq. of motion for scalar and tensor perturbation eq

$$
\begin{aligned}
v''(\vec{x}, \eta) - \frac{z''}{z} v - c_s^2\, \partial_i \partial^i v = 0 \\
v_T''(\vec{x}, \eta) - \frac{z_T''}{z_T} v_T - c_T^2\, \partial_i \partial^i v_T = 0
\end{aligned}
$$

- Coupling functions:

$$f_1(\eta) = \begin{cases} \left(\frac{a(\eta)}{a_f(\eta_f)}\right)^2 & \eta \leq \eta_f \\ 1 & \eta \geq \eta_f \end{cases}$$

$$f_2(\eta) = \left(\frac{a_i(\eta_i)}{a(\eta)}\right)^m \quad ; \quad f_3(\eta) = \left(\frac{a_i(\eta_i)}{a(\eta)}\right)^r$$

$$f_4(\eta) = \left(\frac{a_i(\eta_i)}{a(\eta)}\right)^s \quad ; \quad h(\eta) = h_0\left(\frac{a_i(\eta_i)}{a(\eta)}\right)^2$$

- Fourier expansion

$$\hat{A}_i(\vec{x},\eta) = \int \frac{d^3k}{(2\pi)^3} \sum_{p=+,-} \epsilon_i^{(p)} \Big[\hat{b}_p(\vec{k}) A_p(k,\eta) e^{i\vec{k}.\vec{x}} + \hat{b}_p^\dagger(\vec{k}) A_p^*(k,\eta) e^{-i\vec{k}.\vec{x}}\Big]$$

- Power spectrums for EM field and the helicity of magnetic field

$$P^{(E)}(k,\eta) = \left\{\frac{f_1(\eta)}{a^4} + \frac{6f_2(\eta)}{a^6}\right\} \sum_{p=+,-} \frac{k^3}{2\pi^2}\left|A'_p(k,\eta)\right|^2$$

$$P^{(B)}(k,\eta) = \frac{f_1(\eta)}{a^4} \sum_{p=+,-} \frac{k^5}{2\pi^2}\left|A_p(k,\eta)\right|^2$$

$$P^{(h)} = \frac{\partial\rho_h}{\partial \ln k} = \left(\frac{k^4}{2\pi^2 a^3}\right)\left\{\left|A_+(k,\eta)\right|^2 - \left|A_-(k,\eta)\right|^2\right\}$$

- Substituting $x = k\eta$, $\bar{A}_\pm(k,\eta) = \sqrt{k}A_\pm(k,\eta)$ we get the Eq.

$$\mathcal{A}_1 \frac{d^2\bar{A}_\pm}{dx^2} + \mathcal{A}_2 \frac{d\bar{A}_\pm}{dx} \pm \mathcal{A}_3\bar{A}_\pm = \mathcal{S}_\pm$$

- The coefficients are

$$\mathcal{A}_1 = \frac{1}{a_f^2}\left(\frac{k}{H}\right)^{4+m} + 2(-1)^m x^{4+m}$$

$$\mathcal{A}_2 = -\frac{2}{x^2 a_f^2}\left(\frac{k}{H}\right)^{4+m} + (4+2m)(-1)^m x^{3+m} \qquad (1)$$

$$
\begin{aligned}
\mathcal{A}_3 &= \frac{1}{a_f^2}\left(\frac{k}{H}\right)^{4+m} + 2(-1)^r(2+r)\left(\frac{k}{H}\right)^{m-r} x^{3+r} \\
&- 8s(-1)^s\left(\frac{k}{H}\right)^{m-s+2} x^{s+1} \\
\mathcal{S}_\pm &= \pm 2\sqrt{2}\left(\frac{h_0 x^5}{H}\right)\left(\frac{k}{H}\right)^{m-1}\Bigg[(5+2\gamma)\left(\frac{dV}{dx}+\left(\frac{1+\gamma}{x}\right)V\right) \\
&+ x\left(\frac{d^2V}{dx^2}-\frac{(2+3\gamma)}{x^2}V\right)\Bigg]
\end{aligned}
$$

