GRAVITATIONAL WAVES IN NEUTRINO PLASMA AND NANOGRAV SIGNAL

DEPARTMENT OF PHYSICS AND ASTROPHYSICS,
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Arun Kumar Pandey

REFERENCES

This talk is based on two works

- **Arun Kumar Pandey**, P. K. Natwariya, J. R. Bhatt,
  Magnetic fields in a hot dense neutrino plasma and the Gravitational Waves,
  Phys. Rev. D 101, 023531 (2020) 10.1103/PhysRevD.101.023531

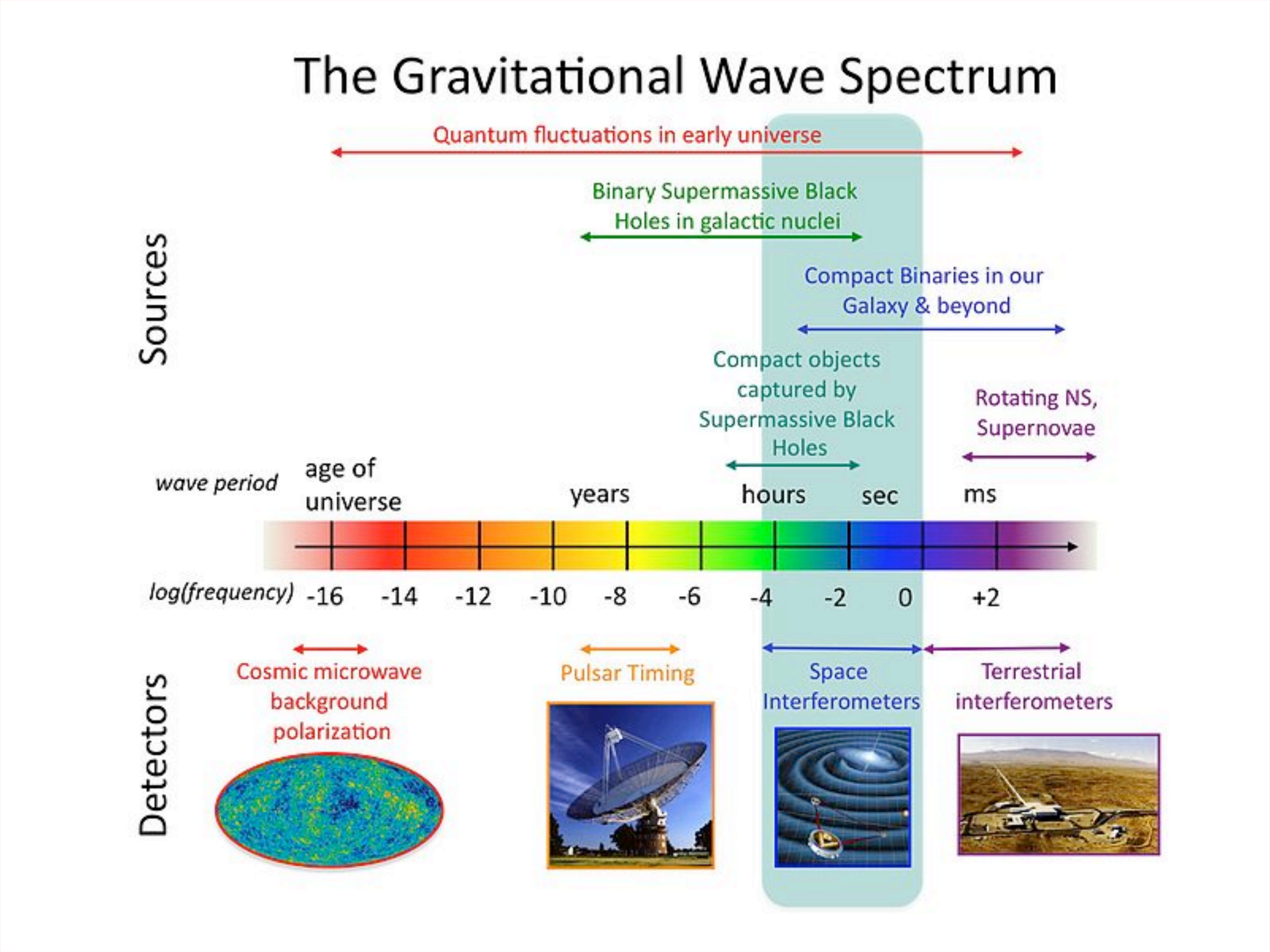
- **Arun Kumar Pandey**
  Gravitational waves in neutrino plasma and NANOGrav signal
  EPJC (2021) 10.1140/epjc/s10052-021-09190-w
  arXiv: 2011.05821 (astro-ph.co)

CONTENT

- Origin of cosmic magnetic fields
- Gravitational waves due to magnetic fields
- Discussions
- New ideas

Gravitational waves

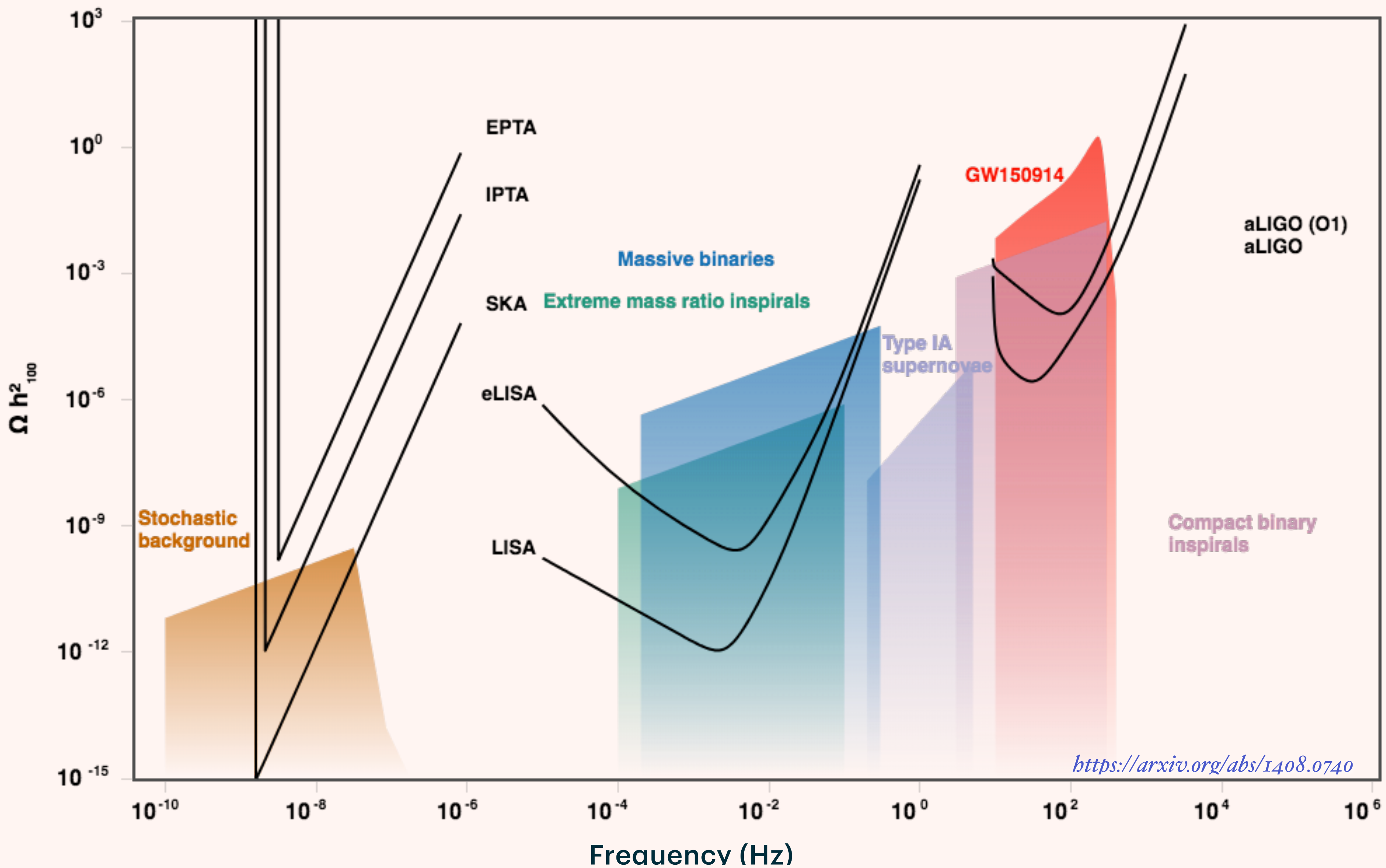
GRAVITATIONAL WAVES



- Predicted by Einstein's General Theory of Relativity
- A ripple in the fabric of space time.

