Perturbations In Some Dark Energy Models

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The Universe

- The Universe is described by a spatially flat metric

$$ds^2 = \underbrace{\boxed{a^2(\tau)}}_{}\!\!{}^{\text{scale factor}} \left(-d\tau^2 + dx^2 + dy^2 + dz^2\right) \qquad \boxed{a_0 = 1}$$

- Large scales $\rightarrow$ larger than the large scale structures $\Rightarrow$ Universe is spatially homogeneous and isotropic
- Small scales $\Rightarrow$ Universe is not so uniform $\rightarrow$ start seeing the structures — galaxies, galaxy clusters, voids ...
- Large scale structures evolved from some initial fluctuations
- Evolution of fluctuations depend on background dynamics

Goal: If some dark energy models can provide a congenial environment for structure formation

Perturbations In A Scalar Field Model With Virtues Of ΛCDM

Scalar Field Model

(P. J. E. Peebles, B. Ratra, ApJL 1988, V. Sahni and A. A. Starobinsky, IJMPD 2000)

Assume: Cold dark matter (CDM) is like the perfect fluid distribution & a scalar field (φ) with a potential $V(\varphi)$ is acting as dark energy with

- Energy density $\Rightarrow \rho_\varphi = \underbrace{\boxed{\frac{1}{2a^2}\varphi'^2}}_{\text{Kinetic: } E_K} + \underbrace{\boxed{V(\varphi)}}_{\text{Potential: } E_P}$
- Pressure $\Rightarrow p_\varphi = \frac{1}{2a^2}\varphi'^2 - V(\varphi)$
- EoS parameter $\Rightarrow w_\varphi = \frac{p_\varphi}{\rho_\varphi} = \frac{\frac{1}{2a^2}\varphi'^2 - V(\varphi)}{\frac{1}{2a^2}\varphi'^2 + V(\varphi)}$
- When $E_K \gg E_P \Longrightarrow$ scalar field behaves as a stiff fluid with $w_\varphi = 1$
- when $E_P \gg E_K \Longrightarrow$ scalar field behaves a cosmological constant with $w_\varphi = -1$

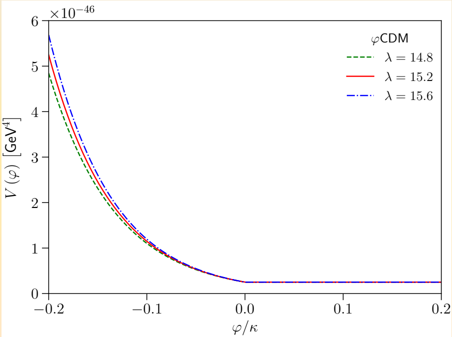
Klein-Gordon equation $\Longrightarrow \varphi'' + 2\mathscr{H}\varphi' + a^2\frac{dV}{d\varphi} = 0$

Potential

$$V(\varphi) = V_0 e^{-\lambda\kappa\varphi}\Theta(-\varphi) + V_0\Theta(\varphi),$$

$$\Theta(\varphi-\varphi_0) = \begin{cases} 0 & \text{for} \quad \varphi < \varphi_0 \\ 1 & \text{for} \quad \varphi \geq \varphi_0 \end{cases}$$

- **Free Parameter-** 1 $\Rightarrow \lambda \rightarrow$ slope
- λ constrained by BBN condition $\Omega_\varphi\left(a\sim 10^{-10}\right) \lesssim 0.09$ (C. Wetterich, NPB 1988, E. J. Copeland *et al.*, PRD 1998)
- V_0 depends on $\Omega_b h^2$, $\Omega_c h^2$, H_0
- $\Omega_{\varphi 0}$ depends on the height of the slow-roll region $\rightarrow V_0$
- At late time $w_\varphi = -1 \longrightarrow$ independent of V_0 or λ or initial conditions
- **Free Parameter-**2 $\Rightarrow \varphi_0 \rightarrow$ transition point
- Once in tracking region, evolution of ρ_φ is independent of $\varphi_0 \longrightarrow$ used $\varphi_0 = 0$



$\lambda = 15.6 \longrightarrow \Omega_\varphi(a\sim 10^{-10}) = 0.01642$

$V_0 = 2.510\times 10^{-47}$ Gev^4 $\longrightarrow \Omega_\varphi(a=1) = 0.6840$

S. Sinha & N. Banerjee, JCAP **04** 060 (2021)

Potential

$$V(\varphi) = V_0 e^{-\lambda\kappa(\varphi-\varphi_0)}\Theta(-\varphi+\varphi_0) + V_0\Theta(\varphi-\varphi_0),$$

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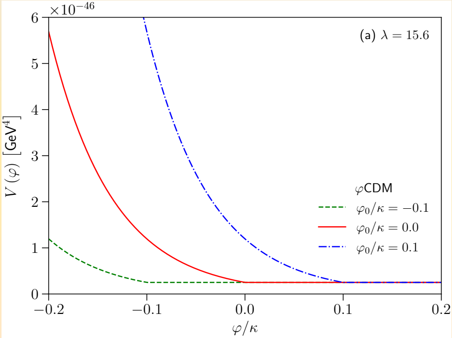

$V_0 e^{-\lambda\kappa\varphi} \longrightarrow V_0 e^{-\lambda\kappa(\varphi-\varphi_0)}$ to accommodate for the continuity of $V(\varphi)$

