Accounting for scale dependent scalar non-Gaussianity in secondary gravitational waves

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Talk based on *arXiv:2108.04193 [astro-ph.CO]*

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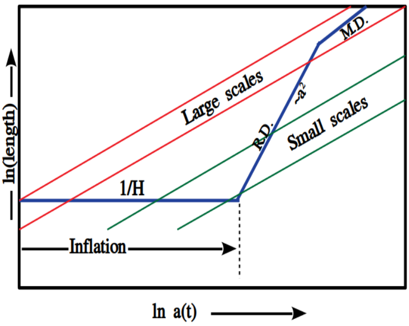
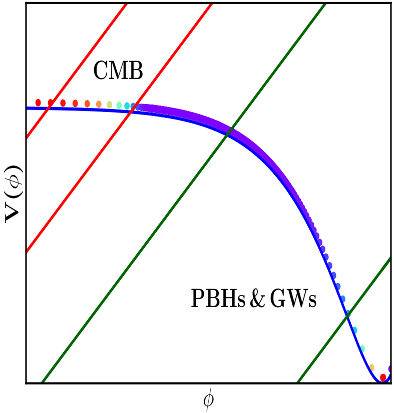
Overview

- Introduction
- Scalar non-Gaussianity parameter - definition and extension
- Non-Gaussian correction to the power spectrum
- Models for illustration
- Results - analytical and numerical
- Summary and Outlook

Inflation

The epoch of inflation is constrained through the observables that are derived from the correlations of the primordial perturbations.





Power and bi-spectra

The curvature perturbation $\mathcal{R}$ can be written in terms of the Fourier component as

$$\mathcal{R}(\boldsymbol{x}, \eta) = \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^{3/2}} \mathcal{R}_{\boldsymbol{k}}(\eta)\, \mathrm{e}^{i\boldsymbol{k}\cdot\boldsymbol{x}}\,.$$

The corresponding two point and three point correlations are defined as

$$\begin{aligned}
\langle \hat{\mathcal{R}}_{\boldsymbol{k}_1} \hat{\mathcal{R}}_{\boldsymbol{k}_2} \rangle &= \frac{2\pi^2}{k_1^3} \mathcal{P}_{_{\mathcal{R}}}(k_1)\, \delta^{(3)}(\boldsymbol{k}_1 + \boldsymbol{k}_2)\,, \\
\langle \hat{\mathcal{R}}_{\boldsymbol{k}_1} \hat{\mathcal{R}}_{\boldsymbol{k}_2} \hat{\mathcal{R}}_{\boldsymbol{k}_3} \rangle &= (2\pi)^{-3/2}\, G(k_1, k_2, k_3)\, \delta^{(3)}(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3)\,.
\end{aligned}$$

Scalar non-Gaussianity parameter - Definition

The scalar non-Gaussianity parameter $f_{_{\rm NL}}$ is defined through the following relation[1]

$$\mathcal{R}(\boldsymbol{x},\eta) = \mathcal{R}^{^{\rm G}}(\boldsymbol{x},\eta) - \frac{3}{5} f_{_{\rm NL}} \left[\mathcal{R}^{^{\rm G}}(\boldsymbol{x},\eta)\right]^2.$$

On computing the power and bi-spectra using this definition, we obtain[2]

$$f_{_{\rm NL}} = -\frac{10}{3}\,\frac{1}{(2\,\pi)^4}\,k_1^3\,k_2^3\,k_3^3\,G(k_1,k_2,k_3) \times \left[k_1^3\,\mathcal{P}_{_{\mathcal{R}}}(k_2)\,\mathcal{P}_{_{\mathcal{R}}}(k_3) + \text{two permutations}\right]^{-1}.$$

[1] See, for instance, *J. Maldacena, JHEP* **0305**, *013 (2003); X. Chen, R. Easther and E. A. Lim, JCAP* **0706**, *023 (2007).*

[2] For instance, *J. Martin and L. Sriramkumar, JCAP* **01**, *008 (2012)*

Scalar non-Gaussianity parameter - Templates

$$G^{\rm local}(k_1, k_2, k_3) \quad = \quad -\frac{3}{10}\,(2\,\pi)^4\, f_{_{\rm NL}}^{\rm local} \left[\frac{\mathcal{P}_{_{\mathcal{R}}}(k_1)\,\mathcal{P}_{_{\mathcal{R}}}(k_2)}{k_1^3\,k_2^3} + \text{two permutations}\right]$$

[3] *P. Creminelli, A. Nicolis, L. Senatore, M. Tegmark, and M. Zaldarriaga, JCAP,* **05**, *004 (2006)*

