Reheating: Characteristics and Constraints

based on arXiv:1811.11173[astro-ph.CO] and arXiv:2005.01874[astro-ph.CO]

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- Introduction.
- Preheating and its characteristics.
- Constraints on reheating.
- Summary.

Introduction

What is Inflation?

- ∗ Inflation[1] is a period of exponential expansion of the universe in a state of negative pressure. i.e.,$\ddot{a} > 0$,
- ∗ We require a 'sufficient amount of inflation' to produce a homogeneous and isotropic universe that acts as an initial condition for hot big bang evolution.
- ∗ The quantum fluctuations in the inflaton generates the seed for large scale structures.
- The inflationary expansion leaves a universe void of any matter or radiation with most of its energy in the (homogeneous) inflaton field.
- The universe defrost in a phase of reheating when the inflation decay to produce all the matter and radiation content of the universe

[1] Albrecht, Andreas and Paul J. Steinhardt (1982). *Phys. Rev. Lett.* 48, pp. 1220–1223; Guth, Alan H. (1981). *Phys. Rev.* D23, pp. 347–356; Linde, Andrei D. (1982). *Phys. Lett.* 108B, pp. 389–393; Starobinsky, Alexei A. (1980). *Phys. Lett.* B91, pp. 99–102.

Initial idea to reheat the universe

- The original idea of reheating was introduces just after the proposal of an inflationary phase.
- They considered a perturbative decay of inflaton into radiation component with a phenomenological decay term $\Gamma_\phi \dot{\phi}$ in the inflaton equation[2].
- In a pioneering paper[3], the authors pointed out that the perturbative decay do not correctly describe the inflaton decay in the initial phase.

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[2] Abbott, L. F. et al. (1982). *Phys. Lett.* 117B, p. 29; Albrecht, Andreas et al. (1982). *Phys. Rev. Lett.* 48, p. 1437; Dolgov, A. D. and Andrei D. Linde (1982). *Phys. Lett.* 116B, p. 329.

[3] Kofman, Lev et al. (1994). *Phys. Rev. Lett.* 73, pp. 3195–3198.

Preheating after Inflation[4]

- After inflation, the homogeneous inflaton field oscillates coherently.
- It decays to other field depending upon their coupling to inflaton via parametric resonance.

$$\mathcal{L} = \underbrace{\frac{1}{2}(\partial_\mu \phi)^2 - V(\phi)}_{\text{Inflaton}} + \underbrace{\frac{1}{2}(\partial_\mu \chi)^2 - \frac{1}{2}m_\chi^2 \chi^2}_{\text{Scalar(daughter) field}} - \underbrace{\frac{1}{2}g^2\phi^2\chi^2}_{\text{Interaction term}} \quad (1)$$

- Quantum particle production in the presence of time dependent classical background.
- Parametric resonance: The mode functions for scalar field($a^{6/(n+2)}\chi_k = X_k$) satisfy Hill/Mathieu equation:

$$\ddot{X}_k + \omega_k^2 X_k = 0; \quad (2)$$

$$\omega_k^2 \equiv \frac{k^2}{a^2} + g^2\phi^2 \quad (3)$$

- The Hill/Mathieu equations

$$\ddot{X} + (\kappa^2 + q\phi^2)X = 0$$

shows parametric growth depending upon the parameter (q,κ^2).

[4] with D. Maity, JCAP **07** (2019) 018

The inflationary model[5]

- We worked with the following class inflationary models:

$$V(\phi) = \begin{cases} \frac{m^{4-n}}{n}\frac{\phi^n}{1+\left(\frac{\phi}{\phi_*}\right)^n} & n \neq 4 \\ \frac{\lambda}{4}\frac{\phi^4}{1+\left(\frac{\phi}{\phi_*}\right)^4} & n = 4 \end{cases} \tag{4}$$

- Shows plateau for large field values, while behaves as $V(\phi) \propto \phi^n$ around $\phi = 0$.
- ϕ_* controls the width of the potential for a fixed n.

