Gravitational waves in a viable scenario of inflationary magnetogenesis

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Based on:
Arxiv: 1708.08119, 1912.12089, 2102.09358

Outline of my talk

- Observational evidences of magnetic fields
- Generation mechanisms of the magnetic fields
- Generation of magnetic field during inflation
- Viable model of magnetic field generation
- Helical magnetic field generation
- Production of stochastic background of Gravitational Waves from magnetic fields anisotropic stresses
- Gravitational waves in our model and NANOGrav signal

Observational evidences of Magnetic Fields

- Magnetic field over galactic scales (ordered on kpc) $\sim$ order of 10μG : Both coherent and stochastic

  Beck 2001; Beck and Wielebinski 2013

- Observed in clusters with a few μG strength, coherence length of the order of 10-20 kpc

  Clarke et al. 2001, Govoni and Feretti 2004

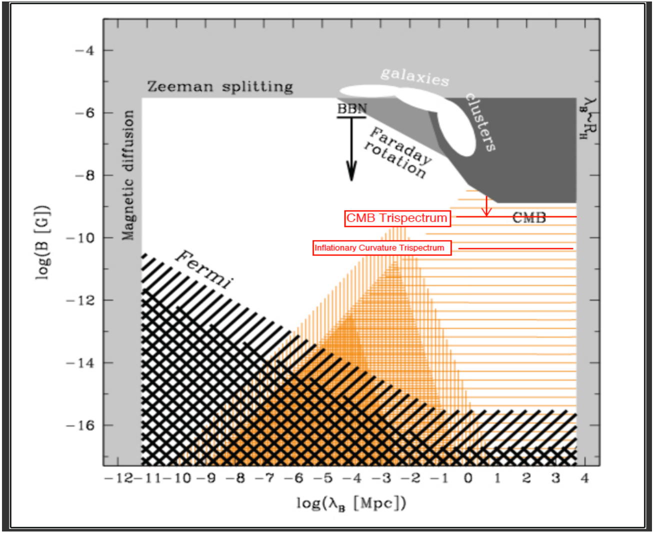
- Evidence for equally strong magnetic field in high redshift ($z \sim 1.3$) galaxies

  Bernet et al. 08; Malik et al. 2020

- FERMI/*LAT* observations of GeV photons from Blazars
  - Lower limit: $\vec{B} \geq 10^{-16}$ G on intergalactic magnetic field at scale above 1 Mpc

  Neronov & Vovk, *Science* 10

Summary of Observational Constraints



Neronov & Vovk, *Science* 10

Origin and Growth: Broad Picture

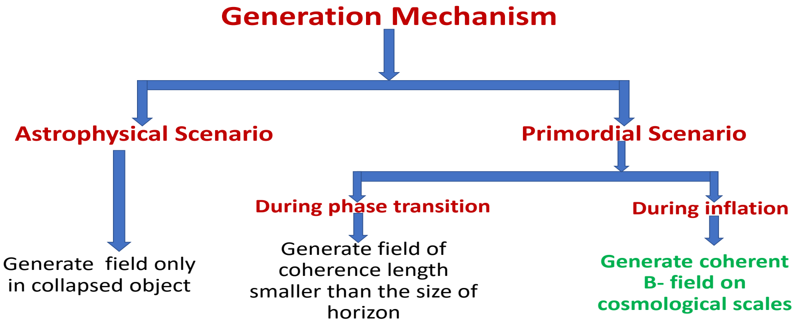
- **Amplification $\longrightarrow$ growth (flux freezing, Dynamo mechanism)**
  Magnetic induction equation,

$$\frac{\partial \vec{B}}{\partial \tau} = \vec{\nabla} \times (\vec{V} \times \vec{B} - \frac{1}{\sigma} \vec{\nabla} \times \vec{B})$$

  Here τ and σ are the time parameter and plasma conductivity, respectively.

- However dynamo requires an initial seed field $\approx 10^{-20}$ G.
  E. N. Parker 1979, Zel'dovich et al. 1983

Generation Mechanism of magnetic field



Astrophysical origin of seeds may not be able to explain B field in voids.