- In the superhorizon scale the EoM turns out as

$$
\frac{d^2\bar{A}_\pm}{dx^2}+\left(\frac{2+m}{x}\right)\frac{d\bar{A}_\pm}{dx} \pm (-1)^{r-m}\left\{\left(\frac{k}{H}\right)^{m-r}\left(\frac{2+r}{x^{1+m-r}}\right)\right\}\bar{A}_\pm
$$

$$
\mp(-1)^{r-m}\left\{4s\,(-1)^{-2-r}\left(\frac{k}{H}\right)^{2+m-s} x^{3+m-s}\right\}\bar{A}_\pm = 0
$$

- For $m = r \geq s$ the equation transforms as

$$\frac{d^2\bar{A}_\pm}{dx^2} + \left(\frac{2+m}{x}\right)\frac{d\bar{A}_\pm}{dx} \pm \left(\frac{2+m}{x}\right)\bar{A}_\pm = 0$$

- The solution of the above equations are

$$\begin{aligned} \bar{A}_+(k,\eta) &= (-k\eta)^{\frac{1-m}{2}}\Big\{C_1 \mathrm{I}_{-m-1}\left[\sqrt{-(8+4m)x}\right] \\ &+ D_1 \mathrm{I}_{m+1}\left[\sqrt{-(8+4m)x}\right]\Big\} \\ \bar{A}_-(k,\eta) &= (-k\eta)^{\frac{1-m}{2}}\Big\{C_2 \mathrm{I}_{-m-1}\left[\sqrt{(8+4m)x}\right] \\ &+ D_2 \mathrm{I}_{m+1}\left[\sqrt{(8+4m)x}\right]\Big\} \end{aligned}$$

- The power spectrums in terms of these we get

$$P^{(E)}(k,\eta) = \frac{m^2}{2\pi^2}\{|C_1|^2+|C_2|^2\}\left[f_1(\eta)+\frac{6f_2(\eta)}{a^2}\right]H^4\big(-k\eta\big)^{2-2m}$$

$$P^{(B)}(k,\eta) = \frac{f_1(\eta)}{2\pi^2}\{|C_1|^2+|C_2|^2\}H^4\big(-k\eta\big)^{4-2m}$$

- The helicity power spectrum

$$P^{(h)}(k,\eta) = \left(\frac{k^3}{2\pi^2 a^3}\right)\big(-k\eta\big)^{-2m}\left\{\big|C_1\big|^2-\big|C_2\big|^2\right\}$$

Possible scenarios and the backreaction problem

- Scale invariant electric field $m = 1$

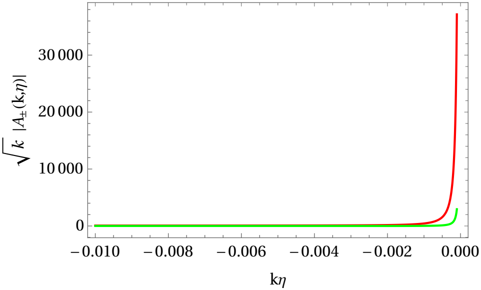
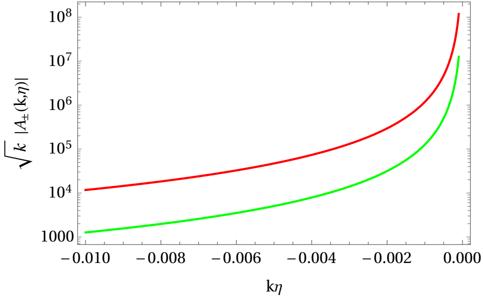


Figure: The red line represents $\sqrt{k}A_+(k,\eta)$ versus $k\eta$ and the green line represents $\sqrt{k}A_-(k,\eta)$ versus $k\eta$. Both the plots are for $k = 0.05\mathrm{Mpc}^{-1} \approx 2 \times 10^{-40}\mathrm{GeV}$, $H = 10^{14}\mathrm{GeV}$ and $m = r = s = 1$ respectively.

- This case does not match with current observation.
- Free from back reaction problem.

