

# Quantum evolution of cosmological perturbations: single and two field inflation

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Based on Ravindran, Parattu, Sriramkumar, GRG (2022), arxiv:2206.05760

# How did the universe originate?

**Where did all these galaxies come from?**

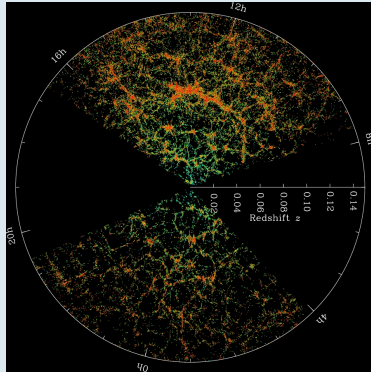
# Where did all the galaxies come from?

## The Creation of the Milky Way (Cherokee)

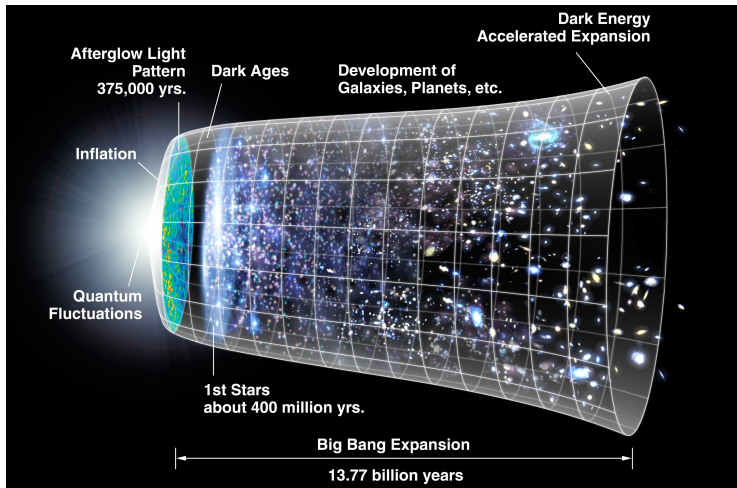


# Largest structures in the universe

## SDSS map of the universe



# How did it all begin?



# Structure from vacuum fluctuations of the inflaton?

Proposed soon after introduction of inflation:

- Hawking (1982)
- Starobinsky (1982)
- Guth and Pi (1982)

# Structure from vacuum fluctuations of the inflaton?

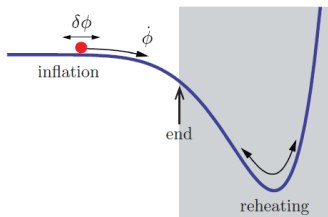
Some agreement between this assumption and observations.  
Starting with inflaton in its ground state:

- Fluctuations at the end of inflation follow a Gaussian distribution
- Fluctuations are nearly scale-invariant



# Vacuum fluctuations become fluctuations in the universe

- Variation in values of inflation at various points- various points inflate differently  $\Rightarrow$  Different densities



# Two questions

- Why don't astronomers need quantum methods to make sense of observations of the universe? Why does the universe appear largely classical?
- Are there any features in the structure in the universe that will prove that it originated as quantum fluctuations?

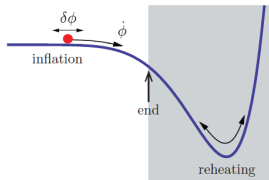
# Formalism

The mathematics of cosmological perturbations

# The Action: Einstein's gravity and a scalar field

- Single field inflation:

$$\mathcal{A} = \frac{1}{2} \int d^4x \sqrt{-g} \left[ \frac{1}{2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right]$$



# Background metric and perturbed metric

- Background FRW metric, spatially flat:

$$ds^2 = -dt^2 + a^2(t) \delta_{ij} dx^i dx^j = a^2(\eta) \left( -d\eta^2 + \delta_{ij} dx^i dx^j \right),$$

# Background metric and perturbed metric

- Background FRW metric, spatially flat:

$$ds^2 = -dt^2 + a^2(t) \delta_{ij} dx^i dx^j = a^2(\eta) \left( -d\eta^2 + \delta_{ij} dx^i dx^j \right),$$

- Perturbations:

$$1)\delta\phi \quad 2)\delta g_{\eta\eta} \quad 3)\delta g_{\eta i} \quad 5)\delta g_{ij},$$

11 degrees of freedom.