Worth considering primordial origin possibly during inflationary process.

Durrer and Neronov 2013; K. Subramamnian, 2010, 2016

Inflation

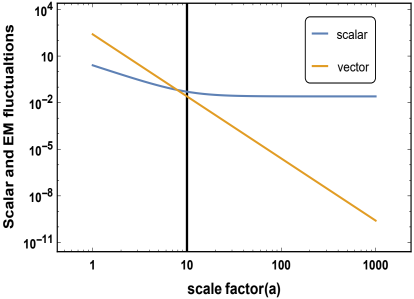
- An era of exponential expansion of space in the early universe.
- Introduced to solve Horizon and Flatness problems.
- Also provides a natural explanation to initial density fluctuations.
- These initial density fluctuations arise due to the quantum mechanical nature of the field which causes inflation or some other field present during inflation.
- As different modes cross the horizon, the nature of fluctuations over these modes becomes classical.

L. Sriramkumar 2009, Daniel Baumann 2009

Motivation for inflationary magnetogenesis

- Inflation offers a natural setting to explain features over a wide range of scales.
- During inflation the universe is devoid of charged plasma, so magnetic flux may be amplified.

Scalar field fluctuation vs EM field fluctuations



EM field fluctuations
$= \langle 0|B_i(\vec{x},\eta)B^i(\vec{y},\eta)|0\rangle \approx \Delta_B|_{k\sim 1/L}$

For inflationary scale, $H = 10^{14}$ GeV

Amplitude of the 1 Mpc mode at horizon crossing $\approx 10^{-10}$ G

Amplitude at the end of inflation $\approx 10^{-10} \times 10^{-46}$ G

contd..

Scalar fluctuations:

$$S_\phi = \frac{-1}{2}\int d^4x\sqrt{-g}(\partial^\nu\phi\partial_\nu\phi - V(\phi))$$

$$(a\,\delta\phi(k,\eta))'' + \left(k^2 - \frac{a''}{a}\right) a\,\delta\phi(k,\eta) = 0$$

EM fluctuations:

$$S_{EM} = -\int \sqrt{-g}\, d^4x\, \frac{1}{4} F_{\mu\nu}F^{\mu\nu}$$

$$(a\,A(k,\eta))'' + k^2 a\,A(k,\eta) = 0$$

Problems with the generation of magnetic field during inflation

- Standard EM action

$$S_{EM} = -\frac{1}{16\pi}\int d^4x\sqrt{-g}\; F_{\mu\nu}F^{\mu\nu}$$

- If we make a conformal transformation $g^*_{\mu\nu} = a^{-2}(\eta)g_{\mu\nu}$

$$\sqrt{-g^*} = a^{-4}(\eta)\sqrt{-g}, \qquad g^{*\mu\nu} = a^2(\eta)g^{\mu\nu}$$

on taking,

$$A^*_\mu = A_\mu \implies S^* = S$$

- Re-scaled magnetic field evolves as in Minkowski space.

$$\mathcal{B} = a^2 B$$

- Hence during inflation $B \propto \dfrac{1}{a^2}$ (irrespective of the plasma effects)

Breaking conformal invariance

Possible ways to achieve this:

- Explicitly breaking the conformal invariance via gravitational couplings.
- Through the coupling of the photons to the axion.
- Couple the photon to a scalar field which is not conformally invariant.

Turner and Widrow 1989, Ratra B. 1992,
J. Martin and Yokoyama 2008, K. Subramanian 2010

Breaking conformal invariance

- **Action:** Modified electromagnetic action + interaction with charged particles/current

$$S_{EM} = -\int \sqrt{-g} d^4x [f^2(\phi) \frac{1}{16\pi} F_{\mu\nu} F^{\mu\nu} + j^\mu A_\mu]$$

- In Coulomb Gauge, $A_i'' + 2\frac{f'}{f} A_i' - a^2 \partial_j \partial^j A_i = 0.$