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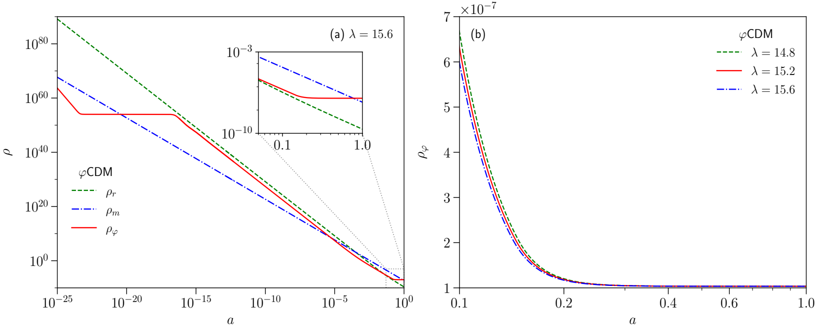
Evolution Of Scalar Field



(1) φ rolls down the potential, $E_K \gg E_P \Longrightarrow \rho_\varphi \propto a^{-6} \longrightarrow \rho_\varphi$ is dominated by E_K

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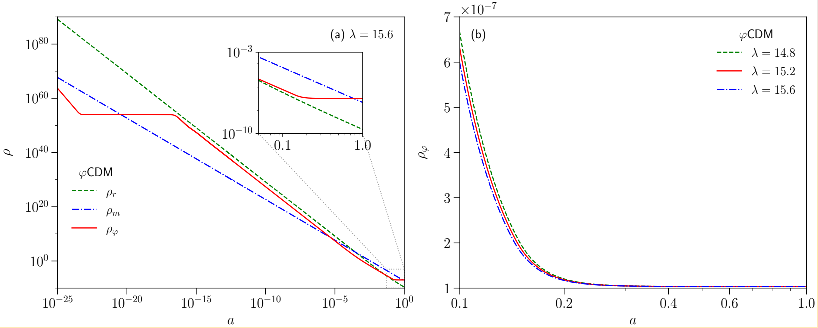

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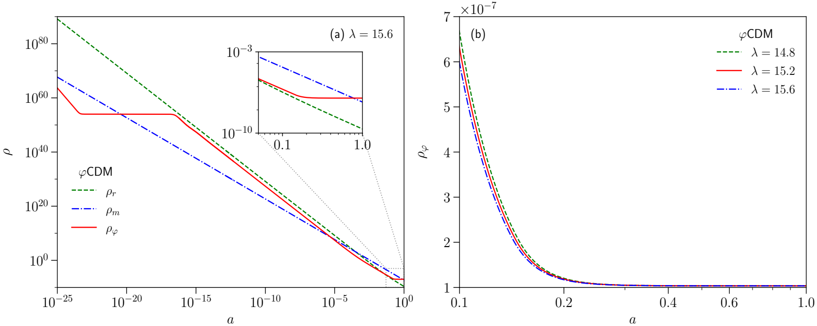
(2) φ rolls down very slowly $\Longrightarrow \varphi''$ inconsequential $\Longrightarrow \rho_\varphi$ becomes flat $\longrightarrow \rho_\varphi$ is determined by E_P

Evolution Of Scalar Field



(1) φ rolls down the potential, $E_K \gg E_P \implies \rho_\varphi \propto a^{-6} \longrightarrow \rho_\varphi$ is dominated by E_K

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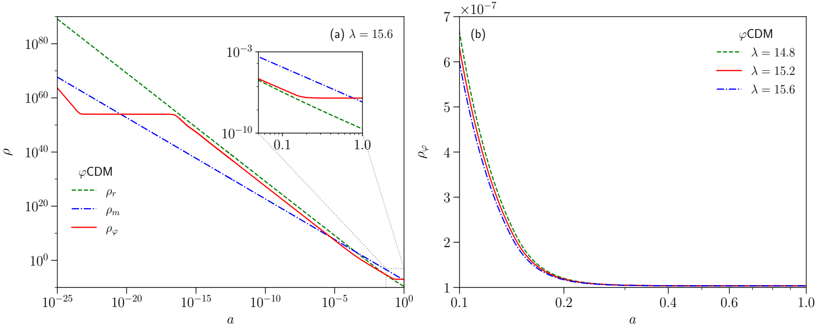
(3) ρ_r reaches the flat $\rho_\varphi \longrightarrow$ both start evolving together

Evolution Of Scalar Field



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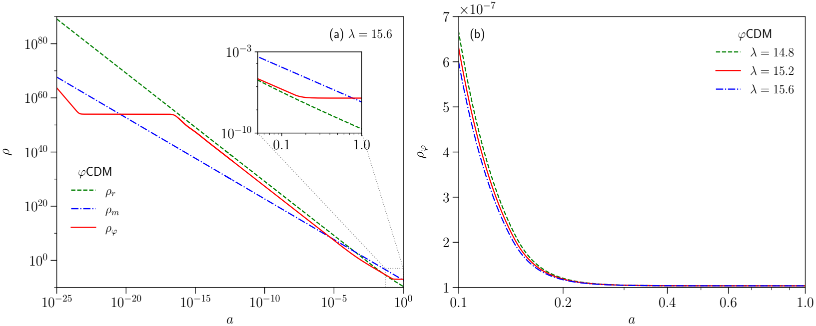

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(5) V_0 takes over $\longrightarrow \rho_\varphi$ behaves like the cosmological constant

Evolution Of Perturbations

- In synchronous gauge, perturbed metric takes the form (H. Kodama, M. Sasaki, PTPS 1984, C.-P. Ma, E. Bertschinger, ApJ 1995, K. A. Malik *et al.*, PRD 2003)

$$ds^2 = a^2(\tau)\left\{-d\tau^2 + \left[(1-2\psi)\delta_{ij} + 2\partial_i\partial_j E\right]dx^i dx^j\right\}$$

