Primordial correlation of gravitons with gauge fields

Based on
Rajeev Kumar Jain, P. Jishnu sai and Martin S. Sloth JCAP03(2022)054

P. Jishnu Sai

Department of Physics
Indian Institute of Science, Bangalore, India

भारतीय विज्ञान संस्थान

Weekly meeting on cosmology, IIT Madras

26 March 2022

Outline

Dynamics of primordial tensors and gauge field during inflation

- Inflationary magnetogenesis
- Quantum fluctuations of metric perturbations
- Quantum fluctuations of gauge field

Cross-correlation of inflationary tensor perturbation with primordial gauge fields

- The in-in formalism
- Leading order correction terms
- Consistency relations for primordial gauge fields
- Semi-classical derivation of the consistency relations

A novel correlation of tensor and curvature perturbations

Summary

Dynamics of primordial tensors and gauge field during inflation

Inflationary magnetogenesis

Typical model

Inflationary magnetogenesis

Typical model

- Large scale magnetic fields are present in all structures in our Universe.

Inflationary magnetogenesis

Typical model

- Large scale magnetic fields are present in all structures in our Universe.
- If magnetic fields are produced during inflation, they are likely to be correlated with the primordial perturbations.

Inflationary magnetogenesis

Typical model

- Large scale magnetic fields are present in all structures in our Universe.
- If magnetic fields are produced during inflation, they are likely to be correlated with the primordial perturbations.
- Here we will be interested in a particular model of inflationary magnetogenesis

Inflationary magnetogenesis

Typical model

- Large scale magnetic fields are present in all structures in our Universe.
- If magnetic fields are produced during inflation, they are likely to be correlated with the primordial perturbations.
- Here we will be interested in a particular model of inflationary magnetogenesis

$$S_{em} = -\frac{1}{4}\int d^4x\sqrt{-g}\,\lambda(\phi)F_{\mu\nu}F^{\mu\nu} \text{ with } \quad \lambda(\phi(a)) = \lambda_I\left(\frac{a}{a_I}\right)^{2n}$$

where $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$

Inflationary magnetogenesis

Typical model

- Large scale magnetic fields are present in all structures in our Universe.
- If magnetic fields are produced during inflation, they are likely to be correlated with the primordial perturbations.
- Here we will be interested in a particular model of inflationary magnetogenesis

$$S_{em} = -\frac{1}{4}\int d^4x\sqrt{-g}\,\lambda(\phi)F_{\mu\nu}F^{\mu\nu} \text{ with } \quad \lambda(\phi(a)) = \lambda_I\left(\frac{a}{a_I}\right)^{2n}$$

where $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$

- The perturbed metric:
$ds^2 = -dt^2 + a^2(t)\, e^{2\zeta(t,\mathbf{x})}[e^{\gamma(t,\mathbf{x})}]_{ij}dx^i dx^j$

Quantum Fluctuations

Metric perturbations and gauge field

- The power spectra associated with metric perturbations

$$\langle \zeta(\mathbf{k},\tau)\zeta(\mathbf{k}',\tau)\rangle = (2\pi)^3\delta^{(3)}(\mathbf{k}+\mathbf{k}')P_\zeta(k,\tau)$$

Quantum Fluctuations

Metric perturbations and gauge field

- The power spectra associated with metric perturbations

$$\langle \zeta(\mathbf{k},\tau)\zeta(\mathbf{k}',\tau)\rangle = (2\pi)^3\delta^{(3)}(\mathbf{k}+\mathbf{k}')P_\zeta(k,\tau)$$

$$\langle \gamma^s(\mathbf{k},\tau)\gamma^{s'}(\mathbf{k}',\tau)\rangle = (2\pi)^3\delta^{(3)}(\mathbf{k}+\mathbf{k}')P_\gamma(k,\tau)\delta_{ss'}$$

Quantum Fluctuations

Metric perturbations and gauge field

- The power spectra associated with metric perturbations

$$\langle\zeta(\mathbf{k},\tau)\zeta(\mathbf{k}',\tau)\rangle = (2\pi)^3\delta^{(3)}(\mathbf{k}+\mathbf{k}')P_\zeta(k,\tau)$$

$$\langle\gamma^s(\mathbf{k},\tau)\gamma^{s'}(\mathbf{k}',\tau)\rangle = (2\pi)^3\delta^{(3)}(\mathbf{k}+\mathbf{k}')P_\gamma(k,\tau)\delta_{ss'}$$

- The two point correlation function of gauge fields A_i,

Quantum Fluctuations

Metric perturbations and gauge field

- The power spectra associated with metric perturbations

$$\langle \zeta(\mathbf{k},\tau)\zeta(\mathbf{k}',\tau)\rangle = (2\pi)^3\delta^{(3)}(\mathbf{k}+\mathbf{k}')P_\zeta(k,\tau)$$

$$\langle \gamma^s(\mathbf{k},\tau)\gamma^{s'}(\mathbf{k}',\tau)\rangle = (2\pi)^3\delta^{(3)}(\mathbf{k}+\mathbf{k}')P_\gamma(k,\tau)\delta_{ss'}$$

- The two point correlation function of gauge fields A_i,

$$\left\langle A_i(\tau,\mathbf{k})A_j(\tau,\mathbf{k}')\right\rangle = (2\pi)^3\delta^{(3)}(\mathbf{k}+\mathbf{k}')\left(\delta_{ij}-\frac{k_i k_j}{k^2}\right)|A_k(\tau)|^2$$

Quantum Fluctuations

Metric perturbations and gauge field

- The power spectra associated with metric perturbations

$$\langle\zeta(\mathbf{k},\tau)\zeta(\mathbf{k}',\tau)\rangle = (2\pi)^3\delta^{(3)}(\mathbf{k}+\mathbf{k}')P_\zeta(k,\tau)$$

$$\langle\gamma^s(\mathbf{k},\tau)\gamma^{s'}(\mathbf{k}',\tau)\rangle = (2\pi)^3\delta^{(3)}(\mathbf{k}+\mathbf{k}')P_\gamma(k,\tau)\delta_{ss'}$$

- The two point correlation function of gauge fields A_i,

$$\left\langle A_i(\tau,\mathbf{k})A_j(\tau,\mathbf{k}')\right\rangle = (2\pi)^3\delta^{(3)}(\mathbf{k}+\mathbf{k}')\left(\delta_{ij}-\frac{k_ik_j}{k^2}\right)|A_k(\tau)|^2$$

with the mode function $A_k(\tau)$ is given by

$$A_k(\tau) = \frac{1}{\sqrt{\lambda_I}}\frac{\sqrt{\pi}}{2}e^{i\pi(n+1)/2}\sqrt{-\tau}\left(\frac{\tau}{\tau_I}\right)^n H^{(1)}_{n+\frac{1}{2}}(-k\tau)$$

Electric and magnetic fields

- One can covariantly define the electric field E_μ and magnetic field B_μ with respect to an observer having four-velocity u^ν.