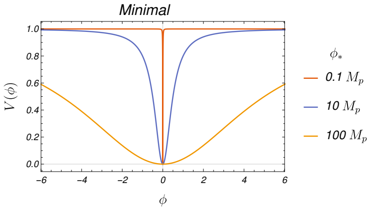


Figure: An illustration of the potential shape

[5] D Maity and PS (2019). *Class. Quant. Grav.* 36, p. 045010.

The instability diagram

Before studying the full-nonlinear dynamics numerically, let us try to understand what we may expect form the simulation by studying the instability diagram for Hill/Mathieu equation.

- The bands show the region in the (q,κ^2) space where the solution will have exponential growth.
- As we increase n, we will have strong resonance shown as increasing Floquet cofficient $\Re(\mu)$ $\Rightarrow$ Increased Thermalization
- For a fixed n, decreasing ϕ_* will have the same effect.

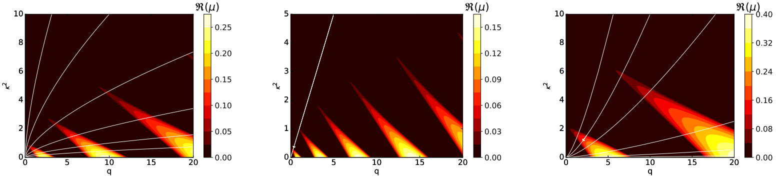


Figure: The instability bands for the χ quanta for $n = 2, 4,$ and 6 and $\phi_* = 10M_\mathrm{p}$

Preheating: Lattice Simulation

- The energy density of the universe just after inflation is in the form of homogeneous inflaton field.
- This energy starts decaying into fluctuations of the inflaton and other fields at the onset of preheating.
- The initial stage of preheating is marked by exponential growth of decay product due to resonance.
- The production of these highly inhomogeneous non-thermal products continues until back-reaction effects renders the preheating inefficient.
- The system is highly non-linear and we have to resort to numerical schemes. Also, the effects of back-reaction can be incorporated numerically.
- We will use `LATTICEEASY`[6], the standard workhorse for calculating nonlinear field dynamics in an expanding FRW universe

[6] Felder, Gary N. and Igor Tkachev (2008). *Comput. Phys. Commun.* 178, pp. 929–932.

Basic equations and quantities

- The field equations $f = (\phi, \chi)$:

$$\ddot{f}_i + 3H\dot{f}_i - \frac{1}{a^2}\nabla^2 f_i + \frac{\partial V(f_i)}{\partial f_i} = 0, \tag{5}$$

- The Friemann equation:

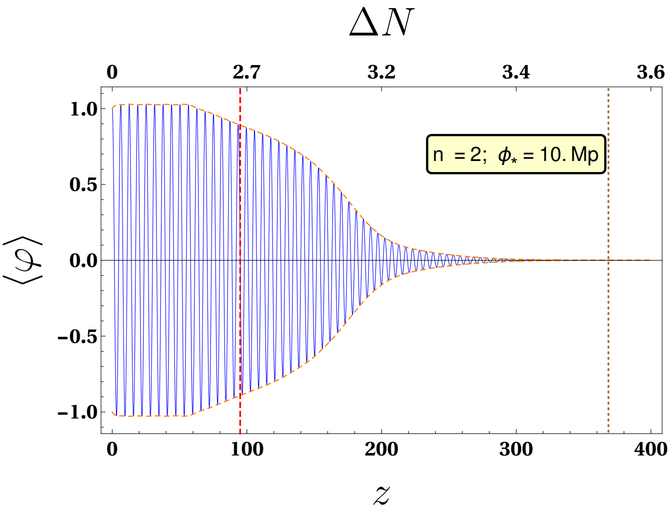
$$3\mathrm{M}_{\mathrm{p}}^2 H^2 = \sum_i \frac{1}{2}\dot{f}_i^2 + \frac{1}{2a^2}|\nabla f_i|^2 + V(f_i)$$

- with

$$n_k = \frac{1}{\omega_k}\left|\dot{f}_k\right|^2 + \frac{\omega_k}{2}|f_k|^2, \quad \omega_k \equiv \sqrt{k^2 + m_{\mathrm{eff}}^2} \quad m_{\mathrm{eff}}^2 \equiv \frac{\partial^2 V}{\partial f^2}$$

Results: Preheating phases

- Episodes in preheating: : 1. Resonance phase, 2. Non-linear regime, and, 3. Stationary phase.