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$$\begin{aligned} G^{\rm equil}(k_1,k_2,k_3) = {} & \frac{9}{10}\,(2\,\pi)^4\; f_{_{\rm NL}}^{\rm equil}\left\{\frac{\mathcal{P}_{_{\mathcal{R}}}(k_1)\mathcal{P}_{_{\mathcal{R}}}(k_2)}{k_1^3\,k_2^3} + \frac{\mathcal{P}_{_{\mathcal{R}}}(k_2)\mathcal{P}_{_{\mathcal{R}}}(k_3)}{k_2^3\,k_3^3}\right. \\ & + \frac{\mathcal{P}_{_{\mathcal{R}}}(k_1)\mathcal{P}_{_{\mathcal{R}}}(k_3)}{k_1^3\,k_3^3} + \frac{[\mathcal{P}_{_{\mathcal{R}}}(k_1)\mathcal{P}_{_{\mathcal{R}}}(k_2)\mathcal{P}_{_{\mathcal{R}}}(k_3)]^{2/3}}{k_1^2 k_2^2 k_3^2} \\ & \left. - \left[\frac{\mathcal{P}_{_{\mathcal{R}}}(k_1)^{1/3}\mathcal{P}_{_{\mathcal{R}}}(k_2)^{2/3}\mathcal{P}_{_{\mathcal{R}}}(k_3)}{k_1 k_2^2 k_3^3} + 5 \text{ permutations}\right]\right\} \end{aligned}$$ [3]

[3] *P. Creminelli, A. Nicolis, L. Senatore, M. Tegmark, and M. Zaldarriaga, JCAP,* **05**, *004 (2006)*

Scalar non-Gaussianity parameter - Templates

$$G^{\rm local}(k_1, k_2, k_3) = -\frac{3}{10}\,(2\,\pi)^4\, f_{\rm NL}^{\rm local} \left[\frac{\mathcal{P}_{\mathcal{R}}(k_1)\,\mathcal{P}_{\mathcal{R}}(k_2)}{k_1^3\,k_2^3} + \text{two permutations}\right]$$

$$G^{\rm ortho}(k_1, k_2, k_3) = \frac{27}{10}\,(2\,\pi)^4\, f_{\rm NL}^{\rm ortho} \left\{\frac{\mathcal{P}_{\mathcal{R}}(k_1)\mathcal{P}_{\mathcal{R}}(k_2)}{k_1^3\,k_2^3} + \frac{\mathcal{P}_{\mathcal{R}}(k_2)\mathcal{P}_{\mathcal{R}}(k_3)}{k_2^3\,k_3^3} + \frac{\mathcal{P}_{\mathcal{R}}(k_1)\mathcal{P}_{\mathcal{R}}(k_3)}{k_1^3\,k_3^3} + \frac{8\,[\mathcal{P}_{\mathcal{R}}(k_1)\mathcal{P}_{\mathcal{R}}(k_2)\mathcal{P}_{\mathcal{R}}(k_3)]^{2/3}}{3\,k_1^2 k_2^2 k_3^2} - \left[\frac{\mathcal{P}_{\mathcal{R}}(k_1)^{1/3}\mathcal{P}_{\mathcal{R}}(k_2)^{2/3}\mathcal{P}_{\mathcal{R}}(k_3)}{k_1 k_2^2 k_3^3} + 5\ \text{permutations}\right]\right\}$$ [4]

[4] *P. Creminelli, A. Nicolis, L. Senatore, M. Tegmark, and M. Zaldarriaga, JCAP,* **05**, *004 (2006); L. Senatore, K. M. Smith, and M. Zaldarriaga, JCAP,* **01**, *028 (2010)*

[5] *Planck collaboration, A&A.* **641**, *A9 (2020) [arXiv:1905.05697 [astro-ph.CO]]*

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$$G^{\rm local}(k_1,k_2,k_3) = -\frac{3}{10}\,(2\,\pi)^4\, f_{\rm NL}^{\rm local}\left[\frac{\mathcal{P}_{\mathcal{R}}(k_1)\,\mathcal{P}_{\mathcal{R}}(k_2)}{k_1^3\,k_2^3} + \text{two permutations}\right]$$

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$$f_{\rm NL}^{\rm local} = -0.9 \pm 5.1,\ \ f_{\rm NL}^{\rm equil} = -26 \pm 47,\ \ f_{\rm NL}^{\rm ortho} = -38 \pm 24$$ [5]

[4] *P. Creminelli, A. Nicolis, L. Senatore, M. Tegmark, and M. Zaldarriaga, JCAP,* **05**, *004 (2006); L. Senatore, K. M. Smith, and M. Zaldarriaga, JCAP,* **01**, *028 (2010)*

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Scalar non-Gaussianity parameter - Extension

We try to extend the definition of $f_{\rm NL}$ to include a generic scale dependence[6]. Hence, instead of

$$\mathcal{R}(\boldsymbol{x},\eta) \quad = \quad \mathcal{R}^{\rm G}(\boldsymbol{x},\eta) - \frac{3}{5} f_{\rm NL} [\mathcal{R}^{\rm G}(\boldsymbol{x},\eta)]^2 \,,$$

we define $f_{\rm NL}(k_1,k_2,k_3)$ through

$$\mathcal{R}_{\boldsymbol{k}}(\eta) \quad = \quad \mathcal{R}^{\rm G}_{\boldsymbol{k}}(\eta) - \frac{3}{5}\int \frac{{\rm d}^3\boldsymbol{k}_1}{(2\,\pi)^{3/2}} \mathcal{R}^{\rm G}_{\boldsymbol{k_1}}(\eta) \mathcal{R}^{\rm G}_{\boldsymbol{k}-\boldsymbol{k_1}}(\eta)\, f_{\rm NL}[\boldsymbol{k},(\boldsymbol{k}_1-\boldsymbol{k}),-\boldsymbol{k}_1]\,.$$

[6] *F. Schmidt and M. Kamionkowski, Phys. Rev.* **D 82**, *103002 (2010); I. Agullo, D. Kranas, and V. Sreenath arXiv:2105.12993 [gr-qc].*