- $\psi = \eta$ & $k^2 E = -\mathrm{h}/2 - 3\eta \longrightarrow (\eta, \mathrm{h})$ are synchronous gauge fields in the Fourier space, $k \to$ comoving wavenumber
- DM density contrasts $\Rightarrow \delta_c = \delta\rho_c/\rho_c$, DM velocity perturbation $\Rightarrow v_c$
- Scalar field density contrasts $\Rightarrow \delta_\varphi = \delta\rho_\varphi/\rho_\varphi$
- Perturbed energy and momentum conservation equations are

$$\delta_c' + k v_c + \frac{\mathrm{h}'}{2} = 0$$

$$v_c' + \mathscr{H} v_c = 0$$

Evolution Of Perturbations

- The perturbation $\delta\varphi$ in the scalar field has the equation of motion (J. Martin, D. J. Schwarz, PRD 1998, P. Brax *et al.*, PRD 2000)

$$\delta\varphi'' + 2\mathcal{H}\delta\varphi' + k^2\delta\varphi + a^2\,\frac{d^2V}{d\varphi^2}\delta\varphi + \frac{1}{2}\varphi' \mathrm{h}' = 0$$

where, $\dfrac{d^2V}{d\varphi^2} \approx \dfrac{3}{2}\dfrac{\mathcal{H}^2}{a^2}\left[-\dfrac{1}{2}\left(c_{s,\varphi}^2 - 1\right)\left(3c_{s,\varphi}^2 + 5\right) + \dfrac{\mathcal{H}'}{\mathcal{H}}\left(c_{s,\varphi}^2 - 1\right)\right]$

- Adiabatic sound speed sq. $\rightarrow c_{s,\varphi}^2 = 1 + \frac{2a^2}{3\mathcal{H}\varphi'}\frac{dV}{d\varphi}$

- The perturbation in energy density $\delta\rho_\varphi$ and pressure δp_φ are given as

$$\delta\rho_\varphi = -\delta T^0_{0(\varphi)} \;=\; \frac{\varphi'\delta\varphi'}{a^2} + \delta\varphi\frac{dV}{d\varphi},$$

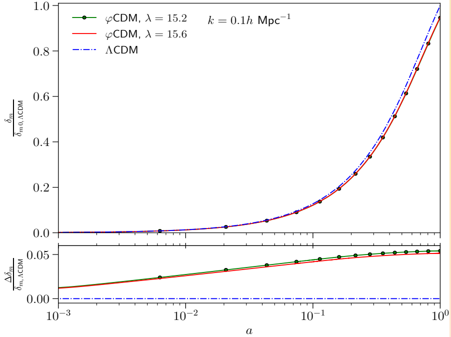
$$\delta T^j_{0(\varphi)} = -\frac{\imath k_j\,\varphi'\,\delta\varphi}{a^2}, \qquad \imath \equiv \sqrt{-1}$$

$$\delta p_\varphi \delta^i_j = \delta T^i_{j(\varphi)} \;=\; \left(\frac{\varphi'\delta\varphi'}{a^2} - \delta\varphi\frac{dV}{d\varphi}\right)\delta^i_j,$$

Evolution Of Perturbations

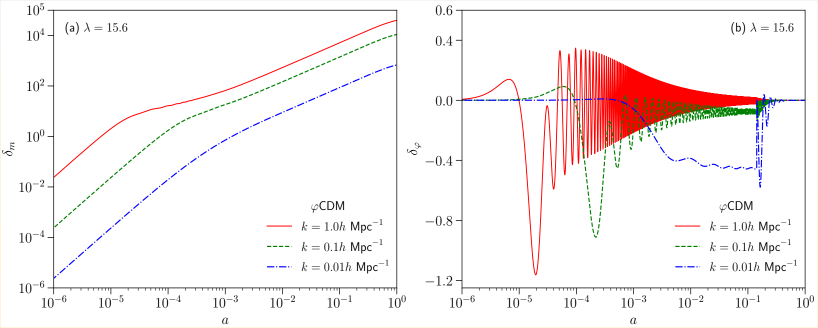
- Solved with adiabatic initial conditions
- Matter density contrast $\Rightarrow \delta_m = \frac{\delta\rho_m}{\rho_m} = \frac{(\delta_c\rho_c + \delta_b\rho_b)}{(\rho_c + \rho_b)}$

- Evolution of δ_m for φCDM and ΛCDM
- δ_m for both φCDM and ΛCDM have been scaled by $\delta_{m0} = \delta_m(a = 1)$ of ΛCDM
- δ_m for $\lambda = 15.2$ takes a slightly smaller value compared to that of δ_m for $\lambda = 15.6$
- Growth of δ_m decreases with decrease in λ



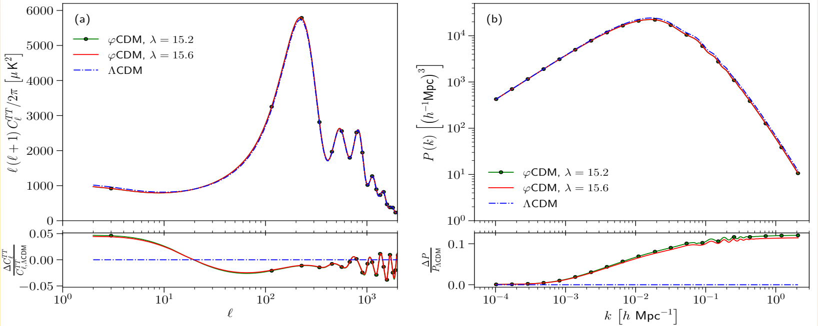
Note: For fractional change, $\Delta\delta_m = (\delta_{m,\Lambda\mathrm{CDM}} - \delta_m)$

