# Reconstruction of potentials of hybrid inflation in the light of primordial black hole formation 

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## Motivation



Nearly scale invariant + peak

$$
\begin{aligned}
& \text { Maximum } \approx 10^{-2} \\
& \frac{P(\text { maximum })}{P(C M B)} \approx 10^{7}
\end{aligned}
$$

Single field models: point of inflection (1712.09750), USR (2008.12202) bump/dip in the potential (1911.00057)

Two field models : with non canonical KE term (2004.08369, 2005.02895) resonant amplification (2010.03537)
Hybrid inflation (1501.07565)

## Motivation

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## Massive primordial black holes from hybrid inflation as dark matter and the seeds of galaxies

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$V(\phi, \psi)=\Lambda\left[\left(1-\frac{\psi^{2}}{M^{2}}\right)^{2}+\frac{\left(\phi-\phi_{\mathrm{c}}\right)}{\mu_{1}}-\frac{\left(\phi-\phi_{\mathrm{c}}\right)^{2}}{\mu_{2}^{2}}+\frac{2 \phi^{2} \psi^{2}}{M^{2} \phi_{\mathrm{c}}^{2}}\right]$

$$
\mathcal{P}_{\zeta}(k) \simeq \frac{\Lambda M^{2} \mu_{1} \phi_{\mathrm{c}}}{192 \pi^{2} M_{\mathrm{P}}^{6} \chi_{2} \psi_{k}^{2}}
$$



Hybrid inflation : Potential


We consider waterfall period $N \approx 20$

## $\delta N$ formalism

Misao Sasaki, Ewan D. Stewart Prog.Theor.Phys.95:71-78,1996


$$
N\left(t_{e}, t_{*}, \mathbf{x}\right) \equiv \int_{t_{*}}^{t_{e}} H \mathrm{~d} t \quad \text { In super-Hubble. regime }
$$

$$
\delta N=\mathcal{R} \text { at } t=t_{e}
$$

$\delta N \equiv N(\phi, \phi+\delta \phi)-N(\phi) \quad$ In slow roll approximation

$$
\delta N=\frac{\partial N}{\partial \phi_{*}} \delta \phi_{*}=\mathcal{R}
$$

If we know $N(\phi)$ and $\delta \phi_{*}$ then we can compute $\mathcal{R}$ at the end of inflation

## $\delta N$ formalism : Two field

Misao Sasaki, Ewan D. Stewart Prog.Theor.Phys.95:71-78,1996
Juan Garcia-Bellido, David Wands, Phys. Rev. D 53, 5437 (1996)
Misao Sasaki, Takahiro Tanaka Prog.Theor.Phys.99:763-782,1998

$$
\begin{aligned}
& \delta N=\frac{\partial N}{\partial \phi_{*}} \delta \phi_{*}+\frac{\partial N}{\partial \psi_{*}} \delta \psi_{*} \\
& \qquad \mathcal{P}_{\mathcal{R}}=\frac{H_{*}^{2}}{4 \pi^{2}}\left[\left(\frac{\partial N}{\partial \phi_{*}}\right)^{2}+\left(\frac{\partial N}{\partial \psi_{*}}\right)^{2}\right]
\end{aligned}
$$

For non slow roll $N=N(\phi, \dot{\phi})$

## Small field hybrid inflation

$$
V=V_{0}[1+f(\phi)+g(\phi) h(\psi)]
$$

$\phi$ is dominant during inflation- inflation

$\psi$ is responsible for the ending of inflation - waterfall field

Assume slow roll in both directions

$$
\begin{gathered}
N=N\left(\phi_{*}, \phi_{e}\left(\psi_{*}\right)\right)=N\left(\phi_{*}, \psi_{*}\right)=-\frac{1}{M_{\mathrm{P}}^{2}} \int_{\phi_{*}}^{\phi_{e}} \frac{V}{V_{\phi}} d \phi \\
\phi_{e}=\phi_{e}\left(\psi_{*}\right) \\
\mathcal{P}_{\zeta}(k)=\mathcal{P}_{s}(k)+\mathcal{P}_{p}(k)
\end{gathered}
$$

$$
\begin{aligned}
& \mathcal{P}_{s}=\frac{H_{*}^{2}}{4 \pi^{2} M_{\mathrm{P}}^{4}}\left[\frac{1}{f_{\phi_{*}}}\left(1-\frac{g_{*}}{g_{e}}\right)\right]^{2} \\
& \mathcal{P}_{p}=\frac{H_{*}^{2}}{4 \pi^{2} M_{\mathrm{P}}^{4}}\left[\frac{1}{g_{e} h_{\psi_{*}}}\right]^{2}
\end{aligned}
$$