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If we compute the scalar bispectrum using the above relation, we obtain

$$\begin{aligned} f_{\rm NL}(k_1,k_2,k_3) \quad &= \quad -\frac{10}{3}\,\frac{1}{(2\,\pi)^4}\; k_1^3\,k_2^3\,k_3^3\; G(k_1,k_2,k_3) \\ &\qquad \times \left[k_1^3\,\mathcal{P}_{\rm S}^{\rm G}(k_2)\,\mathcal{P}_{\rm S}^{\rm G}(k_3) + \text{two permutations}\right]^{-1}. \end{aligned}$$

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Non-Gaussian correction to the power spectrum

We proceed to calculate the power spectrum of $\mathcal{R}_{\boldsymbol{k}}$ including extended definition of $f_{_{\rm NL}}$ and so obtain[7]

$$\mathcal{P}_{_{\mathcal{R}}}(k) \quad = \quad \mathcal{P}_{_{\rm S}}^{^{\rm G}}(k) + \frac{9}{50\pi} k^3 \int {\rm d}^3\boldsymbol{k}_1 \, \frac{\mathcal{P}_{_{\rm S}}^{^{\rm G}}(k_1)}{k_1^3} \frac{\mathcal{P}_{_{\rm S}}^{^{\rm G}}(|\boldsymbol{k}-\boldsymbol{k}_1|)}{|\boldsymbol{k}-\boldsymbol{k}_1|^3} \, f_{_{\rm NL}}^2[k, |\boldsymbol{k}_1-\boldsymbol{k}|, k_1] \,,$$

[7] For similar attempts, see *R.-g. Cai, S. Pi, and M. Sasaki, Phys. Rev. Lett.* **122**, *201101 (2019); C. Unal, Phys. Rev. D* **99**, *041301 (2019); P. Adshead, K. D. Lozanov, and Z. J. Weiner, arXiv:2105.01659 [astro-ph.CO].*

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$\mathcal{P}_{_{\rm C}}(k) > \mathcal{P}_{_{\rm S}}^{^{\rm G}}(k)$ implies large non-Gaussian correction to the original power spectrum.

[7] For similar attempts, see *R.-g. Cai, S. Pi, and M. Sasaki, Phys. Rev. Lett.* **122**, *201101 (2019); C. Unal, Phys. Rev. D* **99**, *041301 (2019); P. Adshead, K. D. Lozanov, and Z. J. Weiner, arXiv:2105.01659 [astro-ph.CO].*

Models for illustration

We shall consider two models to illustrate the computation of $\mathcal{P}_{\rm C}(k)$.

1. Potential of Starobinsky model with a dip added by hand[8]:

$$V(\phi) = V_0 \left[1 - \exp\left(-\sqrt{\frac{2}{3}}\frac{\phi}{M_{\rm Pl}}\right)\right]^2 \left\{1 - \lambda \exp\left[-\frac{1}{2}\left(\frac{\phi-\phi_0}{\Delta\phi}\right)^2\right]\right\}$$

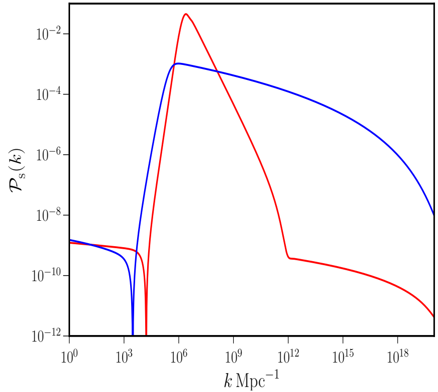
2. Model of critical-Higgs inflation[9]:

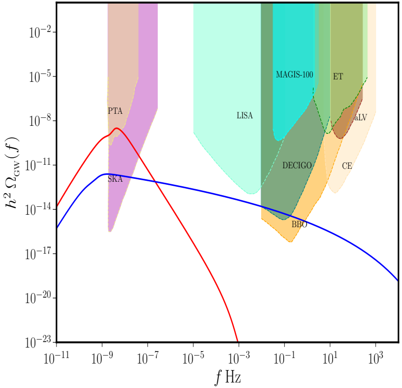
$$V(\phi) = V_0 \frac{\left[1 + a\,\ln^2\left(\frac{\phi}{\mu}\right)\right]\left(\frac{\phi}{\mu}\right)^4}{\left\{1 + c\left[1 + b\,\ln\left(\frac{\phi}{\mu}\right)\right]\left(\frac{\phi}{\mu}\right)^2\right\}^2}$$

[8] *V. Atal, J. Garriga, and A. Marcos-Caballero, JCAP* **09**, *073; S. S. Mishra and V. Sahni, JCAP* **04**, *007*

[9] *J. M. Ezquiaga, J. Garcia-Bellido, and E. Ruiz Morales, Phys. Lett.* **B776**, *345 (2018); F. Bezrukov, M. Pauly, and J. Rubio, JCAP* **02**, *040; M. Drees and Y. Xu, Eur. Phys. J. C* **81**, *182 (2021)*