- Define $\bar{A} \equiv aA(k, \eta)$ $\bar{A}'' + 2\frac{f'}{f} \bar{A}' + k^2 \bar{A} = 0.$

- Define $\mathcal{A} \equiv f\bar{A}(k, \eta)$ $\mathcal{A}''(k, \eta) + \left(k^2 - \frac{f''}{f}\right) \mathcal{A}(k, \eta) = 0.$

Energy density of the EM field

- Energy momentum tensor

$$T_{\mu\nu} = f^2 \left[g^{\alpha\beta} F_{\mu\alpha} F_{\nu\beta} - g_{\mu\nu} \frac{F_{\alpha\beta} F^{\alpha\beta}}{4} \right]$$

- Energy density of the EM field, $\rho = \langle 0 | T_{\mu\nu} u^\mu u^\nu | 0 \rangle$
- $T_{\mu\nu} u^\mu u^\nu = \frac{f^2}{2} B^i B_i + \frac{f^2}{2} E^i E_i$

Magnetic energy density

$$\langle 0 | \frac{f^2}{2} B^i B_i | 0 \rangle = \frac{1}{2\pi^2} \frac{k^5}{a^4} |\mathcal{A}(k, \eta)|^2$$

Electric energy density

$$\langle 0 | \frac{f^2}{2} E^i E_i | 0 \rangle = \frac{f^2}{2\pi^2} \frac{k^3}{a^4} \left| \left[\frac{\mathcal{A}(k, \eta)}{f} \right]' \right|^2$$

Magnetic energy density spectrum for $f \propto a^{\alpha}$ model

- For, $f = f_i\ a^{\alpha}$ and $a = -\frac{1}{H_f \eta}$

$$\mathcal{A}''(k,\eta) + \left(k^2 - \frac{\alpha(\alpha+1)}{\eta^2}\right)\mathcal{A}(k,\eta) = 0.$$

- **Spectral magnetic energy density**:

$$\frac{d\rho_B}{d\ln k} \approx \frac{\mathcal{F}(n)}{2\pi^2} H_f^4 (-k\eta)^{4+2n} \approx \frac{\mathcal{F}(n)}{2\pi^2} H_f^4 \left(\frac{k}{aH}\right)^{4+2n}$$

$$n = -\alpha \text{ if } \alpha \geq -1/2 \text{ and } n = 1+\alpha \text{ if } \alpha \leq -1/2$$

$$\mathcal{F}(n) = \frac{\pi}{2^{2n+1}\Gamma^2(n+1/2)\cos^2(\pi n)}.$$

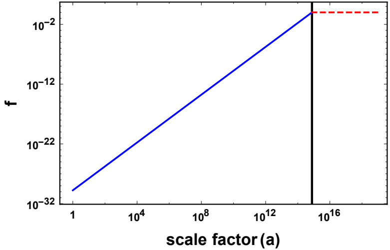
- **For scale invariant magnetic energy density spectrum,**

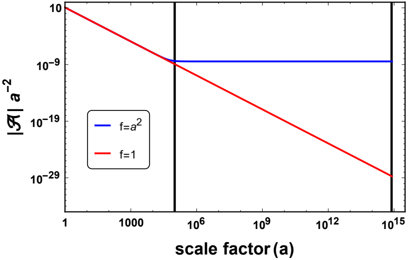
$$\alpha = 2, -3$$

contd..

$$\boxed{\frac{d\rho_B(k,\eta)}{d\ln k} = \frac{1}{2\pi^2}\frac{k^5}{a^4}|\mathcal{A}(k,\eta)|^2}$$

Evolution of a mode $k = 10^5 H_f$





For scale invariant magnetic energy density spectrum,
$\alpha = 2, -3$

Present strength of generated magnetic field

- For scale invariant spectrum

$$\frac{d\rho_B}{d\ln k} \approx \frac{9}{4\pi^2} H_f^4$$

- Magnetic energy density varies with time as $\rho_B \propto 1/a^4$. Magnetic energy strength at present

$$\rho_{B_0} = \rho_{B_f} \left(\frac{a_f}{a_0}\right)^4$$

- Corresponding magnetic field strength

$$B_0 = 2\sqrt{\rho_{B_f}} \left(\frac{a_f}{a_0}\right)^2 \sim 5 \times 10^{-10} G \left[\frac{H_f}{10^{-5} M_{pl}}\right]$$

Back reaction: constraints needed to avoid it

- For $\alpha = -3$, Electric energy density spectrum $\propto \left(\frac{k}{aH}\right)^{-2}$

  • Electric energy density diverges towards the end of inflation.