- Scale invariant magnetic field.



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- This does not matches with current observation.
- It suffers from back reaction problem.

- The evolution of helicity during inflationary era

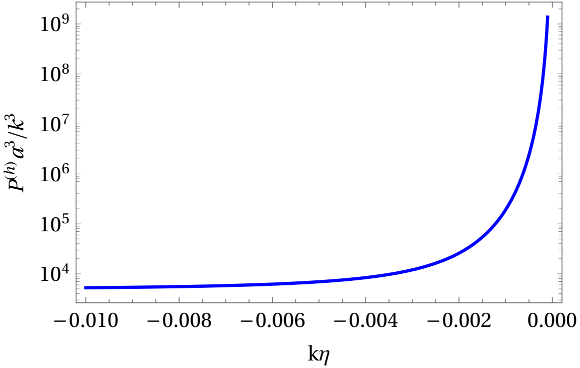


Figure: $P^{(h)}a^3/k^3$ versus $k\eta$ during inflationary epoch.

Post inflationary evolution

- To boost the magnetic field strength we consider a finite reheating era
- The mode function solution during the reheating era

$$A_{\pm}^{(re)}(k,\eta) = \frac{1}{\sqrt{2k}}\left[\alpha_{\pm}(k)\ e^{-ik(\eta-\eta_f)} + \beta_{\pm}(k)\ e^{ik(\eta-\eta_f)}\right]$$

-

$$\begin{aligned}\alpha_{\pm}(k) &= \sqrt{\frac{k}{2}}\,A_{\pm}(k,\eta_f) + \frac{i}{\sqrt{2k}}\,A'_{\pm}(k,\eta_f)\\ \beta_{\pm}(k) &= \sqrt{\frac{k}{2}}\,A_{\pm}(k,\eta_f) - \frac{i}{\sqrt{2k}}\,A'_{\pm}(k,\eta_f)\end{aligned}$$

- The power spectrum of EM field

$$\begin{aligned}\frac{\partial\rho(\vec{B})}{\partial \ln k} &= \frac{1}{2\pi^2}\sum_{r=\pm}\left(\frac{k^4}{a^4}\right)\Big[|\alpha_r|^2 + |\beta_r|^2\\ &+ 2|\alpha_r|\,|\beta_r|\ \cos\left\{\theta_1^{(r)} - \theta_2^{(r)} - 2k(\eta-\eta_f)\right\}\Big]\end{aligned}$$

Reheating Dynamics

Case-1

- For this we follow the reheating dynamics proposed in Kamionkowski et al. Phys. Rev. Lett. 113, 041302 (2014), where inflaton energy is assumed to converted into radiation instantaneously at the end of reheating.
- The dynamics is parametrized by T_{re}, N_{re} and ω_{eff}.
- $$N_{re} = \frac{4}{3\omega_{\rm eff}-1}\left[\frac{1}{4}ln(\rho_f) - \frac{1}{4}ln\left(\frac{\pi^2 g_{re}}{30}\right) - \frac{1}{3}\left(\frac{43}{11 g_{s,re}}\right) - ln\left(\frac{a_0 T_0}{k}\right) - ln(H_*) + N_f\right]$$
  $$T_{re} = \left(\frac{43}{11 g_s, re}\right)^{\frac{1}{3}}\left(\frac{a_0 T_0}{k}\right) H_* e^{-N_f} e^{-N_{re}}$$
- For this case
  $$\eta - \eta_f = \frac{2}{(1+3\omega_{\rm eff})}\left[\frac{1}{aH} - \frac{1}{a_f H_f}\right]$$

- The power spectrum at present day

$$\frac{\partial \rho(\vec{B})}{\partial \ln k} = \frac{1}{\pi^2}\left(\frac{k^4}{a^4}\right) \sum_{r=+,-} |\beta_r|^2 \left\{ \mathrm{Arg}[\alpha_r \ \beta_r^*] - \pi - \left(\frac{4k}{3\omega_{\rm eff}+1}\right)\left(\frac{1}{aH} - \frac{1}{a_f H_f}\right)\right\}^2$$