Electric and magnetic fields

- One can covariantly define the electric field E_μ and magnetic field B_μ with respect to an observer having four-velocity u^ν.

$$E_\mu = F_{\mu\nu} u^\nu , \qquad B_\mu = {}^*F_{\mu\nu} u^\nu \,.$$

Electric and magnetic fields

- One can covariantly define the electric field E_μ and magnetic field B_μ with respect to an observer having four-velocity u^ν.

$$E_\mu = F_{\mu\nu} u^\nu , \qquad B_\mu = {}^*F_{\mu\nu} u^\nu .$$

- Then the corresponding power spectra are,

Electric and magnetic fields

- One can covariantly define the electric field E_μ and magnetic field B_μ with respect to an observer having four-velocity u^ν.

$$E_\mu = F_{\mu\nu} u^\nu , \qquad B_\mu = {}^*F_{\mu\nu} u^\nu .$$

- Then the corresponding power spectra are,

$$\begin{aligned}
\left\langle B_\mu(\tau, \mathbf{k}) B^\mu(\tau, \mathbf{k}') \right\rangle &= (2\pi)^3 \delta^{(3)}(\mathbf{k} + \mathbf{k}') P_B(k, \tau), \\
\left\langle E_\mu(\tau, \mathbf{k}) E^\mu(\tau, \mathbf{k}') \right\rangle &= (2\pi)^3 \delta^{(3)}(\mathbf{k} + \mathbf{k}') P_E(k, \tau).
\end{aligned}$$

Electric and Magnetic fields

- One can covariantly define the electric field E_μ and magnetic field B_μ with respect to an observer having four-velocity u^ν.

$$E_\mu = F_{\mu\nu} u^\nu , \qquad B_\mu = {}^*F_{\mu\nu} u^\nu \, .$$

- Then the corresponding power spectra are,

$$\begin{aligned}
\left\langle B_\mu(\tau, \mathbf{k}) B^\mu(\tau, \mathbf{k}') \right\rangle &= (2\pi)^3 \delta^{(3)}(\mathbf{k} + \mathbf{k}') P_B(k, \tau), \\
\left\langle E_\mu(\tau, \mathbf{k}) E^\mu(\tau, \mathbf{k}') \right\rangle &= (2\pi)^3 \delta^{(3)}(\mathbf{k} + \mathbf{k}') P_E(k, \tau).
\end{aligned}$$

Thus,

$$\begin{aligned}
P_B(k, \tau) &= 2 \frac{k^2}{a^4} |A_k(\tau)|^2 \\
P_E(k, \tau) &= \frac{2}{a^4} |A_k'(\tau)|^2
\end{aligned}$$

Cross-correlation of inflationary tensor perturbation with primordial gauge fields

The in-in formalism

- In order to compute the correlation function during inflation, we adopt a very useful and powerful tool of the in-in formalism. In this formalism, the expectation value of an operator $\mathcal{O}$ at time τ_I is given by

The in-in formalism

- In order to compute the correlation function during inflation, we adopt a very useful and powerful tool of the in-in formalism. In this formalism, the expectation value of an operator $\mathcal{O}$ at time τ_I is given by

$$\langle \mathcal{O}(\tau_I) \rangle = \langle 0 | \, \bar{\mathrm{T}} \left(e^{i \int_{-\infty}^{\tau_I} d\tau H_{\mathrm{int}}} \right) \mathcal{O}(\tau_I) \mathrm{T} \left(e^{-i \int_{-\infty}^{\tau_I} d\tau H_{\mathrm{int}}} \right) | 0 \rangle$$

The in-in formalism

- In order to compute the correlation function during inflation, we adopt a very useful and powerful tool of the in-in formalism. In this formalism, the expectation value of an operator $\mathcal{O}$ at time τ_I is given by

$$\langle \mathcal{O}(\tau_I) \rangle = \langle 0 | \, \bar{\mathrm{T}} \left(e^{i \int_{-\infty}^{\tau_I} d\tau H_{\rm int}} \right) \mathcal{O}(\tau_I) \mathrm{T} \left(e^{-i \int_{-\infty}^{\tau_I} d\tau H_{\rm int}} \right) |0\rangle$$

- The leading order interaction Hamiltonian is

$$H_{\rm int}(\tau) = \frac{1}{2} \int d^3x \, \lambda(\tau) \Big(\gamma^{ij} A_i' A_j' - \gamma^{ij} \delta^{kl} (\partial_i A_k \partial_j A_l + \partial_k A_i \partial_l A_j) + 2\gamma^{ij} \delta^{kl} \partial_i A_k \partial_l A_j \Big)$$

The in-in formalism

- In order to compute the correlation function during inflation, we adopt a very useful and powerful tool of the in-in formalism. In this formalism, the expectation value of an operator $\mathcal{O}$ at time τ_I is given by

$$\langle \mathcal{O}(\tau_I) \rangle = \langle 0 | \, \bar{\mathrm{T}} \left(e^{i \int_{-\infty}^{\tau_I} d\tau H_{\mathrm{int}}} \right) \mathcal{O}(\tau_I) \mathrm{T} \left(e^{-i \int_{-\infty}^{\tau_I} d\tau H_{\mathrm{int}}} \right) | 0 \rangle$$

- The leading order interaction Hamiltonian is

$$H_{\mathrm{int}}(\tau) = \frac{1}{2} \int d^3x \, \lambda(\tau) \Big(\gamma^{ij} A_i' A_j' - \gamma^{ij} \delta^{kl} (\partial_i A_k \partial_j A_l + \partial_k A_i \partial_l A_j) + 2\gamma^{ij} \delta^{kl} \partial_i A_k \partial_l A_j \Big)$$

- Using in-in formalism, we have calculated $\langle \gamma A_\mu A^\mu \rangle$,$\langle \gamma B_\mu B^\mu \rangle$ and $\langle \gamma E_\mu E^\mu \rangle$ perturbatively

$\langle \gamma A_\mu A^\mu \rangle$

- Let us first compute the cross-correlation of a tensor mode with two gauge field modes, i.e., a correlator of the form $\langle \gamma A_\mu A^\mu \rangle$.