  • Electrical energy density may dominate over inflation energy density.

  This is known as **back reaction problem**. Demozzi et al. 2009

- **For magnetic field energy density to be scale invariant and electric field not to dominate, α is constrained to be $\alpha = 2$**

- Scale invariant energy density spectrum and no back reaction problem

$$f = f_i \ a^2.$$

Note that at the beginning of inflation $a_i = 1$ and $\eta_i = -H_f^{-1}$

Strong coupling problem

- In the usual approach with conformal breaking, the final value of f is made unity to match with the standard EM theory.
- Since f grows as $a^2 \implies$ initial value of f is very small.
- Effective coupling parameter $e_N = e/f^2$ becomes very large.
- For an inflation having

$$H_f = 10^{14}\text{GeV} \qquad a_f/a_i \approx exp(60)$$

If $f = 1$ at the end of inflation, the effective EM coupling is $e_N = e$ (standard)

However, at the onset of inflation,

$$f = exp(-120) \qquad \Rightarrow \qquad e_N \approx e \times 10^{56}$$

Addressing the strong coupling problem

- In our model, we bring the system back to the standard form not at end of inflation but some time after it before reheating.

  $f_i = 1 \implies, \quad f = a^2 > 1$(during inflation) & $f \gg 1$ at end of inflation.

  Hence no strong coupling problem.

- As coupling parameter is very small at the end of inflation. Hence, f need to be brought back to unity post inflation.

$$\text{During Inflation}, \quad f = a^2$$

$$\text{Post Inflation}, \quad f = f_f \left(a/a_f\right)^{-\beta}$$

- Models are constrained by the requirement of how fast the factor f falls to 1 from a large value.

Post Inflationary era

- We assume a matter dominated universe after inflation till reheating.
- For $f \propto a^{-\beta}$, we solved vector potential by demanding the continuity of vector potential and its time derivative at the end of inflation.
- Energy density in magnetic and electric field can be calculated as before.

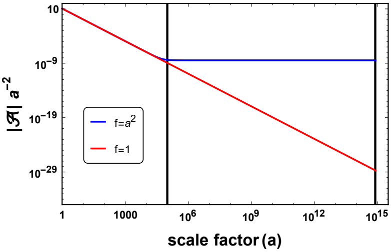
  At reheating, magnetic energy spectrum for super horizon modes $\propto k^4$ for $\alpha = 2$

Magnetic field energy spectrum

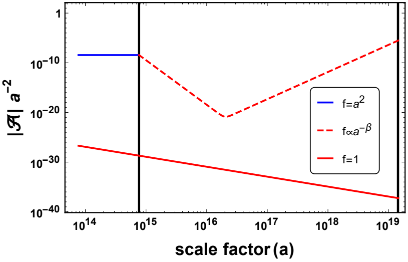
$$\boxed{\frac{d\rho_B(k,\eta)}{d\ln k} = \frac{1}{2\pi^2}\frac{k^5}{a^4}|\mathcal{A}(k,\eta)|^2}$$

Evolution of a mode $k = 10^5 H_f$

During inflation



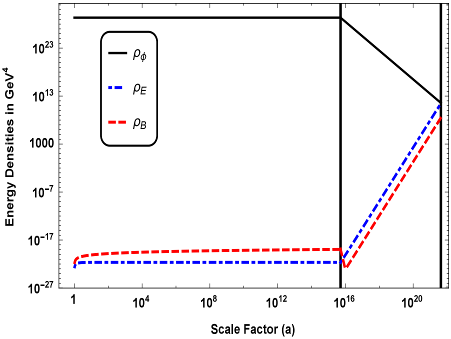
Post Inflation



Constraints from Post Inflationary Pre-reheating phase

Total energy in electric and magnetic field should be less that in inflation field at reheating.