# Degrees of Freedom

- After removing all redundant degrees of freedom,

1 physical scalar mode, no vector modes, 2 tensor modes

- 1 scalar degrees of freedom- represented by a conveniently normalized gauge-invariant variable: Mukhanov-Sasaki Variable  $v_M$

# The Mukhanov-Sasaki Variable

- Second-order action for the variable in conformal time:

$$\frac{1}{2} \int d\eta \int d^3\mathbf{x} \left[ v_{\text{MS}}'^2 - (\partial_i v_{\text{MS}})^2 - \frac{2z'}{z} v_{\text{MS}}' v_{\text{MS}} + \left(\frac{z'}{z}\right)^2 v_{\text{MS}}^2 \right],$$

with  $z = (a\dot{\phi})/H$ .

- Upto a total derivative, a scalar field in Minkowski with a time-dependent mass.



# Quantization

- Corresponding momentum

$$p_{\text{MS}} = v'_{\text{MS}} - \frac{z'}{z} v_{\text{MS}} .$$

- Promote to operators and postulate

$$\begin{aligned} [\hat{v}_{\text{MS}}(\mathbf{x}, \eta), \hat{p}_{\text{MS}}(\mathbf{y}, \eta)] &= i\delta^{(3)}(\mathbf{x} - \mathbf{y}); \\ [\hat{v}_{\text{MS}}(\mathbf{x}, \eta), \hat{v}_{\text{MS}}(\mathbf{y}, \eta)] &= [\hat{p}_{\text{MS}}(\mathbf{x}, \eta), \hat{p}_{\text{MS}}(\mathbf{y}, \eta)] = 0 . \end{aligned}$$

# The Fourier modes

- Fourier decomposition in the flat 3-space:

$$v_{\text{MS}}(\eta, \mathbf{x}) = \int \frac{d^3 \mathbf{k}}{(2\pi)^{3/2}} v_{\mathbf{k}}(\eta) e^{i\mathbf{k}\cdot\mathbf{x}} .$$

# The Fourier modes

- Fourier decomposition in the flat 3-space:

$$v_{\text{MS}}(\eta, \mathbf{x}) = \int \frac{d^3 \mathbf{k}}{(2\pi)^{3/2}} v_{\mathbf{k}}(\eta) e^{i\mathbf{k}\cdot\mathbf{x}} .$$

- Reality of  $v_{\text{MS}}$ :

$$v_{-\mathbf{k}}(\eta) = v_{\mathbf{k}}^*(\eta) .$$

# The Fourier modes

- Action, restricting to independent modes using  $v_{-\mathbf{k}}(\eta) = v_{\mathbf{k}}^*(\eta)$ :

$$\int d\eta \int_{\frac{\mathbb{R}^3}{2}} d^3\mathbf{k} v'_{\mathbf{k}} v'_{-\mathbf{k}} - \left(k^2 - \frac{z'^2}{z^2}\right) v_{-\mathbf{k}}^* v_{\mathbf{k}} - \frac{z'}{z} \left(v'_{\mathbf{k}} v_{-\mathbf{k}} + v'_{-\mathbf{k}} v_{\mathbf{k}}\right)$$

- The Fourier modes evolve independently at linear order.

# Hermitian variables

- Real and imaginary parts:

$$v_{\mathbf{k}}^R \equiv \frac{1}{\sqrt{2}} (v_{\mathbf{k}} + v_{\mathbf{k}}^*); \quad v_{\mathbf{k}}^I \equiv \frac{1}{i\sqrt{2}} (v_{\mathbf{k}} - v_{\mathbf{k}}^*)$$

- Action:

$$\frac{1}{2} \int_{\mathbb{R}/2} d\eta d^3\mathbf{k} \left[ v_{\mathbf{k}}^{R'2} - \frac{2z'}{z} v_{\mathbf{k}}^R v_{\mathbf{k}'}^{R'} - \left( k^2 - \frac{z'^2}{z^2} \right) v_{\mathbf{k}}^{R2} \right. \\ \left. + v_{\mathbf{k}}^{I'2} - \frac{2z'}{z} v_{\mathbf{k}}^I v_{\mathbf{k}'}^{I'} - \left( k^2 - \frac{z'^2}{z^2} \right) v_{\mathbf{k}}^{I2} \right]$$

# Hermitian variables

- The real and imaginary parts evolve independently and identically.
- Conjugate momenta:

$$p_k^R = v_k^{R'} - \frac{z'}{z} v_k^R, \quad p_k^I = v_k^{I'} - \frac{z'}{z} v_k^I$$

- Now on,  $(\hat{v}_k, \hat{p}_k)$  stands for either of these pairs

# Evolution

- Classical Hamiltonian:

$$H = \int_{\mathbb{R}/2} d\eta d^3\mathbf{k} \left( \frac{p_{\mathbf{k}}^2}{2} + \frac{z'}{z} p_{\mathbf{k}} v_{\mathbf{k}} + \frac{k^2}{2} v_{\mathbf{k}}^2 \right)$$