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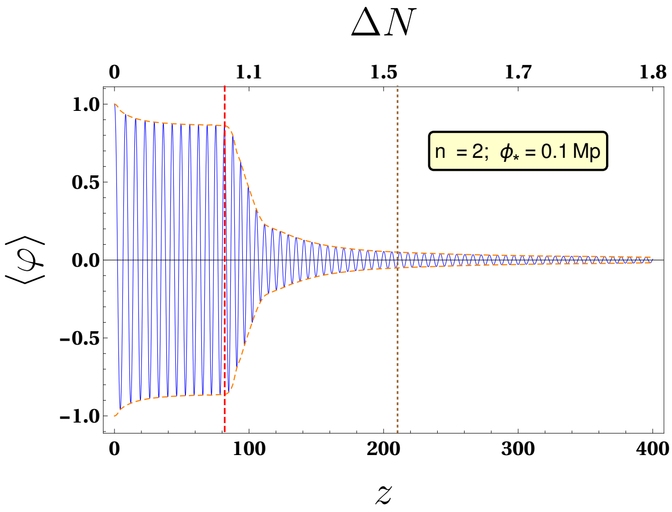
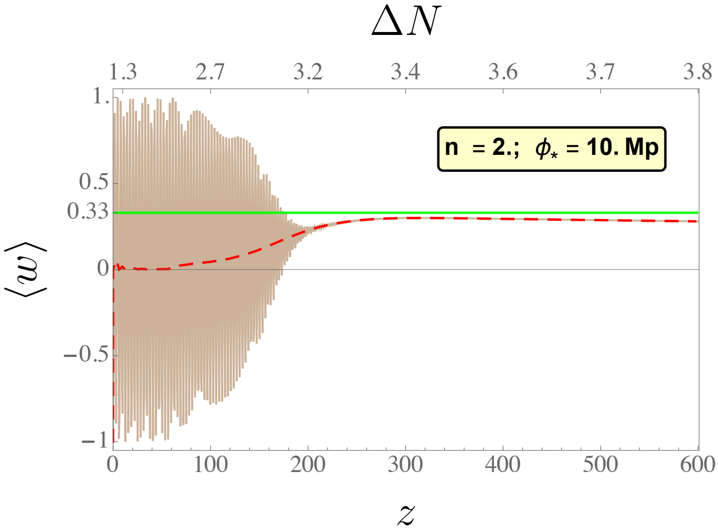


Results: EOS

- For $n = 2$, eos do not reach radiation eos.



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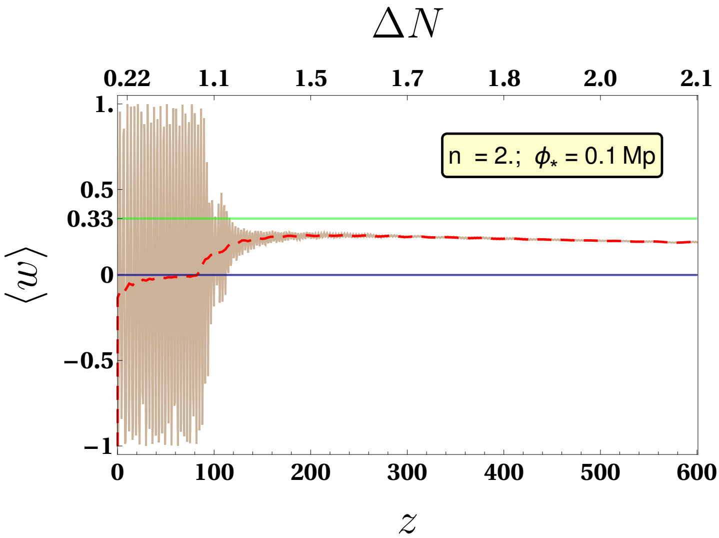
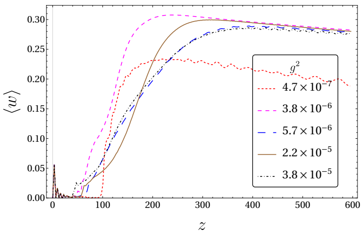
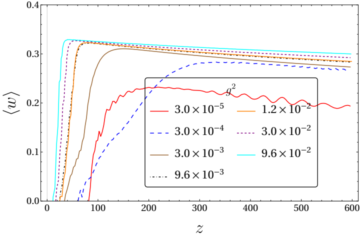