$\langle \gamma A_\mu A^\mu \rangle$

- Let us first compute the cross-correlation of a tensor mode with two gauge field modes, i.e., a correlator of the form $\langle \gamma A_\mu A^\mu \rangle$.

$$\begin{aligned}\langle \gamma(\mathbf{k}_1, \tau_I) A_\mu(\mathbf{k}_2, \tau_I) A^\mu(\mathbf{k}_3, \tau_I) \rangle &= \langle 0 | \gamma(\mathbf{k}_1, \tau_I) A_\mu(\mathbf{k}_2, \tau_I) A^\mu(\mathbf{k}_3, \tau_I) | 0 \rangle \\ &+ i \int_{-\infty}^{\tau_I} d\tau \; \langle 0 | [H_{\rm int}(\tau), \gamma(\mathbf{k}_1, \tau_I) A_\mu(\mathbf{k}_2, \tau_I) A^\mu(\mathbf{k}_3, \tau_I)] | 0 \rangle ,\end{aligned}$$

$\langle \gamma A_\mu A^\mu \rangle$

- Let us first compute the cross-correlation of a tensor mode with two gauge field modes, i.e., a correlator of the form $\langle \gamma A_\mu A^\mu \rangle$.

$$\begin{aligned}\langle \gamma(\mathbf{k}_1, \tau_I) A_\mu(\mathbf{k}_2, \tau_I) A^\mu(\mathbf{k}_3, \tau_I) \rangle &= \langle 0 | \gamma(\mathbf{k}_1, \tau_I) A_\mu(\mathbf{k}_2, \tau_I) A^\mu(\mathbf{k}_3, \tau_I) | 0 \rangle \\ &+ i \int_{-\infty}^{\tau_I} d\tau \ \ \langle 0 | [H_{\rm int}(\tau), \gamma(\mathbf{k}_1, \tau_I) A_\mu(\mathbf{k}_2, \tau_I) A^\mu(\mathbf{k}_3, \tau_I)] | 0 \rangle \,,\end{aligned}$$

- The first term requires more attention, $\gamma A_\mu A^\mu$ isn't a three point function!

$\langle \gamma A_\mu A^\mu \rangle$

- Let us first compute the cross-correlation of a tensor mode with two gauge field modes, i.e., a correlator of the form $\langle \gamma A_\mu A^\mu \rangle$.

$$\begin{aligned} \langle \gamma(\mathbf{k}_1, \tau_I) A_\mu(\mathbf{k}_2, \tau_I) A^\mu(\mathbf{k}_3, \tau_I) \rangle &= \langle 0 | \, \gamma(\mathbf{k}_1, \tau_I) A_\mu(\mathbf{k}_2, \tau_I) A^\mu(\mathbf{k}_3, \tau_I) \, | 0 \rangle \\ &+ i \int_{-\infty}^{\tau_I} d\tau \;\; \langle 0 | \left[H_{\rm int}(\tau), \gamma(\mathbf{k}_1, \tau_I) A_\mu(\mathbf{k}_2, \tau_I) A^\mu(\mathbf{k}_3, \tau_I) \right] | 0 \rangle \,, \end{aligned}$$

- The first term requires more attention, $\gamma A_\mu A^\mu$ isn't a three point function!

$$\gamma A_\mu A^\mu \;\; = \;\; \gamma g^{\mu\nu} A_\mu A_\nu = \gamma \left(g_0^{\mu\nu} + \delta g^{\mu\nu} \right) A_\mu A_\nu$$

$\langle \gamma A_\mu A^\mu \rangle$

- Let us first compute the cross-correlation of a tensor mode with two gauge field modes, i.e., a correlator of the form $\langle \gamma A_\mu A^\mu \rangle$.

$$\begin{aligned} \langle \gamma(\mathbf{k}_1, \tau_I) A_\mu(\mathbf{k}_2, \tau_I) A^\mu(\mathbf{k}_3, \tau_I) \rangle &= \langle 0 | \, \gamma(\mathbf{k}_1, \tau_I) A_\mu(\mathbf{k}_2, \tau_I) A^\mu(\mathbf{k}_3, \tau_I) \, | 0 \rangle \\ &+ i \int_{-\infty}^{\tau_I} d\tau \;\; \langle 0 | \left[H_{\rm int}(\tau), \gamma(\mathbf{k}_1, \tau_I) A_\mu(\mathbf{k}_2, \tau_I) A^\mu(\mathbf{k}_3, \tau_I) \right] | 0 \rangle \,, \end{aligned}$$

- The first term requires more attention, $\gamma A_\mu A^\mu$ isn't a three point function!

$$\begin{aligned} \gamma A_\mu A^\mu &= \gamma g^{\mu\nu} A_\mu A_\nu = \gamma \left(g_0^{\mu\nu} + \delta g^{\mu\nu} \right) A_\mu A_\nu \\ &= \frac{1}{a^2} \gamma A_i A_i - \frac{1}{a^2} \gamma \gamma_{ij} A_i A_j + \mathcal{O}(\gamma^3) \end{aligned}$$

Kinematical correction terms

- There exist some leading order correction terms in the correlator which are arising from VEV.

