Bouncing cosmology in a curved braneworld

Tanmoy Paul

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The Randall-Sundrum model

- A 5D spacetime with two 3-brane scenario. The spacetime within the branes is known as bulk.
- If ϕ is the extra dimensional coordinate, then the branes are location of constant ϕ hypersurfaces. In particular, $\phi = 0$ and $\phi = \pi$ are two branes: hidden and visible branes respectively.
- The visible brane is identified with our visible universe.
- The bulk contains 5D cosmological constant (Λ) and the branes contain certain amount of tension.
- therefore there are three matter parts: (1) bulk cosmological constant (Λ), (2) hidden brane tension (V_{hid}) and (3) visible brane tension (V_{vis}).

The Randall-Sundrum model- Continued

- The action:

$$S_5 = \int d^5x\sqrt{-g_5}\left[2M^3R^{(5)} - \Lambda - V_{hid}\delta(\phi) - V_{vis}\delta(\phi-\pi)\right]$$

- The Randall-Sundrum (RS) metric becomes,

$$ds^2 = e^{-2k_0 r_c \phi}\eta_{\mu\nu} + r_c^2 d\phi^2$$

where r_c is the inter-brane separation and $k_0 = \sqrt{\frac{-\Lambda}{24M^3}}$.

- Here $V_{hid} = -V_{vis} = 24M^3k_0$.

- Note: the induced metric of the branes are flat. This is due to, Λ, V_{hid} and V_{vis} exactly cancel their effects on the branes and thus the effective brane energy is zero.

The Randall-Sundrum model- Continued

- Due to the intervening gravity, the branes should collapse and the model becomes unstable.
- Thus, we need some stabilizing agent which acts as a repeller in-between the branes and make them stable at a fixed separation.
- According to Goldberger-Wise mechanism, a bulk scalar field can act as suitable stabilizing agent. However the source of scalar field is unknown.

The Randall-Sundrum model- Continued

- The stabilization procedure goes as follows:
  (1) make r_c vary, i.e $r_c \rightarrow T(x)$ where $T(x)$ is radion field.

  (2) The stabilizing agent generates a potential for $T(x)$ in 4D effective action.

  (3) The radion potential has a stable minimum (say at $\langle T \rangle$).

  (4) The radion field gets stable or the brane separation gets fixed at $\langle T \rangle$ - stabilized brane separation.

The Randall-Sundrum model- Continued

- Two problems of original RS model:
  (1) The branes are flat, i.e Λ, V_{hid} and V_{vis} exactly cancel their effects on the branes to make the effective brane energy zero. One may think it as extremely fine tuned model.

  (2) In stabilization, one needs a bulk scalar field. However the source of scalar field is unknown.

Generalized RS scenario- Our model

- Here Λ, V_{hid} and V_{vis} do not exactly cancel their effects on the branes, thus they are not fine tuned.
- Thereby, the effective brane energy density is not zero and the branes are curved.
- In stabilization, we do not need any extra bulk scalar field.
- Actually the non-zero effective brane energy density generates a potential term for the radion in 4D action.
- The radion potential has a stable point where the radion or the brane separation gets stabilized.

Generalized RS scenario- Our model

- The action:

$$S_5 = \int d^5x\sqrt{-g_5}\left[2M^3R^{(5)} - \Lambda - V_{hid}\delta(\phi) - V_{vis}\delta(\phi-\pi)\right]$$

- The spacetime metric:

$$ds^2 = e^{-2A(r_c,\phi)}g_{\mu\nu}dx^\mu dx^\nu + r_c^2 d\phi^2$$

where,

$$e^{-A} = \omega \sinh\left(\ln\frac{c_2}{\omega} - k_0 r_c \phi\right)$$

- The effective brane energy density $\propto \omega$. For $\omega = 0$, we recover the original RS model.

- The induced metric on the branes are not flat. This is due to $\omega \neq 0$.

Generalized RS scenario- Our model

- In regard to stabilization, let us go with the stabilization procedure as mentioned earlier.
- Make $r_c \rightarrow T(x)$ which is known as radion field.
- Next, we need to find the 4D effective action.
- The we have to investigate whether any stable radion potential generate in the 4D action.
- In this case, we will see that the effective brane energy $(\propto \omega)$ generates a radion potential which is indeed stable.

4D effective action

- Plugging the 5D metric into the 5D action and integrating over the extra dimensional coordinate ϕ will yield the 4D effective action - Goldberger-Wise mechanism.