- Quantum Hamiltonian:

$$\hat{H} = \int_{\mathbb{R}/2} d\eta d^3\mathbf{k} \left( \frac{\hat{p}_{\mathbf{k}}^2}{2} + \frac{z'}{2z} (\hat{p}_{\mathbf{k}} \hat{v}_{\mathbf{k}} + \hat{v}_{\mathbf{k}} \hat{p}_{\mathbf{k}}) + \frac{k^2}{2} \hat{v}_{\mathbf{k}}^2 \right)$$

# Schrodinger equation

- Schrodinger equation:

$$i \frac{\partial \Psi}{\partial \eta} = -\frac{1}{2} \frac{\partial^2 \Psi}{\partial v^2} - \frac{i}{2} \frac{z'}{z} \left( \Psi + 2v \frac{\partial \Psi}{\partial v} \right) + \frac{k^2}{2} v^2 \Psi .$$

- Solution through the Gaussian ansatz:

$$\Psi(v, \eta) = \mathcal{N}(\eta) \exp \left[ -\frac{\Omega(\eta) v^2}{2} \right],$$

- Bunch-Davies initial conditions



# Quadratic correlation functions

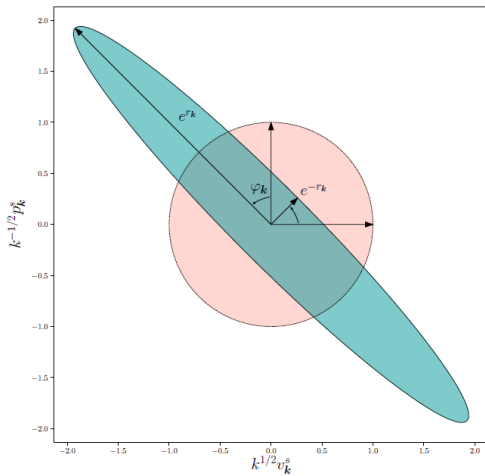
- The covariance matrix:

$$V = \begin{bmatrix} \langle \hat{v}^2 \rangle & \frac{1}{2} \langle \hat{v} \hat{p} + \hat{p} \hat{v} \rangle \\ \frac{1}{2} \langle \hat{v} \hat{p} + \hat{p} \hat{v} \rangle & \langle \hat{p}^2 \rangle \end{bmatrix}.$$

- Define  $Z^T = (\tilde{v}, \tilde{p})$

$$\frac{Z^T V^{-1} Z}{2} = 1.$$

## Error ellipse



# Quadratic correlation functions

- We obtain

$$\begin{aligned}\langle \hat{v}_{\mathbf{k}} \hat{v}_{\mathbf{k}'} \rangle &= \frac{1}{2k} [\cosh(2r) + \sinh(2r) \cos(2\phi)] \delta^{(3)}(\mathbf{k} - \mathbf{k}'), \\ \langle \hat{p}_{\mathbf{k}} \hat{p}_{\mathbf{k}'} \rangle &= \frac{k}{2} [\cosh(2r) - \sinh(2r) \cos(2\phi)] \delta^{(3)}(\mathbf{k} - \mathbf{k}'). \\ \frac{\langle \hat{v}_{\mathbf{k}} \hat{p}_{\mathbf{k}'} + \hat{p}_{\mathbf{k}'} \hat{v}_{\mathbf{k}} \rangle}{2} &= \frac{1}{2} [\sinh(2r) \sin(2\phi)] \delta^{(3)}(\mathbf{k} - \mathbf{k}').\end{aligned}$$

- Compare

$$[\hat{v}_{\mathbf{k}}, \hat{p}_{\mathbf{k}}] = i\delta^{(3)}(\mathbf{k} - \mathbf{k}')$$

# Quantum and classical nature of cosmological perturbations

## Where is the quantum?

Many papers written on how these quantum perturbations became classical with the evolution of the universe:

- The large scale structure appears to be classical
- Astronomers are not known to use quantum techniques to analyze their data

# 1980's and 1990's: How did these quantum perturbations become classical?

Many papers written on how these quantum perturbations became classical with the evolution of the universe:

- Guth and Pi (1985)
- Albrecht et al. (1994)
- Polarski, Starobinsky (1996). Polarski, Starobinsky, Kiefer etc.(1995-2010).
- A good review: Kiefer, Polarski- Why do cosmological perturbations look classical to us? (2009)