Case-II

- In this reheating model we consider perturbative reheating model where effective equation of state is time-dependent.
- Governing equations

$$\frac{d\Phi}{d\xi} + \frac{\sqrt{3} M_{Pl}\Gamma_\phi}{H_f^2}(1+\omega_\phi)\frac{\xi^{1/2}\Phi}{\Phi\xi^{-3\omega_\phi} + R\xi^{-1}} = 0$$

$$\frac{dR}{d\xi} - \frac{\sqrt{3} M_{Pl}\Gamma_\phi}{H_f^2}(1+\omega_\phi)\frac{\xi^{\frac{3(1-2\omega_\phi}{2}}\Phi}{\Phi\xi^{-3\omega_\phi} + R\xi^{-1}} = 0$$

- Where

$$\Phi = \left(\frac{\rho_\phi}{H_{\rm f}^4}\right)\xi^{3\left(1+\omega_\phi\right)} \; ; \; R = \left(\frac{\rho_R}{H_{\rm f}^4}\right)\xi^4 \; ; \; \xi = \left(\frac{a}{a_f}\right)$$

- Initial conditions

$$\Phi(\xi = 1) = \rho_f / H_f^4 \qquad \text{and} \qquad R(\xi = 1) = 0$$

- Reheating temperature is identified from radiation temperature at the point of $H(t_{re}) = \Gamma_\phi$, when maximum inflaton energy density transfer into radiation.
- The reheating temperature in terms of radiation temperature:

$$T_{re} = T_{rad}^{end} = \left(\frac{30}{\pi^2 g_{re}}\right)^{1/4} \left[\rho_R(\Gamma_\phi, \xi_{re}, n_s)\right]^{1/4}$$

- From entropy conservation:

$$T_{re} = \left(\frac{43}{11 g_{re}}\right)^{\frac{1}{3}} \left(\frac{a_0 T_0}{k}\right) H_* e^{-N_{\rm f}} e^{-N_{\rm re}}$$

- The present day magnetic field power spectrum

$$\frac{\partial\rho(\vec{B})}{\partial \ln k} = \frac{1}{\pi^2}\left(\frac{k^4}{a^4}\right) \sum_{r=\pm} \left|\beta_r\right|^2 \left\{\mathrm{Arg}\left[\alpha_r\beta_r^*\right] - \pi - 2\left(\frac{k}{a_f H_\mathrm{f}}\right)\int_1^{\xi} \frac{d\xi}{\xi^2}\frac{\sqrt{3}M_{Pl}H_\mathrm{f}}{\sqrt{\rho_r(\xi)+\rho_\phi(\xi)}}\right\}^2$$

Numerical results

Case-1: Kamionkowski Model

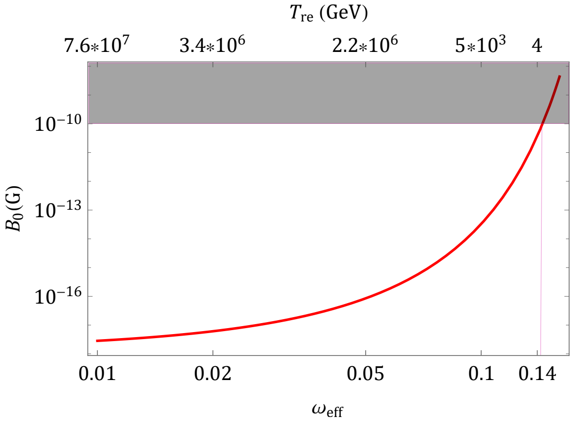
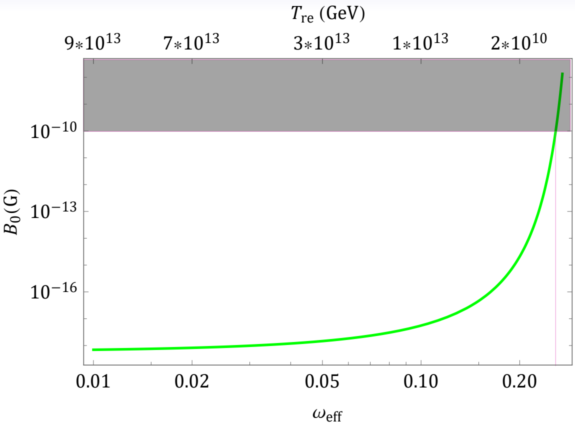