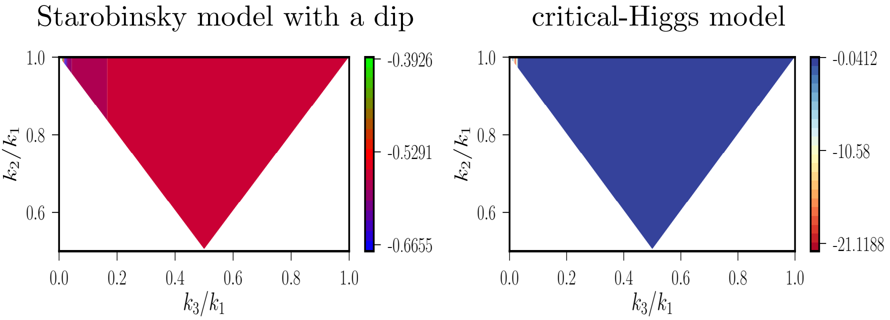
$\mathcal{P}_{\mathrm{S}}^{\mathrm{G}}(k)$ and $\Omega_{\mathrm{GW}}(f)$





$\mathcal{P}_{\mathrm{S}}^{\mathrm{G}}(k)$ and $\Omega_{\mathrm{GW}}(f)$ for the models of Starobinsky potential with a dip (in red) and critical-Higgs inflation (in blue)

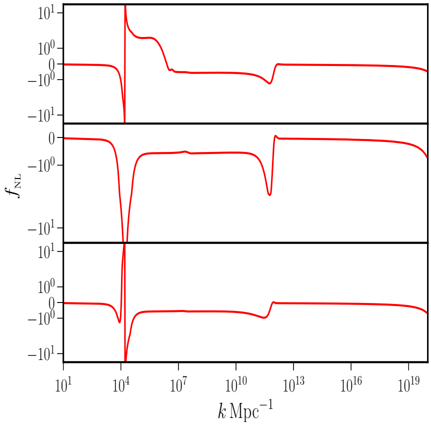
Behavior of $f_{\rm NL}$ - around the peak



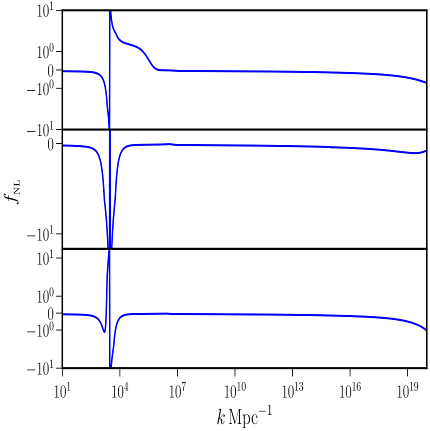
$f_{\rm NL}$ is presented as a density plot around k_1. We have set $k_1 = 10^6\,{\rm Mpc}^{-1}$ corresponding to the peak in $\mathcal{P}_{\rm S}^{\rm G}(k)$.

Behavior of $f_{\rm NL}$ - different limits

Starobinsky model with a dip



critical-Higgs model



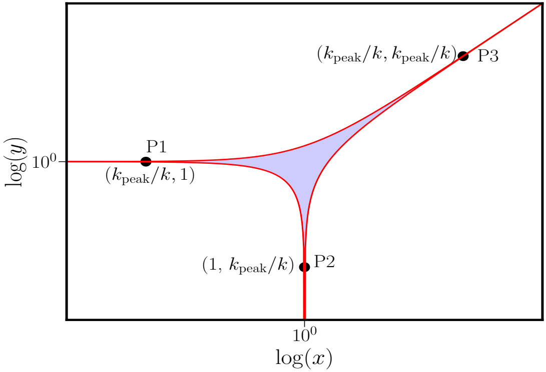
$f_{\rm NL}$ in various limits, namely squeezed limit $[k_1 \simeq k_2\,, k_3 \to 0]$ (on top), equilateral limit $[k_1 = k_2 = k_3]$ (in the middle) and flattened limit $[k_1 = k_2 = 2k_3]$ (in the bottom)

Behavior of the integrand

Recall that the quantity to be computed is

$$\mathcal{P}_{\rm C}(k) = \frac{9}{25}\int_0^\infty {\rm d}x \int_{|1-x|}^{|1+x|} {\rm d}y \frac{\mathcal{P}_{\rm S}^{\rm G}(kx)}{x^2}\,\frac{\mathcal{P}_{\rm S}^{\rm G}(ky)}{y^2}\, f_{\rm NL}^2\,[k, kx, ky]\,.$$

The range of the integrals can be plotted as follows.

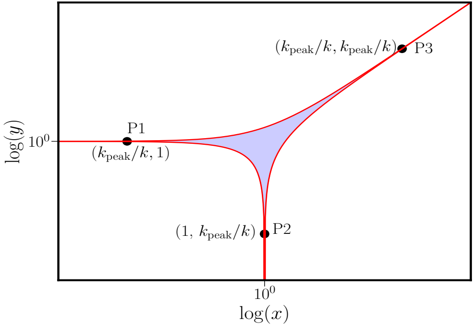


Behavior of the integrand

If we approximate the power spectra with localized peaks using a Dirac delta function, we may write

$$\mathcal{P}_{\mathcal{R}}(k) \simeq \mathcal{P}_{\mathrm{S}}^{\mathrm{G}}(k_{\mathrm{peak}})\,\delta(\ln k - \ln k_{\mathrm{peak}})\,.$$

For such localized peaks, the integrand receives maximum contribution around the points where $x = k_{\mathrm{peak}}/k$ or $y = k_{\mathrm{peak}}/k$ or $x = y = k_{\mathrm{peak}}/k$.