- $\hat{S}_{eff} = S_1 + S_2 + S_3$.
  with,

$$\begin{aligned} S_1 &= \frac{2M^3}{k_0} \int d^4x \, \sqrt{-\hat{g}} \; h(\xi) \, \hat{R} \\ S_2 &= -2M^3 k_0 \int d^4x \, \sqrt{-\hat{g}} \; \hat{V}(\xi) \\ S_3 &= \int d^4x \, \sqrt{-g} \, \left(\frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi \right) \hat{G}(\xi) \end{aligned} \tag{1}$$

- $\xi = e^{-k_0 \pi T(x)}$.

4D effective action- Continued

- S_1 : the on-brane gravity part. S_2 : the potential term for the radion field. S_3: the kinetic term for the radion field.
- The forms of various functions in $\hat{S}_{eff}$ come as,

$$\begin{aligned} h(\xi) &= \left\{ \frac{c_2^2}{4} + \omega^2 \ln \xi + \frac{\omega^4}{4c_2^2} \left(\frac{1}{\xi^2} \right) - \frac{\omega^4}{4c_2^2} - \frac{c_2^2}{4} \xi^2 \right\} \\ \hat{V}(\xi) &= 6\omega^2 h(\xi) \\ \hat{G}(\xi) &= 1 + \frac{4}{3} \frac{\omega^2}{c_2^2} \left(\frac{1}{\xi^2} \right) \ln \xi - \frac{\omega^4}{c_2^4} \left(\frac{1}{\xi^4} \right) \end{aligned} \tag{2}$$

4D effective action- Continued

- The $\hat{S}_{eff}$ is in Jordan frame, because the coefficient of Ricci scalar is not unity.

- We transform the effective action in Einstein frame by the transformation

$$\hat{g}_{\mu\nu} \longrightarrow g_{\mu\nu} = h(\xi)\hat{g}_{\mu\nu} \tag{3}$$

4D effective action- Continued

- With this transformation, the Einstein frame effective action becomes,

$$S_{eff} = \int d^4x\sqrt{-g}\left[\frac{R}{2\kappa^2} + \frac{1}{2}G(\xi)\partial^\mu\Phi\partial_\mu\Phi - 8M^3\kappa^2 V(\xi)\right] \quad (4)$$

$$(5)$$

- The quantity with an overhat is in Jordan frane and without hat is in Einstein frame.

- $V(\xi)$ is the radion potential in Einstein frame:

$$V(\xi) = \frac{\hat{V}(\xi)}{h(\xi)^2} = \frac{6\omega^2}{h(\xi)}$$

- $G(\xi)$ is the non-minimal kinetic coupling of radion in Einstein frame:

$$G(\xi) = \frac{\hat{G}(\xi)}{h(\xi)} + \frac{1}{c_2^2}\left[\frac{h'(\xi)}{h(\xi)}\right]^2$$

4D effective action- Continued

- $V(\xi)$ has an inflection point at $\langle\xi\rangle = \omega/c_2$ where the radion or the brane separation gets stabilized.

- Note: In the Einstein frame action -
  (1) the coefficient of the Ricci scalar becomes unity.

  (2) The radion field still has a non-minimal kinetic coupling.

  (3) The $V(\xi)$ is proportional to ω^2 , i.e the radion potential vanishes for Minkowskian brane where $\omega^2 = 0$.

- In order to better understand, we give the plots of $V(\xi)$ and $G(\xi)$:

4D effective action- Continued

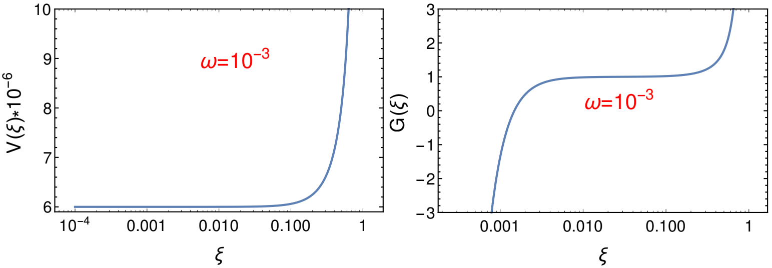

Figure: The above figure depicts the variation of (a) the radion potential $V(\xi)$ and (b) the non-canonical kinetic coupling $G(\xi)$, for $\omega = 10^{-3}$.

Incorporating cosmolgical dynamics

- $V(\xi)$ increases with $\xi \Rightarrow$ during the cosmological evolution, the radion field decreases with cosmic time.