$$\boxed{\rho_E + \rho_B < \rho_\phi \,|_{reheat} = g_r \frac{\pi^2}{30} T_r^4}$$



Magnetic field strength and coherence length at reheating

Coherence length at reheating is given by,

$$L_c = a_r \frac{\int_0^{k_r} \frac{2\pi}{k} \frac{d\rho_B(k,\eta)}{d\ln k} d\ln k}{\int_0^{k_r} \frac{d\rho_B(k,\eta)}{d\ln k} d\ln k}.$$

Given this coherence length, we can estimate the magnetic field strength at reheating using,

$$B[L_c] = \sqrt{8\pi \frac{d\rho_B(k,\eta)}{d\ln k}}\Bigg|_{k=\frac{2\pi a_r}{L_c}}.$$

Post reheating evolution of magnetic field

Coherence length and magnetic field today:

$$B_0^{NL}[L_{c0}^{NL}] = B(L_c)\left(\frac{a_0}{a_r}\right)^{-2}\left(\frac{a_m}{a_r}\right)^{-p}, \quad L_{c0}^{NL} = L_c\left(\frac{a_0}{a_r}\right)\left(\frac{a_m}{a_r}\right)^{q},$$

where $a_m \implies$ scale factor at radiation-matter equality,

$$p \equiv (n+3)/(n+5) \quad \text{and} \quad q \equiv 2/(n+5)$$

here n is defined in such a way that

$$\frac{d\rho_B}{d\ln k} \propto k^{n+3}$$

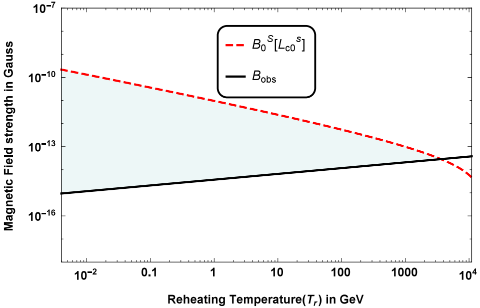
Banerjee and Jedamzik, 2004; K. Subramanian, 2016)

Results taking nonlinear effects into account

After incorporating the results of magnetic field evolution suggested by simulation,

$$B_0^S[L_{c0}^S] = B_0[L_{c0}]\,(a_m/a_r)^{-0.5}\,, \quad L_{c0}^S = L_{c0}\,(a_m/a_r)^{0.5}$$

Brandenburg et al. 2015, 2017 and Zrake et al. 2014



For $T_r = 150$ MeV, $B_0 \approx 4 \times 10^{-11}$ G and $L_{c_0} \approx 0.01$ Mpc

For $T_r = 100$ GeV, $B_0 \approx 6 \times 10^{-13}$ G and $L_{c_0} \approx 7 \times 10^{-4}$ Mpc

EM action for the generation of helical magnetic field

Action

$$S_{EM} \quad = -\int \sqrt{-g}\, d^4x \left[\frac{f^2(\phi)}{16\pi} \left(F_{\mu\nu}F^{\mu\nu} + F_{\mu\nu}\tilde{F}^{\mu\nu} \right) + j^\mu A_\mu \right]$$

Modified Maxwell's Equation

$$A_i'' + 2\frac{f'}{f}\left(A_i' + \epsilon_{ijk}\partial_j A_k\right) - a^2 \partial_j \partial^j A_i = 0$$

In terms of circular polarisation basis

$$\bar{A}_h'' + 2\frac{f'}{f}\left(\bar{A}_h' + hk\bar{A}_h\right) + k^2 \bar{A}_h = 0$$

