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POTENTIAL RECONSTRUCTION FROM GENERAL POWER SPECTRUM IN INFLATION

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INTRODUCTION

The zoology of the viable models of slow-roll inflation is vast . However, there is no a priori reason why one model is more favourable than others. Exactly speaking, certain models may enjoy theoretical advantages. Such models would be theoretically more aesthetic than others. But regarding observational viability, as long as observational constraints are satisfied within the same confidence level, every model is equivalent.

Therefore, an attractive alternative to the model building of slow-roll inflation is, directly from the given power spectrum, to reconstruct the potential.

We suggest a method to reconstruct, within canonical single-field inflation, the inflaton potential directly from the primordial power spectrum which may deviate significantly from near scale-invariance.

Our approach relies on a more Generalized Slow-Roll (GSR) approximation than the standard one

EVOLUTION OF PERTURBATIONS

$$v'' + \left(k^2 - \frac{z''}{z}\right) v = 0$$

$$x \equiv k\xi \qquad y \equiv \sqrt{2k}v \qquad \xi \equiv -\int \frac{dt}{a} = \frac{1}{aH}\left[1 + \mathcal{O}(\epsilon)\right]$$

$$\frac{d^2y}{dx^2} + \left(1 - \frac{2}{x^2}\right) y = \frac{1}{x^2} g y$$

$$g = (\xi a H)^2 \left(2 - \epsilon_1 + \frac{3}{2}\epsilon_2 - \frac{1}{2}\epsilon_1\epsilon_2 + \frac{1}{4}\epsilon_2^2 + \frac{1}{2}\epsilon_2\epsilon_3\right) - 2$$

The function g represents all the possible deviations from the scale-invariance of the power spectrum.

g=CONSTANT

$$\frac{d^2y}{dx^2} + \left(1 - \frac{2-\epsilon_1}{x^2(1-\epsilon_1)^2}\right) y = 0$$

$$P_{\mathcal{R}}(k) = \frac{1}{\pi}|\Gamma(\nu_y)|^2 \left(\frac{k\eta}{2}\right)^{3-2\nu_y} \times \frac{H(\eta)^2}{2\pi^2\epsilon_1(\eta)}$$

$$\nu_y = \frac{3+\epsilon}{2(1-\epsilon)}$$

$$\boxed{\mathcal{P}_{\mathcal{R}}(k) \approx x^{-2\epsilon_1}\frac{H^2}{4\pi^2\epsilon_1}}$$

Power law spectrum

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GREEN'S FUNCTION METHOD

$$\frac{d^2y}{dx^2} + \left(1 - \frac{2}{x^2}\right) y = \frac{1}{x^2} g y$$

$$y(x) = y_0(x) - \int_x^\infty \frac{du}{u^2} g(u) y_0(u) \, \mathrm{Im}[y_0^*(u) y_0(x)]$$

$$y_0(x) = \left(1 + \frac{i}{x}\right) e^{ix}$$

$$\boxed{\lim_{x\to 0} P_{\mathcal{R}} \approx x^{-2g/3} \frac{H^2}{4\pi^2\epsilon_1} = x^{-2\epsilon_1} \frac{H^2}{4\pi^2\epsilon_1}}$$

Power law spectrum

GSR POWER SPECTRUM

$$\log \mathcal{P}_{\mathcal{R}}(k) = \int_0^\infty \frac{d\xi}{\xi}\left[-k\xi W'(k\xi)\right]\left[\log\left(\frac{1}{f^2}\right) + \frac{2}{3}\frac{f'}{f} + \mathcal{O}(g^2)\right]$$