- $G(\xi)$ has a zero crossing from positive to negative as ξ decreases $\Rightarrow$ the kinetic energy of ξ becomes negative $\Rightarrow$ violation of energy condition (may) $\Rightarrow$ possibility of a non-singular bounce.

Two questions

1. Does the cosmological evolution of radion field solve the gauge hierarchy problem ?

2. Does the cosmological evolution of metric lead to a non-singular bounce ?

Background equations

- Metric ansatz:

$$ds^2 = -dt^2 + a(t)^2\left[dx^2 + dy^2 + dz^2\right] \tag{6}$$

- Background equations:

$$\begin{aligned} H^2 &= \frac{\kappa^2}{3}\rho(t) = \frac{c_2^2}{4}G(\xi)\dot{\xi}^2 + \frac{k_0^2}{6}V(\xi) \\ \dot{H} &= -\frac{\kappa^2}{2}(\rho + p) = -\frac{3}{4}c_2^2 G(\xi)\dot{\xi}^2 \\ 0 &= \ddot{\xi} + 3H\dot{\xi} + \frac{G'(\xi)}{2G(\xi)}\dot{\xi}^2 + \frac{k_0^2}{3c_2^2}\frac{V'(\xi)}{G(\xi)} \end{aligned}$$

Background solutions

- Recall, our investigation- whether the model admits a bounce scenario.
- If the bounce occurs, then it must occur during $G(\xi) < 0$, because during this regime, the energy condition is violated.
- The regime of $G(\xi) < 0$ is $\xi \sim \omega$, so first we determine the background solutions near $\xi \sim \omega$, in particular,

$$\xi(t) = \frac{\omega}{c_2}\left(1 + \delta(t)\right)$$

with $\delta(t) < 1$.

Background solutions- Continued

- During the regime $\xi(t) = \frac{\omega}{c_2}(1+\delta(t))$, the background equations become (leading order of δ):

$$\dot{H} + 3H^2 - \frac{12k_0^2\omega^2}{c_2^2} = 0$$

$$\dot{\delta}^2 = \frac{c_2^2}{\omega^2}\frac{\dot{H}}{4ln\left(\frac{c_2}{\omega}\right)}\left[1+\delta\left\{\frac{4+2ln\left(\frac{c_2}{\omega}\right)}{ln\left(\frac{c_2}{\omega}\right)}\right\}\right]$$

- Condition: $\lim_{t\to\infty}\xi(t) \to \frac{\omega}{c_2}$ - due to stability of radion field.

- Solutions of above equations:

$$H(t) = 2k_0\frac{\omega}{c_2}tanh\left[6\frac{\omega}{c_2}k_0 t\right]$$

$$\delta(t) = \frac{2}{A}\left[exp\left\{-\frac{A}{6}\frac{\omega}{c_2}\sqrt{\frac{3}{ln\left(\frac{c_2}{\omega}\right)}}\left(tan^{-1}tanh\left(\frac{3\omega}{c_2}k_0 t\right)-\frac{\pi}{4}\right)\right\}-1\right]$$

Background solutions- Continued

- The solution of $H(t)$ indicates a non-singular bounce at $t = 0$.
- Therefore the bounce occurs in a curved braneworld scenario during $G(\xi) < 0$.
- The $\xi(t)$ decreases with time and asymptotically reaches to the value which solves the gauge hierarchy problem concomitantly.

Background solutions- Continued

- In the earlier case, we determined the solutions during the phantom regime.
- However it is unlikely that the radion field starts its journey from phantom regime, the radion should start its journey from normal regime where $G(\xi) > 0$.
- One immediate question: How does the radion field, starting from normal regime, reach to the phantom regime by its dynamical evolution ?
- To answer this, we solve the background equations for wide range of ξ, numerically.
- In numerical analysis, the boundary conditions are provided from the analytic solutions that we just determined. In particular, $H(0) = 0$ and $\xi(0) = 6.0041 \times 10^{-4}$ for $\omega = 10^{-3}$.

Background solutions- Continued

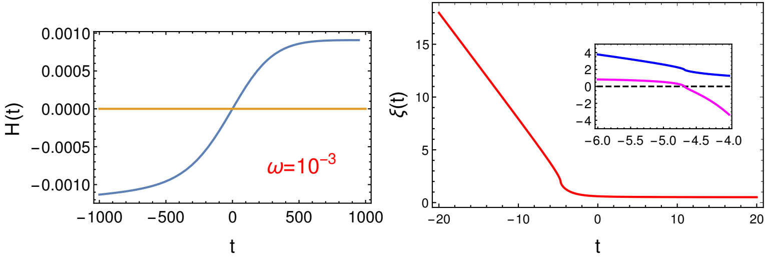

Figure: The above figure depicts the time evolution of (a) $H(t)$ and (b) $\xi(t) \times 1000$. The inset depicts the non-canonical kinetic term $G(\xi)$ (magenta curve) and $\xi(t) \times 1000$ (blue curve) near the zero crossing of $G(\xi)$.