Credit: *NASA Goddard Space Flight Center*

GRAVITATIONAL WAVES



Ground based detectors:
LIGO/VIRGO collaborations (~10-10^3 Hz)
Pulsar Timing Arrays (PTA)

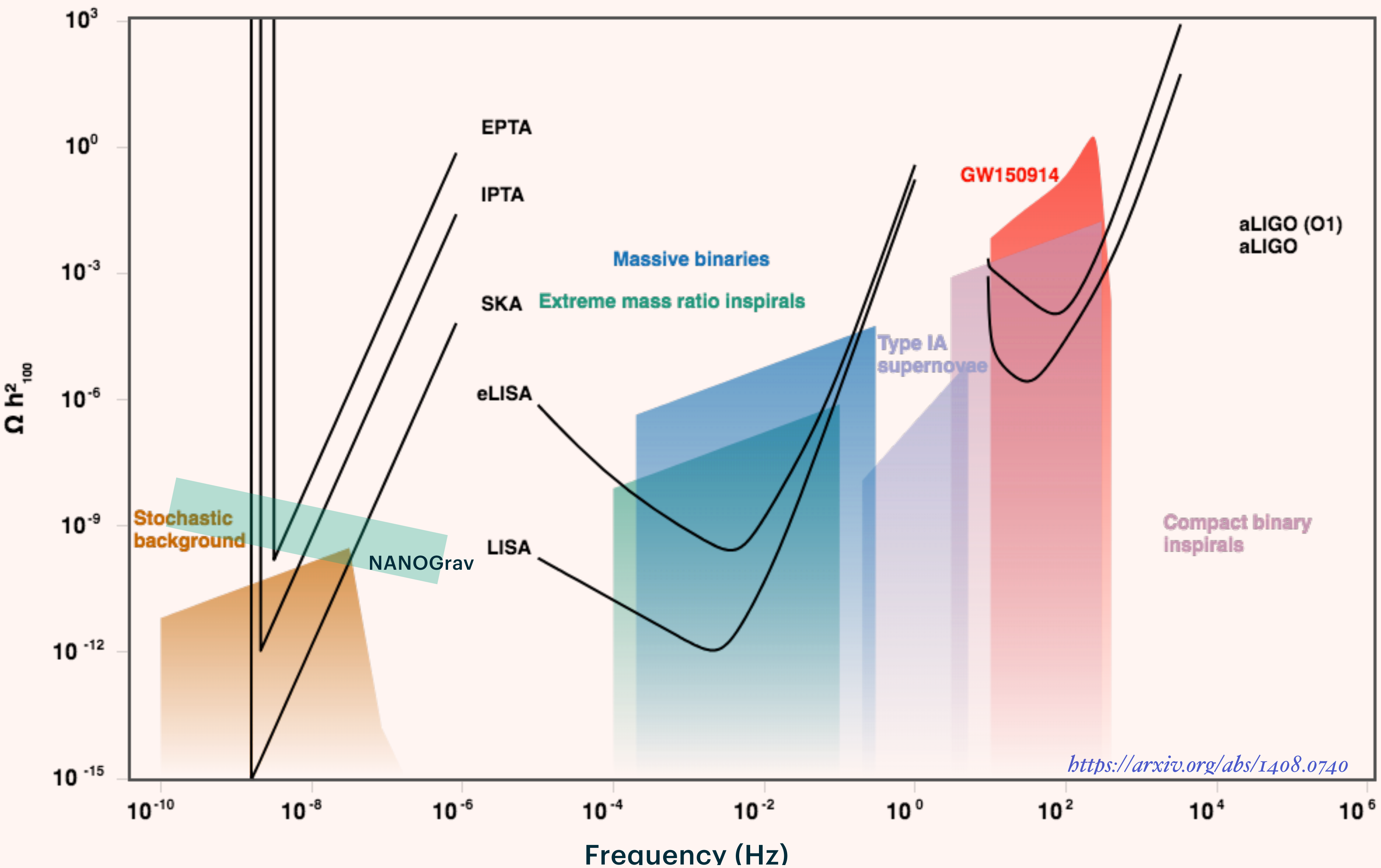
Space based detectors:

LISA

PTAs → 1). European Pulsar Timing Array (EPTA)
2). North American Nanohertz Observatory for Gravitational Waves (NANOGrav)
3). Parkes Pulsar Timing Array (PPTA)

The individual groups are also the constituents of an international collaboration, known as the International Pulsar Timing Array (IPTA)

GRAVITATIONAL WAVES



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NANOGrav: North American Nanohertz Observatory for Gravitational Waves

GRAVITATIONAL WAVES

- Einstein tensor

$$G_{\mu\nu} := R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

- Energy momentum tensor

- Here $G_{\mu\nu}$ and $R_{\mu\nu}$ are function of metric $g_{\mu\nu}$

- Metric perturbations due to small fluctuations in the matter part can be given as

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} + h_{\mu\nu} + \mathcal{O}(h^2)$$

Perturbed metric

Background

$\eta_{\mu\nu}$ = Flat space metric

Small perturbations

This small perturbation describes the propagation of ripples in spacetime curvature

GRAVITATIONAL WAVES

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GRAVITATIONAL WAVES

- Using Einstein's field equation and the perturbed metric, we can write linear field equation for $h_{\mu\nu}$ in compact form (in terms of redefined variable $\bar{h}_{\mu\nu}$) as

$(|h_{\mu\nu}| \ll 1)$

$$\Box \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu}$$

we need to find the source

- Here $\bar{h}_{\mu\nu} = h_{\mu\nu} + \frac{1}{2} g_{\mu\nu} \bar{h}$ and using gauge condition $\partial_\mu \bar{h}^{\mu\nu} = 0$

- In vacuum,

$$\left(-\frac{\partial^2}{\partial t^2} + \nabla^2\right) h_{\mu\nu} = 0$$

- Linear wave solution,

$$h^{\mu\nu} = A^{\mu\nu} \exp(ik_\alpha x^\alpha) .$$

Amplitude

Propagation vector
Gives the propagation direction

Components=10, (as it is symmetric tensor of rank 2)
Using TT-gauge conditions: $\partial_\mu \bar{h}^{\mu\nu} = 0$,
$h^i_i = 0 = h^{0\mu}$, the number of independent components reduce to 2, which is represented by h_+ and $h_\times$.

GRAVITATIONAL WAVES

- Solution, when we have source, can be written in terms of Green's function as above equation is linear in

$$\bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4}\int d^4x'\, G(x-x')\, T_{\mu\nu}(x')$$

- Here $G(x-x')$ is the Green's function and is a solution of $\Box_x G(x-x') = \delta^4(x-x')$.