Analytical estimate of $\mathcal{P}_{\text{C}}(k)$

For $k < k_{\text{peak}}$, we evaluate the integral at the point P3 and obtain

$$\begin{aligned} \mathcal{P}_{\text{C}}(k) &= \frac{18}{25}\left(\frac{k}{k_{\text{peak}}}\right)^3 \left[\mathcal{P}_{\text{S}}^{\text{G}}(k_{\text{peak}}) f_{\text{NL}}(k, k_{\text{peak}}, k_{\text{peak}})\right]^2, \\ &= \frac{1}{8}\left(\frac{k}{k_{\text{peak}}}\right)^3 \left\{\mathcal{P}_{\text{S}}^{\text{G}}(k_{\text{peak}}) \left[n_{\text{S}}(k_{\text{peak}}) - 1\right]\right\}^2, \end{aligned}$$

where we have used the consistency relation[10]

$$f_{\text{NL}}(k, k_{\text{peak}}, k_{\text{peak}}) = \frac{5}{12}\left[n_{\text{S}}(k_{\text{peak}}) - 1\right],$$

with $n_{\text{S}}(k) - 1 = \mathrm{d}\ln \mathcal{P}_{\mathcal{R}}(k)/\mathrm{d}\ln k$.

[10] For related discussion, see, *H. V. Ragavendra, P. Saha, L. Sriramkumar, and J. Silk, Phys. Rev.* **D 103**, *083510 (2021) [arXiv:2008.12202 [astro-ph.CO]].*

Analytical estimate of $\mathcal{P}_{\rm C}(k)$

For $k > k_{\rm peak}$, we evaluate the integral at the point P1 and obtain

$$\begin{aligned}\mathcal{P}_{\rm C}(k) &= \frac{18}{25}\,\mathcal{P}_{\rm S}^{\rm G}(k_{\rm peak})\mathcal{P}_{\rm S}^{\rm G}(k)\,f_{\rm NL}^{2}(k,k,k_{\rm peak})\,,\\ &= \frac{1}{8}\mathcal{P}_{\rm S}^{\rm G}(k_{\rm peak})\mathcal{P}_{\rm S}^{\rm G}(k)\,[n_{\rm S}(k)-1]^2\,,\end{aligned}$$

where we have used $f_{\rm NL}(k,k,k_{\rm peak}) = (5/12)[n_{\rm S}(k)-1]\,.$

The contribution from P2 is same as P1, hence the total estimate shall be

$$\mathcal{P}_{\rm C}(k) = \frac{1}{4}\mathcal{P}_{\rm S}^{\rm G}(k_{\rm peak})\mathcal{P}_{\rm S}^{\rm G}(k)\,[n_{\rm S}(k)-1]^2\,.$$

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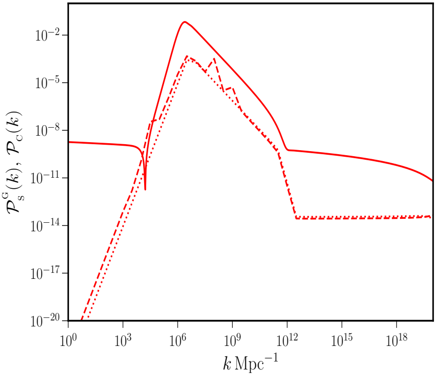
$$\mathcal{P}_{\rm C}(k) = \frac{1}{4}\mathcal{P}_{\rm S}^{\rm G}(k_{\rm peak})\mathcal{P}_{\rm S}^{\rm G}(k)\left[n_{\rm S}(k)-1\right]^2\,.$$

Therefore the analytical expressions for $\mathcal{P}_{\rm C}(k)$ over the complete range will be

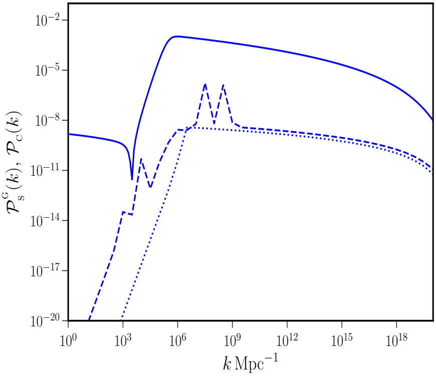
$$\mathcal{P}_{\rm C}(k) = \begin{cases} \dfrac{1}{8}\left(\dfrac{k}{k_{\rm peak}}\right)^3\left\{\mathcal{P}_{\rm S}^{\rm G}(k_{\rm peak})[n_{\rm S}(k_{\rm peak})-1]\right\}^2\,, & \text{for } k < k_{\rm peak}\,, \\ \dfrac{1}{4}\,\mathcal{P}_{\rm S}^{\rm G}(k_{\rm peak})\mathcal{P}_{\rm S}^{\rm G}(k)\left[n_{\rm S}(k)-1\right]^2\,, & \text{for } k > k_{\rm peak}\,. \end{cases}$$

$\mathcal{P}_{\rm C}(k)$ against $\mathcal{P}_{\rm S}^{\rm G}(k)$

$$\mathcal{P}_{\rm C}(k) \quad = \quad \frac{9}{25}\int_0^\infty {\rm d}x \int_{|1-x|}^{|1+x|} {\rm d}y \, \frac{\mathcal{P}_{\rm S}^{\rm G}(kx)}{x^2}\, \frac{\mathcal{P}_{\rm S}^{\rm G}(ky)}{y^2}\, f_{\rm NL}^2\,[k, kx, ky]\,.$$



Starobinsky model with a dip



critical-Higgs model

$\mathcal{P}_{\rm C}(k)$ computed numerically (dashed lines) and analytically (dotted lines) is presented against the original $\mathcal{P}_{\rm S}^{\rm G}(k)$.