Background solutions- Continued

- The plot of $H(t)$:
  (1) admits a non-singular bounce at $t = 0$. Note, from the inset, that the bounce occurs during $G(\xi) < 0$.

  (2) $H(t)$ leads to a late time acceleration at both sides of bounce.

  (3) The bounce is slightly asymmetric.

- The plot of $\xi(t)$:
  (1) monotonically decreases with the time.

  (2) Finally reach to the stabilized value $= \omega/c_2$.

III. Evolution of perturbation variables

Scalar perturbation

- The scalar perturbation over FRW metric,

$$ds^2 = a^2(\eta)\Big[\big(1+2\Psi\big)d\eta^2 - \big(1-2\Psi\big)\delta_{ij}dx^i dx^j\Big]$$

- The radion field perturbation is,

$$\Phi(\eta,\vec{x}) = \Phi_0(\eta) + \delta\Phi(\eta,\vec{x})$$

- The scalar perturbation equation,

$$\begin{aligned}
\nabla^2\Psi - 3\mathcal{H}\Psi' - 3\mathcal{H}\Psi &= \frac{\kappa^2}{2}a^2\delta T^0_0 \\
\big(\Psi' + \mathcal{H}\Psi\big)_{,i} &= \frac{\kappa^2}{2}a^2\delta T^0_i \\
\Big[\Psi'' + 3\mathcal{H}\Psi' + \big(2\mathcal{H}' + \mathcal{H}^2\big)\Psi\Big]\delta^i_j &= -\frac{\kappa^2}{2}a^2\delta T^i_j
\end{aligned}$$

Scalar perturbation- Continued

- In the present context, the scalar perturbation equations,

$$
\begin{aligned}
\nabla^2\Psi &- 3\mathcal{H}\Psi' - 3\mathcal{H}\Psi = \frac{\kappa^2}{2}\Big[G(\Phi_0)\Phi_0'\delta\Phi' \\
&+ \frac{1}{2}G'(\Phi_0)(\Phi_0')^2\delta\Phi + 2a^2M^3k_0V'(\Phi_0)\delta\Phi\Big]
\end{aligned}
$$

$$
\Psi' + \mathcal{H}\Psi = \frac{\kappa^2}{2}\Phi_0'\delta\Phi
$$

$$
\begin{aligned}
\Psi'' + 3\mathcal{H}\Psi' &+ \left(2\mathcal{H}' + \mathcal{H}^2\right)\Psi = \frac{\kappa^2}{2}\Big[G(\Phi_0)\Phi_0'\delta\Phi' \\
&+ \frac{1}{2}G'(\Phi_0)(\Phi_0')^2\delta\Phi - 2a^2M^3k_0V'(\Phi_0)\delta\Phi\Big]
\end{aligned}
$$

Scalar perturbation- Continued

- The second equation can be used to obtain $\delta\Phi$ in terms of Ψ and Ψ', substituting which into the other two equations $\Rightarrow$

$$\Psi'' - \nabla^2\Psi + 6\mathcal{H}\Psi' + \left(2\mathcal{H}' + 4\mathcal{H}^2\right)\Psi = -4a^2 M^3 k_0 \left(\frac{V'(\Phi_0)\left(\Psi' + \mathcal{H}\Psi\right)}{G(\Phi_0)\Phi_0'}\right)$$

- Or, in terms of cosmic time,

$$\ddot{\Psi} - \frac{1}{a^2}\nabla^2\Psi + \left[7H + \frac{2k_0^2\ V'(\xi_0)}{3c_2^2 G(\xi_0)\dot{\xi}_0}\right]\dot{\Psi} + \left[2\dot{H} + 6H^2 + \frac{2k_0^2 H\ V'(\xi_0)}{3c_2^2 G(\xi_0)\dot{\xi}_0}\right]\Psi = 0$$

Scalar perturbation- Continued

- The present model leads to a late time acceleration at both sides of bounce.
- Thus the comoving Hubble radius decreases and asymptotically goes to zero at both sides of bounce.
- However near the bounce, the comoving Hubble radius has an infinite size.
- Thereby all the perturbation modes near the bounce lie within the Hubble horizon.
- Due to this reason, we solve the perturbation equations near the bounce where the Bunch-Davies condition is consistent.