Kinematical terms

- There exist some leading order correction terms in the correlator which are arsing from VEV.

$$\gamma A_\mu A^\mu = \frac{1}{a^2}\gamma A_i A_i - \frac{1}{a^2}\gamma\gamma_{ij} A_i A_j + \mathcal{O}(\gamma^3)$$

Kinematical terms

- There exist some leading order correction terms in the correlator which are arising from VEV.

$$\gamma A_\mu A^\mu = \frac{1}{a^2}\gamma A_i A_i \quad \underbrace{-\frac{1}{a^2}\gamma\gamma_{ij}A_iA_j}_{\text{Kinematical correction term}} \quad +\mathcal{O}(\gamma^3)$$

Kinematical terms

- There exist some leading order correction terms in the correlator which are arising from VEV.

$$\gamma A_\mu A^\mu = \frac{1}{a^2}\gamma A_i A_i \quad \underbrace{-\frac{1}{a^2}\gamma\gamma_{ij}A_iA_j}_{\text{Kinematical correction term}} \quad +\mathcal{O}(\gamma^3)$$

- Then the contribution from this kinematical correction can easily calculate,

Kinematical terms

- There exist some leading order correction terms in the correlator which are arising from VEV.

$$\gamma A_\mu A^\mu = \frac{1}{a^2}\gamma A_i A_i \quad \underbrace{-\frac{1}{a^2}\gamma\gamma_{ij}A_i A_j}_{\text{Kinematical correction term}} \quad +\mathcal{O}(\gamma^3)$$

- Then the contribution from this kinematical correction can easily calculate,

$$\langle 0|\,\gamma(\mathbf{k}_1)A_\mu(\mathbf{k}_2)A^\mu(\mathbf{k}_3)\,|0\rangle = \frac{1}{2}(2\pi)^3\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\epsilon_{ij}\frac{k_{2i}k_{2j}}{\tilde{k}_2^2}P_\gamma(k_1)P_A(\tilde{k}_2)$$

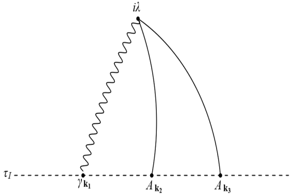
Here, $\tilde{\mathbf{k}}_2 = \mathbf{k}_2 + \frac{1}{2}\mathbf{k}_1$.

$\langle \gamma A_\mu A^\mu \rangle$

- Let's get back to our in-in formula

$$\begin{aligned}\langle \gamma(\mathbf{k}_1,\tau_I)A_\mu(\mathbf{k}_2,\tau_I)A^\mu(\mathbf{k}_3,\tau_I)\rangle &= \langle 0|\,\gamma(\mathbf{k}_1,\tau_I)A_\mu(\mathbf{k}_2,\tau_I)A^\mu(\mathbf{k}_3,\tau_I)\,|0\rangle \\ &+i\int_{-\infty}^{\tau_I} d\tau\ \ \langle 0|\,[H_{\rm int}(\tau),\gamma(\mathbf{k}_1,\tau_I)A_\mu(\mathbf{k}_2,\tau_I)A^\mu(\mathbf{k}_3,\tau_I)]\,|0\rangle\end{aligned}$$

- The second term in the above equation can calculate using the standard methods



Correction terms for $\langle \gamma B_\mu B^\mu \rangle$ and $\langle \gamma E_\mu E^\mu \rangle$

- There exist similar kinematical correction terms for $\gamma B_\mu B^\mu$

Correction terms for $\langle \gamma B_\mu B^\mu \rangle$ and $\langle \gamma E_\mu E^\mu \rangle$

- There exist similar kinematical correction terms for $\gamma B_\mu B^\mu$

$$\gamma B_\mu B^\mu = \frac{1}{2a^4} \gamma F_{ij} F_{ij} - \frac{1}{a^4} \gamma \gamma_{ij} F_{il} F_{jl} + \mathcal{O}(\gamma^3)$$

Correction terms for $\langle \gamma B_\mu B^\mu \rangle$ and $\langle \gamma E_\mu E^\mu \rangle$

- There exist similar kinematical correction terms for $\gamma B_\mu B^\mu$

$$\gamma B_\mu B^\mu = \frac{1}{2a^4}\gamma F_{ij}F_{ij} \quad \underbrace{-\frac{1}{a^4}\gamma\gamma_{ij}F_{il}F_{jl}}_{\text{kinematical correction term}} \quad +\mathcal{O}(\gamma^3)$$

Correction terms for $\langle \gamma B_\mu B^\mu \rangle$ and $\langle \gamma E_\mu E^\mu \rangle$

- There exist similar kinematical correction terms for $\gamma B_\mu B^\mu$

$$\gamma B_\mu B^\mu = \frac{1}{2a^4}\gamma F_{ij}F_{ij} \quad \underbrace{-\frac{1}{a^4}\gamma\gamma_{ij}F_{il}F_{jl}}_{\text{kinematical correction term}} \quad +\mathcal{O}(\gamma^3)$$

- But the correction terms in the electric fields are bit more subtile. Since we know that

$$E_\mu(\mathbf{x},\tau) \propto i[H_{\rm tot}, A_\mu] = i[H_0, A_\mu] + i[H_{\rm int}, A_\mu]$$

- From the definition of electric field, we can see that

Correction terms for $\langle \gamma B_\mu B^\mu \rangle$ and $\langle \gamma E_\mu E^\mu \rangle$

- There exist similar kinematical correction terms for $\gamma B_\mu B^\mu$

$$\gamma B_\mu B^\mu = \frac{1}{2a^4}\gamma F_{ij}F_{ij} \quad \underbrace{-\frac{1}{a^4}\gamma\gamma_{ij}F_{il}F_{jl}}_{\text{kinematical correction term}} \quad +\mathcal{O}(\gamma^3)$$

- But the correction terms in the electric fields are bit more subtile. Since we know that

$$E_\mu(\mathbf{x},\tau) \propto i[H_{\rm tot}, A_\mu] = i[H_0, A_\mu] + i[H_{\rm int}, A_\mu]$$

- From the definition of electric field, we can see that

$$\gamma E_\mu E^\mu = \frac{1}{a^4}\left[\gamma \frac{dA_i}{d\tau}\frac{dA_i}{d\tau} - \gamma\gamma_{ij}\frac{dA_i}{d\tau}\,\frac{dA_j}{d\tau} + i\gamma\left(\frac{dA_i}{d\tau}[H_{\rm int}, A_i] + [H_{\rm int}, A_i]\frac{dA_i}{d\tau}\right)\right] + \mathcal{O}(\gamma^3)$$

Correction terms for $\langle \gamma B_\mu B^\mu \rangle$ and $\langle \gamma E_\mu E^\mu \rangle$

- There exist similar kinematical correction terms for $\gamma B_\mu B^\mu$

$$\gamma B_\mu B^\mu = \frac{1}{2a^4}\gamma F_{ij}F_{ij} \quad \underbrace{-\frac{1}{a^4}\gamma\gamma_{ij}F_{il}F_{jl}}_{\text{kinematical correction term}} \quad +\mathcal{O}(\gamma^3)$$