Evolution Of Perturbations



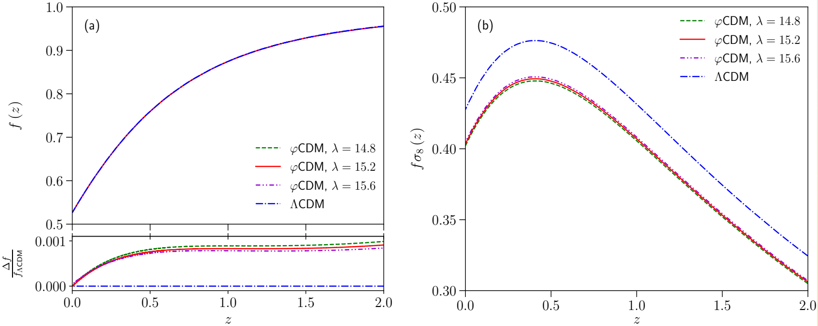
- In the matter dominated era, the modes of δ_m grow in a very similar fashion
- The modes of δ_φ oscillate rapidly with decreasing amplitude after entering the horizon

Power Spectra



- C_ℓ^{TT} are almost independent of λ
- Less matter content $\Rightarrow$ higher oscillation amplitudes in C_ℓ^{TT}
- Smaller $\lambda \Rightarrow$ slightly lower low-ℓ modes
- Larger $\lambda \Rightarrow$ marginally lower $P(k)$ at small scales

Growth Rate



- $f = \frac{d\ln\delta_m}{d\ln a}$ is almost same for all the models at low redshift ($z = \frac{1}{a} - 1$)
- Smaller $\lambda \Rightarrow$ lower f
- Substantial difference in $f\sigma_8$ for φCDM and ΛCDM
- A low $f\sigma_8 \rightarrow$ characteristic distinguishing feature

Model	λ	σ_8
	14.8	0.7638
φCDM	15.2	0.7664
	15.6	0.7687
ΛCDM	—	0.8123

Differentiating Interaction In The Dark Sector With Perturbation

Motivation

- Interaction in the dark sector may not be ruled out *a priori*
- Question: When is the interaction significant in the evolution history of the Universe?
- Possibilities: **(a)** Interaction was there from the beginning of the Universe and exists through its evolution, **(b)** Interaction is a recent phenomenon **(c)** Interaction was entirely an early phenomenon and not at all present today
- An evolving coupling parameter instead of being a constant may answer

To assess if there is any stage of evolution when the interaction is significant

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Interaction In The Dark Sector

$$\rho_c' + 3\mathscr{H} \boxed{\rho_c} = -a\,Q$$

cold dark matter

$Q > 0$: DM $\rightarrow$ DE

$$\rho_{de}' + 3\mathscr{H}(1 + w_{de}) \boxed{\rho_{de}} = a\,Q$$

dark energy

$Q < 0$: DE $\rightarrow$ DM

Interaction In The Dark Sector

$$\rho_c' + 3\mathscr{H}\boxed{\rho_c} = -a\,Q$$

cold dark matter

$$\rho_{de}' + 3\mathscr{H}(1+w_{de})\boxed{\rho_{de}} = a\,Q$$

dark energy

$$Q = \frac{\mathscr{H}\rho_{de}\beta(a)}{a}$$

- EoS of DE: $w_{de} = w_0 + w_1(1-a)$
- Ansatz for coupling parameter $\beta(a)$ are :

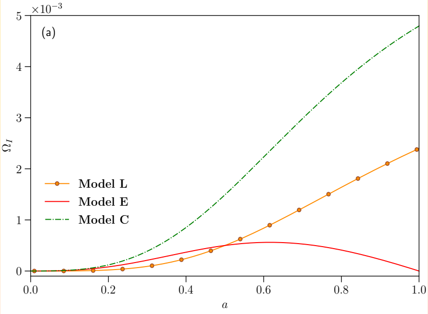
Model L: significant at late time $\Rightarrow$

$$\beta(a) = \beta_0\left(\frac{2a}{1+a}\right)$$

Model E: significant at early time $\Rightarrow$

$$\beta(a) = \beta_0\left(\frac{1-a}{1+a}\right)$$

Model C: a constant $\Rightarrow$ $\beta(a) = \beta_0$



$\Omega_I = \frac{Q}{3H^3/\kappa}$, $w_0 = -0.9995$, $w_1 = 0.005$, $\beta_0 = 0.007$

Evolution of Perturbations

- Perturbed energy and momentum conservation equations are

$$\delta_c' + kv_c + \frac{h'}{2} = \mathscr{H}\beta(a)\frac{\rho_{de}}{\rho_c}(\delta_c - \delta_{de})$$

$$v_c' + \mathscr{H}v_c = 0$$

$$\delta_{de}' + 3\mathscr{H}\left(c_{s,de}^2 - w_{de}\right)\delta_{de} + (1 + w_{de})\left(kv_{de} + \frac{h'}{2}\right)$$
$$+3\mathscr{H}\left[3\mathscr{H}(1 + w_{de})\left(c_{s,de}^2 - w_{de}\right)\right]\frac{v_{de}}{k} + 3\mathscr{H}w_{de}'\frac{v_{de}}{k}$$
$$= 3\mathscr{H}^2\beta(a)\left(c_{s,de}^2 - w_{de}\right)\frac{v_{de}}{k}$$

$$v_{de}' + \mathscr{H}\left(1 - 3c_{s,de}^2\right)v_{de} - \frac{k\,\delta_{de}\,c_{s,de}^2}{(1 + w_{de})} = \frac{\mathscr{H}\,\beta(a)}{(1 + w_{de})}\left[v_c - \left(1 + c_{s,de}^2\right)v_{de}\right]$$