So we need to find $T_{\mu\nu}$

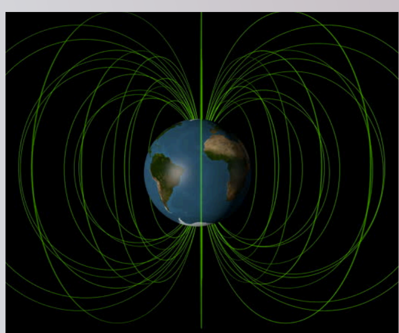
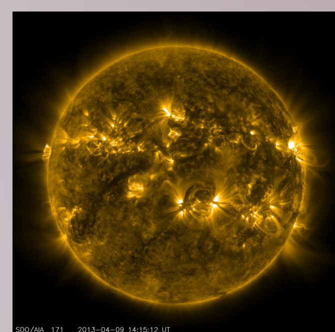
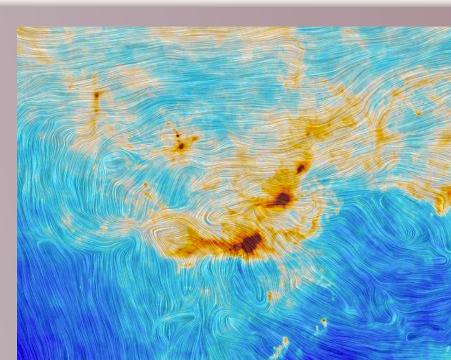
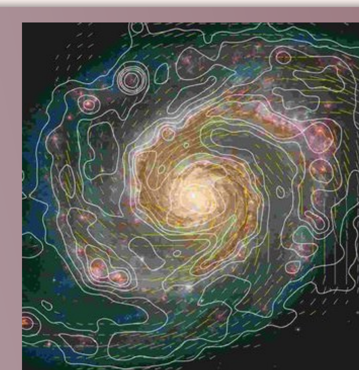
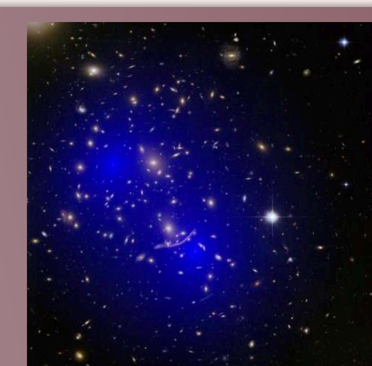
- When sourced by a magnetic fields,

$$T_{ij}(\mathbf{k}) = \frac{1}{2(2\pi)^4 a^2}\int d^3q \left[B_i(\mathbf{q})B_j^*(\mathbf{q}-\mathbf{k}) - \frac{1}{2}B_l(\mathbf{q})B_l^*(\mathbf{q}-\mathbf{k})\delta_{ij}\right]$$

Question is from where does these magnetic fields come?

Magnetic fields in early universe

The magnetised universe

$B \approx 0.5\,G$	$B \approx 1 - 10^{3}\,G$	$B \approx 10^{-5}\,G$	$B \approx 10^{-5}\,G$	$B \approx 10^{-8} - 10^{-7}\,G$
				
Planets: [here: Earth **credit**: Science Picture company]	**Stars:** [here:Sun **credit:** NASA/SDO]	**Interstellar medium:** [here: Molecular cloud **credit**: SESA and Planck collaboration]	**Galaxies:** [here: M51 **credit**: Beck 2011]	**Galaxies Clusters:** **[here:** **credit**: NRAO and NASA]

Inter Galactic Medium

- Magnetic fields are everywhere

Generally: {
Small scale → Large scale
Strong fields ← Weak fields

OBSERVATIONAL BOUNDS

- Observational upper bound

  CMB: B < 5 nG at 1 Mpc scale (PLANCK results 2015: XIX)

- Observational lower bound
  High energy gamma rays: $B > 10^{-16}\,G$
  (Neronov&Vovk 2010, Taylor et al 2011, Takahashi et al 2011….)

⟹ Debatable

Big question of origin?

Still unsettled

ORIGIN OF MAGNETIC FIELD

- Astrophysical Origin
- Primordial Origin

ORIGIN OF MAGNETIC FIELD

- **Primordial mechanism:**

  * Inflation: breaking of conformal invariance of electromagnetic interaction during inflation $B \sim 10^{-65} - 10^{-9}\, G$ (Turner et al. PRD 37, 2743, 1983, Ratra 1992)

  * Phase transition: First order phase transitions in the early universe producing bubbles of new phase inside the old one $B \sim 10^{-29} - 10^{-20}\, G$ (Hogan 1983, Sigel et al. 1997)

  * Recombination: rotating plasma blobs interacting with background radiation $B \sim 10^{-20}\, G$ [Ramesh Narayan, Naoz, 2001]

  * Asymmetries in the early universe

- **Post Recombination mechanism:**
  * Biermann Battery mechanism $B \sim 10^{-21}\, G$ at length scale of few pc

$$\frac{\partial \vec{B}}{\partial t} = \alpha \frac{\vec{\nabla} n_e \times \vec{\nabla} p_e}{n_e^2}$$