- But the correction terms in the electric fields are bit more subtile. Since we know that

$$E_\mu(\mathbf{x},\tau) \propto i[H_{\rm tot}, A_\mu] = i[H_0, A_\mu] + i[H_{\rm int}, A_\mu]$$

- From the definition of electric field, we can see that

$$\gamma E_\mu E^\mu = \frac{1}{a^4}\left[\gamma \frac{dA_i}{d\tau}\frac{dA_i}{d\tau} \quad \overbrace{-\gamma\gamma_{ij}\frac{dA_i}{d\tau}\,\frac{dA_j}{d\tau}}^{\text{Kinematical correction term}} + i\gamma\left(\frac{dA_i}{d\tau}[H_{\rm int}, A_i] + [H_{\rm int}, A_i]\frac{dA_i}{d\tau}\right)\right] + \mathcal{O}(\gamma^3)$$

Correction terms for $\langle \gamma B_\mu B^\mu \rangle$ and $\langle \gamma E_\mu E^\mu \rangle$

- There exist similar kinematical correction terms for $\gamma B_\mu B^\mu$

$$\gamma B_\mu B^\mu = \frac{1}{2a^4}\gamma F_{ij}F_{ij} \quad \underbrace{-\frac{1}{a^4}\gamma\gamma_{ij}F_{il}F_{jl}}_{\text{kinematical correction term}} \quad +\mathcal{O}(\gamma^3)$$

- But the correction terms in the electric fields are bit more subtile. Since we know that

$$E_\mu(\mathbf{x},\tau) \propto i[H_{\rm tot}, A_\mu] = i[H_0, A_\mu] + i[H_{\rm int}, A_\mu]$$

- From the definition of electric field, we can see that

$$\gamma E_\mu E^\mu = \frac{1}{a^4}\Bigg[\gamma \frac{dA_i}{d\tau}\frac{dA_i}{d\tau} \quad \overbrace{-\gamma\gamma_{ij}\frac{dA_i}{d\tau}\frac{dA_j}{d\tau}}^{\text{Kinematical correction term}} \underbrace{+i\gamma\left(\frac{dA_i}{d\tau}[H_{\rm int}, A_i] + [H_{\rm int}, A_i]\frac{dA_i}{d\tau}\right)}_{\text{Dynamical correction term}}\Bigg] + \mathcal{O}(\gamma^3)$$

The magnetic and electric non-linearity parameters

The magnetic and electric non-linearity parameters

- The bispectra associated with $\langle\gamma B_\mu B^\mu\rangle$ and $\langle\gamma E_\mu E^\mu\rangle$ are,

The magnetic and electric non-linearity parameters

- The bispectra associated with $\langle \gamma B_\mu B^\mu \rangle$ and $\langle \gamma E_\mu E^\mu \rangle$ are,

$$\langle \gamma(\mathbf{k}_1) B_\mu(\mathbf{k}_2) B^\mu(\mathbf{k}_3) \rangle \equiv (2\pi)^3 \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)\, \mathcal{B}_{\gamma BB}(k_1, k_2, k_3)$$
$$\langle \gamma(\mathbf{k}_1) E_\mu(\mathbf{k}_2) E^\mu(\mathbf{k}_3) \rangle \equiv (2\pi)^3 \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)\, \mathcal{B}_{\gamma EE}(k_1, k_2, k_3)$$

The magnetic and electric non-linearity parameters

- The bispectra associated with $\langle\gamma B_\mu B^\mu\rangle$ and $\langle\gamma E_\mu E^\mu\rangle$ are,

$$\langle\gamma(\mathbf{k}_1)B_\mu(\mathbf{k}_2)B^\mu(\mathbf{k}_3)\rangle \equiv (2\pi)^3\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\,\mathcal{B}_{\gamma BB}(k_1,k_2,k_3)$$
$$\langle\gamma(\mathbf{k}_1)E_\mu(\mathbf{k}_2)E^\mu(\mathbf{k}_3)\rangle \equiv (2\pi)^3\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\,\mathcal{B}_{\gamma EE}(k_1,k_2,k_3)$$

- The strength of these bispectra can be characterised by defining the non-linearity parameters b_{NL}^{γ} and e_{NL}^{γ} as follows

$$\mathcal{B}_{\gamma BB}(k_1,k_2,k_3) = \frac{1}{2}b_{NL}^{\gamma}P_\gamma(k_1)\left[P_B(k_2)+P_B(k_3)\right]$$
$$\mathcal{B}_{\gamma EE}(k_1,k_2,k_3) = \frac{1}{2}e_{NL}^{\gamma}P_\gamma(k_1)\left[P_E(k_2)+P_E(k_3)\right]$$

- If the two non-linearity parameters b_{NL}^{γ} and e_{NL}^{γ} are momentum independent, they correspond to a *local* shape of the bispectra

The Magnetic and electric non-linearity parameters

- The bispectra associated with $\langle \gamma B_\mu B^\mu \rangle$ and $\langle \gamma E_\mu E^\mu \rangle$ are,

$$\langle \gamma(\mathbf{k}_1) B_\mu(\mathbf{k}_2) B^\mu(\mathbf{k}_3) \rangle \equiv (2\pi)^3 \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) \mathcal{B}_{\gamma BB}(k_1, k_2, k_3)$$
$$\langle \gamma(\mathbf{k}_1) E_\mu(\mathbf{k}_2) E^\mu(\mathbf{k}_3) \rangle \equiv (2\pi)^3 \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) \mathcal{B}_{\gamma EE}(k_1, k_2, k_3)$$

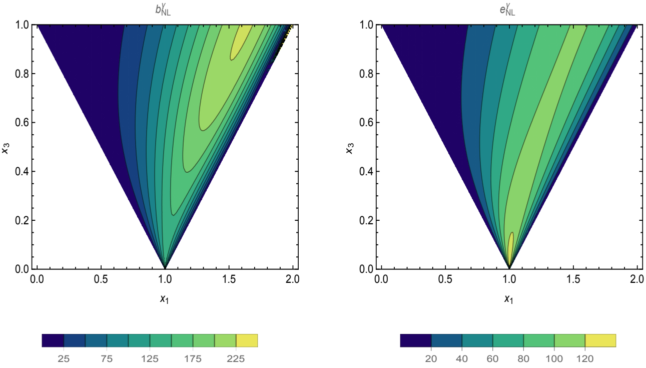
- The strength of these bispectra can be characterised by defining the non-linearity parameters b_{NL}^γ and e_{NL}^γ as follows

$$\mathcal{B}_{\gamma BB}(k_1, k_2, k_3) = \frac{1}{2} b_{NL}^\gamma P_\gamma(k_1) \left[P_B(k_2) + P_B(k_3)\right]$$
$$\mathcal{B}_{\gamma EE}(k_1, k_2, k_3) = \frac{1}{2} e_{NL}^\gamma P_\gamma(k_1) \left[P_E(k_2) + P_E(k_3)\right]$$