Results: EOS

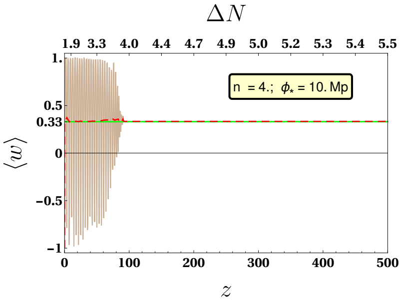
- For $n = 2$, changing the coupling parameters do not improve the situation.

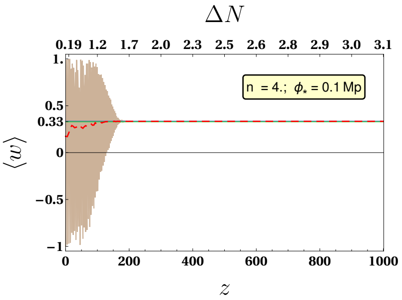




Results: EOS

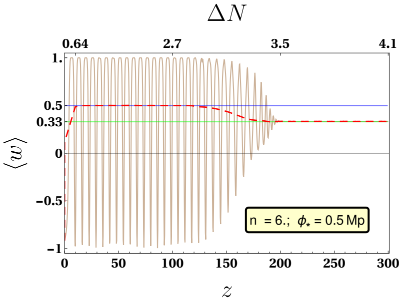
- For $n = 4$, $w_{\rm co}$ too is $1/3$.

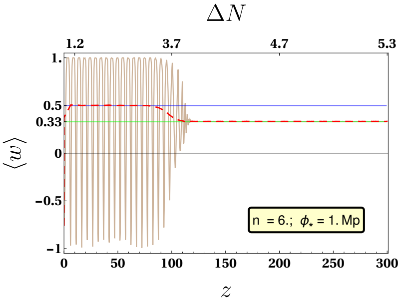




Results: EOS

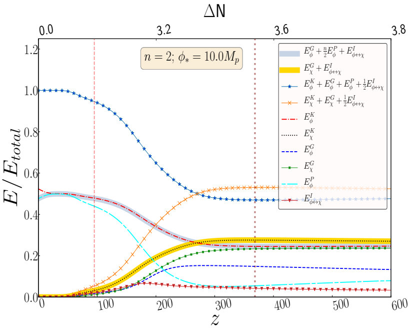
- For $n = 6$, eos transits for $w_{\rm co} = 1/2$ to $w_{\rm rad} = 1/3$ (thermalization?).

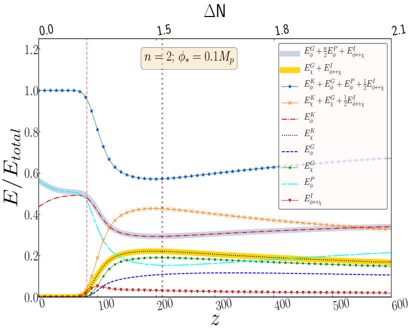




Results: Energy fractions

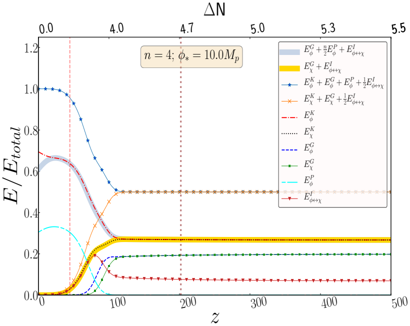
- For $n = 2$, energy is not equally distributed among componets.