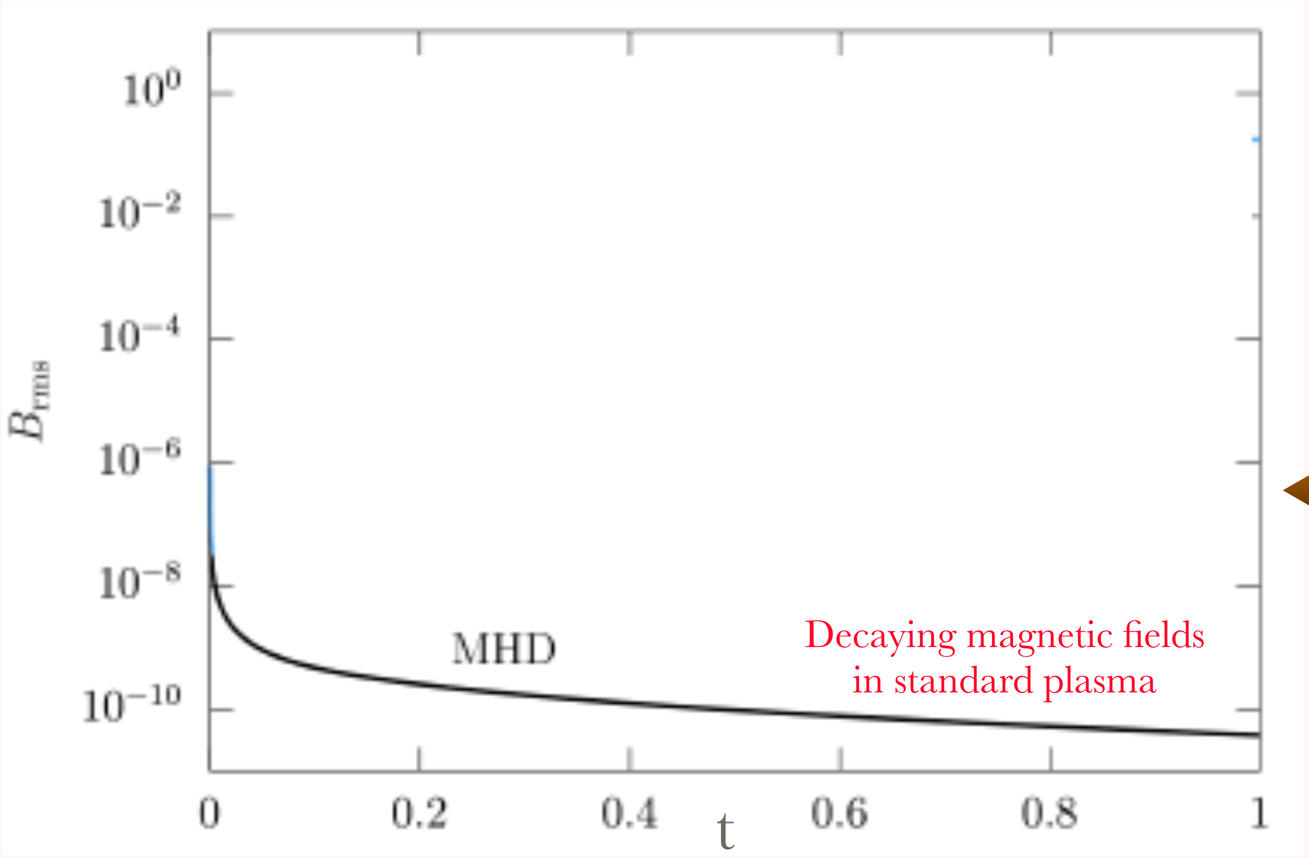
DYNAMICS OF MAGNETIC FIELDS

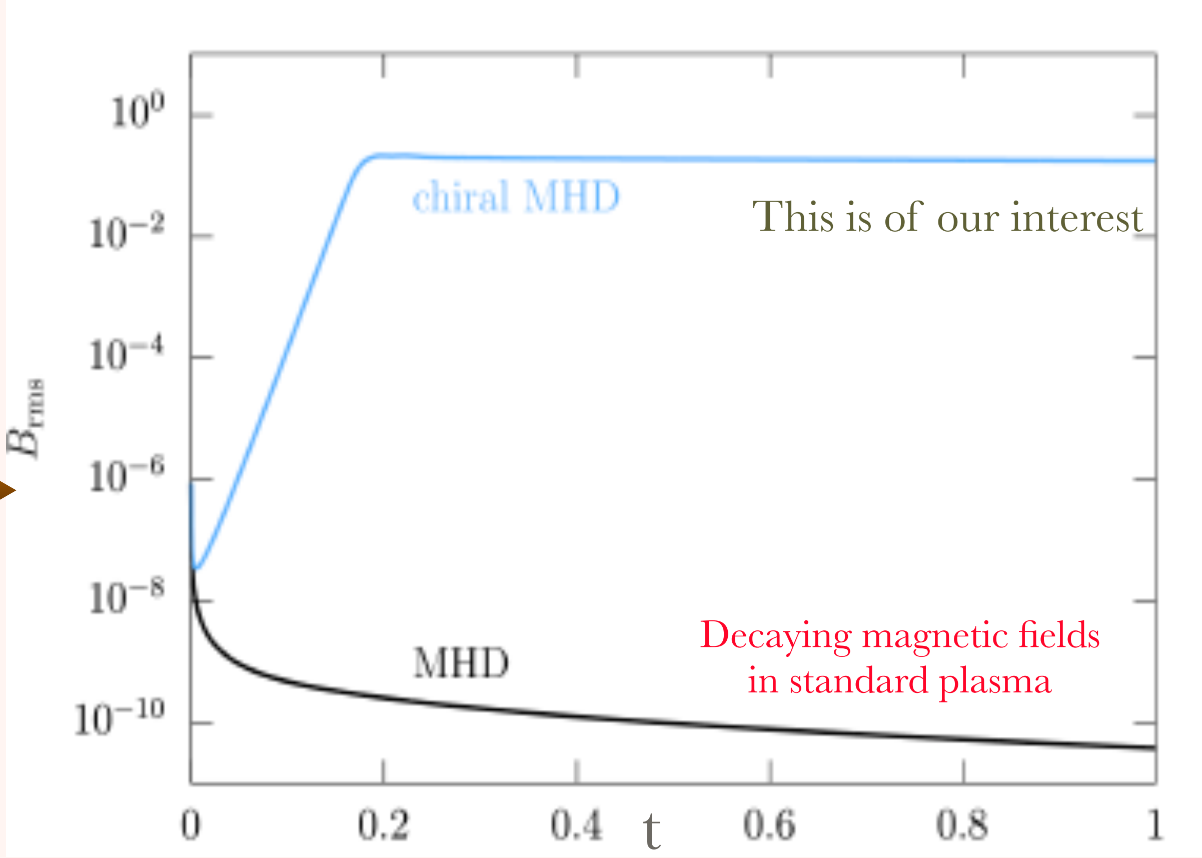
- The evolution of magnetic fields is controlled by induction equation:

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times [\mathbf{U} \times \mathbf{B} - \eta(\nabla \times \mathbf{B})]$$

advection

Dissipation





Magnetic fields in a hot dense neutrino plasma

NEUTRINOS IN HOT DENSE PLASMA MEDIUM

- Neutrinos behave differently in a dense matter than in vacuum.

- The lepton-neutrino interaction is given by the effective current-current Lagrangian [Dvornikova & Semikoz 2014]

$$\mathscr{L}_{\text{eff}} = -\sqrt{2} G_F \sum_{\rho=\nu_e,\nu_\tau,\nu_\mu} j_\nu^\rho \, j_{l_\rho}$$

(G_F is the Fermi const)

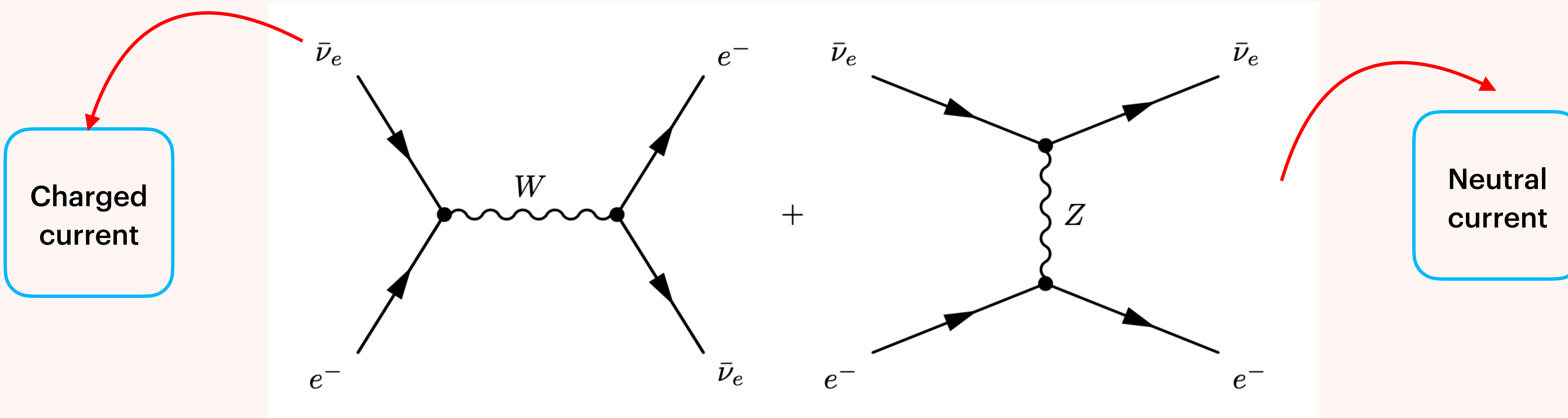
- where $j_{l_\rho} = \bar{\psi}\gamma_\rho (a_L^{(\alpha)} P_L + a_R^{(\alpha)} P_R)\bar{\psi}$ and $j_{\nu_\alpha}^\rho = \bar{\nu}_\alpha \gamma^\rho (1-\gamma^5)\nu_\alpha$ (here γ^μ is the Dirac matrices and $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$).

- Summation run over all neutrino species. The coefficients $a_{L,R}$ is related to Weinberg angle.