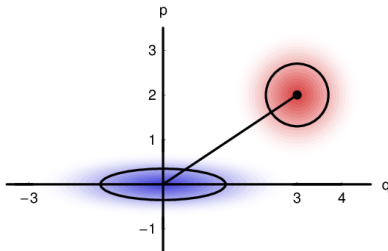
## Two things to note

1. No absolute single criterion of classicality!



## Two things to note

2. We compare the system of cosmological perturbations with a *classical distribution*, not a classical particle state.





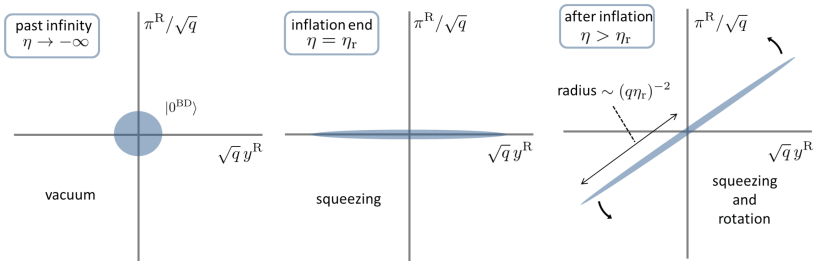
## Two main mechanisms

1. Squeezing
2. Decoherence

## Squeezing

- Now see the Hamiltonian

$$\hat{H}_k = \frac{\hat{p}_k^2}{2} + \frac{z'}{2z} (\hat{p}_k \hat{v}_k + \hat{v}_k \hat{p}_k) + \frac{k^2}{2} \hat{v}_k^2$$



# Squeezing and classicality

For example,

$$\frac{\langle \hat{v}_{\mathbf{k}} \hat{p}_{\mathbf{k}'} + \hat{p}_{\mathbf{k}'} \hat{v}_{\mathbf{k}} \rangle}{2} = \frac{1}{2} [\sinh(2r) \sin(2\phi)] \delta^{(3)}(\mathbf{k} - \mathbf{k}')$$

Compare

$$[\hat{v}_{\mathbf{k}}, \hat{p}_{\mathbf{k}}] = i\delta^{(3)}(\mathbf{k} - \mathbf{k}')$$

## Squeezing and classicality

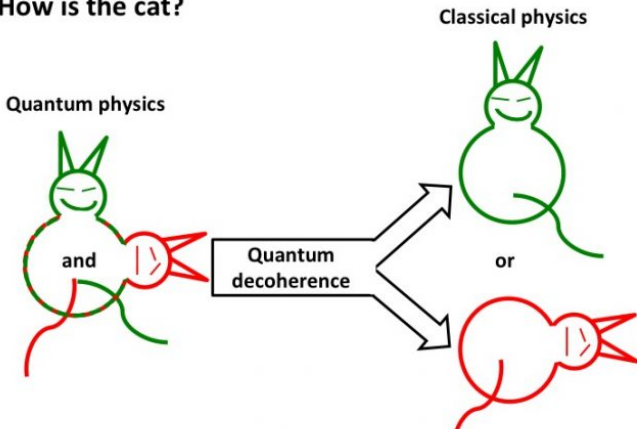
Table from Martin, Vennin (2016):

Correlation function	Classical state		Stochastic distribution	
	Generic	Super-Hubble	Generic	Super-Hubble (squeezed)
$\mathcal{P}_\zeta \propto \langle \zeta_k \zeta_{-k} \rangle$	Succeeds		Succeeds	
Other non-Hermitian two-point correlators	Fails	Strongly fails	Fails	Succeeds
Other Hermitian two-point correlators			Succeeds	
Higher order Hermitian correlators	Fails	Strongly fails	Fails	Succeeds

# Decoherence

Due to interaction with other fields in the universe, coherence (superpositions) are lost

**How is the cat?**

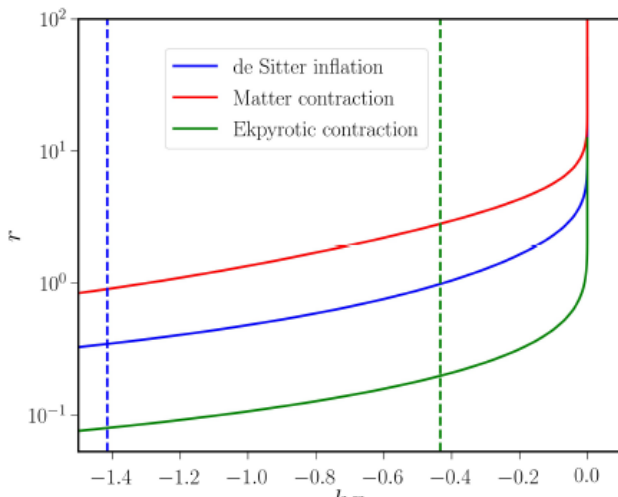


# Decoherence

- Mathematically, the reduced density matrix of the system, after tracing out the environment, gets diagonalized.
- Measured by the entanglement entropy