- If the two non-linearity parameters b_{NL}^γ and e_{NL}^γ are momentum independent, they correspond to a *local* shape of the bispectra

$$\mathbf{B} = \mathbf{B}^G + \frac{1}{2} b_{NL}^\gamma \gamma^G \mathbf{B}^G ,$$
$$\mathbf{E} = \mathbf{E}^G + \frac{1}{2} e_{NL}^\gamma \gamma^G \mathbf{E}^G ,$$

The in-in results



The extent of the non-linearity parameters b_{NL}^{γ} (left) and e_{NL}^{γ} (right) corresponding to different triangular configuration are plotted for the case of $n = 2$. Here, we defined $x_1 = \frac{k_1}{k_2}$ and $x_3 = \frac{k_3}{k_2}$ while k_2 is set at an arbitrary scale. The colour legends representing the magnitude are also shown below each plot.

Squeezed/Soft limit and new consistency relations

Squeezed/Soft limit and new consistency relations

In this limit, we have $\mathbf{k}_1 \to 0$ and $\mathbf{k}_2 \to -\mathbf{k}_3 \equiv \mathbf{k}$. The primed correlator $\langle ... \rangle'$ indicate that we have suppressed the factor $(2\pi)^3 \delta^{(3)}(\mathbf{k_1} + \mathbf{k_2} + \mathbf{k_3})$.

Squeezed/Soft limit and new consistency relations

In this limit, we have $\mathbf{k}_1 \to 0$ and $\mathbf{k}_2 \to -\mathbf{k}_3 \equiv \mathbf{k}$. The primed correlator $\langle ... \rangle'$ indicate that we have suppressed the factor $(2\pi)^3 \delta^{(3)}(\mathbf{k_1} + \mathbf{k_2} + \mathbf{k_3})$.

$$\langle \gamma(\mathbf{k}_1) A_\mu(\mathbf{k}_2) A^\mu(\mathbf{k}_3) \rangle' = \begin{cases} \epsilon_{ij} \frac{k_i k_j}{k^2} \left(n + \frac{1}{2}\right) P_\gamma(k_1) P_A(k), \text{ if } n > -\frac{1}{2} \\ \\ -\epsilon_{ij} \frac{k_i k_j}{k^2} \left(n - \frac{1}{2}\right) P_\gamma(k_1) P_A(k), \text{ if } n < -\frac{1}{2} \end{cases}$$

Squeezed/Soft limit and new consistency relations

In this limit, we have $\mathbf{k}_1 \to 0$ and $\mathbf{k}_2 \to -\mathbf{k}_3 \equiv \mathbf{k}$. The primed correlator $\langle ... \rangle'$ indicate that we have suppressed the factor $(2\pi)^3 \delta^{(3)}(\mathbf{k_1} + \mathbf{k_2} + \mathbf{k_3})$.

$$\langle \gamma(\mathbf{k}_1) A_\mu(\mathbf{k}_2) A^\mu(\mathbf{k}_3) \rangle' = \begin{cases} \epsilon_{ij} \frac{k_i k_j}{k^2} \left(n + \frac{1}{2}\right) P_\gamma(k_1) P_A(k), \text{ if } n > -\frac{1}{2} \\ \\ -\epsilon_{ij} \frac{k_i k_j}{k^2} \left(n - \frac{1}{2}\right) P_\gamma(k_1) P_A(k), \text{ if } n < -\frac{1}{2} \end{cases}$$

$$\langle \gamma(\mathbf{k}_1) B_\mu(\mathbf{k}_2) B^\mu(\mathbf{k}_3) \rangle' = \begin{cases} \epsilon_{ij} \frac{k_i k_j}{k^2} \left(n - \frac{1}{2}\right) P_\gamma(k_1) P_B(k), \text{ if } n > -\frac{1}{2} \\ \\ -\epsilon_{ij} \frac{k_i k_j}{k^2} \left(n + \frac{1}{2}\right) P_\gamma(k_1) P_B(k), \text{ if } n < -\frac{1}{2} \end{cases}$$

Squeezed/Soft limit and new consistency relations

In this limit, we have $\mathbf{k}_1 \to 0$ and $\mathbf{k}_2 \to -\mathbf{k}_3 \equiv \mathbf{k}$. The primed correlator $\langle ...\rangle'$ indicate that we have suppressed the factor $(2\pi)^3\delta^{(3)}(\mathbf{k_1}+\mathbf{k_2}+\mathbf{k_3})$.

$$\langle\gamma(\mathbf{k}_1)A_\mu(\mathbf{k}_2)A^\mu(\mathbf{k}_3)\rangle' = \begin{cases} \epsilon_{ij}\frac{k_ik_j}{k^2}\left(n+\frac{1}{2}\right)P_\gamma(k_1)P_A(k), \text{ if } n > -\frac{1}{2} \\ \\ -\epsilon_{ij}\frac{k_ik_j}{k^2}\left(n-\frac{1}{2}\right)P_\gamma(k_1)P_A(k), \text{ if } n < -\frac{1}{2} \end{cases}$$

$$\langle\gamma(\mathbf{k}_1)B_\mu(\mathbf{k}_2)B^\mu(\mathbf{k}_3)\rangle' = \begin{cases} \epsilon_{ij}\frac{k_ik_j}{k^2}\left(n-\frac{1}{2}\right)P_\gamma(k_1)P_B(k), \text{ if } n > -\frac{1}{2} \\ \\ -\epsilon_{ij}\frac{k_ik_j}{k^2}\left(n+\frac{1}{2}\right)P_\gamma(k_1)P_B(k), \text{ if } n < -\frac{1}{2} \end{cases}$$

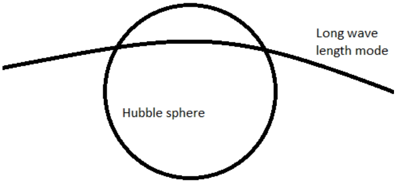
$$\langle\gamma(\mathbf{k}_1)E_\mu(\mathbf{k}_2)E^\mu(\mathbf{k}_3)\rangle' = \begin{cases} \epsilon_{ij}\frac{k_ik_j}{k^2}\left(n-\frac{1}{2}\right)P_\gamma(k_1)P_E(k), \text{ if } n > \frac{1}{2} \\ \\ -\epsilon_{ij}\frac{k_ik_j}{k^2}\left(n+\frac{1}{2}\right)P_\gamma(k_1)P_E(k), \text{ if } n < \frac{1}{2} \end{cases}$$

Semi-classical derivation of the consistency relations

- The presence of long wavelength mode can be studied as modified background. Since inflationary perturbations are conserved in super horizon scale we can absorb the effect of long wavelength perturbation in to coordinates.