- The average over the neutrino ensemble of the neutrino current is $\langle \bar{\nu}_\alpha \gamma^\rho (1-\gamma^5)\nu_\alpha \rangle \approx n_{\nu_\alpha} - n_{\bar{\nu}_\alpha} = \Delta n_{\nu_\alpha}$

- Net charge for leptons are given $j_{l_L}^0 - j_{l_R}^0 = \sqrt{2}\, G_F \, (\Delta n_{\nu_e} - \Delta n_{\nu_\mu} - \Delta n_{\nu_\tau})$

EFFECTIVE LAGRANGIAN



The two tree-level Feynman diagrams for the elastic scattering process $\bar{\nu}_e + e^- \to \bar{\nu}_e + e^-$

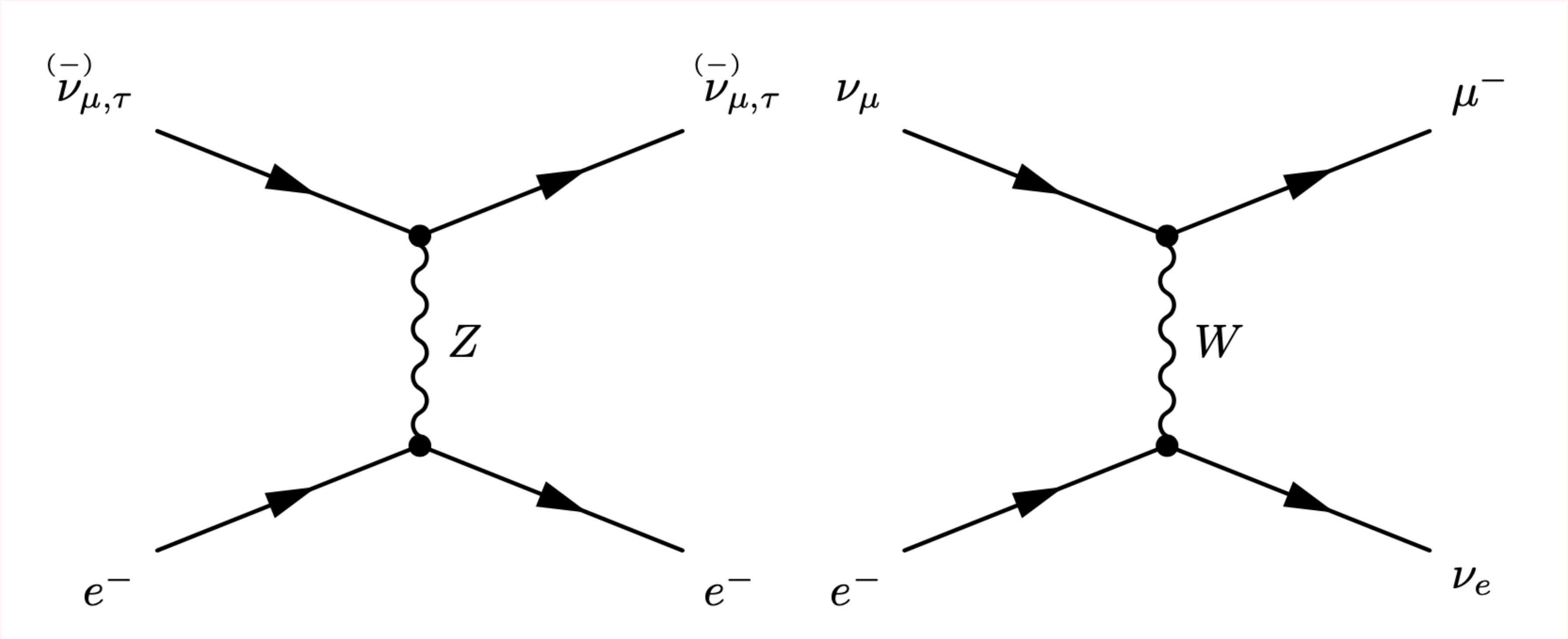
- The effective low energy Lagrangian for the elastic scattering processes is

$$\mathscr{L}_{\text{eff}}(\bar{\nu}_e + e^- \to \bar{\nu}_e + e) = -\frac{G_F}{\sqrt{2}}\left\{\underbrace{[\bar{\nu}_e\gamma^\rho(1-\gamma^5)e][\bar{e}\gamma_\rho(1-\gamma^5)\nu_e]}_{\text{charge current contribution}} + \underbrace{[\bar{\nu}_e\gamma^\rho(1-\gamma^5)\nu_e][\bar{e}\gamma_\rho(g_V^l - g_A^l\gamma^5)e]}_{\text{neutral current contribution}}\right\}$$

- Using Fierz transformation, this can be rearranged as

$$\mathscr{L}_{\text{eff}}(\bar{\nu}_e + e^- \to \bar{\nu}_e + e^-) = -\frac{G_F}{\sqrt{2}}\left\{\underbrace{[\bar{\nu}_e\gamma^\rho(1-\gamma^5)\nu_e]}_{\text{neutral current contribution}} \quad \underbrace{[\bar{e}\gamma_\rho((1+g_V^l) - (1+g_A^l)\gamma^5)e]}_{\text{neutral current contribution}}\right\}$$

EFFECTIVE LAGRANGIAN



The two tree-level Feynman diagrams for the elastic scattering process (a). $\bar{\nu}_{\mu,\tau} + e^- \to \bar{\nu}_{\mu,\tau} + e^-$,

(b). Tree-level Feynman diagram for the charged-current $\bar{\nu}_\mu + e^- \to \bar{\nu}_e + \mu^-$

- Lagrangian for the process, given above, in a compact form as $(\alpha = \mu, \tau)$

$$\mathscr{L}_{\text{eff}}(\bar{\nu}_\alpha + e^- \to \bar{\nu}_\alpha + e^-) = -\frac{G_F}{\sqrt{2}}\left\{\underbrace{[\bar{\nu}_\alpha\gamma^\rho(1-\gamma^5)\nu_\alpha][\bar{e}\gamma_\rho(g_V^l - g_A^l\gamma^5)e]}_{\text{neutral current contribution}}\right\}$$

- In the case of $\nu\bar{\nu}$ gas embedded in leptons, the current which is written in terms of polarisation tenor is written as $j_\mu = \Pi_{\mu\nu}A^\nu$ (here A^μ is any gauge fields).

- Here $\Pi_{\mu\nu}(k) = (-g_{\mu\nu} + \frac{k_\mu k_\nu}{k^2})\Pi_T + \frac{k_\mu k_\nu}{k^2}\Pi_L + i\varepsilon_{\mu\nu\alpha\beta}k^\alpha(f_L^\beta - f_R^\beta)\Pi_P$ (where $f_{L,R}^\beta$ represents the external neutrino macroscopic currents in νl interacting gas under the mean field approximation via the external neutrino macroscopic currents)

MAGNETIC FIELD GENERATION

- In the case of the present scenario, it has been shown that, the lepton current has following set of terms [pandey et al. 2020].

$$\vec{j} = \sigma(\vec{E} + \vec{v} \times \vec{B}) + \Sigma_2 \vec{B} + \Sigma_\omega \vec{\omega}$$

- Here the transport coefficients Σ_2 and Σ_ω depends on the asymmetries in the number densities of the various neutrino species and temperature.