Summary

- Extending the definition of $f_{_{\rm NL}}$ helps us account for the complete bispectrum in the calculation of non-Gaussian correction to the power spectrum.
- For spectra with localized peaks, the correction is largely dependent on the squeezed limit of $f_{_{\rm NL}}$ and the consistency relation helps us to obtain an analytical estimate easily.
- The correction in case of canonical single field models with ultra slow roll seems to be negligible compared to the original power spectrum. Therefore, there is no discernible imprint of $f_{_{\rm NL}}$ on the spectrum of $\Omega_{_{\rm GW}}$.

Outlook

- Models of inflation with large $f_{\rm NL}$, due to, say, excited initial states, are of interest to study in this context of correction to the power spectrum[11]. However, this scenario is plagued by severe backreaction on the energy density of inflation.
- Non-canonical models with large $f_{\rm NL}$ over CMB scales could be of interest as they may lead to significant corrections to scalar power and hence be possibly constrained indirectly against the data.

[11] *H. V. Ragavendra, L. Sriramkumar and J. Silk, JCAP* **05**, *010 (2021) [arXiv:2011.09938 [astro-ph.CO]]*

Conclusion

Thank you for your attention.

Reference:

H. V. Ragavendra, *Accounting for scalar non-Gaussianity in secondary gravitational waves*, arXiv:2108.04193 [astro-ph.CO].

Appendix - I

The equations of motion governing the scalar perturbation $\mathcal{R}(\boldsymbol{x},\eta)$ and the tensor perturbation $\gamma^{ij}(\boldsymbol{x},\eta)$ in Fourier space are[12]

$$\mathcal{R}_k'' + 2\frac{z'}{z}\mathcal{R}_k' + k^2\,\mathcal{R}_k = 0\,,$$
$$\gamma_k^{\lambda\,\prime\prime} + 2\frac{a'}{a}\gamma_k^{\lambda\,\prime} + k^2\,\gamma_k^{\lambda} = 0\,,$$

where $z = a\sqrt{2\,\epsilon_1}\,M_{_{\mathrm{Pl}}}$ and overprime denotes derivative with respect to η.

[12] $\mathrm{d}\eta = \mathrm{d}t/a(t)$; overprime means $\mathrm{d}/\mathrm{d}\eta$; overdot means $\mathrm{d}/\mathrm{d}t$.

Appendix - II

The equation of motion governing the secondary tensor modes $h_{\boldsymbol{k}}^{\lambda}(\eta)$ is given by[13]

$$h_k^{\lambda\,\prime\prime} + 2\frac{a'}{a}h_k^{\lambda\,\prime} + k^2 h_k^{\lambda} = S_k^{\lambda}(\eta)\,,$$

where the source term $S_k^{\lambda}(\eta)$ contains terms quadratic in$\mathcal{R}_k$.
The time averaged power spectrum of the secondary tensor perturbations is given by

$$\begin{aligned}\overline{\mathcal{P}_h(k,\eta)} \quad = \quad & \frac{2}{81\,k^2\,\eta^2}\int_0^{\infty}\mathrm{d}v\int_{|1-v|}^{1+v}\mathrm{d}u\,\left[\frac{4\,v^2-(1+v^2-u^2)^2}{4\,u\,v}\right]^2\mathcal{P}_{\mathcal{R}}(k\,v)\,\mathcal{P}_{\mathcal{R}}(k\,u)\\ & \times\,\left[\mathcal{I}_c^2(u,v)+\mathcal{I}_s^2(u,v)\right],\end{aligned}$$

where $\mathcal{I}_c$ and $\mathcal{I}_s$ arise from the transfer functions.
The dimensionless energy density of the secondary GWs today is

$$\mathrm{h}^2\,\Omega_{\mathrm{GW}}(k) = \frac{1}{24}\,\left(\frac{g_{*,k}}{g_{*,0}}\right)^{-1/3}\Omega_{\mathrm{r}}\,\mathrm{h}^2\,\left(\frac{k}{aH}\right)^2\overline{\mathcal{P}_h(k,\eta)}\,.$$

[13] See, for instance, *K. Kohri, and T. Terada, Phys. Rev.* **D 97**, *123532 (2018)*.