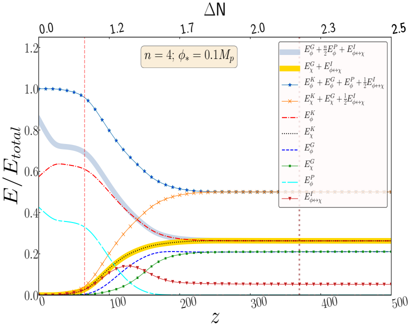




Results: Energy fractions

- For $n = 4$, energy is democratically distributed among componets.





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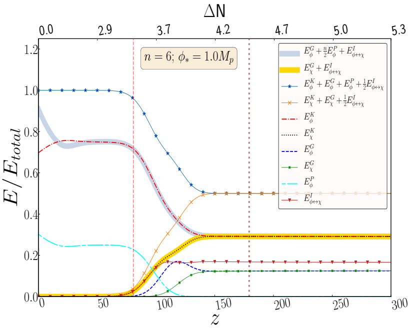
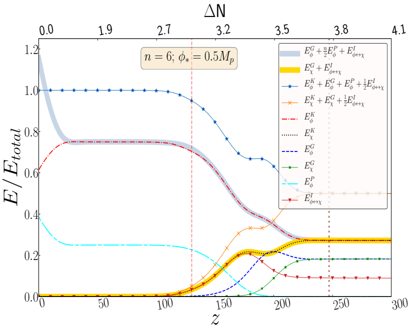


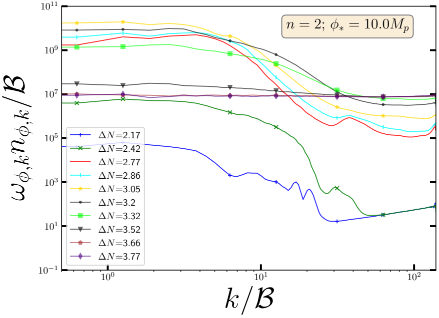


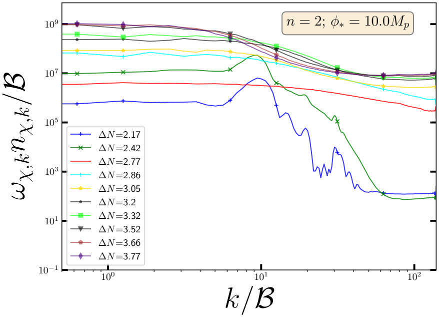
A sign of thermalization

We find the thermalization across modes by considering the Rayleigh-Jeans spectrum defined by the product $n_k \omega_k = T$.

A sign of thermalization

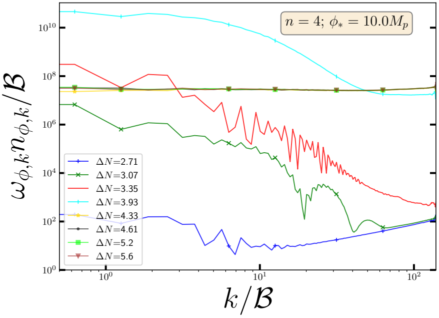
- For $n = 2$, no sign of thermalization.