$$S = -\text{Tr}(\rho_r \ln \rho_r)$$

## Single-field power law models



## Ultra slow-roll models

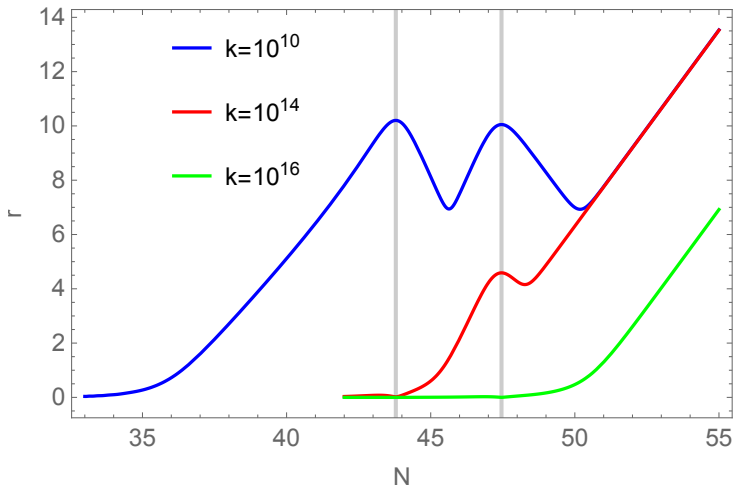
- There were some indications in the literature that the discussion of classicality would be different in models with an ultra slow-roll (USR) phase.
- Usual claim: quantum nature is hard to probe because of information lost in the decaying mode. But this decaying mode grows in USR case!
- de Putter, Dore (2019): the usual decaying mode grows, but then the other mode gets highly suppressed in the late universe. Thus, effectively the same conclusion. High squeezing, but they worked without squeezing parameters.



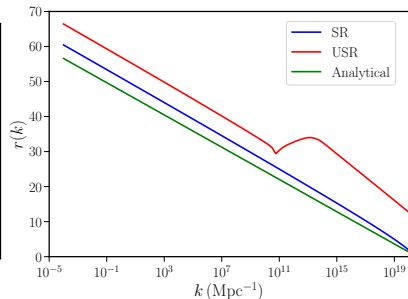
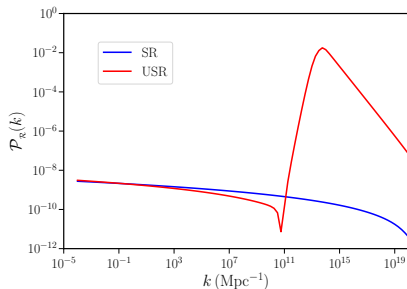
## Ultra slow-roll model:

$$V(\phi) = V_0 \left\{ \tanh \left( \frac{\phi}{\sqrt{6} M_{\text{Pl}}} \right) + A \sin \left[ \frac{1}{f_\phi} \tanh \left( \frac{\phi}{\sqrt{6} M_{\text{Pl}}} \right) \right] \right\}^2.$$

- $V_0 = 2 \times 10^{-10} M_{\text{Pl}}^4$ ,  $A = 0.130383$  and  $f_\phi = 0.129576$ .
- The point of inflection in the potential:  $\phi_0 = 1.05 M_{\text{Pl}}$ .
- Initial value of the field:  $\phi_i = 6.1 M_{\text{Pl}}$ ,  $\epsilon_{1i} = 10^{-4}$ .
- 66 e-folds of inflation

Ultra slow-roll models: squeezing amplitude  $r$ 

## Ultra slow-roll models: Power spectrum



## Two-field models

- Provide a richer dynamics than single-field models.

## Two-field models

- Provide a richer dynamics than single-field models.
- In models where isocurvature perturbations decay, natural to consider them as an environment to integrate out and consider entanglement entropy and quantum discord of the subsystem of the remaining curvature perturbations.