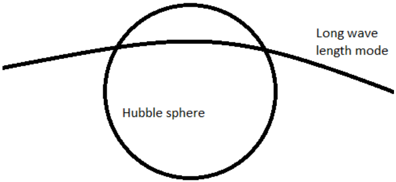
Semi-classical derivation of the consistency relations

- The presence of long wavelength mode can be studied as modified background. Since inflationary perturbations are conserved in super horizon scale we can absorb the effect of long wavelength perturbation in to coordinates.



Semi-classical derivation of the consistency relations

- The presence of long wavelength mode can be studied as modified background. Since inflationary perturbations are conserved in super horizon scale we can absorb the effect of long wavelength perturbation in to coordinates.



- Then the rescaled background will be: $ds^2 = -dt^2 + a^2(t)d\tilde{x}^2$ with $d\tilde{x}^2 \to dx^2 + \gamma_{ij}^B dx^i dx^j$.

Semi-classical derivation of the consistency relations

- In the squeezed limit, due to the rescaled background by the long wavelength graviton mode, one can write a three point correlation function in terms of the modified two point function as,

Semi-classical derivation of the consistency relations

- In the squeezed limit, due to the rescaled background by the long wavelength graviton mode, one can write a three point correlation function in terms of the modified two point function as,

$$\lim_{k_1 \to 0} \langle \gamma(\tau_I, \mathbf{k}_1) Y_\mu(\tau_I, \mathbf{k}_2) Y^\mu(\tau_I, \mathbf{k}_3) \rangle = \left\langle \gamma(\tau_I, \mathbf{k}_1) \left\langle Y_\mu(\tau_I, \mathbf{k}_2) Y^\mu(\tau_I, \mathbf{k}_3) \right\rangle_B \right\rangle$$

with $Y_\mu = \{A_\mu, B_\mu, E_\mu\}$

Semi-classical derivation of the consistency relations

- In the squeezed limit, due to the rescaled background by the long wavelength graviton mode, one can write a three point correlation function in terms of the modified two point function as,

$$\lim_{k_1 \to 0} \langle \gamma(\tau_I, \mathbf{k}_1) Y_\mu(\tau_I, \mathbf{k}_2) Y^\mu(\tau_I, \mathbf{k}_3) \rangle = \left\langle \gamma(\tau_I, \mathbf{k}_1) \langle Y_\mu(\tau_I, \mathbf{k}_2) Y^\mu(\tau_I, \mathbf{k}_3) \rangle_B \right\rangle$$

where $Y_\mu = \{A_\mu, B_\mu, E_\mu\}$

- The two point function in the modified background:

$$\langle Y_\mu(\mathbf{x}) Y^\mu(\mathbf{x}) \rangle_B = \langle Y_\mu(\mathbf{x}) Y^\mu(\mathbf{x}) \rangle_0 + \gamma^B_{ij} \frac{\partial}{\partial \gamma^B_{ij}} \langle Y_\mu(\tilde{\mathbf{x}}) Y^\mu(\tilde{\mathbf{x}}) \rangle|_{\gamma^B = 0} + \dots$$

Semi-classical derivation of consistency relations

- We show that the semi-classical derivation for the graviton magnetic fields cross-correlator can only be trusted or $n > 1/2$

Semi-classical derivation of consistency relations

- We show that the semi-classical derivation for the graviton magnetic fields cross-correlator can only be trusted or $n > 1/2$

$$\lim_{k_1 \to 0} \langle \gamma(\tau_I, \mathbf{k}_1) A_\mu(\tau_I, \mathbf{k}_2) A^\mu(\tau_I, \mathbf{k}_3) \rangle' = \left(n + \frac{1}{2} \right) \epsilon_{ij} \frac{k_{2i} k_{2j}}{k_2^2} P_\gamma(k_1) P_A(k_2),$$

$$\lim_{k_1 \to 0} \langle \gamma(\tau_I, \mathbf{k}_1) B_\mu(\tau_I, \mathbf{k}_2) B^\mu(\tau_I, \mathbf{k}_3) \rangle' = \left(n - \frac{1}{2} \right) \epsilon_{ij} \frac{k_{2i} k_{2j}}{k_2^2} P_\gamma(k_1) P_B(k_2),$$

Semi-classical derivation of consistency relations

- We show that the semi-classical derivation for the graviton magnetic fields cross-correlator can only be trusted or $n > 1/2$

$$\lim_{k_1\to 0} \langle \gamma(\tau_I, \mathbf{k}_1) A_\mu(\tau_I, \mathbf{k}_2) A^\mu(\tau_I, \mathbf{k}_3)\rangle' = \left(n + \frac{1}{2}\right) \epsilon_{ij} \frac{k_{2i} k_{2j}}{k_2^2} P_\gamma(k_1) P_A(k_2),$$

$$\lim_{k_1\to 0} \langle \gamma(\tau_I, \mathbf{k}_1) B_\mu(\tau_I, \mathbf{k}_2) B^\mu(\tau_I, \mathbf{k}_3)\rangle' = \left(n - \frac{1}{2}\right) \epsilon_{ij} \frac{k_{2i} k_{2j}}{k_2^2} P_\gamma(k_1) P_B(k_2),$$

- Similarly for graviton electric fields cross-correlator can only be trusted for $n < -1/2$