Evolution of Perturbations

- Solved with adiabatic initial conditions
- To avoid the instability in dark energy perturbations $\Rightarrow c^2_{s,de} = 1$
- Matter density contrast $\Rightarrow \delta_m = \frac{\delta\rho_m}{\rho_m} = \frac{(\delta_c\rho_c + \delta_b\rho_b)}{(\rho_c + \rho_b)}$

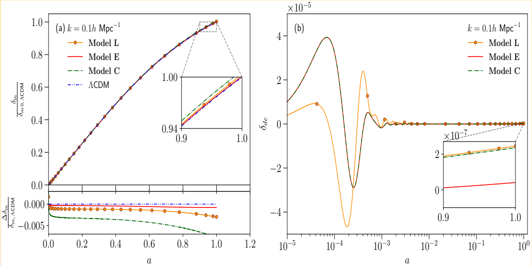
➤ δ_m for Model E evolves close to the ΛCDM model

➤ δ_m for Model L & Model C grow to a little higher value

➤ At early time, δ_{de} oscillates and then decays to very small values

➤ Early time evolution of δ_{de} in Model E is similar to Model C

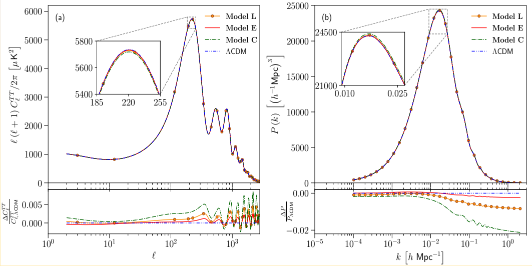
➤ Late time evolution of δ_{de} in Model L is similar to Model C



The origin on the x-axis is actually 10^{-5}

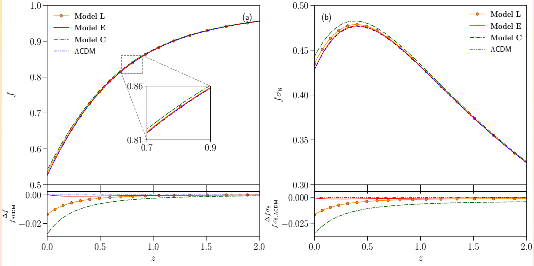
Note: For fractional change, $\Delta\delta_m = (\delta_{m,\Lambda CDM} - \delta_m)$

Power Spectra & Growth Rate



- Lower oscillation amplitudes in C_ℓ^{TT} → Model C < Model L < Model E < ΛCDM
- Less dark energy ⇒ less ISW effect → Model C < Model L < Model E < ΛCDM
- Higher $P(k)$ → Model C > Model L > Model E > ΛCDM

- Model L & Model C have slightly higher values of f and $f\sigma_8$ at $z = 0$
- Model E & ΛCDM have same values of f and $f\sigma_8$ at $z = 0$
- Model E had a slightly larger value of f and $f\sigma_8$ than ΛCDM, in the recent past



Priors & Datasets

$$\mathscr{P} \equiv \{\Omega_b h^2, \Omega_c h^2, 100\theta_{MC}, \tau, \beta_0, w_0, w_1, \ln\left(10^{10} A_s\right), n_s\}$$

Parameter	Prior
$\Omega_b h^2$	$[0.005, 0.1]$
$\Omega_c h^2$	$[0.001, 0.99]$
$100\theta_{MC}$	$[0.5, 10]$
τ	$[0.01, 0.8]$
β_0	$[-1.0, 1.0]$
w_0	$[-0.9999, -0.3333]$
w_1	$[0.005, 1.0]$
$\ln\left(10^{10} A_s\right)$	$[1.61, 3.91]$
n_s	$[0.8, 1.2]$

Priors & Datasets

model parameters

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***Planck*:** CMB anisotropies measurements from *Planck* 2018 collaboration (*Planck* TT, TE, EE + lowE + lensing) (N. Aghanim *et al.*(Planck Collaboration), A&A 2020)

BAO: distance measurements from **(a)** 6dFGS at $z = 0.106$ (F. Beutler, MNRAS 2011), **(b)** SDSS-MGS at $z = 0.15$ (A. J. Ross, MNRAS 2015) & **(c)** DR12 of BOSS-SDSS III at $z = 0.38$, 0.51 and 0.61 (S. Alam *et al.*, MNRAS 2017)

Pantheon: 'Pantheon' catalogue for the luminosity distance measurements of the Type Ia supernovae (SNe Ia) (D. M. Scolnic *et al.*, ApJ 2018)

RSD: $f\sigma_8$ data compilation (S. Nesseris, PRD 2017, B. Sagredo *et al.*, PRD 2018, F. Skara & L. Perivolaropoulos, PRD 2020)