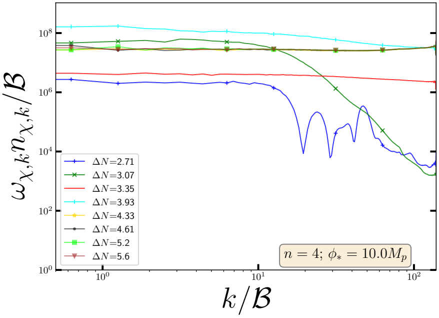




A sign of thermalization

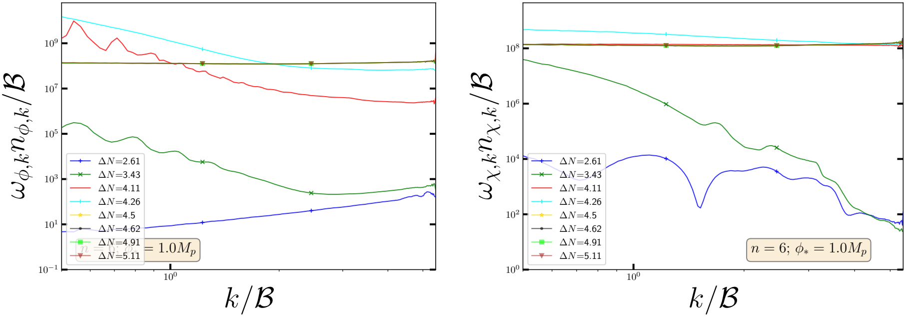
- For $n = 4$, shows a tendency to thermalize.





A sign of thermalization

- For $n = 6$, much more tendency to thermalize.



General results for preheating

The preheating phase consist of three phase

- The initial parametric resonance phase when the back-reaction is negligible. The comoving amplitude of the inflaton zero mode is almost constant.
- The next stage is the non-linear phase. The back-reaction of the produced quanta work on the inflaton condensate. The gradient energy component increase significantly indicating the qrowth of inhomogeneity.
- The final stage is the stationary phase when the inflaton decay ceases. The comoving energy is nearly constant.
- Inflation decay is incomplete for traditional $g^2\phi^2\chi^2$, $M\phi\chi^2$ couplings for inflation with quadratic type potentials.

Constraints on reheating: issues

- Inflationary phase is well constrained from data from CMB observations. (scalar power spectrum $\mathcal{P}_{\mathcal{R}}$, spectral tilt $\mathrm{n_s}$, upper-bound on tensor-to-scalar ratio r. and other complementary data from PBH and GWs)
- The inflationary energy scale assumed to be around 10^{16} GeV while the BBN requires a radiation dominated universe at 10 MeV.
- There is a huge gap in energy scales of several orders in cosmological history from inflation to BBN (Primordial Dark Ages) that has no observables.
- Thermalization erases previous history of the components.

Connecting reheating and CMB

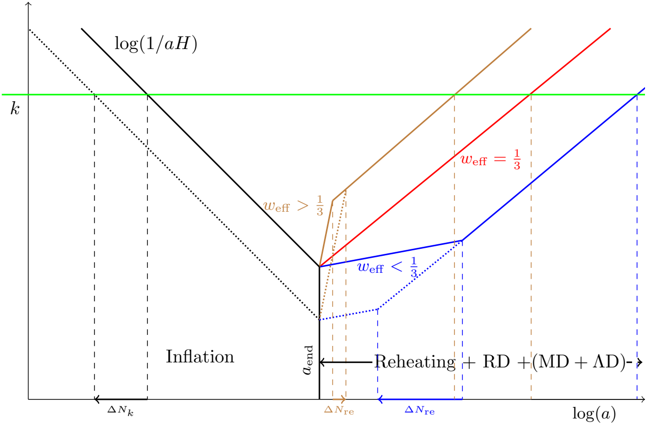


Figure: The comoving scales connects the inflationary phase with the CMB.

$$N_{\rm re} = \frac{4}{3w_{\rm eff} - 1}\left[N_k + N_{\rm co} - 61.6 - \ln(H_k) + \frac{1}{4}\ln(\rho_k)\right]$$

Accounting for time-evolution of the reheating equation of the state[7]

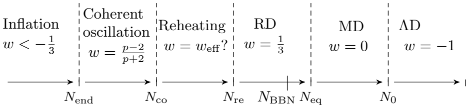
- CMB and reheating:

$$T_{\rm re} = \left(\frac{43}{11 g_{\rm re}}\right)^{\frac{1}{3}} \left(\frac{a_0 T_0}{k}\right) H_k e^{-N_k} e^{-N_{\rm re}}$$

- CMB measurements can constrain reheating phase provided we know the effective equation of state during reheating.

$$w_{\rm eff} = \frac{1}{N_{\rm re}} \int^{\rm N_{re}} w(N')\mathrm{d}N'$$

- Each epoch in cosmic evolution is characterized by its equation of state.