Appendix - III

The source function in the evolution of mode function of the secondary tensor perturbation is given by

$$S_{\boldsymbol{k}}^{\lambda}(\eta) = 4\int \frac{\mathrm{d}^3\boldsymbol{p}}{(2\,\pi)^{3/2}}\, e^{\lambda}(\boldsymbol{k},\boldsymbol{p}) \Biggl\{ 2\,\Psi_{\boldsymbol{p}}(\eta)\,\Psi_{\boldsymbol{k}-\boldsymbol{p}}(\eta) + \frac{4}{3\,(1+w)\,\mathcal{H}^2}\,\left[\Psi_{\boldsymbol{p}}'(\eta)+\mathcal{H}\,\Psi_{\boldsymbol{p}}(\eta)\right]\,\left[\Psi_{\boldsymbol{k}-\boldsymbol{p}}'(\eta)+\mathcal{H}\,\Psi_{\boldsymbol{k}-\boldsymbol{p}}(\eta)\right]\Biggr\}^{[14]}\,.$$

The Bardeen potential is related to the primordial scalar perturbation as

$$\Psi_{\boldsymbol{k}}(\eta) = \frac{2}{3}\,\mathcal{T}(k\,\eta)\,\mathcal{R}_{\boldsymbol{k}}\,,$$

where $\mathcal{T}(k\,\eta)$ is the transfer function

$$\mathcal{T}(k\,\eta) = \frac{9}{(k\,\eta)^2}\left[\frac{\sin\left(k\,\eta/\sqrt{3}\right)}{k\,\eta/\sqrt{3}} - \cos\left(k\,\eta/\sqrt{3}\right)\right].$$

[14] $\mathcal{H} = aH$

Appendix - IV

The action cubic order in $\mathcal{R}$ that contributes to the scalar bispectrum is given by

$$\begin{aligned}\delta S_3[\mathcal{R}] &= M_{\rm Pl}^2 \int_{\eta_{\rm i}}^{\eta_{\rm e}} {\rm d}\eta \int {\rm d}^3\boldsymbol{x} \biggl[a^2\,\epsilon_1^2\,\mathcal{R}\,\mathcal{R}'^2 + a^2\,\epsilon_1^2\,\mathcal{R}\,(\partial\mathcal{R})^2 - 2\,a\,\epsilon_1\,\mathcal{R}'\,(\partial\mathcal{R})\,(\partial\chi) \\ &\quad + \frac{a^2}{2}\,\epsilon_1\,\epsilon_2'\,\mathcal{R}^2\,\mathcal{R}' + \frac{\epsilon_1}{2}\,(\partial\mathcal{R})\,(\partial\chi)\,\partial^2\chi + \frac{\epsilon_1}{4}\,\partial^2\mathcal{R}\,(\partial\chi)^2 + 2\,\mathcal{F}(\mathcal{R})\,\frac{\delta\mathcal{L}_2}{\delta\mathcal{R}}\biggr],\end{aligned}$$ [15]

$$\begin{aligned}\delta S_3^{\rm B}[\mathcal{R}] &= M_{\rm Pl}^2 \int_{\eta_{\rm i}}^{\eta_{\rm e}} {\rm d}\eta \int {\rm d}^3\boldsymbol{x}\, \frac{\rm d}{{\rm d}\eta}\biggl\{-9\,a^3\,H\,\mathcal{R}^3 + \frac{a}{H}\,(1-\epsilon_1)\,\mathcal{R}\,(\partial\mathcal{R})^2 \\ &\quad - \frac{1}{4\,a\,H^3}\,(\partial\mathcal{R})^2\,\partial^2\mathcal{R} - \frac{a\,\epsilon_1}{H}\,\mathcal{R}\,\mathcal{R}'^2 - \frac{a\,\epsilon_2}{2}\,\mathcal{R}^2\,\partial^2\chi \\ &\quad + \frac{1}{2\,a\,H^2}\,\mathcal{R}\,\left(\partial_i\partial_j\mathcal{R}\,\partial_i\partial_j\chi - \partial^2\mathcal{R}\,\partial^2\chi\right) \\ &\quad - \frac{1}{2\,aH}\,\mathcal{R}\,\left[\partial_i\partial_j\chi\,\partial_i\partial_j\chi - (\partial^2\chi)^2\right]\biggr\}.\end{aligned}$$ [16]

[15] See, for instance, *J. Maldacena, JHEP* **05**, *013 (2003).*
[16] *F. Arroja, and T. Tanaka, JCAP* **1105** *005 (2011)*

Appendix - V

The full expressions of all the contributions to the scalar bispectrum are listed below.

$$
\begin{aligned}
G(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) &= \sum_{C=1}^{9} G_C(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) \\
&= M_{\mathrm{Pl}}^2 \sum_{C=1}^{6} \Big[f_{k_1}(\eta_{\mathrm{e}})\, f_{k_2}(\eta_{\mathrm{e}})\, f_{k_3}(\eta_{\mathrm{e}}) \\
&\quad \times \mathcal{G}_C(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) + \text{complex conjugate} \Big] \\
&\quad + G_7(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) \\
&\quad + G_8(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) + G_9(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3)\,.
\end{aligned}
$$

Bulk terms:

$$
\begin{aligned}
\mathcal{G}_1(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) &= 2\,i \int_{\eta_{\mathrm{i}}}^{\eta_{\mathrm{e}}} \mathrm{d}\eta\; a^2\, \epsilon_1^2\, \left(f_{k_1}^{*}\, f_{k_2}^{\prime *}\, f_{k_3}^{\prime *} + \text{two permutations} \right), \\
\mathcal{G}_2(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) &= -2\,i\; (\boldsymbol{k}_1 \cdot \boldsymbol{k}_2 + \text{two permutations}) \int_{\eta_{\mathrm{i}}}^{\eta_{\mathrm{e}}} \mathrm{d}\eta\; a^2\, \epsilon_1^2\, f_{k_1}^{*}\, f_{k_2}^{*}\, f_{k_3}^{*}, \\
\mathcal{G}_3(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) &= -2\,i \int_{\eta_{\mathrm{i}}}^{\eta_{\mathrm{e}}} \mathrm{d}\eta\; a^2\, \epsilon_1^2\, \left(\frac{\boldsymbol{k}_1 \cdot \boldsymbol{k}_2}{k_2^2}\, f_{k_1}^{*}\, f_{k_2}^{\prime *}\, f_{k_3}^{\prime *} + \text{five permutations} \right),
\end{aligned}
$$