Redshift Space Distortion Data

Survey	z	$f\sigma_8(z)$	Ω_m	Refs.
6dFGS+SnIa	0.02	0.428 ± 0.0465	0.3	(D. Huterer *et al.*, JCAP 2017)
SnIa+IRAS	0.02	0.398 ± 0.065	0.3	(S. J. Turnbull *et al.*, MNRAS 2017, M. J. Hudson *et al.*, ApJL 2012)
2MASS	0.02	0.314 ± 0.048	0.266	(M. Davis *et al.*, MNRAS 2011, M. J. Hudson *et al.*, ApJL 2012)
SDSS-veloc	0.10	0.370 ± 0.130	0.3	(M. Feix *et al.*, PRL 2015)
SDSS-MGS	0.15	0.490 ± 0.145	0.31	(C. Howlett *et al.*, MNRAS 2015)
2dFGRS	0.17	0.510 ± 0.060	0.3	(Y.-S. Song *et al.*, JCAP 2009)
GAMA	0.18	0.360 ± 0.090	0.27	(C. Blake *et al.*, MNRAS 2013)
GAMA	0.38	0.440 ± 0.060		(C. Blake *et al.*, MNRAS 2013)
SDSS-LRG-200	0.25	0.3512 ± 0.0583	0.25	(L. Samushia *et al.*, MNRAS 2012)
SDSS-LRG-200	0.37	0.4602 ± 0.0378		(L. Samushia *et al.*, MNRAS 2012)
BOSS-LOWZ	0.32	0.384 ± 0.095	0.274	(A. G. Sánchez *et al.*, MNRAS 2014)
SDSS-CMASS	0.59	0.488 ± 0.060	0.307115	(C.-H. Chuang *et al.*, MNRAS 2016)
WiggleZ	0.44	0.413 ± 0.080	0.27	(C. Blake *et al.*, MNRAS 2012)
WiggleZ	0.60	0.390 ± 0.063	$\mathbf{C}_{\text{WiggleZ}}$	(C. Blake *et al.*, MNRAS 2012)
WiggleZ	0.73	0.437 ± 0.072		(C. Blake *et al.*, MNRAS 2012)
VIPERS PDR-2	0.60	0.550 ± 0.120	0.3	(A. Pezzotta *et al.*, A&A 2017)
VIPERS PDR-2	0.86	0.400 ± 0.110		(A. Pezzotta *et al.*, A&A 2017)
FastSound	1.40	0.482 ± 0.116	0.27	(T. Okumura *et al.*, PASJ 2016)
SDSS-IV	0.978	0.379 ± 0.176	0.31	(G.-B. Zhao *et al.*, MNRAS 2018)
SDSS-IV	1.23	0.385 ± 0.099	$\mathbf{C}_{\text{SDSS-IV}}$	(G.-B. Zhao *et al.*, MNRAS 2018)
SDSS-IV	1.526	0.342 ± 0.070		(G.-B. Zhao *et al.*, MNRAS 2018)
SDSS-IV	1.944	0.364 ± 0.106		(G.-B. Zhao *et al.*, MNRAS 2018)
VIPERS PDR2	0.60	0.49 ± 0.12	0.31	(F. G. Mohammad *et al.*, A&A 2018)
VIPERS PDR2	0.86	0.46 ± 0.09		(F. G. Mohammad *et al.*, A&A 2018)
BOSS DR12 voids	0.57	0.501 ± 0.051	0.307	(S. Nadathur *et al.*, PRD 2019)
2MTF 6dFGSv	0.03	0.404 ± 0.0815	0.3121	(F. Qin *et al.*, MNRAS 2019)
SDSS-IV	0.72	0.454 ± 0.139	0.31	(M. Icaza-Lizaola *et al.*, MNRAS 2019)

$\Omega_m \longrightarrow$ corresponding fiducial cosmology used to convert redshift to distance

Redshift Space Distortion

- The anisotropic red-shift space clustering of galaxies along the line-of-sight due to non-negligible galaxy peculiar velocities $\Rightarrow$ **Redshift-space distortion (RSD)**
- Likelihood $\mathscr{L} \propto e^{-\chi^2/2}$, where $\chi^2 = V^i\, \mathbf{C}_{ij}^{-1}\, V^j$
- For RSD data $\rightarrow \chi^2_{f\sigma_8} = V^i_{f\sigma_8}\, \mathbf{C}^{-1}_{ij,f\sigma_8}\, V^j_{f\sigma_8}$, where

$$\mathbf{C}_{ij,f\sigma_8} = \begin{pmatrix} \sigma_1^2 & 0 & \cdots & 0 & \cdots & 0 \\ 0 & \sigma_2^2 & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & \mathbf{C}_{\mathrm{WiggleZ}} & \cdots & 0 \\ 0 & 0 & \cdots & 0 & \mathbf{C}_{\mathrm{SDSS\text{-}IV}} & 0 \\ 0 & 0 & \cdots & 0 & \cdots & \sigma_N^2 \end{pmatrix}$$

$$\mathbf{C}_{\mathrm{WiggleZ}} = 10^{-3} \begin{pmatrix} 6.400 & 2.570 & 0.000 \\ 2.570 & 3.969 & 2.540 \\ 0.000 & 2.540 & 5.184 \end{pmatrix},$$

$$\mathbf{C}_{\mathrm{SDSS\text{-}IV}} = 10^{-2} \begin{pmatrix} 3.098 & 0.892 & 0.329 & -0.021 \\ 0.892 & 0.980 & 0.436 & 0.076 \\ 0.329 & 0.436 & 0.490 & 0.350 \\ -0.021 & 0.076 & 0.350 & 1.124 \end{pmatrix}$$