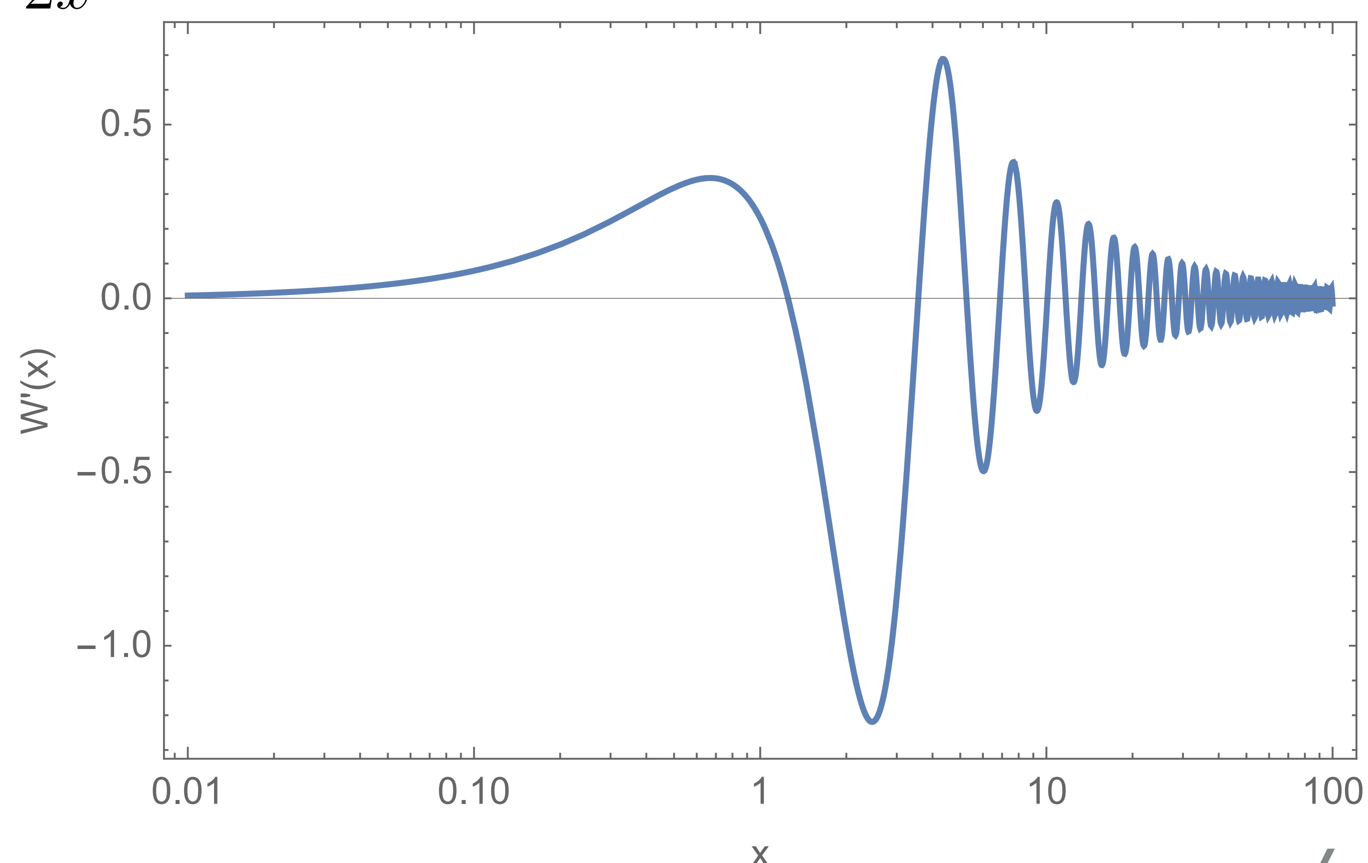
$$f(\log\xi) \equiv \frac{2\pi x z}{k} = \frac{2\pi a \xi \dot{\phi}}{H}$$

The Window function

$$W(x) = \frac{3\sin(2x)}{2x^3} - \frac{3\cos(2x)}{x^2} - \frac{3\sin(2x)}{2x} - 1$$