- Using Maxwell's equation and current expression given above

$$\frac{\partial \vec{B}}{\partial \tau} = \frac{1}{\sigma} \nabla^2 \vec{B} + \nabla \times (\vec{v} \times \vec{B}) + \frac{\Sigma_2}{\sigma} \nabla \times \vec{B} + \frac{\Sigma_\omega}{\sigma} \nabla \times \vec{\omega}$$

- In the absence of any external magnetic fields, first three terms on the right hand side will vanish

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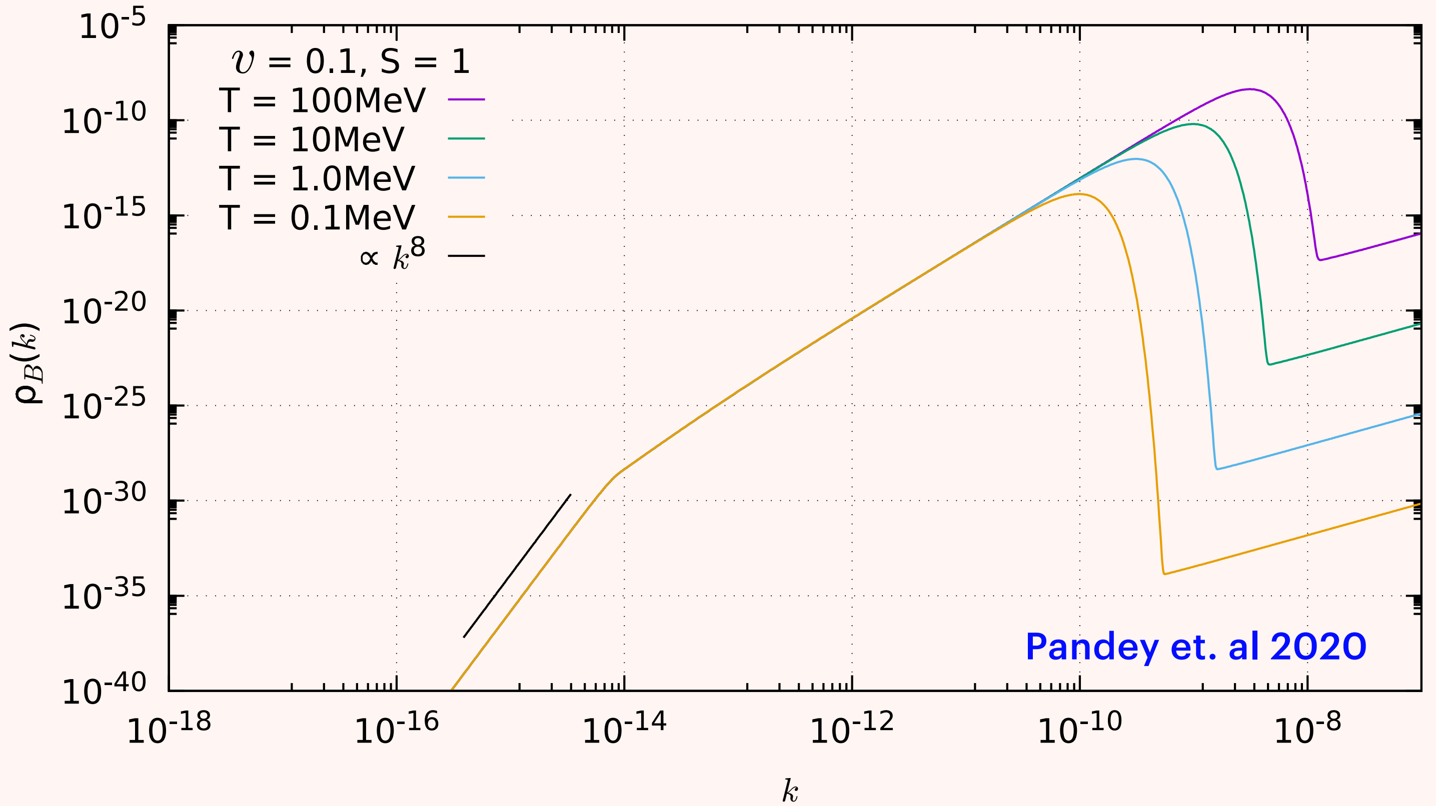
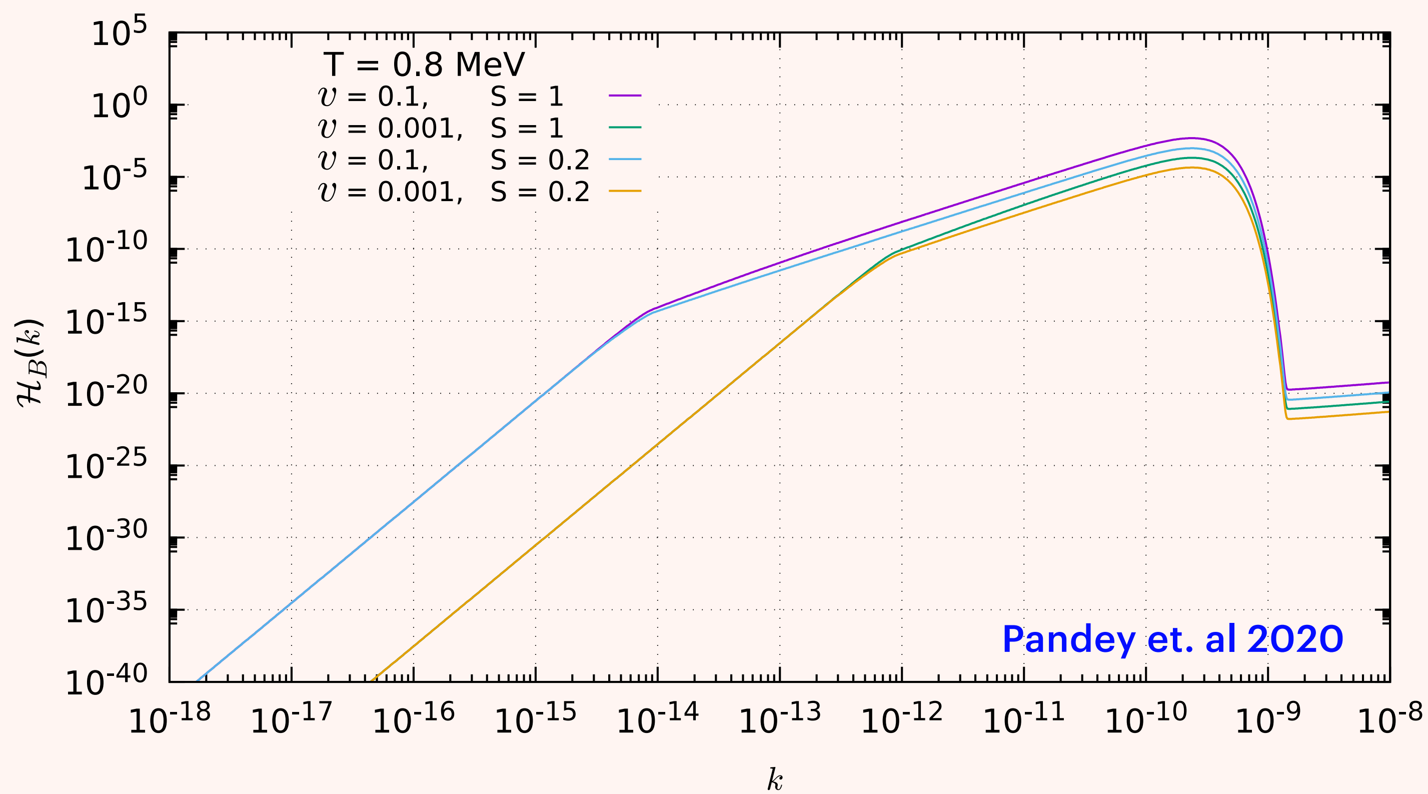




Origin of the Gravitational waves

GRAVITATIONAL WAVES

- The transverse-traceless part of the stress-energy tensor can be obtained by taking projection of energy momentum tensor by $\mathscr{P}_{iljm}(\mathbf{k})$: $\Pi_{ij}(\mathbf{k}) = \mathscr{P}_{iljm}(\mathbf{k})\, T_{lm}(k)$. Here

$$T_{ij}(\mathbf{k}) = \frac{a^{-2}}{2(2\pi)^4}\int d^3q \left[B_i(\mathbf{q})\mathbf{B}_\mathbf{j}^*(\mathbf{q}-\mathbf{k}) - \frac{\mathbf{1}}{\mathbf{2}}\mathbf{B}_\mathbf{l}(\mathbf{q})\mathbf{B}_\mathbf{l}^*(\mathbf{q}-\mathbf{k})\delta_{\mathbf{ij}}\right]$$