- For vector $V^i_{f\sigma_8} \rightarrow$ the theoretical predictions ($f\sigma_8^{\mathrm{th}}$) are divided by a correction term $\mathscr{R}$

$$V^i_{f\sigma_8}(z_i, \mathscr{P}) \equiv f\sigma_{8,i}^{\mathrm{obs}} - \frac{f\sigma_8^{\mathrm{th}}(z_i, \mathscr{P})}{\mathscr{R}(z_i)}, \qquad \mathscr{R} \longrightarrow \text{Alcock-Paczyński (AP) correction}$$

Alcock-Paczyński (AP) Effect

- The anisotropies due to incorrect fiducial cosmology while converting the relative redshifts to comoving coordinates $\Rightarrow$ **Alcock-Paczyński (AP) effect** (Alcock & Paczyński, Nature 1979, E. Macaulay *et al.*, PRL 2013)
- Distance between two galaxies for true model

$$dL_{\perp} = (1+z)D_A(z)d\theta, \qquad dL_{\parallel} = \frac{cdz}{H(z)}$$

- Distance between two galaxies for fiducial model

$$dL_{\perp}^{\rm fid} = (1+z)D_A^{\rm fid}(z)d\theta = \left(\frac{D_A^{\rm fid}}{D_A}\right)dL_{\perp}, \qquad dL_{\parallel}^{\rm fid} = \frac{cdz}{H^{\rm fid}(z)} = \left(\frac{H}{H^{\rm fid}}\right)dL_{\parallel}$$

- Amount of anisotropy included is

$$F = \left(\frac{H^{\rm fid}}{H}\right)\left(\frac{D_A^{\rm fid}}{D_A}\right)$$

- The corrected observed quantity is (B. Sagredo *et al.*, PRD 2018, L. Kazantzidis & L. Perivolaropoulos, PRD 2018, F. Skara & L. Perivolaropoulos, PRD 2020)

$$f\sigma_8(z) \simeq \frac{H(z)D_A(z)}{H^{\rm fid}(z)D_A^{\rm fid}(z)} f\sigma_8^{\rm fid}(z) \equiv \mathscr{R}\left(z, \Omega_{0m}, \Omega_{0m}^{\rm fid}\right) f\sigma_8^{\rm fid}(z)$$

Observational constraints

Presence of interaction for a brief period in the evolutionary history $\Longrightarrow$ **Model E** $\longrightarrow$ describes the evolutionary history of the Universe better than Model L & Model C

Observational constraints

Parameter	*Planck*	*Planck* + $f\sigma_8$	*Planck* + BAO	*Planck* + BAO + Pantheon	*Planck* + BAO + Pantheon + $f\sigma_8$
$\Omega_b h^2$	0.022358 ± 0.000165	0.022490 ± 0.000162	0.022489 ± 0.000156	0.022500 ± 0.000152	0.022546 ± 0.000151
$\Omega_c h^2$	0.12008 ± 0.00126	0.11848 ± 0.00117	0.11850 ± 0.00101	0.118405 ± 0.000970	0.117845 ± 0.000909
$100\theta_{MC}$	1.040769 ± 0.000324	1.040941 ± 0.000318	1.040941 ± 0.000313	1.040945 ± 0.000315	1.040999 ± 0.000313
τ	$0.05466^{+0.00699}_{-0.00779}$	$0.05630^{+0.00703}_{-0.00797}$	$0.05704^{+0.00704}_{-0.00792}$	0.05697 ± 0.00749	$0.05778^{+0.00700}_{-0.00790}$
β_0	0.0339 ± 0.0372	0.0395 ± 0.0381	0.0432 ± 0.0376	0.0448 ± 0.0377	0.0446 ± 0.0370
w_0	< -0.914	< -0.977	< -0.969	< -0.981	< -0.985
w_1	< 0.168	< 0.0645	< 0.0707	< 0.0604	< 0.0489
$\ln(10^{10}A_s)$	3.0486 ± 0.0147	3.0488 ± 0.0148	3.0509 ± 0.0148	3.0507 ± 0.0144	3.0511 ± 0.0146
n_s	0.96315 ± 0.00453	0.96681 ± 0.00434	0.96652 ± 0.00419	0.96672 ± 0.00418	0.96802 ± 0.00404
$H_0 \left[\mathrm{km\ s^{-1} Mpc^{-1}}\right]$	$64.12^{+2.40}_{-1.39}$	$67.00^{+1.02}_{-0.702}$	$66.787^{+0.775}_{-0.600}$	$67.200^{+0.577}_{-0.516}$	67.631 ± 0.516
Ω_m	$0.3492^{+0.0149}_{-0.0282}$	$0.31569^{+0.00834}_{-0.0114}$	$0.31765^{+0.00678}_{-0.00812}$	$0.31353^{+0.00590}_{-0.00658}$	0.30842 ± 0.00588
σ_8	$0.7836^{+0.0221}_{-0.0138}$	$0.80265^{+0.00992}_{-0.00800}$	$0.8019^{+0.0102}_{-0.00866}$	0.80539 ± 0.00830	0.80573 ± 0.00774

- 1-D marginalised values with errors at 1σ (68% Confidence Level) for **Model E**

Observational constraints

Parameter	*Planck*	*Planck* + $f\sigma_8$	*Planck* + BAO	*Planck* + BAO + Pantheon	*Planck* + BAO + Pantheon + $f\sigma_8$
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- $\beta_0 > 0 \Rightarrow$ Energy flows from DM $\rightarrow$ DE
- For *Planck* data, $\beta_0 = 0$ lies within the 1σ error region
- For other datasets, $\beta_0 = 0$ lies outside the 1σ error region
- w_0 and w_1 are unconstrained