has the property

$$\int_0^\infty \frac{\mathrm{d}x}{x}\left[-x\,W'(x)\right] = 1$$



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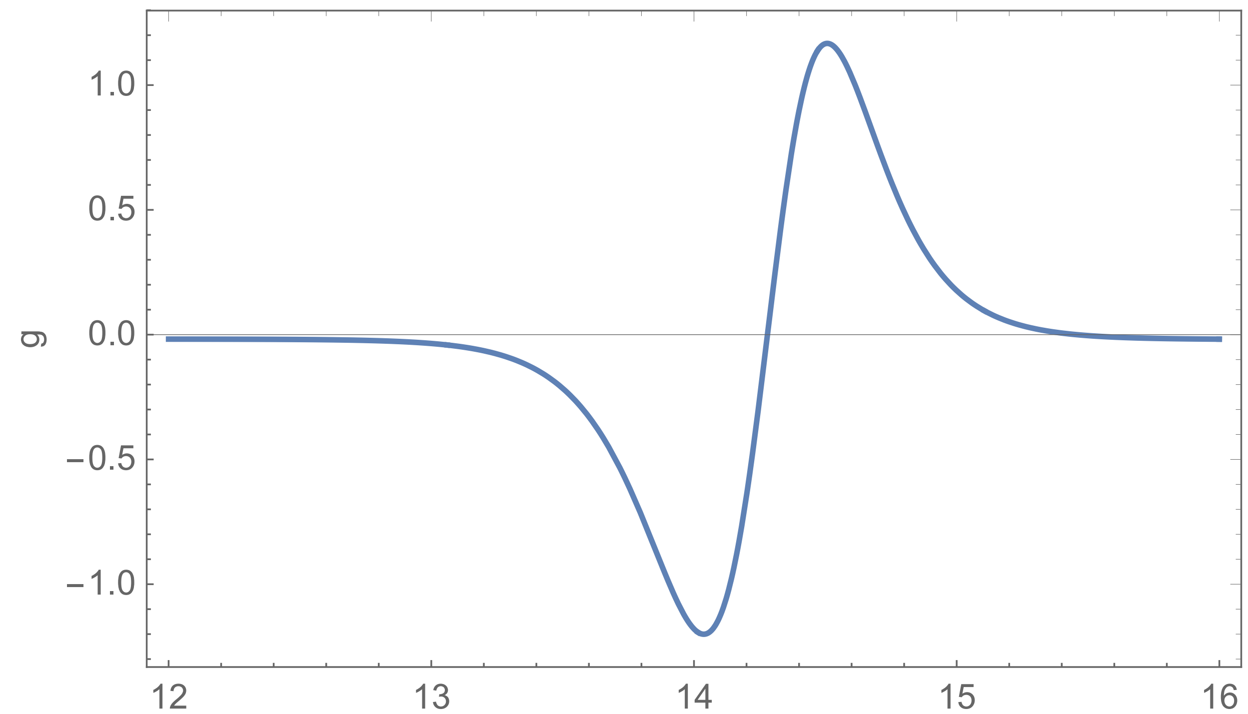
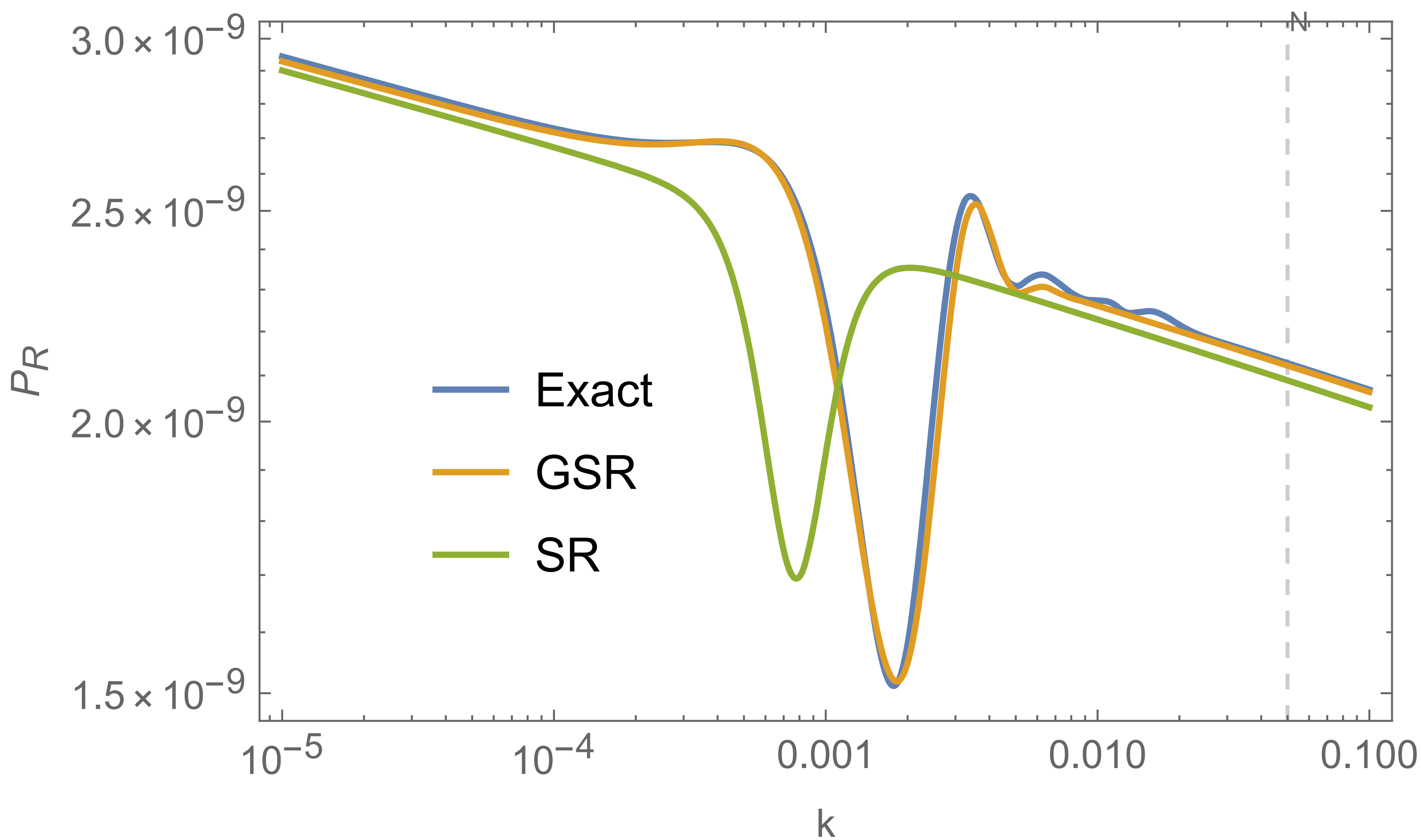
POTENTIAL WITH A STEP