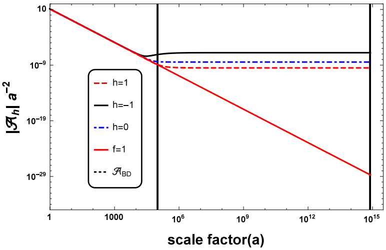
here $h = \pm 1$

Magnetic field energy spectrum

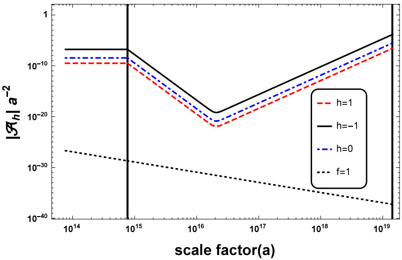
$$\boxed{\frac{d\rho_B(k,\eta)}{d\ln k} = \frac{1}{(2\pi)^2}\frac{k^5}{a^4}\left(|\mathcal{A}_+(k,\eta)|^2 + |\mathcal{A}_-(k,\eta)|^2\right)}$$

Evolution of a mode $k = 10^5 H_f$

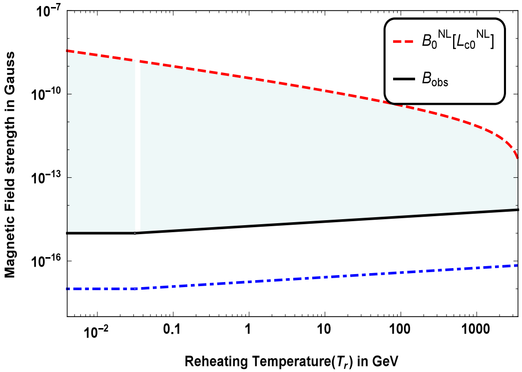
During inflation



Post Inflation



Present strength of magnetic fields



$$L_{c0}^{NL} = L_{c0}\left(\frac{a_m}{a_r}\right)^{2/3}$$

$$B_0^{NL}[L_{c0}^{NL}] = B_0[L_{c0}]\left(\frac{a_m}{a_r}\right)^{-1/3}$$

Banerjee and Jedamzik 2004,
Brandenburg and Kahniashvilli 2016

For $T_r = 150$ MeV, $B_0 \approx 10^{-9}$ G and $L_{c_0} = 0.5$ Mpc
For $T_r = 100$ GeV, $B_0 \approx 4 \times 10^{-11}$ G and $L_{c_0} = 0.07$ Mpc

Gravitational wave

Gravitational waves → Represented by the transverse traceless part of the space-time metric perturbation.

- The metric for homogeneous, isotropic and spatially flat universe (FLRW metric) with tensor perturbations.

$$\boldsymbol{ds^2 = a^2(\eta)(-d\eta^2 + (\delta_{ij} + 2\, h_{ij}\,)dx^i dx^j)}$$

where h_{ij} satisfies: $\partial^i h_{ij} = 0$ and $h^i_i = 0$.

- The energy density of the stochastic GW in terms of tensor perturbations,

$$\rho_{GW} = \frac{1}{16\pi G}\frac{\langle h'_{ij} h'^{ij}\rangle}{a^2} \equiv \int d\ln k \frac{d\rho_{GW}}{d\ln k}$$

where

$$\frac{d\rho_{GW}}{d\ln k} = \frac{k^3}{4(2\pi)^3 G a^2}\sum_{\aleph}\left(\left|\frac{dh^{\aleph}(k,\eta)}{d\eta}\right|^2\right)$$

with $(\aleph = T, \times)$ or $(\aleph = +, -)$ for linear and circular polarisation basis respectively.

Evolution of Gravitational waves

- The evolution equation for h_{ij} in presence of a source

$$h''_{ij} + \frac{2a'}{a} h'_{ij} + k^2 h_{ij} = 8\pi G \, a^2 \overline{T}_{ij} \, .$$

$a^2 \overline{T}_{ij}$: transverse traceless part of the energy-momentum tensor of the source.