## Two-field models

- Action:

$$S[\phi, \chi] = \int d^4x \sqrt{-g} \left[ -\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{e^{2b(\phi)}}{2} \partial_\mu \chi \partial^\mu \chi - V(\phi, \chi) \right].$$

- Our choice:

$$V(\phi, \chi) = \frac{m^2}{2} (\phi^2 + \chi^2),$$
$$b(\phi) = \frac{b_1}{2} \left\{ 1 + \tanh [\alpha (\phi - \phi_0)] \right\}.$$

# Case 1: Slow roll scenario

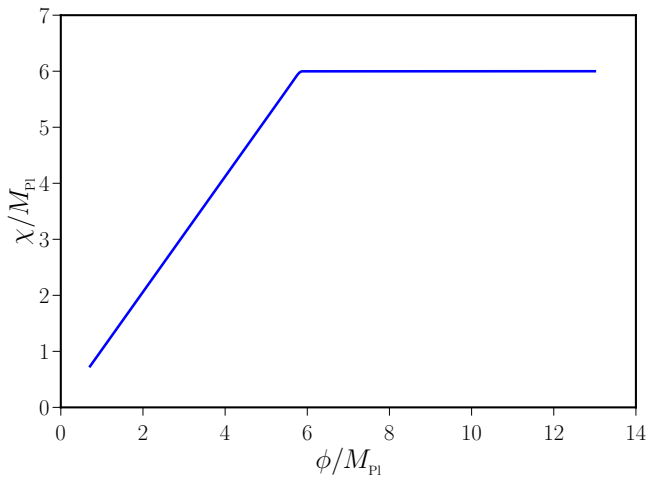
- $b_1 = 0 \Rightarrow$  Two uncoupled, canonical scalar fields.
- $m = 9 \times 10^{-6} M_{\text{pl}}$ .
- Initial values:  $\phi_i = \chi_i = 11.5 M_{\text{pl}}$  and  $\dot{\phi}_i = \dot{\chi}_i = -3.68 \times 10^{-6} M_{\text{pl}}^2$ .
- Inflation lasts for about 66 e-folds
- Essentially two fields evolving identically,  $\phi - \chi = 0 \Rightarrow$  Effectively single-field evolution.

## Case 2: PBH scenario

- $b_1 = 15$ ,  $\alpha = 10 M_{\text{Pl}}^{-1}$  and  $\phi_0 = 6 M_{\text{Pl}}$
- $m = 1.03 \times 10^{-5} M_{\text{Pl}}$ .
- Initial values:  $(\phi_i, \chi_i) = (13 M_{\text{Pl}}, 6 M_{\text{Pl}})$  and  $(\dot{\phi}_i, \dot{\chi}_i) = (-5.44 \times 10^{-6} M_{\text{Pl}}^2, 0)$ .
- Inflation lasts for about 66 e-folds
- One can see a sharp turn in the trajectory in field space.



## Case 2: PBH scenario



# Perturbations

- Action for real or imaginary parts of any Fourier component of the MS variables:

$$S = \frac{1}{2} \int d\eta \left( v_\sigma'^2 + v_s'^2 - 2 \frac{z'}{z} v_\sigma' v_\sigma - 2 \frac{a'}{a} v_s' v_s - 2 \xi v_\sigma' v_s + 2 \xi \frac{z'}{z} v_\sigma v_s - m_\sigma^2 v_\sigma^2 - m_s^2 v_s^2 \right).$$

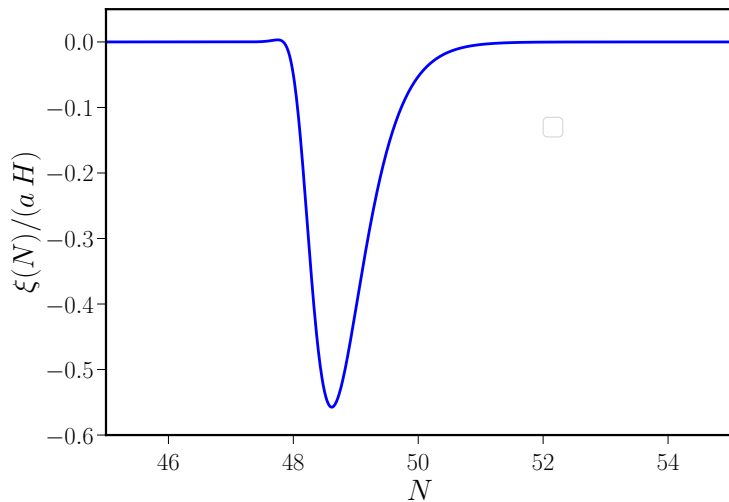
# Perturbations

- Action for real or imaginary parts of any Fourier component of the MS variables:

$$S = \frac{1}{2} \int d\eta \left( v_\sigma'^2 + v_s'^2 - 2 \frac{z'}{z} v_\sigma' v_\sigma - 2 \frac{a'}{a} v_s' v_s - 2 \xi v_\sigma' v_s + 2 \xi \frac{z'}{z} v_\sigma v_s - m_\sigma^2 v_\sigma^2 - m_s^2 v_s^2 \right).$$

- $\xi$  controls the interaction between  $v_\sigma$  and  $v_s$ .