$$V(\phi) = \frac{1}{2} m^2 \phi^2 \left(1 + c \tanh \frac{\phi - b}{d} \right).$$





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RECONSTRUCTION :GSR

$$\boxed{\log\left(\frac{1}{f^2}\right) = \int_0^\infty \frac{dk}{k} m(k\xi) \log \mathcal{P}_{\mathcal{R}}(k)}$$

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$$m(x) = \frac{2}{\pi}\left[\frac{1}{x} - \frac{\cos(2x)}{x} - \sin(2x)\right]$$

$$\dot{H} = -\frac{\dot{\phi}^2}{2m_{\mathrm{Pl}}^2} = -\frac{H^2 f^2}{2(2\pi)^2 m_{\mathrm{Pl}}^2 a^2 \xi^2}$$

$$H^{-3}\frac{dH}{d\xi} = \frac{1}{2(2\pi)^2 m_{\mathrm{Pl}}^2}\frac{f^2}{\xi}$$

$$\frac{d\phi}{d\log\xi} = -\frac{fH}{2\pi}$$

$$V = 3m_{\mathrm{Pl}}^2 H^2 - \frac{1}{2}\dot{\phi}^2 = 3m_{\mathrm{Pl}}^2 H^2\left[1 - \frac{f^2 H^2}{6(2\pi)^2 m_{\mathrm{Pl}}^2}\right]$$

RECONSTRUCTION : POWER LAW SPECTRUM

$$\mathcal{P}_{\mathcal{R}}(k) = A_s \left(\frac{k}{k_*}\right)^{n_s - 1} \longrightarrow f^2 = \frac{(k_* \xi)^{n_s-1}}{A_s e^{\alpha(n_s-1)}}$$

$$\alpha \equiv 2 - \log 2 - \gamma$$

$$V(\phi) = \frac{3 m_{\rm Pl}^2 H_i^2 \beta}{\beta - (k_* \xi_i)^{n_s-1}} \frac{1 - \frac{1}{6}(1-n_s)\tanh^2\left[\sqrt{1-n_s}\frac{\phi-\phi_0}{2m_{\rm Pl}}\right]}{1 + \sinh^2\left[\sqrt{1-n_s}\frac{\phi-\phi_0}{2m_{\rm Pl}}\right]}$$

$$V(\phi) = 3 m_{\rm Pl}^2 H_i^2 \beta \exp\left(\sqrt{1-n_s}\frac{\phi-\phi_0}{m_{\rm Pl}}\right)$$

$\Delta\phi > \mathrm{M_P}$

$$V(\phi) \approx 3 m_{\rm Pl}^2 H_i^2 \left[1 - \frac{1-n_s}{4}(\Delta\phi)^2\right]$$

$\Delta\phi < \mathrm{M_P}$

FEATURED POWER SPECTRUM

$$\log \mathcal{P}_{\mathcal{R}}(k) = \log \mathcal{P}^0_{\mathcal{R}}(k) + \mathcal{I}_0(k) + \log\left[1 + \mathcal{I}_1^2(k)\right]$$

Planck 2018: Astron.Astrophys. 641 (2020), A10

$$\begin{aligned}
\mathcal{I}_0(k) &= \mathcal{A}_s \mathcal{W}_0(k/k_s)\mathcal{D}\left(\frac{k/k_s}{x_s}\right), \\
\mathcal{I}_1(k) &= \frac{1}{\sqrt{2}}\left[\frac{\pi}{2}(1-n_s) + \mathcal{A}_s \mathcal{W}_1(k/k_s)\mathcal{D}\left(\frac{k/k_s}{x_s}\right)\right], \\
\mathcal{W}_0(x) &= \frac{1}{2x^4}\left[(18x - 6x^3)\cos(2x) + (15x^2 - 9)\sin(2x)\right], \\
\mathcal{W}_1(x) &= -\frac{3}{x^4}(x\cos x - \sin x)\left[3x\cos x + (2x^2 - 3)\sin x\right], \\
\mathcal{D}(x) &= \frac{x}{\sinh x}.
\end{aligned}$$