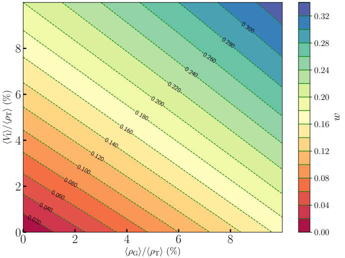


[7] with S. Anand and L. Sriramkumar, Phys. Rev. **D 102**, 103511 (2020)

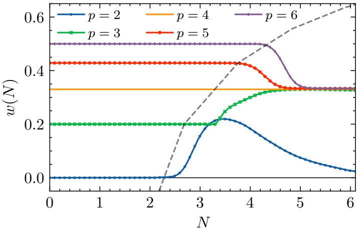
Accounting for time-evolution of the reheating equation of the state: Results from simulation

- Inflationary potentials around the minima behaves as $V(\phi) \propto \phi^p$.
- The EoS for models $p > 2$, behaves like radiation at the end of simulation.

$$w = \frac{1}{3} + \left(\frac{p-4}{6}\right)\left(\frac{p+2}{4} + \frac{\langle \rho_G \rangle}{\langle V(\phi) \rangle} + \frac{3}{2}\frac{\langle V_I(\phi, \mathcal{F}) \rangle}{\langle V(\phi) \rangle}\right)$$



(a)



(b)

Parameterizing the equation of state during reheating

"With four parameters I can fit an elephant, and with five I can make him wiggle his trunk"–Johnny von Neumann

- **Case A:** exponential form

$$w(N) = w_0 + w_1 \exp\left(-\frac{1}{\Delta}\frac{N}{N_{\rm re}}\right)$$

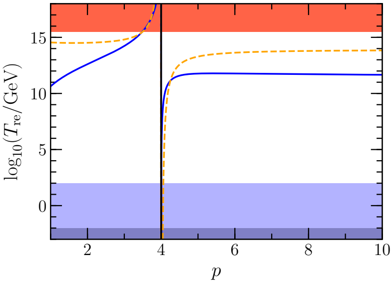
- **Case B:** tan-hyperbolic form

$$w(N) = w_0 + w_1 \tanh\left(\frac{1}{\Delta}\frac{N}{N_{\rm re}}\right)$$

- We fix the parameters from the physical conditions:
  1. Equation of state at the end of coherent oscillation is $w_{\rm co} = \frac{p-2}{p+2}$
  2. Equation of state asymptotically reaches radiation like equation of state $w_{\rm rad} = \frac{1}{3}$ and is bracketed by the values $w_{\rm co}$ and $w_{\rm re}$.
  3. The reheating ends when the equation reaches 90% of the radiation-like equation of state.

What is equation of state during reheating?

p	$w_p = \frac{(p-2)}{(p+2)}$	$w_{\rm eff}^{\rm exp}$	$w_{\rm eff}^{\rm tanh}$
1	-1/3	0.12	0.09
2	0	0.20	0.19
4	1/3	1/3	1/3
6	1/2	0.41	0.42
8	3/5	0.44	0.45
$p \to \infty$	1	0.53	0.56



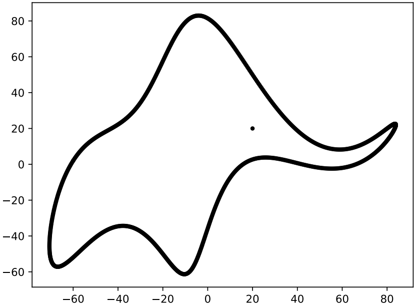
- Equation of state is completely specified by the inflationary potential.
- Deviates significantly from ‘zeroth order approximation’.

Summary

- The primordial dark ages is the least explored phases of the early universe.
- The thermalization process and how the standard model particles are generated are not completely understood.
- We need a lot of computational and theoretical developments to understand this phase.

Thank You!

The 6 parameter ‘Elephant’



Thank You, again!