## Reconstruction

$$
\begin{aligned}
\mathcal{P}_{s}(k) & \equiv A_{s}\left(\frac{k}{k_{p}}\right)^{n_{s}-1} \\
f(\phi) & =-\left(\frac{\phi}{\mu}\right)^{2} \\
N_{t} & =N_{t}(\phi)
\end{aligned}
$$



$$
\mathcal{P}_{p}=\frac{H_{*}^{2}}{4 \pi^{2}}\left[\delta+\beta e^{-\alpha\left(N_{t}-N_{t c}\right)^{2}}-e^{\lambda\left(N_{t}-N_{t 1}\right)}\right]^{2}
$$

$$
\text { Assume } \quad h(\psi)=\frac{1}{n}\left(\frac{\psi}{\kappa}\right)^{n}
$$

$$
g(\phi)=-\frac{H^{2}}{V_{0} h_{\psi}\left(N_{t}\right)}\left[\frac{d^{2} \psi}{d N_{t}^{2}}+\left(3-\epsilon_{H}\right) \frac{d \psi}{d N_{t}}\right]
$$



## Background

$$
\frac{d^{2} \phi}{d N_{t}^{2}}+\left(3-\epsilon_{H}\right) \frac{d \phi}{d N_{t}}+\frac{V_{\phi}}{H^{2}}=0
$$




## Power spectrum

$$
\begin{gathered}
v_{\sigma}^{\prime \prime}+\left(k^{2}-\frac{z^{\prime \prime}}{z}\right) v_{\sigma}=\frac{2}{z}\left(a z \dot{\theta} v_{s}\right)^{\prime} \\
v_{s}^{\prime \prime}+\left(k^{2}-\frac{a^{\prime \prime}}{a}+a^{2} \mu_{s}^{2}\right) v_{s}=-2 a z \dot{\theta}\left(\frac{v_{\sigma}}{z}\right)^{\prime} \\
\mu_{s}^{2}=V_{s s}-\dot{\theta}^{2} \\
\hat{v}_{\sigma}=f_{\sigma} \hat{a}+f_{\sigma}^{\star} \hat{a}^{\dagger}+g_{\sigma} \hat{b}+g_{\sigma}^{\star} \hat{b}^{\dagger} \\
\hat{v}_{s}=f_{s} \hat{a}+f_{s}^{\star} \hat{a}^{\dagger}+g_{s} \hat{b}+g_{s}^{\star} \hat{b}^{\dagger}
\end{gathered}
$$

$$
\mathcal{P}_{\mathcal{R}}=\frac{k^{3}}{2 \pi^{2}} \frac{\left|f_{\sigma}\right|^{2}+\left|g_{\sigma}\right|^{2}}{z^{2}}
$$

## Hyperbolic peak

$$
\mathcal{P}_{p}=\frac{H_{*}^{2}}{4 \pi^{2}}\left[\delta+\beta \operatorname{sech}\left[\gamma\left(N_{t}-N_{t c}\right)\right]-e^{\lambda\left(N_{t}-N_{t 1}\right)}\right]^{2}
$$



$$
\begin{gathered}
v_{s}^{\prime \prime}+\left(k^{2}-\frac{a^{\prime \prime}}{a}+a^{2} \mu_{s}^{2}\right) v_{s}=-2 a z \dot{\theta}\left(\frac{v_{\sigma}}{z}\right)^{\prime} \\
\mu_{s}^{2}=V_{s s}-\dot{\theta}^{2}
\end{gathered}
$$

In our model $v_{s s}<0$, isocurvature increases and transfer the power to curvature perturbation at the end

There are model with $\dot{\theta}^{2}>V_{s s}$
G. A. Palma, S. Sypsas and C. Zenteno, Phys. Rev. Lett. 125 (2020) no.12, 121301
M. Braglia, D. K. Hazra, F. Finelli, G. F. Smoot, L. Sriramkumar and A. A. Starobinsky, JCAP 08 (2020), 001
J. Fumagalli, S. Renaux-Petel, J. W. Ronayne and L. T. Witkowski, [arXiv:2004.08369 [hep-th]]

## On going

- Large field models
- Non canonical kinetic term
- Constructing models for steeper peaks like $k^{4}$