Appendix - V

$$\mathcal{G}_4(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) = i\int_{\eta_\mathrm{i}}^{\eta_\mathrm{e}} \mathrm{d}\eta\; a^2\,\epsilon_1\,\epsilon_2'\,\left(f_{k_1}^*\,f_{k_2}^*\,f_{k_3}'^* + \text{two permutations}\right),$$

$$\mathcal{G}_5(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) = \frac{i}{2}\int_{\eta_\mathrm{i}}^{\eta_\mathrm{e}} \mathrm{d}\eta\; a^2\,\epsilon_1^3\,\left(\frac{\boldsymbol{k}_1\cdot\boldsymbol{k}_2}{k_2^2}\,f_{k_1}^*\,f_{k_2}'^*\,f_{k_3}'^* + \text{five permutations}\right),$$

$$\mathcal{G}_6(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) = \frac{i}{2}\int_{\eta_\mathrm{i}}^{\eta_\mathrm{e}} \mathrm{d}\eta\, a^2\,\epsilon_1^3\,\left(\frac{k_1^2\,(\boldsymbol{k}_2\cdot\boldsymbol{k}_3)}{k_2^2\,k_3^2}\,f_{k_1}^*\,f_{k_2}'^*\,f_{k_3}'^* + \text{two permutations}\right).$$

$$\begin{aligned} G_7(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) = & -i\,M_{\mathrm{Pl}}^2\,\left[f_{k_1}(\eta_\mathrm{e})\,f_{k_2}(\eta_\mathrm{e})\,f_{k_3}(\eta_\mathrm{e})\right] \\ & \times\left[a^2\epsilon_1\epsilon_2\,f_{k_1}^*(\eta)\,f_{k_2}^*(\eta)\,f_{k_3}'^*(\eta) + \text{two permutations}\right]_{\eta_i}^{\eta_\mathrm{e}} \\ & + \text{complex conjugate}. \end{aligned}$$ [17]

[17] See, for instance, *J. Martin, and L. Sriramkumar, JCAP* **1201**, *008 (2012).*

Appendix - V

Boundary terms:

$$G_8(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) = i\, f_{k_1}(\eta_{\rm e})\, f_{k_2}(\eta_{\rm e})\, f_{k_3}(\eta_{\rm e}) \left[\frac{a}{H}\, f_{k_1}^{*}(\eta)\, f_{k_2}^{*}(\eta)\, f_{k_3}^{*}(\eta)\right]_{\eta_{\rm i}}$$

$$\times \Bigg\{54\,(a\,H)^2 + 2\,(1-\epsilon_1)\,(\boldsymbol{k}_1\cdot\boldsymbol{k}_2 + \boldsymbol{k}_1\cdot\boldsymbol{k}_3 + \boldsymbol{k}_2\cdot\boldsymbol{k}_3)$$

$$+ \frac{1}{2\,(a\,H)^2}\left[(\boldsymbol{k}_1\cdot\boldsymbol{k}_2)\,{k_3}^2 + (\boldsymbol{k}_1\cdot\boldsymbol{k}_3)\,{k_2}^2 + (\boldsymbol{k}_2\cdot\boldsymbol{k}_3)\,{k_1}^2\right]\Bigg\}_{\eta_{\rm i}}$$

$$+ \text{complex conjugate},$$

$$G_9(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) = i\, f_{k_1}(\eta_{\rm e})\, f_{k_2}(\eta_{\rm e})\, f_{k_3}(\eta_{\rm e}) \Bigg\{\frac{\epsilon_1}{2\,H^2}\, f_{k_1}^{*}(\eta)\, f_{k_2}^{*}(\eta)\, f_{k_3}^{\prime *}(\eta)$$

$$\times \left[k_1^2 + k_2^2 - \left(\frac{\boldsymbol{k}_1\cdot\boldsymbol{k}_3}{k_3}\right)^2 - \left(\frac{\boldsymbol{k}_2\cdot\boldsymbol{k}_3}{k_3}\right)^2\right]$$

$$- \frac{a\,\epsilon_1}{H}\, f_{k_1}^{*}(\eta)\, f_{k_2}^{\prime *}(\eta)\, f_{k_3}^{\prime *}(\eta) \left[2 - \epsilon_1 + \epsilon_1 \left(\frac{\boldsymbol{k}_2\cdot\boldsymbol{k}_3}{k_2\,k_3}\right)^2\right]\Bigg\}_{\eta_{\rm i}}^{\eta_{\rm e}}$$

$$+ \text{two permutations} + \text{complex conjugate}.$$ [18]

[18] *H. V. Ragavendra, D. Chowdhury, and L. Sriramkumar, arXiv:2003.01099 [astro-ph.CO]*