## Interaction



# Schrodinger equation

- Schrodinger equation solved through the Gaussian ansatz:

$$\Psi(v_\sigma, v_s, \eta) \propto \exp \left[ -\frac{1}{2} \Omega_{\sigma\sigma}(\eta) v_\sigma^2 - \frac{1}{2} \Omega_{ss}(\eta) v_s^2 - \Omega_{\sigma s}(\eta) v_\sigma v_s \right]$$

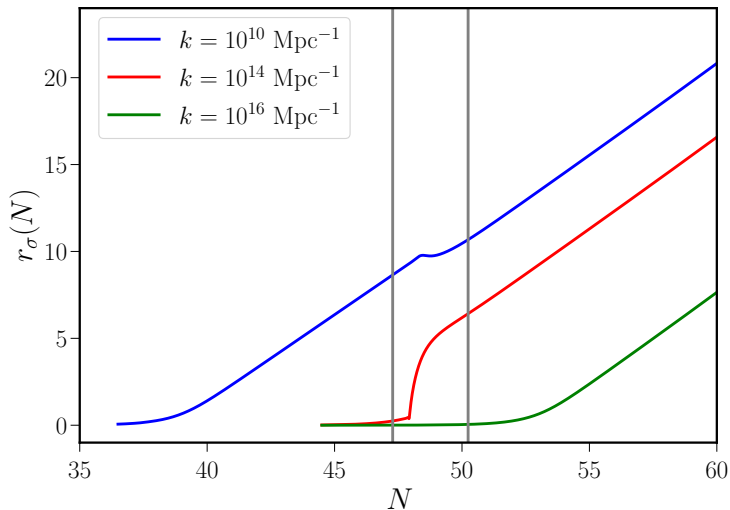
- Bunch-Davies initial conditions for both  $v_\sigma$  and  $v_s$  at early times when they are decoupled.

# Quadratic correlation functions

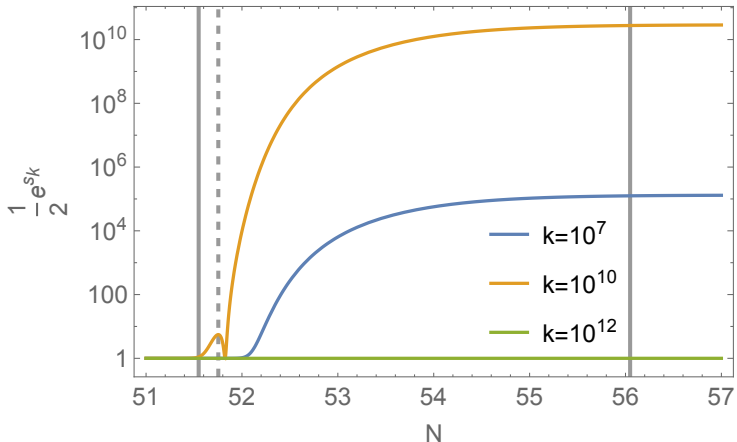
- We obtain

$$\begin{aligned}\langle \hat{v}_\sigma^2 \rangle &= \frac{(2 N_\sigma + 1)}{2 k} [\cosh (2 r_\sigma) + \sinh (2 r_\sigma) \cos (2 \phi_\sigma)] , \\ \langle \hat{p}_\sigma^2 \rangle &= \frac{(2 N_\sigma + 1) k}{2} [\cosh (2 r_\sigma) - \sinh (2 r_\sigma) \cos (2 \phi_\sigma)] . \\ \frac{\langle \hat{v}_\sigma \hat{p}_\sigma + \hat{p}_\sigma \hat{v}_\sigma \rangle}{2} &= \frac{(2 N_\sigma + 1)}{2} [\sinh (2 r_\sigma) \sin (2 \phi_\sigma)]\end{aligned}$$