- Stress tensor power spectrum:

$$\langle \Pi_{\rm ab}(\mathbf{k})\Pi^*_{\rm cd}(\mathbf{k}')\rangle = \frac{1}{4a^4}[\mathscr{M}_{abcd}f(k) + i\mathscr{A}_{abcd}g(k)]\delta(\mathbf{k}-\mathbf{k}'),$$

here

$$f(k) = \frac{1}{4}\frac{1}{(4\pi)^2}\int d^3p\,[(1+\gamma^2)(1+\beta^2)\,S(p)S(k-p) + 4\,\gamma\,\beta\,\mathscr{H}(p)\mathscr{H}(k-p)\,]$$

$$g(k) = \frac{1}{2}\frac{1}{(4\pi)^2}\int d^3p\,\left[(1+\gamma^2)\,\beta\,S(p)\,\mathscr{H}(k-p)\right].$$

GRAVITATIONAL WAVES

- We consider flat FLRW metric and the tensor perturbation

$$ds^2 = a^2(\eta)[-d\eta^2 + (\delta_{ij} + 2h_{ij})dx^i dx^j],$$

- In this gauge, these tensor perturbations describe the GW whose evolution equation can be obtained by solving Einstein's equation which, to the linear order in h_{ij}, is given as:

$$h''_{ij} + 2\,H\,h'_{ij} + k^2\,h_{ij} = 16\,\pi\,G\,a^2\,\Pi_{ij},$$

where prime denotes the derivative with respect to the conformal time η and $H = \frac{1}{a(\eta)}\frac{\partial a(\eta)}{\partial \eta}$.

- The energy density of the gravitational wave is given as

$$\frac{d\,S_{GW}(k)}{d\,\log k} = \frac{k^3}{2\,M_*^4\,a^2\,(2\pi)^6 G}(|h'_+|^2 + |h'_\times|^2)\,.$$

GRAVITATIONAL WAVES

- Here h'_+ and $h'_\times$ are
$$h'_{+,\times}(x) = -\frac{90}{\pi^2 g_{\rm eff}}\sqrt{\frac{f \mp g}{3}}\, j'_0(x)\log(x_{in})$$
- Using these, we can write the power spectrum at the time of generation:

$$\frac{d\Omega_{GW,s}}{d\log k} = \frac{1}{(\rho_{c,s}/M_*^4)}\frac{dS_{GW,s}}{d\log k} = \frac{16\pi k^3}{3(2\pi)^6 a_s^2}\left(\frac{90}{\pi^2 g_{\rm eff}}\right)^2 \frac{f(k)}{H_s^2}\left[j'_0(x)\log(x_{in})\right]^2.$$

- Once gravitational waves are produced, they are decoupled from rest of the Universe. This implies that the energy density of the gravitational waves will fall as a^{-4} and frequency red shifts as a^{-1} . Hence, the power spectrum at today's epoch can be given as

$$\frac{d\Omega_{S_{GW,0}}}{d\log k} \equiv \frac{d\Omega_{S_{GW,s}}}{d\log k}\left(\frac{a_s}{a_0}\right)^4\left(\frac{\rho_{c,s}}{\rho_{c,0}}\right)$$

GRAVITATIONAL WAVES

- Assuming that the Universe has expanded adiabatically which implies that the entropy per comoving volume is conserved ($a^3 s =$ constant). Hence

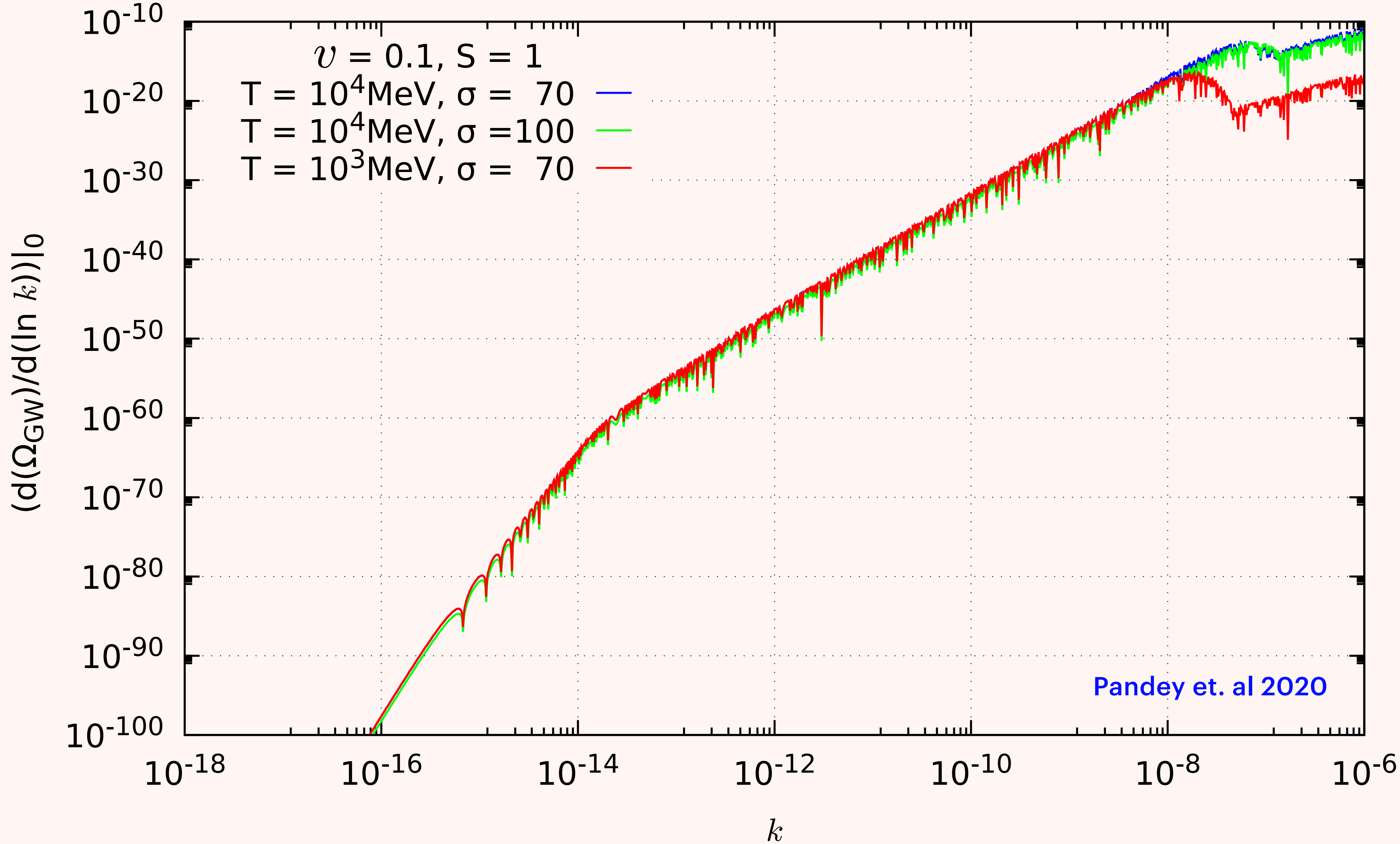
$$\frac{a_s}{a_0} = \left(\frac{g_{\text{eff},0}}{g_{\text{eff,s}}}\right)^{1/3} \left(\frac{T_0}{T_s}\right)$$

where we have used g_{eff} for the effective degrees of freedom that contributes to the entropy density also.