- In terms of dimensionless variable $x \equiv k\eta$ and $\Pi_{ij} \equiv [1/(\rho + p)] T_{ij}$, the above equation, in radiation dominated Universe, reduces to

$$\frac{d^2 h_{ij}}{dx^2} + \frac{2}{x} \frac{dh_{ij}}{dx} + h_{ij} = \frac{4}{x^2} \Pi_{ij}.$$

- After solving the above equation, we get the following by taking sub-horizon limit,

$$\left| \frac{dh^{\aleph}(k,x)}{dx} \right|^2 = \frac{8}{x^2} \int_{x_R}^{x_{\nu d}} \int_{x_R}^{x_{\nu d}} \frac{dx_1 dx_2}{x_1 x_2} \cos(x_2 - x_1) |\Pi^{\aleph}|^2 (k, x_1, x_2)$$

Evolution of Gravitational waves

- By taking primordial magnetic field as a source for the gravitational wave, we get

$$|\Pi^{\aleph}|^2(\boldsymbol{k},\boldsymbol{\eta},\boldsymbol{\eta}') = \left(\frac{1}{\tilde{\rho}+\tilde{p}}\right)^2 \frac{1}{8(2\pi)^5}\int \boldsymbol{d}^3\boldsymbol{q}\Big[\boldsymbol{P_B(q,\eta)}\ \boldsymbol{P_B}(|\vec{\boldsymbol{k}}-\vec{\boldsymbol{q}}|,\boldsymbol{\eta})(1+\gamma^2+\beta^2+\gamma^2\beta^2)\Big]$$
$$\Big[\boldsymbol{C_B(q,\eta,\eta')}\ \boldsymbol{C_B}(|\vec{\boldsymbol{k}}-\vec{\boldsymbol{q}}|,\boldsymbol{\eta},\boldsymbol{\eta}')\Big]$$

- In summary, the generated GW spectrum depends upon the following:
  - **Magnetic field power spectrum (P_B)**
  - **Unequal time correlation function (C_B)**
  - **Green function ($\cos(x_2-x_1)/x_2x_1$)**

Generated GW spectrum (Results)

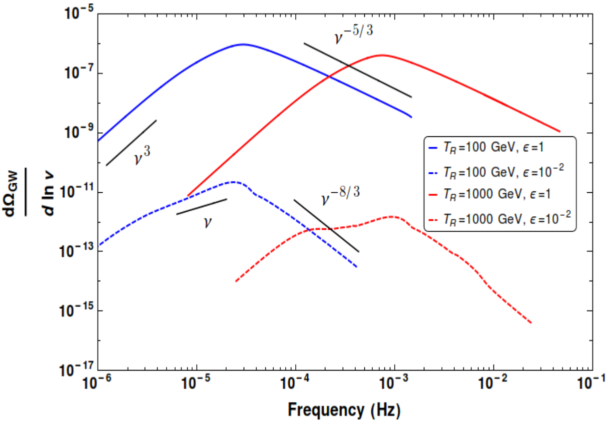
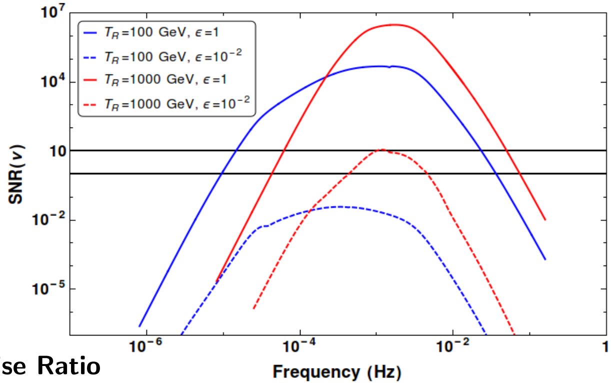


Figure: *GW energy spectrum for the reheating scale $T_R = 100$ GeV and $T_R = 1000$ GeV and also for the different fraction (ϵ) of EM field energy density to the background energy density at reheating along with the LISA sensitivity curve.*