Scalar perturbation- Continued

- Near the bounce (i.e near $t = 0$), the perturbation equation in Fourier space (upto the leading order in t),

$$\begin{aligned}\ddot{\Psi}_k &+ \left[-\sqrt{\alpha}p + (q+14)\alpha t\right]\dot{\Psi}_k \\ &+ \left[k^2 + 4\alpha - 2\alpha\sqrt{\alpha}p\ t\right]\Psi_k(t) = 0\end{aligned}$$

where α, p and q are constants and depend on ω.

- Solution of $\Psi_k(t)$:

$$\Psi_k(t) \propto e^{[p\sqrt{\alpha}\ t\ -7\alpha t^2 - \frac{q}{2}\alpha t^2]}$$
$$H\left[-1 + \frac{k^2 + 4\alpha}{\alpha(q+14)}, \frac{-p + (q+14)\sqrt{\alpha}\ t}{\sqrt{2(q+14)}}\right]$$

Scalar perturbation- Continued

- Scalar power spectrum for k th mode:

$$P_{\Psi}(k,t) = \frac{k^3}{2\pi^2}\left|\Psi_k(t)\right|^2$$

Tensor perturbation

- The tensor perturbation over the FRW metric,

$$ds^2 = -dt^2 + a(t)^2 \left(\delta_{ij} + h_{ij}\right) dx^i dx^j$$

- There are two tensor polarization modes, both the modes (in Fourier space) evolve by,

$$\frac{1}{a(t)z_T^2(t)} \frac{d}{dt}\left[a(t)z_T^2(t)\dot{h}_k\right] + \frac{k^2}{a^2} h_k(t) = 0$$

  with $z_T(t) = a(t)/\kappa$.

- The tensor perturbation does not couple with the radion field, as expected.

Tensor perturbation- Continued

- Due to the reason mentioned earlier, we again solve the tensor perturbation equation near the bounce.

- Near the bounce, the tensor perturbation equation (upto the leading order in t),

$$\ddot{h}_k + 6\alpha \dot{h}_k \ t + k^2 h_k(t) = 0$$

- Solution of $h_k(t)$:

$$h_k(t) \propto e^{-3\alpha t^2} \ H\left[-1 + \frac{k^2}{6\alpha}, \sqrt{3\alpha} \ t \right]$$

Tensor perturbation- Continued

- Tensor power spectrum for k th mode:

$$P_h(k,t) = \frac{k^3}{\pi^2}\left|h_k(t)\right|^2$$

IV. Observable quantities

Observable quantities- Continued

- Scalar spectral index,

$$n_s - 1 = \left. \frac{\partial \ln P_\Psi}{\partial \ln k} \right|_{H.C}$$

- Tensor-to-scalar ratio,

$$r = \left. \frac{P_h(k,t)}{P_\Psi(k,t)} \right|_{H.C}$$

- We are interested on large scale modes, in particular $k = 0.05\mathrm{Mpc}^{-1} \approx 10^{-40}\mathrm{GeV}$.

- From the horizon crossing condition $(k = aH) \Rightarrow$, the horizon crossing instant of the $k = 0.05\mathrm{Mpc}^{-1}$ is,

$$t_h \approx 10^{-93}\mathrm{sec}.$$

Observable quantities- Continued

- The horizon crossing of the large scale mode(s) occur very near to the bounce.
- Thereby, using the solutions of $\Psi_k(t)$ and $h_k(t)$ (that we found near the bounce, in the previous section), we determine the expressions of n_s and r in the present context,

$$n_s = n_s(\omega, k_0/M)$$
$$r = r(\omega, k_0/M)$$

recall, k_0 denotes the 5D bulk curvature scale.

Observable quantities- Continued

- For $k_0/M = 0.5$ and within the interval $10^{-6} \lesssim \omega \lesssim 10^{-3}$, we get consistent observables with the Planck results.

TAKE HOME MESSAGE

- We consider a curved braneworld scenario where we incorporate the cosmological dynamics.
- The model admits a non-singular bounce and the radion field or the brane separation gets dynamically stabilized.
- The radion potential- which is essential for bounce, generates from 5D spacetime.
- The bounce is not an ekpyrotic bounce scenario and thus suffers from the BKL instability.

THANK YOU

HAVE A NICE DAY