- Therefore, spectrum of the GW for the present time is

$$\frac{d\Omega_{S_{GW,0}}}{d\log k} = \frac{16\pi k^3}{3(2\pi)^6 a_s^2} \times \left(\frac{90}{\pi^2 g_{\text{eff}}}\right)^2 \left(\frac{g_{\text{eff},0}}{g_{\text{eff,s}}}\right)^{4/3} \left(\frac{T_0}{T_s}\right)^4 \frac{f(k)}{H_0^2} \left[j_0'(x)\log(x_{in})\right]^2$$

GRAVITATIONAL WAVES



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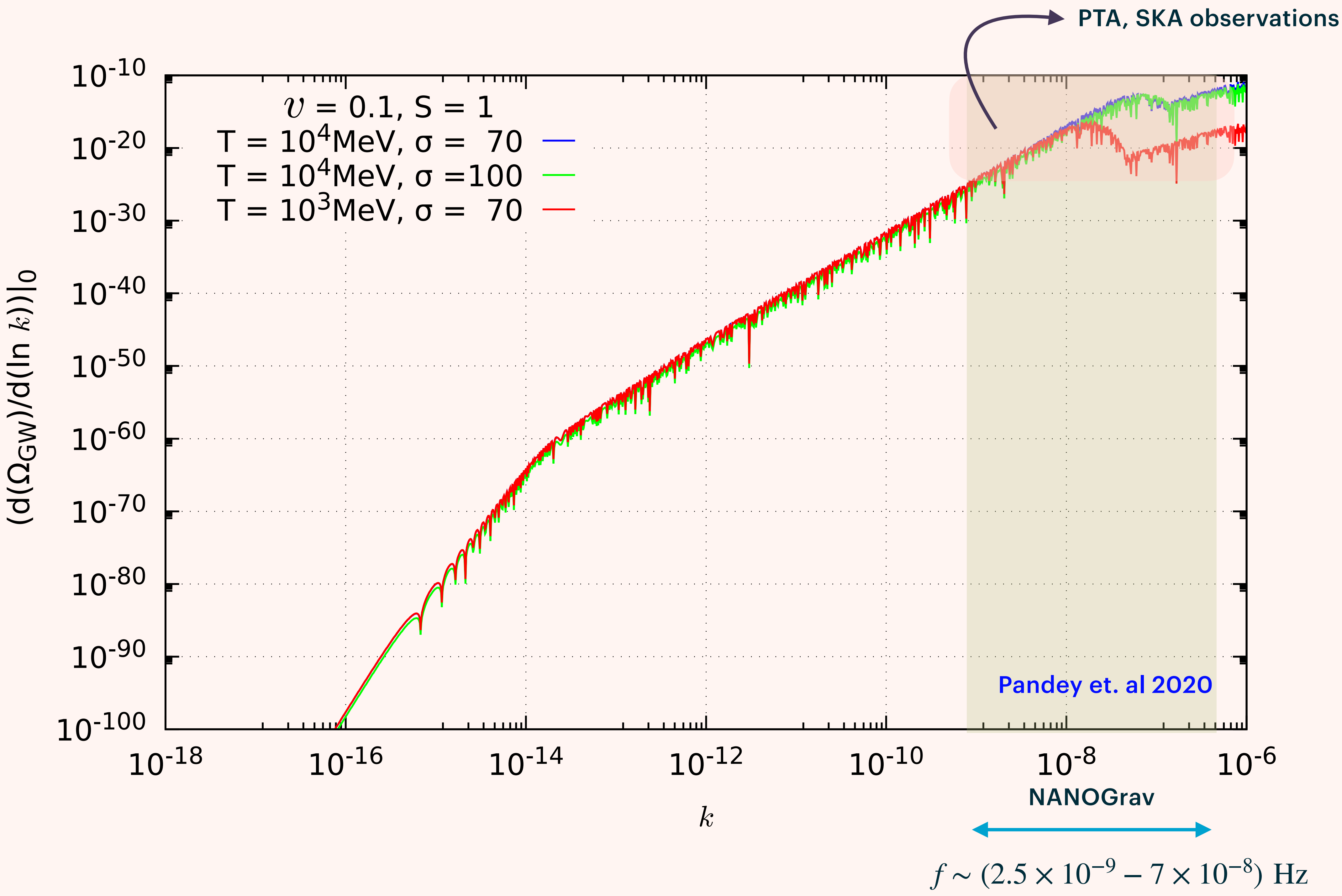
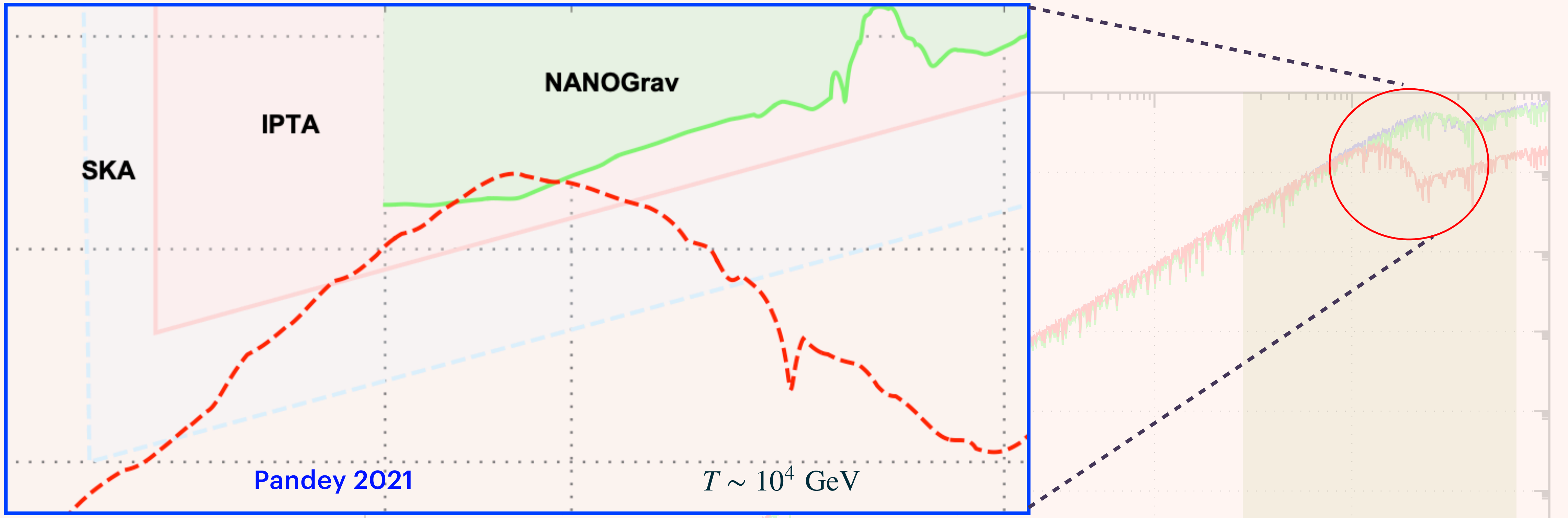


GRAVITATIONAL WAVES



NANOGrav: They were interested in $f \in (2.5 \times 10^{-9} - 7.0 \times 10^{-8})$ Hz and GW amplitude of $\Omega_{\rm GW}(f \sim 5.5 \text{ Hz}) \in (3 \times 10^{-10} - 2 \times 10^{-9})$.

They show that the detected GW have strong power law preference as $\Omega_{\rm GW} \propto f^{\kappa}$ and $\kappa \sim (-1.5, 0.5)$ range.

In our present study, we have calculated slope of the characteristic strain spectrum $h_c(f)$ for typical values of magnetic fields around the peak of the spectrum and compared it to the observed $h_c(f)$ and we have found that it is -0.5, which is well within the range of the NANOGrav experiments.

Favoured strength of the magnetic fields:
To explain the NANOGrav signal, strength of the magnetic fields calculated is $B_0 \sim 10^{-12}$ G at a frequency of $f \sim 10^{-8}$ Hz and a length scale of 1 Mpc.


Thank You