Detection of the generated GW spectrum with LISA



Signal to Noise Ratio

$$\mathrm{SNR} = \sqrt{T \int_{\nu_{min}}^{\nu_{max}} d\nu \left(\frac{d\Omega_{GW}}{d\ln\nu} \Big/ \frac{d\Omega_n}{d\ln\nu} \right)^2} \equiv \sqrt{\int_{\nu_{min}}^{\nu_{max}} d\ln\nu \; (\mathrm{SNR}(\nu))^2}$$

where $\nu_{min}(\nu_{max})$: minimal(maximal) frequencies accessible at the LISA and

$$\mathrm{SNR}(\nu) = \sqrt{\nu T} \left(\frac{d\Omega_{GW}}{d\ln\nu} \Big/ \frac{d\Omega_n}{d\ln\nu} \right).$$

Generated GW spectrum (Results)

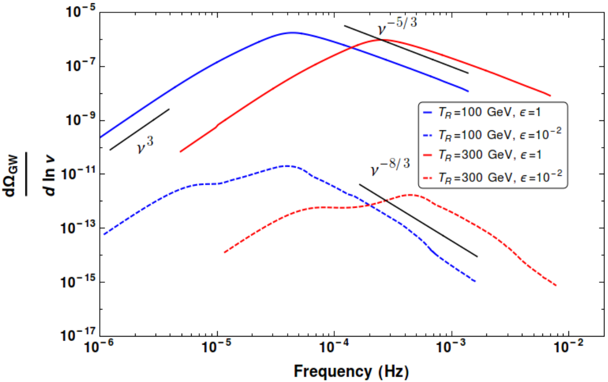


Figure: *GW energy spectrum generated from the EM field anisotropic stresses (helical case).*

The peak value of the generated GW spectrum in this case is of the same order as in non-helical case.

Generated GW spectrum and NANOGrav Signal

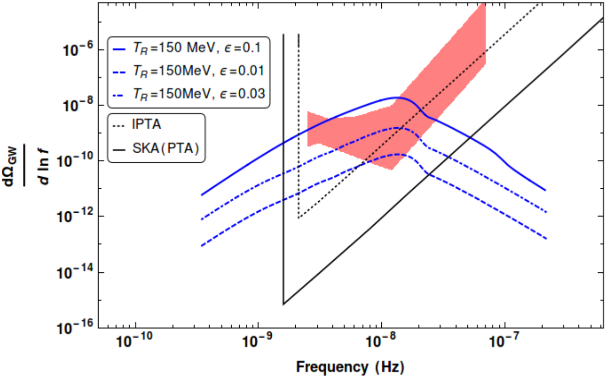


Figure: *In this figure, the blue curves are the GW spectrum obtained in our model of inflationary magnetogenesis for a scenario where reheating temperature T_R=150 MeV for different value of ϵ. Pink colour region is 95% confidence region of the parameter space of the GW signal modelled as broken power law by NANOGrav collaboration (Z.Arzoumanian et al. 2020).*

Summary

- Addressed the strong coupling and back-reaction problems of inflationary magnetogenesis.
- Incorporating inverse transfer effect suggested by numerical simulations for the non-linear processing of the magnetic field, the generated magnetic field satisfies bounds on IGM magnetic fields if $T_r < 4000$ GeV.
- Helical magnetic field $\rightarrow$ satisfy γ-ray observation for all possible reheating scales but reheating scale should be below 4000 GeV.
- A possible way to check the viability of these models is via the detection of produced stochastic background of GWs.
- The generated GWs background lies within the sensitivity of LISA for $T_r \geq 100$ GeV and a possible detection of GW spectrum of the nature calculated here by LISA will provide important probe of the scenarios of magnetogenesis.

Summary

- A similar strength of the GW spectrum is obtained if the magnetic field is of helical nature. However, in the case of helical EM fields, the generated GW spectrum is circularly polarised while it is unpolarised when the generated EM fields are nonhelical.

- For reheating scale around $T_r = 150$ MeV, PTA provide important constraints to our models.
- The resulting GW background can be interpreted as leading to the signal inferred in NANOGrav 12.5 year data when EM field energy density lies in the range of 3% to 10% of the background energy density.

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Thank you