Figure: B_0 (in Gauss units) vs $\omega_{\rm eff}$ with $k = 0.05\mathrm{Mpc}^{-1}$ in the Kamionkowski like reheating scenario. The reheating temperature for different values of $\omega_{\rm eff}$ is shown in the upper label of the x-axis. $H_* = 10^{-5} M_{Pl}$, $n_s = 0.9649$, $\ln\left[10^{10}\mathcal{A}_s\right] = 3.044$, $N_f = 50$



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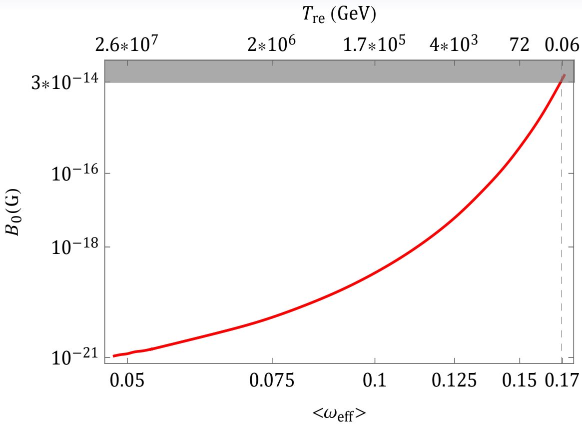
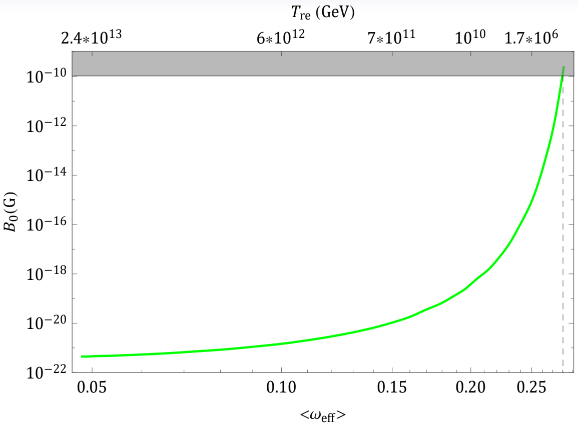


Figure: B_0 (in Gauss units) vs $\langle\omega_{\rm eff}\rangle$ with $k = 0.05\mathrm{Mpc}^{-1}$ in the perturbative reheating scenario. The reheating temperature for different values of $\langle\omega_{\rm eff}\rangle$ is shown in the upper label of the x-axis. $H_* = 10^{-5} M_{Pl}$, $n_s = 0.9649$, $\ln\left[10^{10}\mathcal{A}_s\right] = 3.044$, $N_f = 50$



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Conclusion

- Our detail analysis shows that this is precisely the mechanism which can provide right magnitude of the present magnetic field for the scale invariant electric field scenario.
- It allows one to obtain valuable information about the reheating EoS parameter ($\omega_{\rm eff}$)
- Combining CMB, presently observed constraint on B_0 and the BBN constraint, our analysis restrict the value of of $\omega_{\rm eff}$ as follows:
- $0.01 \lesssim \omega_{\rm eff} \lesssim 0.14$ for $N_f = 50$ and $0.01 \lesssim \omega_{\rm eff} \lesssim 0.25$ for $N_f = 55$ for the reheating scenario where EoS is constant.
- $0.05 \lesssim \langle\omega_{\rm eff}\rangle \lesssim 0.17$ for $N_f = 50$ and $0.04 \lesssim \langle\omega_{\rm eff}\rangle \lesssim 0.27$ for $N_f = 55$ for the perturbative reheating scenario
- This provides a viable constraint on the reheating EoS parameter from CMB observations.

THANK YOU