## Squeezing parameter

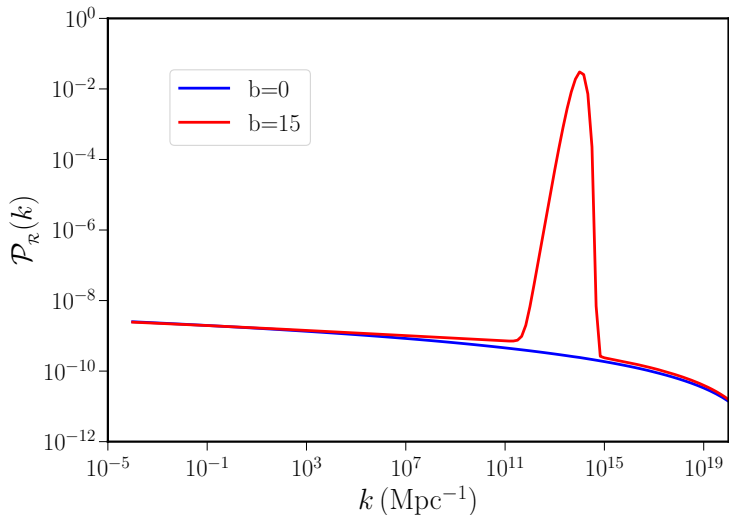


## Entanglement entropy and decoherence





## Power spectrum of the curvature perturbation



## 2000s: can we prove that cosmic structure is quantum?

Classical mechanisms for cosmic structure:

- A. Berera and L.-Z. Fang, Phys. Rev. Lett. 74, 1912 (1995)
- A. Berera, Phys. Rev. Lett. 75, 3218 (1995).
- D. Lopez Nacir, R. A. Porto, L. Senatore, and M. Zaldarriaga, JHEP 01, 075 (2012).
- L. Senatore, E. Silverstein, and M. Zaldarriaga, JCAP 08, 016 (2014).

## 2000s: can we prove that cosmic structure is quantum?

- Bell inequalities: A quintessential quantum property
- Explored in Campo, Parentani (2005, 2006): concluded that they can indeed be violated if decoherence is not too large, but very difficult to verify observationally
- Maldacena (2015): A rather complicated model that can produce observable Bell violation

# Bell inequality violation

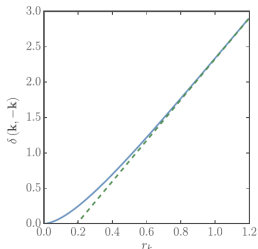
- Especially Martin, Vennin and collaborators (2016-2022): violated in Fourier space. But real-space violation is more physical. In real space, Bell inequality violation for certain operators considering two spatially separated patches does not occur because of decoherence due to interaction with other patches.
- Other type of operators may be studied for Bell inequality violation.
- There are various other Bell-type inequalities

## Quantum discord

- Quantum discord: a quantum property, correlations between different parts of a system, that is more general than entanglement. Used in cosmology by Lim (2015).
- Martin and Vennin (2015): A more extensive analysis to show that this quantity is indeed very large at the end of inflation, placing the perturbations in a very quantum state.

# Quantum discord

- Quantum discord between  $\mathbf{k}$  and  $-\mathbf{k}$  modes in single-field case:  $\cosh^2 r \log \cosh^2 r - \sinh^2 r \log \sinh^2 r$



- Large squeezing leading to classicality  
 $\Rightarrow$  Large quantum discord giving large quantumness!

## Quantum discord for two-field models

- For two-field models: a natural division of the system into curvature and isocurvature modes.
- From quantum information literature: when a pure state is divided into two, quantum discord = entanglement entropy.
- We verified this explicitly for a general Gaussian wavefunction.

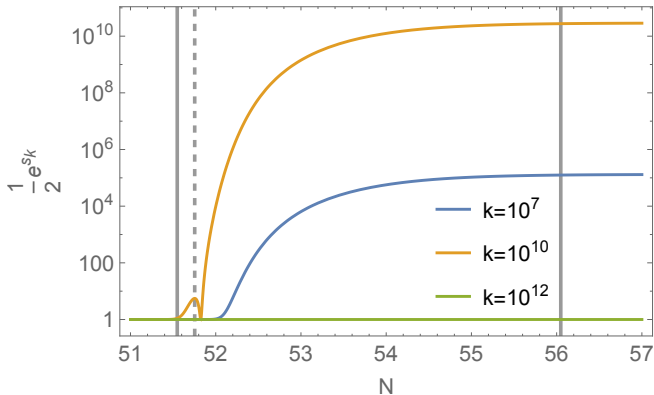
## Quantum discord for two-field models

- While quantum discord is a recent topic in cosmology, entanglement entropy has been around for much longer.
- Entanglement entropy in the case of two-field models has been discussed before: Prokopec, Rigopoulos (2007); Battarra, Lehnert (2014)



## Quantum discord for two-field models

- Thus, I have already shown you plots of discord.



**Thank you!**

Questions welcome...