Observational constraints

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- Derived parameters, H_0, Ω_m and σ_8 are also listed
- For *Planck* data, central value of H_0 is small and error bars are high
- For *Planck* data, σ_8 is skewed towards the galaxy cluster value of $\sigma_8 = 0.77^{+0.04}_{-0.03}$
- Addition of datasets, changes the central values and decreases the error bars

Observational constraints

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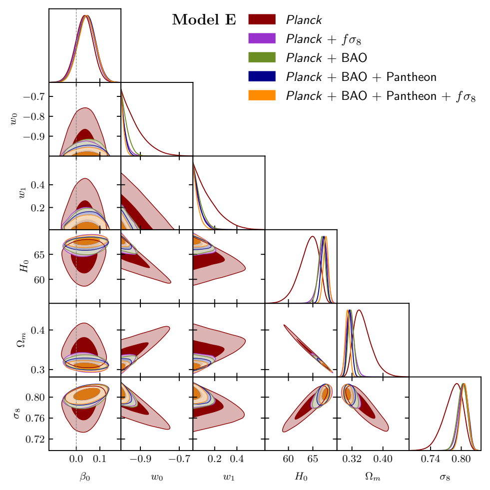
☛ For all the combined datasets, the values shift towards the *Planck* ΛCDM values

Observational constraints



Comparison

Parameter	Model L	Model E	Model C
β_0	0.00788 ± 0.00815	0.0339 ± 0.0372	0.00624 ± 0.00673
w_0	< -0.909	< -0.914	< -0.907
w_1	< 0.174	< 0.168	< 0.174
H_0	$63.98^{+2.45}_{-1.47}$	$64.12^{+2.40}_{-1.39}$	$63.93^{+2.51}_{-1.44}$
Ω_m	$0.3507^{+0.0157}_{-0.0292}$	$0.3492^{+0.0149}_{-0.0282}$	$0.3513^{+0.0153}_{-0.0299}$
σ_8	$0.7825^{+0.0228}_{-0.0141}$	$0.7836^{+0.0221}_{-0.0138}$	$0.7821^{+0.0232}_{-0.0140}$

Compared w.r.t. *Planck* data

- Model L and Model C have very close parameter central values
- Model E has larger β_0 compared to Model L and Model C
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- **Bayesian evidence** $\Rightarrow \ln B_{ij}$ where $B_{ij} = \frac{p(x|M_i)}{p(x|M_j)} \equiv$ ratio of evidences
- $i \equiv \{\text{Model L, Model E, Model C}\}$, $j \equiv \Lambda$CDM
- $\ln B_{ij} < 0 \Rightarrow \Lambda$CDM is preferred

Model	Dataset	$\ln B_{ij}$	$\Delta \ln B_{ij}$
Model L	*Planck*	-8.843	-2.244
	Planck + $f\sigma_8$	-11.410	-2.245
	Planck + BAO	-10.610	-2.187
	Planck + BAO + Pantheon	-11.354	-2.104
	Planck + BAO + Pantheon + $f\sigma_8$	-11.977	-2.328
Model E	*Planck*	-7.233	-0.633
	Planck + $f\sigma_8$	-9.730	-0.566
	Planck + BAO	-9.047	-0.624
	Planck + BAO + Pantheon	-9.733	-0.483
	Planck + BAO + Pantheon + $f\sigma_8$	-10.192	-0.542
Model C	*Planck*	-6.599	0.0
	Planck + $f\sigma_8$	-9.164	0.0
	Planck + BAO	-8.423	0.0
	Planck + BAO + Pantheon	-9.250	0.0
	Planck + BAO + Pantheon + $f\sigma_8$	-9.650	0.0

$0 \le |\ln B_{ij}| < 1 \Rightarrow$ weak, $1 \le |\ln B_{ij}| < 3 \Rightarrow$ Definite/Positive,
$3 \le \ln|\ln B_{ij}| < 5 \Rightarrow$ Strong, $\ln|\ln B_{ij}| \ge 5 \Rightarrow$ Very Strong

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- $\ln B_{ij} < 0 \Rightarrow \Lambda\text{CDM}$ is preferred
- relative differences $\Delta \ln B_{ij}$, w.r.t. Model C show:
  - **(i)** Model L $\rightarrow$ strongly disfavoured
  - **(ii)** Model E $\rightarrow$ weakly disfavoured

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Interaction, if present, is likely to be significant only at some early stage of evolution of the Universe

Summary & Conclusion

- Scalar field with a potential that drives the recent acceleration like the cosmological constant starting from arbitrary initial conditions
  - The evolution of perturbations is similar to the ΛCDM model
- Considered 'evolving' coupling parameter for interaction — **(a)** interaction is a more recent phenomenon & **(b)** interaction is a phenomenon in the distant past
  - Early interaction describes the evolution of the perturbations better than the late interaction

Thank You

Extra

Power Spectra & Growth Rate

$$C_\ell^{TT} = \frac{2}{k}\int k^2 dk\, P_\zeta(k)\Delta^2_{T\ell}(k)$$

$$P(k,a) = A_s k^{n_s} T^2(k) D^2(a)$$

$$f(a) = \frac{d\ln\delta_m}{d\ln a}$$

$$\sigma_8(a) = \sigma_8(1)\frac{\delta_m(a)}{\delta_m(1)}$$

$$f\sigma_8(a) \equiv f(a)\sigma_8(a)$$