Semi-classical derivation of consistency relations

- We show that the semi-classical derivation for the graviton magnetic fields cross-correlator can only be trusted or $n > 1/2$

$$\lim_{k_1 \to 0} \langle \gamma(\tau_I, \mathbf{k}_1) A_\mu(\tau_I, \mathbf{k}_2) A^\mu(\tau_I, \mathbf{k}_3) \rangle' = \left(n + \frac{1}{2} \right) \epsilon_{ij} \frac{k_{2i} k_{2j}}{k_2^2} P_\gamma(k_1) P_A(k_2)$$

$$\lim_{k_1 \to 0} \langle \gamma(\tau_I, \mathbf{k}_1) B_\mu(\tau_I, \mathbf{k}_2) B^\mu(\tau_I, \mathbf{k}_3) \rangle' = \left(n - \frac{1}{2} \right) \epsilon_{ij} \frac{k_{2i} k_{2j}}{k_2^2} P_\gamma(k_1) P_B(k_2)$$

- Similarly for graviton electric fields cross-correlator can only be trusted for $n < -1/2$

$$\lim_{k_1 \to 0} \langle \gamma(\tau_I, \mathbf{k}_1) E_\mu(\tau_I, \mathbf{k}_2) E^\mu(\tau_I, \mathbf{k}_3) \rangle' = - \left(n + \frac{1}{2} \right) \epsilon_{ij} \frac{k_{2i} k_{2j}}{k_2^2} P_\gamma(k_1) P_E(k_2)$$

A direct correlation of tensor and curvature perturbations

- The curvature perturbation induced by any magnetic field is

$$\zeta_B(\tau) = \int_{\tau_0}^{\tau} d \ln \tau' \lambda(\tau') \frac{B_i B^i}{3H^2 \epsilon}$$

A direct correlation of tensor and curvature perturbations

- the curvature perturbation induced by any magnetic field is

$$\zeta_B(\tau) = \int_{\tau_0}^{\tau} d\ln\tau' \lambda(\tau') \frac{B_i B^i}{3H^2\epsilon}$$

- One may naively expect that due to the induced curvature perturbations, there might exist a direct non-trivial correlation of the primordial tensor mode

$$\langle\gamma\zeta\rangle \simeq \langle\gamma\zeta_B\rangle \propto \langle\gamma\mathbf{B}\cdot\mathbf{B}\rangle \neq 0$$

A direct correlation of tensor and curvature perturbations

- The curvature perturbation induced by any magnetic field is

$$\zeta_B(\tau) = \int_{\tau_0}^{\tau} d\ln\tau' \lambda(\tau') \frac{B_i B^i}{3H^2\epsilon}$$

- One may naively expect that due to the induced curvature perturbations, there might exist a direct non-trivial correlation of the primordial tensor mode

$$\langle\gamma\zeta\rangle \simeq \langle\gamma\zeta_B\rangle \propto \langle\gamma\mathbf{B}\cdot\mathbf{B}\rangle \neq 0$$

- But, we explicitly showed that such a correlator actually vanishes due to the statistical isotropy.

$$\langle\gamma\zeta\rangle = 0$$

$\langle\gamma\zeta\rangle$ as a novel observervable

- It is worth noting that isotropy is broken by the long-wavelength modes of vector field.

$\langle\gamma\zeta\rangle$ as a novel observervable

- It is worth noting that isotropy is broken by the long-wavelength modes of vector field.

$$B_i \to B_c \delta_{ix} + \delta B_i \ , \qquad E_i = \delta E_i \ .$$

$\langle \gamma\zeta \rangle$ as a novel observervable

- It is worth noting that isotropy is broken by the long-wavelength modes of vector field.

$$B_i \to B_c \delta_{ix} + \delta B_i \ , \qquad E_i = \delta E_i \ .$$

- This will leads to a direct coupling between γ and ζ.

$$H^{(1)}_{\rm int} = -\frac{1}{2} B_c^2 a \, \xi \int d^3 x a^4 \zeta \gamma_{xx}$$

with

$$\xi(\tau) = \frac{\partial \lambda}{\partial \phi} \frac{\phi'(\tau)}{H} \ .$$

$\langle\gamma\zeta\rangle$ as a novel observervable

- It is worth noting that isotropy is broken by the long-wavelength modes of vector field.

$$B_i \to B_c \delta_{ix} + \delta B_i \ , \qquad E_i = \delta E_i \ .$$

- This will leads to a direct coupling between γ and ζ.

$$H_{\rm int}^{(1)} = -\frac{1}{2} B_c^2 a \, \xi \int d^3 x a^4 \zeta \gamma_{xx}$$

with

$$\xi(\tau) = \frac{\partial \lambda}{\partial \phi} \frac{\phi'(\tau)}{H} \ .$$

- There could be contributions from a quantum gravity induced higher dimensional operators.

$$S = -\frac{1}{4} \int d^4 x \sqrt{-g} \, \lambda(\phi) \left(F_{\mu\nu} F^{\mu\nu} + \frac{1}{4M^4} (F_{\mu\nu} F^{\mu\nu})^2 + \cdots \right)$$

$\langle\gamma\zeta\rangle$ as a novel observervable

- It is worth noting that isotropy is broken by the long-wavelength modes of vector field.

$$B_i \to B_c \delta_{ix} + \delta B_i \ , \qquad E_i = \delta E_i \ .$$

- This will leads to a direct coupling between γ and ζ.

$$H^{(1)}_{\rm int} = -\frac{1}{2} B_c^2 a\, \xi \int d^3x a^4 \zeta \gamma_{xx}$$

with

$$\xi(\tau) = \frac{\partial\lambda}{\partial\phi} \frac{\phi'(\tau)}{H} \ .$$

- There could be contributions from a quantum gravity induced higher dimensional operators.

$$S = -\frac{1}{4} \int d^4x \sqrt{-g}\, \lambda(\phi) \left(F_{\mu\nu} F^{\mu\nu} + \frac{1}{4M^4} (F_{\mu\nu} F^{\mu\nu})^2 + \cdots \right)$$

- Such kind of correction leads to $\langle\gamma B_\mu B^\mu\rangle_{aniso}$ and that will lead to $\langle\gamma\zeta\rangle \neq 0$

Summary

- In a particular model of inflationary magnetogenesis, we defined and calculated the non-Gaussian cross correlation of gauge fields with the tensor perturbations.
- We showed that there exist a leading order correction to these non-Gaussian cross correlations.
- We studied the shape function associated with these non-Gaussian correlators.
- We have derived new set of consistency relations analogous to known consistency relations in the literature.
- We have calculated a direct correlation between one graviton mode and a curvature perturbation mode.

Thank You